FUKA

A new public code for initial data of unequal-mass, spinning compact-object binaries

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https://kadath.obspm.fr/fuka/

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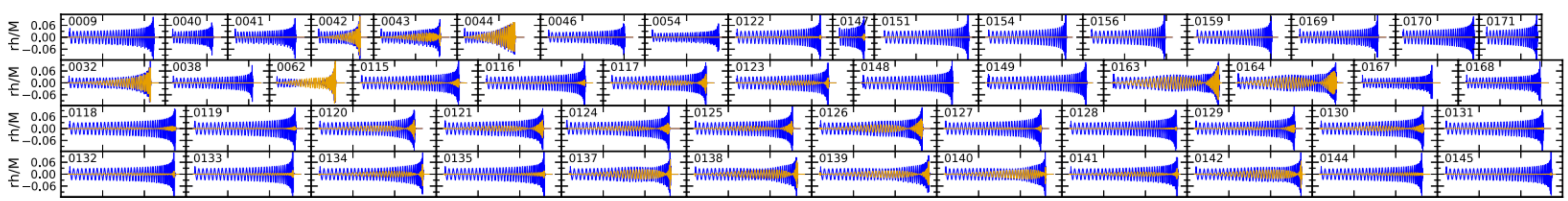
Outline

Motivation

The primary motivation of this code base is to provide consistent initial data solutions to Einstein's field equations

1. Gravitation wave extraction
2. Study of merger dynamics in numerical simulations with configurations such as
   - Binary black holes (BBH) with high mass ratio with mixed spins,
   - Binary neutron stars (BNS) with high mass ratio and mixed spins,
   - Black hole - neutron stars (BHNS) with mass ratio and mixed spins,



https://link.aps.org/doi/10.1103/PhysRevLett.111.241104

XCTS Constraint Equations

By choosing $\partial_t \tilde{\gamma}_{ij} = \partial_t K = K = 0$, where $\tilde{\gamma}_{ij}$ is the flat matric, the XCTS formulation results in the following set of coupled elliptic differential equations[1]

$$\tilde{D}^2\Psi = -\frac{1}{8}\Psi^{-7}\hat{A}_{ij}\hat{A}^{ij} \underbrace{-2\pi\Psi^5 E}$$

$$\tilde{D}^2(\alpha\Psi) = \frac{7}{8}\alpha\Psi^{-7}\hat{A}_{ij}\hat{A}^{ij} \underbrace{+2\pi\alpha\Psi^5(\tilde{E}+2\tilde{S})}$$

$$\tilde{D}^2\beta^i = -\frac{1}{3}\tilde{D}^i\tilde{D}_j\beta^j + 2\hat{A}^{ij}\tilde{D}_j(\alpha\Psi^{-6}) \underbrace{+16\pi\alpha\Psi^4\tilde{j}^i}$$

$$\hat{A}^{ij} \equiv \frac{\Psi^6}{2\alpha}(\tilde{D}^i\beta^j + \tilde{D}^j\beta^i - \frac{2}{3}\tilde{\gamma}^{ij}\tilde{D}_k\beta^k).$$

Where the source terms are defined as:

$$E := \rho h W^2 - p$$

$$S^i_{\ i} := 3p + (E+p)U^2$$

$$j^i := \rho h W^2 U^i$$

[1] PhysRevD.90.064006, eq.7-10

XCTS Constraint Equations

By choosing $\partial_t \tilde{\gamma}_{ij} = \partial_t K = K = 0$, where $\tilde{\gamma}_{ij}$ is the flat matric, the XCTS formulation results in the following set of coupled elliptic differential equations

