

```
In[1]:= Needs["xAct`xCoba`", FileNameJoin[
      {Environment["HOME"], "docker-workspace/repos/Generato/src/Generato.wl"}]];
```

```
-----
Package xAct`xPerm` version 1.2.3, {2015, 8, 23}
```

```
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```

```
Connecting to external mac executable...
```

```
Connection established.
```

```
-----
Package xAct`xTensor` version 1.2.0, {2021, 10, 17}
```

```
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```

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```

```
-----
Package xAct`xCoba` version 0.8.6, {2021, 2, 28}
```

```
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```

```
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```

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```

```
-----
Package Generato`Basic`, {2024, 1, 11}
```

```
-----
Package Generato`ParseMode`, {2024, 7, 06}
```

```
-----
Package Generato`Component`, {2024, 1, 11}
```

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-----
Package Generato`Varlist`, {2024, 1, 11}
```

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-----  
Package Generato`Interface`, {2024, 1, 11}  
-----  
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```

```
Package Generato`Derivation`, {2024, 1, 18}  
-----
```

```
In[2]:= DefManifold[M3, 3, IndexRange[a, z]];
```

```
In[3]:= SetDirectory[NotebookDirectory[]];
```

```
In[4]:= << "wl/Z4c_vars_check.wl"
```

```
In[8]:= << "wl/Z4c_rhs.wl";
```

Check expression

```
In[9]:= rhsOf[var_] := Module[{}, var /. {var[[0]] => RHSOf[ToString[var[[0]]]}];  
(*eqsof[var_] := Module[{}, var = rhsOf[var]];*)  
eqssof[varlist_] := Module[{var}, Table[var = varlist[[ivar]][1];  
var = rhsOf[var] // ScreenDollarIndices, {ivar, 1, Length[varlist]}]];
```

Intermediate

Intermediate

```
In[11]:= eqssof[IntermediateVarlist][[3 ;; -1]] // TableForm
```

```
Out[11]//TableForm=
```

$$\begin{aligned}\gamma^{ij} &= \chi \tilde{\gamma}^{ij} \\ \gamma_{ij} &= \frac{\tilde{\gamma}_{ij}}{\chi} \\ \tilde{\Gamma}_{kij} &= \frac{1}{2} \left(\partial \tilde{\gamma}_{ijk} + \partial \tilde{\gamma}_{jki} - \partial \tilde{\gamma}_{kij} \right) \\ \tilde{\Gamma}_{ki}^j &= \tilde{\Gamma}_{kia} \tilde{\gamma}^{ja} \\ \tilde{\Gamma}_{ij}^k &= \tilde{\Gamma}_{aij} \tilde{\gamma}^{ka} \\ (\tilde{\Gamma}_d)^i &= \tilde{\Gamma}_{ab}^i \tilde{\gamma}^{ab} \\ \partial \gamma_{kij} &= \frac{\partial \tilde{\gamma}_{kij}}{\chi} - \frac{\partial \chi_k \tilde{\gamma}_{ij}}{\chi^2} \\ \Gamma_{kij} &= \frac{1}{2} \left(\partial \gamma_{ijk} + \partial \gamma_{jki} - \partial \gamma_{kij} \right) \\ \Gamma_{ij}^k &= \Gamma_{aij} \gamma^{ka} \\ \tilde{A}^{ij} &= \tilde{A}_{ab} \tilde{\gamma}^{ia} \tilde{\gamma}^{jb}\end{aligned}$$

DD

```
In[12]:= eqssof[DDVarlist] // TableForm
```

```
Out[12]//TableForm=
```

$$\begin{aligned}\tilde{D} \tilde{D} \chi_{ij} &= \partial \partial \chi_{ij} - \partial \chi_a \tilde{\Gamma}^a_{ij} \\ DD\alpha_{ij} &= \partial \partial \alpha_{ij} - \partial \alpha_a \Gamma^a_{ij}\end{aligned}$$

Rs

```
In[13]:= eqssof[RVarlist] // TableForm
```

```
Out[13]//TableForm=
```

$$\begin{aligned}\tilde{R}^x_{ij} &= -\frac{\partial \chi_i \partial \chi_j}{4 \chi^2} - \frac{3 \partial \chi_a \partial \chi_b \tilde{\gamma}_{ij} \tilde{\gamma}^{ab}}{4 \chi^2} + \frac{\tilde{\gamma}_{ij} \tilde{\gamma}^{ab} \tilde{D} \tilde{D} \chi_{ab}}{2 \chi} + \frac{\tilde{D} \tilde{D} \chi_{ij}}{2 \chi} \\ \tilde{R}_{ij} &= \frac{1}{2} \left(\partial \tilde{\Gamma}_j^a \tilde{\gamma}_{ai} + \partial \tilde{\Gamma}_i^a \tilde{\gamma}_{aj} \right) + \tilde{\Gamma}_{ib}^a \tilde{\Gamma}_{aj}^b + \tilde{\Gamma}_{bj}^a \tilde{\Gamma}_{ia}^b + \tilde{\Gamma}_{bi}^a \tilde{\Gamma}_{ja}^b - \frac{1}{2} \partial \partial \tilde{\gamma}_{abij} \tilde{\gamma}^{ab} + \frac{1}{2} \left(\tilde{\Gamma}_{ija} + \tilde{\Gamma}_{jia} \right) \\ R_{ij} &= \tilde{R}_{ij} + \tilde{R}^x_{ij} \\ R &= \gamma^{ab} R_{ab}\end{aligned}$$

Matter

