CS412 Machine Learning
HW 2 – Probabilities – Bayesian Learning
100pts

- **Please TYPE your answer or write legibly by hand (pts off if it is hard to read).**
- **Use this document to type in your answers** (rather than writing on a separate sheet of paper), so as to keep questions, answers and and grades together to facilitate grading.
- **SHOW all your work for partial/full credit.**
- **Allocated spaces should be enough for your answers** (unnecessarily long and irrelevant answers may loose points)

1) 20 pt - Suppose that we have 3 colored boxes r (red), b (blue) and g (green).
Box r contains 9 apples, 5 oranges and 3 limes;
Box b contains 8 apples, 4 oranges and 1 limes;
Box g contains 5 apples, 2 oranges and 6 limes.

Assume a process **where we pick a box first and then pick a fruit from the selected box**. A box is chosen at random according to the following probability of being selected: p(r) = p(b) = 0.3 and p(g)=0.4 and a piece of fruit is selected from the **chosen** box randomly.

a) 10 pt – What is the **probability of selecting an orange**?

p(o) = p(r) * p(o | r) + p(b) * p(o| b) + p(g) * p(o | g)

p(o) = (3/10) * (5/17) + (3/10) * (4/13) + (4/10) * (2/13) = 0.242

b) 10pt - If we **observe that the selected fruit is an orange**, what is the probability that it came from the red box?

p(r | o) = $\frac{(5/17) * (0.3)}{0.242}$ = 0.36

2) 40 pt - **For a 2-dimensional input space, we are given the following class conditional probability densities. Assume that x_1 and x_2 are conditionally independent given class names.**

p(x_1|C1) = 1/3 for 0 <= x_1 <= 4
0 elsewhere

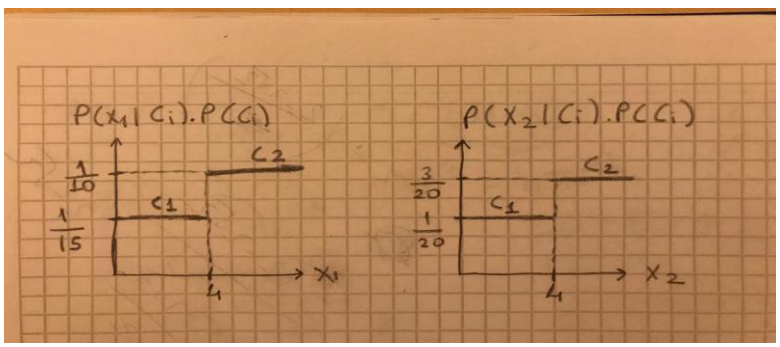
p(x_2|C1) = 1/4 for 0 <= x_2 <= 4
0 elsewhere

p(x_1|C2) = 1/2 for 4 < x_1
0 elsewhere

p(x_2|C2) = 3/4 for 4 < x_2
0 elsewhere

Assume P(C1)=P(C2)=0.2 and P(C3)=0.6.

a) **12pt – Draw the corresponding pdfs for x_1 and x_2 , being as precise as possible** (e.g. **label axes and important points on the axes**). You can draw by hand, take a picture and include here as image.



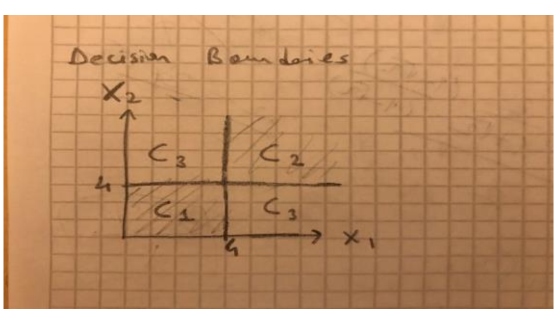
b) **10pts - Develop a classification strategy for given feature values** (just looking at the graph – no formula), just complete the sentence(s):

if x_1 is in the region .....4<X1......., and x_2 is in the region ......4<X2...... , I will classify it as ….........C2..............;

if x_1 is in the region .....0<=X1<=4...., and x_2 is in the region ....0<=X2<=4....., I will classify it as …...........C1............;

Otherwise, I will classify it as …..........C3............;

c) **8pts - Draw the decision regions.**



d) 10pts – Give a one line qualitative answer (no precise numbers/thresholds...) **& reasoning for each case below (**how your decision changes or whether it doesn't).

- Would your decision strategy change if $P(C_1)=0.8$ and $P(C_2)=P(C_3)=0.1$?

No it will not change. Because, there is no intersection between the classes of C_i in the pdf's graph both for X1 and X2.. Hence, the dominant probabilities will not change in the given intervals.

- How about if it was the reverse $P(C_1)=P(C_3)=0.1$ and $P(C_2)=0.8$?

No it will not change. Because, there is no intersection between the classes of C_i in the pdf's graph both for X1 and X2. Therefore, the dominant probabilities will not be affected in the given intervals.

3) 40pts – NAIVE BAYES

a) 10pts – Given that two random variables X and Y are conditionally independent given C, circle True or False (2pts for each correct answer; -1pts each wrong answer):

- $P(X \mid Y) = P(X)$ True / **False**
- $P(X \mid Y, C) = P(X \mid Y)$ True / **False**
- $P(X, C \mid Y) = P(X \mid Y)$ True / **False**
- $P(X, Y \mid C) = P(X \mid C)\, P(Y \mid C)$ **True** / False
- $P(X, Y, C) = P(X \mid C)\, P(Y \mid C)\, P(C)$ **True** / False

b) 24pts - Using the Mammal dataset given below, **how would you classify the animal that give birth, cannot fly, sometimes live in water, and has not legs, using Naive Bayes classifier *without any smoothing*. Show your work** (e.g. indicate class conditional attribute probabilities under the given table in the next page and just transfer them here).

P(mammal| x) = p(birth | mammal)*p(cannot fly | mammal)*p(sometimes live in water | mammal)*p(has not legs | mammal)*p(mammal)

= (6/7) * (6/7) * (0) * …. = 0

P(non-mammals| x) = p(birth | non-mammals)*p(cannot fly | non-mammals)*p(sometimes live in water | non-mammals)*p(has not legs | non-mammals)*p(non-mammals)

= (1/13) * (10/13) * (4/13) * (4/13) * (13/20) ~= 0.0036 (without normalisation)

Decision: ....non-mammals.....................

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

Write here the estimated probabilities (you should only write those related to the question for simplicity):

P(Give Birth=Yes | mammals) = (6/7)

P(Can Fly = No | mammals) = (6/7)

P(Live in water = sometimes | mammals) = 0

P(Have Legs = No | mammals) = (2/7)

P(Give Birth=Yes | non-mammals) = (1/13)

P(Can Fly = No | non-mammals) = (10/13)

P(Live in water = sometimes | non-mammals) = 4/13

P(Have Legs = No | non-mammals) = (4/13)

P(mammals) = (7/20)

P(non-mammals) = (13/20)

c) 6pts - Without re-doing the whole process, calculate the probabilities for P(Live in Water|mammals), P(Have Legs|mammals) and P(Give Birth|non-mammals) using Laplace smoothing:

P(Live in Water=Sometimes|mammals) = (0 + 1) / (7 + 3) = (1 / 10)

P(Have Legs=No|mammals) = (2 + 1) / (7 + 2) = (3/9)

P(Give Birth=Yes|non-mammals) = (1 + 1) / (13 + 2) = (2/15)