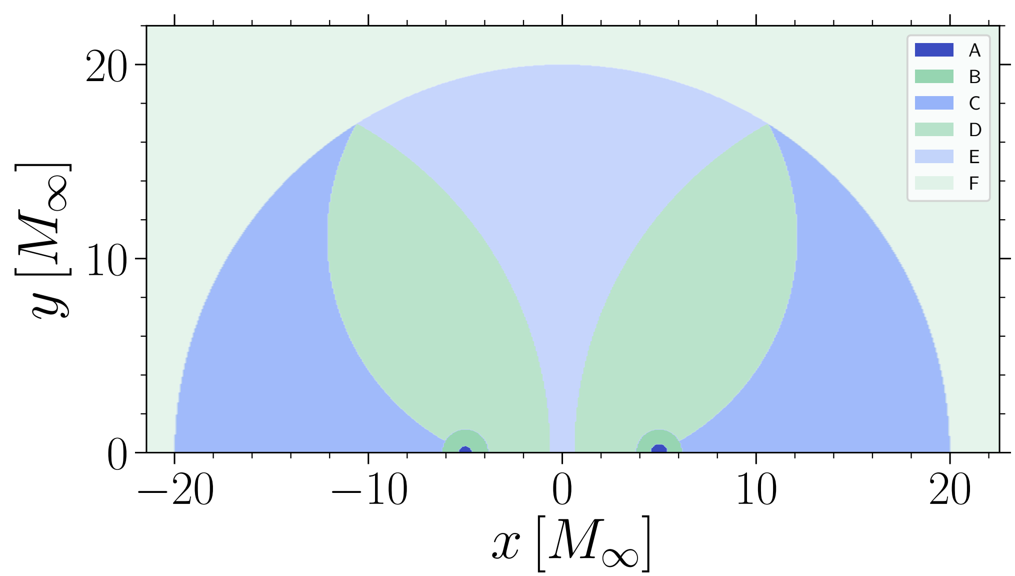
$$
\begin{aligned}
\frac{p}{\rho}\tilde{D}^2\Psi &= \frac{p}{\rho}\Big[-\frac{1}{8}\Psi^{-7}\hat{A}_{ij}\hat{A}^{ij} - 2\pi\Psi^5 E\Big] \\
\frac{p}{\rho}\tilde{D}^2(\alpha\Psi) &= \frac{p}{\rho}\Big[\frac{7}{8}\alpha\Psi^{-7}\hat{A}_{ij}\hat{A}^{ij} + 2\pi\alpha\Psi^5(\tilde{E} + 2\tilde{S})\Big] \\
\frac{p}{\rho}\tilde{D}^2\beta^i &= \frac{p}{\rho}\Big[-\frac{1}{3}\tilde{D}^i\tilde{D}_j\beta^j + 2\hat{A}^{ij}\tilde{D}_j(\alpha\Psi^{-6}) + 16\pi\alpha\Psi^4\tilde{j}^i\Big] \\
\hat{A}^{ij} &\equiv \frac{\Psi^6}{2\alpha}(\tilde{D}^i\beta^j + \tilde{D}^j\beta^i - \frac{2}{3}\tilde{\gamma}^{ij}\tilde{D}_k\beta^k).
\end{aligned}
$$

Where the source terms are defined as:

$$
\begin{aligned}
E &:= \rho h W^2 - p \\
S^i{}_i &:= 3p + (E + p)U^2 \\
j^i &:= \rho h W^2 U^i
\end{aligned}
$$

Implementation

- FUKA is based on the Kadath[2] spectral solver library
  - Equations can be implemented in a LaTeX-like syntax
  - New operators can be added modularly
- ID involving Black Holes are constructed using excision conditions and quasi-equilibrium ($M_{\mathrm{K}} - M_{\mathrm{ADM}} = 0$)
- ID involving Neutron stars can be constructed using piecewise polytrope or tabulated Equations of State