```
In[14]:= eqssof[MatterVarlist] // TableForm
```

```
Out[14]//TableForm=
```

$$\begin{aligned}\rho &= \frac{\beta^a \beta^b T_{ab} - 2 \beta^b T_{tb} + T_{tt}}{\alpha^2} \\ S_i &= -\frac{-\beta^a T_{ai} + T_{ti}}{\alpha} \\ S_{ij} &= T_{ij} \\ S &= \gamma^{ab} S_{ab}\end{aligned}$$

Constraint

```
In[15]:= eqssof[dAtUUVarlist] // TableForm
```

```
Out[15]//TableForm=
```

$$\partial \tilde{A}^i = -\partial \tilde{\gamma}_{abc} \tilde{A}^{ic} \tilde{\gamma}^{ab} - \partial \tilde{\gamma}_{abc} \tilde{A}^{ac} \tilde{\gamma}^{ib} + \partial \tilde{A}_{abc} \tilde{\gamma}^{ac} \tilde{\gamma}^{ib}$$

```
In[16]:= eqssof[ConstraintVarlist] // TableForm
```

```
Out[16]//TableForm=
```

$$\begin{aligned}\tilde{Z}^i &= \frac{1}{2} \left(\tilde{\Gamma}^i - (\tilde{\Gamma}_d)^i \right) \\ H &= \tilde{A}_{ab} \tilde{A}^{ab} - 16 \pi \rho - \frac{2}{3} \left(\hat{K} + 2 \Theta \right)^2 + R \\ \tilde{M}^i &= -\frac{2 \partial \chi_a \tilde{A}^{ia}}{3 \chi} + \tilde{A}^{ab} \tilde{\Gamma}_{ab}^i - \frac{2}{3} \left(\partial \hat{K}_a + 2 \partial \Theta_a \right) \tilde{\gamma}^{ia} - 8 \pi \tilde{\gamma}^{ia} S_a + \partial \tilde{A}^i\end{aligned}$$

EOM

```
In[17]:= eqssof[dtEvolVarlist] // TableForm
```

```
Out[17]//TableForm=
```

$$\begin{aligned}\partial_t \chi &= \frac{2}{3} \chi \left(-\partial \beta_a^a + \alpha \left(\hat{K} + 2 \Theta \right) \right) \\ \partial_t \tilde{\gamma}_{ij} &= -2 \alpha \tilde{A}_{ij} + \partial \beta_j^a \tilde{\gamma}_{ai} + \partial \beta_i^a \tilde{\gamma}_{aj} - \frac{2}{3} \partial \beta_a^a \tilde{\gamma}_{ij} \\ \partial_t \hat{K} &= -DD\alpha_{ab} \gamma^{ab} + \kappa_1 (1 - \kappa_2) \alpha \Theta + \alpha \left(\tilde{A}_{ab} \tilde{A}^{ab} + \frac{1}{3} \left(\hat{K} + 2 \Theta \right)^2 \right) + 4 \pi \alpha (\rho + S) \\ \partial_t \tilde{A}_{ij} &= \partial \beta_j^a \tilde{A}_{ai} + \partial \beta_i^a \tilde{A}_{aj} - \frac{2}{3} \partial \beta_a^a \tilde{A}_{ij} + \chi \left(-DD\alpha_{ij} - \frac{1}{3} \gamma_{ij} \gamma^{ab} \left(-DD\alpha_{ab} + \alpha \left(R_{ab} - 8 \pi S_{ab} \right) \right) + \right. \\ \partial_t \tilde{\Gamma}^i &= -2 \partial \alpha_a \tilde{A}^{ia} + \partial \partial \beta_{ab}^i \tilde{\gamma}^{ab} + \frac{1}{3} \partial \partial \beta_{ab}^b \tilde{\gamma}^{ia} + 2 \alpha \left(-\frac{3 \partial \chi_a \tilde{A}^{ia}}{2 \chi} + \tilde{A}^{ab} \tilde{\Gamma}_{ab}^i - \frac{1}{3} \left(2 \partial \hat{K}_a + \partial \Theta_a \right) \tilde{\gamma}^{ia} - \right. \\ \partial_t \Theta &= -\alpha (8 \pi \rho + \kappa_1 (2 + \kappa_2) \Theta) + \frac{1}{2} \alpha \left(-\tilde{A}_{ab} \tilde{A}^{ab} + \frac{2}{3} \left(\hat{K} + 2 \Theta \right)^2 + R \right) \\ \partial_t \alpha &= -\mu_L \alpha \hat{K} \\ \partial_t \beta^i &= -\eta \beta^i + \mu_S \tilde{\Gamma}^i\end{aligned}$$

Check with static solutions

```
In[18]:= DefChart[cart, M3, {1, 2, 3}, {X[], Y[], Z[]}, ChartColor -> Blue];
```

```

In[19]:= Module[{Mat, invMat},
  Mat = Table[gamt[{ii, -cart}, {jj, -cart}] // ToValues, {ii, 1, 3}, {jj,
    1, 3}];
  invMat = Inverse[Mat] /. {1 / Det[Mat] → (detinvgamt[] // ToValues)};
  SetEQNDelayed[detinvgamt[], 1 / Det[Mat] // Simplify];
  SetEQNDelayed[invgamt[i_, j_], invMat[[i][1], j[1]] // Simplify]
];

```