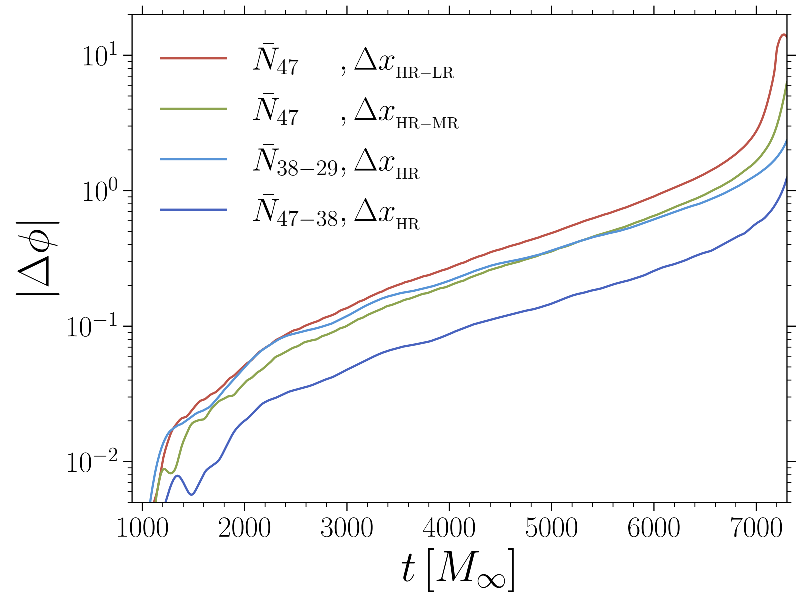
[2] https://kadath.obspm.fr/

Capabilities Map

- ☑ Isolated black hole with arbitrary mass and spin
- ☑ Isolated neutron star with arbitrary mass and spin
- ☑ Binary black holes with high mass ratio with mixed spins
- ☑ Binary neutron stars with high mass ratio and mixed spins
- ☑ Black Hole - Neutron stars with mass ratio and mixed spins
- ☑ Maximal spins (anti-)aligned with orbital axis using conformal flatness approximation
- ☑ Eccentricity reduced binaries
- ☑ Minimize ADM linear momentums
- ☑ Provide an extensible framework for continued development
- ☒ Provide export interfaces to streamline importing into evolution environments

Error Sources

1. The error contribution from the ID solution (e.g. $H|_{t=0} \approx 0$)
2. The error of the evolution scheme
   - Analytical formulation and numerical implementation used
   - Finite resolution of the evolution
3. Note: $\tilde{N} = N^{1/3}$ where N is the total number of colocation points



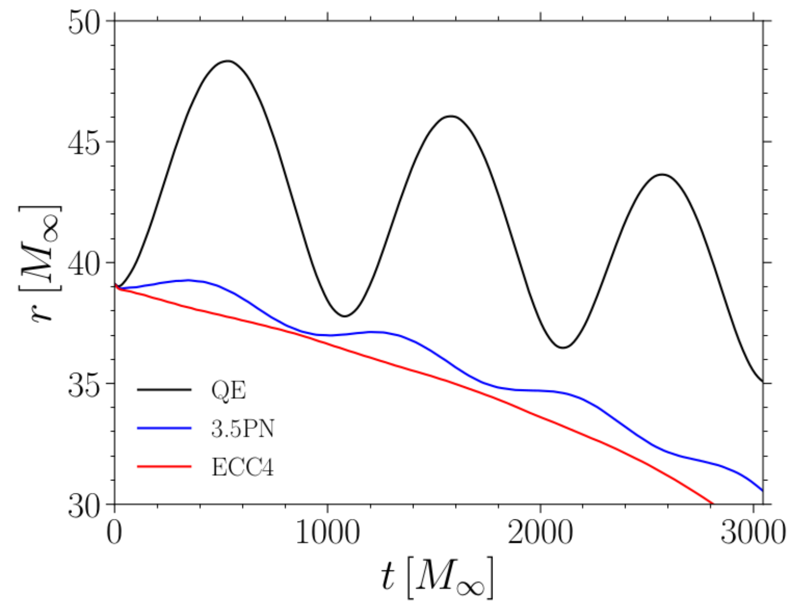
Mb$_{<1/2>} = 1.4946, q = 1$

$\chi_1 = 0, \chi_2 = 0$

EOS = SLy

BNS - Eccentricity Comparison

- QE is necessary starting assumption, but a poor approximation
- 3.5 PN already provides a drastic improvement
- Further improvements can be made using iterative eccentricity reduction



$M_\infty = 2.7, q = 0.6875$

$\chi_1 = 0, \chi_2 = 0.6$

EOS = TNTYST

BHNS - Eccentricity Reduction Example

$$M_{BH} = 2.42 M_{\odot},\ \chi_{BH} = 0.52 \qquad M_{NS} = 1.17 M_{\odot},\ \chi_{NS} = 0.6$$
$$\Delta x_{min} = 0.2,\ \eta = 0.5,\ \kappa = 0.02$$

Solver Layout

- Each solver includes the following:
  - setup: setup numerical space and initialize fields
  - solve: solve the input dataset
  - increase_resolution: restructure the numerical space
  - reader: useful diagnostic code
- Reusing ID is essential
- High resolution is expensive, but can be minimized using resolution increase capabilities

Demo

```
codes/NS:->./bin/Release/setup
Using default Togashi setup.
*****************ns info*****************
                 chi: 0
                 dim: 3
                  hc: 1.2591
                madm: 1.4
                  mb: 1.55
                  nc: 0.00137
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 9
                 rin: 3.1
                rmid: 6.2
                rout: 9.3
             eosfile: togashi.lorene
             eostype: Cold_Table
               h_cut: 0
   interpolation_pts: 2000

codes/NS:->ls -1t
initns.dat
initns.info
sfho_pwp
togashi
build
bin
src
README
CMakeLists.txt
bench
compile
```

- Running "setup" will generate the default dataset: initns.dat/info
- DAT file contains numerical space and scalar fields
- INFO file contains scalar quantities and drives the respective codes

Demo

```
initial
{
    chi 0
    dim 3
    hc 1.0094227447624744
    madm 1.3999999999999999
    mb 1.55
    nc 0.0013699999999999999
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile togashi.lorene
    eostype Cold_Table
    h_cut 0
    interpolation_pts 2000
}
ns
{
    chi 0
    dim 3
    hc 1.0094227447624744
    madm 1.3999999999999999
    mb 1.55
    nc 0.0013699999999999999
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile togashi.lorene
    eostype Cold_Table
    h_cut 0
    interpolation_pts 2000
}
```

- The initial section can maintain a history of iterations
- The NS section describes numerical grid setup and related scalar quantities

Demo

```
ns
{
    chi 0
    dim 3
    hc 1.0094227447624744
    madm 1.3999999999999999
    mb 1.55
    nc 0.0013699999999999999
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile togashi.lorene
    eostype Cold_Table
    h_cut 0
    interpolation_pts 2000
}
```

The fixing parameters consist of:

- "chi", dimensionless spin parameter (χ)
- "madm", Total ADM mass of the isolated NS
- "eosfile", the file that describes the tabulated or polytropic EOS
- "eostype", Cold_PWPoly or Cold_Table

Demo

```
======================================
Iter:     +2
Error:    +0.00154638
Mb:       +1.62137
Madm:     +1.4
Madm_ql:  +1.4 [+3.93971e-13]
Mk:       +1.20369 [+0.140223]
R:        +6.33379 +6.33379

Error init = 4.23875e-05, Eq: firstint Dom: 0 Bounday: -1
DOF / rank = 6598 x 6598 / 128 = 340106
Block size is 32, consider lowering the number of cores.
Load and redistrib Jacobian: 7.0127 / 7.0061 / 7.01712 seconds (avg/min/max)
Inverting the matrix: 0.366115 seconds
Distributing solution: 0.0143809 seconds

codes/NS:->ls -1 togashi/
converged_NS.1.4.0.09.dat
converged_NS.1.4.0.09.info
converged_NS_norot_bc.1.4.0.09.dat
converged_NS_norot_bc.1.4.0.09.info
codes/NS:->cp togashi/converged_NS.1.4.0.09.info  initns.info
codes/NS:->cp togashi/converged_NS.1.4.0.09.dat  initns.dat
codes/NS:->vim initns.info
```

Demo

```
ns
{
    chi 0.3
    dim 3
    hc 1.260774576466819
    madm 1.5
    mb 1.5525575066925488
    nc 0.0013740471269387219
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile togashi.lorene
    eostype Cold_Table
    h_cut 0
    interpolation_pts 2000
}
```

```
Last Stage Enabled: total_bc
=================================
Single star solver 3D
*****************ns info*****************
                 chi: 0.3
                 dim: 3
                  hc: 1.26077
                madm: 1.5
                  mb: 1.55256
                  nc: 0.00137405
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 9
                 rin: 3.03417
                rmid: 6.06834
                rout: 9.10251
             eosfile: togashi.lorene
             eostype: Cold_Table
               h_cut: 0
   interpolation_pts: 2000
=================================
############################
Rotating - with fixed MADM
############################
Error init = 0.675, Eq: integ(intJ) - chi * Madm * Madm = 0 Dom: 3 Bounday: 1
DOF / rank = 12853 x 12853 / 128 = 1.29062e+06
Load and redistrib Jacobian: 44.4222 / 44.3911 / 44.4487 seconds (avg/min/max)
Inverting the matrix: 1.27914 seconds
Distributing solution: 0.0209185 seconds
=======================================
```

Demo

```
ns
{
    chi 0
    dim 3
    hc 1.0094227447624744
    madm 1.3999999999999999
    mb 1.55
    nc 0.0013699999999999999
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile togashi.lorene
    eostype Cold_Table
    h_cut 0
    interpolation_pts 2000
}
```

```
kadath/eos:->ls -1
apr4.polytrope
APR+VQCD_simple.lorene
APR+VQCD_simple_no_phep.lorene
bhblp.lorene
dd2.lorene
gam2.polytrope
poly2_123.6.lorene
poly2.lorene
sfho.lorene
sly4.polytrope
sly_pwp.lorene
togashi.lorene
kadath/eos:->
```

Demo

```
ns
{
    chi 0
    dim 3
    hc 1.0094227447624744
    madm 1.3999999999999999
    mb 1.55
    nc 0.0013699999999999999
    nshells 0
    omega 0
    ql_madm 1.3999999999999999
    qpig 12.566370614359172
    res 9
    rin 3.0341687888025062
    rmid 6.0683375776050124
    rout 9.1025063664075176
    eosfile sly4.polytrope
    eostype Cold_PWPoly
    h_cut 0
    interpolation_pts 2000
}
```

```
codes/NS:->mkdir sfho_pwp
codes/NS:->./bin/Release/setup initns.info
Creating setup from: initns.info
****************ns info****************
                 chi: 0
                 dim: 3
                  hc: 1.23905
                madm: 1.4
                  mb: 1.55
                  nc: 0.00137
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 9
                 rin: 3.03417
                rmid: 6.06834
                rout: 9.10251
             eosfile: sly4.polytrope
             eostype: Cold_PWPoly
               h_cut: 0
   interpolation_pts: 2000

codes/NS:->
```

Demo

```
codes/NS:->./bin/Release/increase_resolution sfho_pwp/converged_NS.1.4.0.09.info initns 11
Resolution of old space: 9 (r), 9 (theta), 8 (phi)
Rmin/max: 6.28166 6.28166
Resolution of new space: 11 (r), 11 (theta), 10 (phi)
Bounds: Array of 1 dimension(s)
3.14083 6.28166 9.42249

*****************ns info*****************
                 chi: 0
                 dim: 3
                  hc: 1.2594
                madm: 1.4
                  mb: 1.5593
                  nc: 0.00143
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 11
                 rin: 3.14083
                rmid: 6.28166
                rout: 9.42249
             eosfile: sly4.polytrope
             eostype: Cold_PWPoly
               h_cut: 0
   interpolation_pts: 2000
```

Demo

```
codes/BNS:->./bin/Release/setup ../NS/sfho_pwp/converged_NS.1.4.0.09.info 50
```

```
***************binary info***************
                 com: 0
            distance: 33.8524
        global_omega: 0.007676
        outer_shells: 0
                   q: 1
                qpig: 12.5664
                 res: 9
                rext: 67.7048
*****************ns info*****************
                 chi: 0
                 dim: 3
                  hc: 1.2594
                madm: 1.4
                  mb: 1.5593
                  nc: 0.00143
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 9
                 rin: 3.14083
                rmid: 6.28166
                rout: 9.42249
             eosfile: sly4.polytrope
             eostype: Cold_PWPoly
               h_cut: 0
   interpolation_pts: 2000
*****************ns info*****************
                 chi: 0
                 dim: 3
                  hc: 1.2594
                madm: 1.4
                  mb: 1.5593
                  nc: 0.00143
             nshells: 0
               omega: 0
             ql_madm: 1.4
                qpig: 12.5664
                 res: 9
                 rin: 3.14083
                rmid: 6.28166
                rout: 9.42249
Rmin/max:
6.28166 6.28166
Bounds:
NS1: 3.14083 6.28166 9.42249
NS2: 3.14083 6.28166 9.42249
xc1: -16.9262
xc2: 16.9262
```

Summary

- How can FUKA ID help your research?
  - Study eccentricity reduced inspirals using iterative reduction or PN estimates
  - Exploration of merger dynamics of a large spectrum of BBH, BHNS, and BNS configuration
- Considerations
  - FUKA is computationally demanding
  - On 128 cores:
    - Isolated objects take 2-4min for low resolution
    - Binaries take 1-2hrs

Future Work

1. Transition to imported boosted objects - if this proves reliable
2. Solver ver.2 is in the works but will not replace ver.1
3. Extend BBH and BHNS codes to include imposing Kerr solution on the horizon to acheive extremal spins $(\chi \approx 0.995)$
4. Improve solvers, documentation, and exporters based on community feedback