

Spritz: general relativistic magnetohydrodynamics with neutrinos

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in collaboration with

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and

TCAN⁺ collaboration



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1st milestone in new GRMHD + neutrinos code development

F Cipolletta et al 2020 Class. Quantum Grav. 37 135010

Available at <https://zenodo.org/record/3689752>

- **STAGGERED Avec:** Accurate evolution of magnetic field

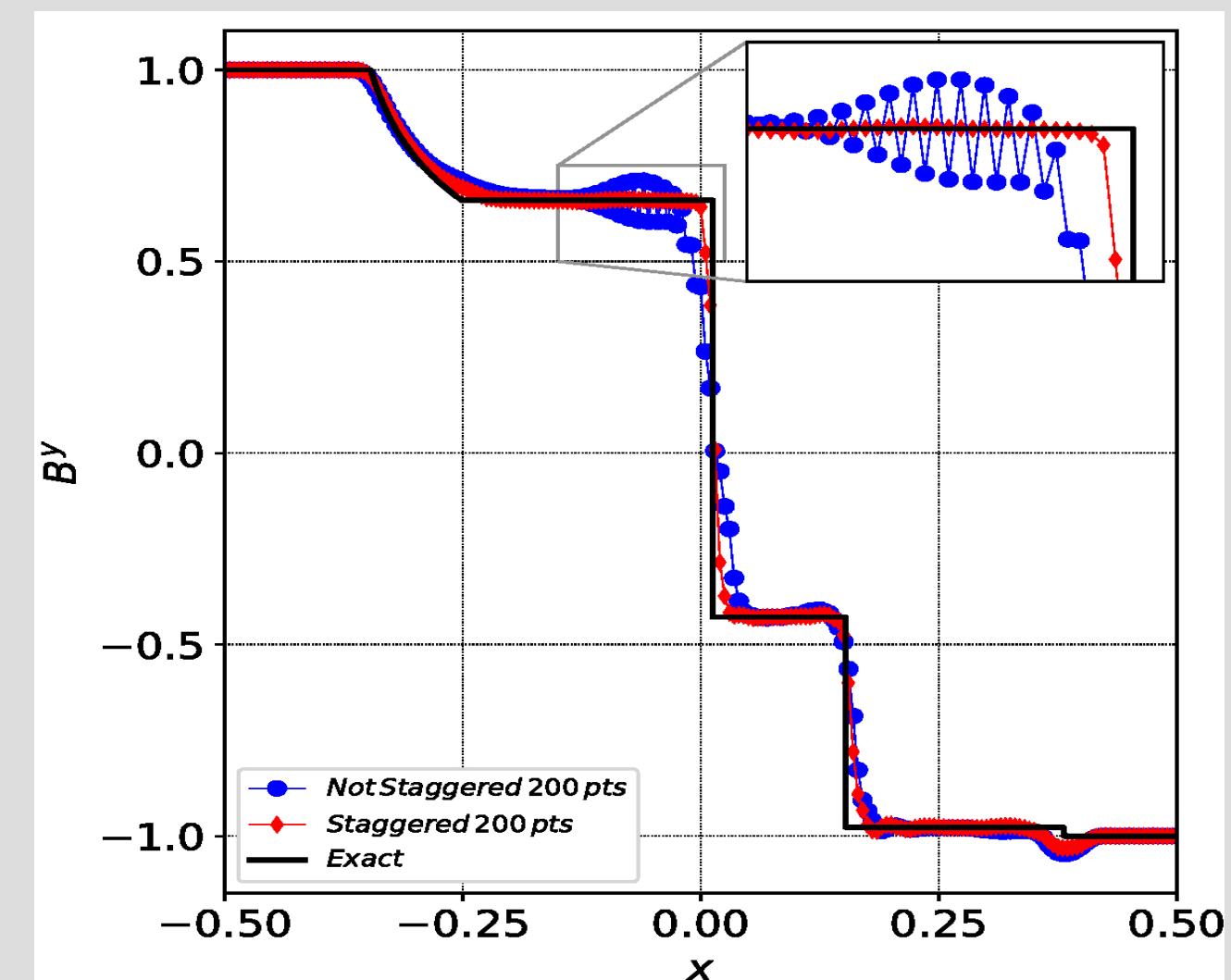
- **Reconstruction orders:** minmod, PPM + WENO-z - *F Cipolletta et al 2021 Class. Quantum Grav. 38 085021*

- **EOS_Omni thorn:** Allows implementation of “general” EOS

- **Extensive testing:** 1D, 2D, 3D

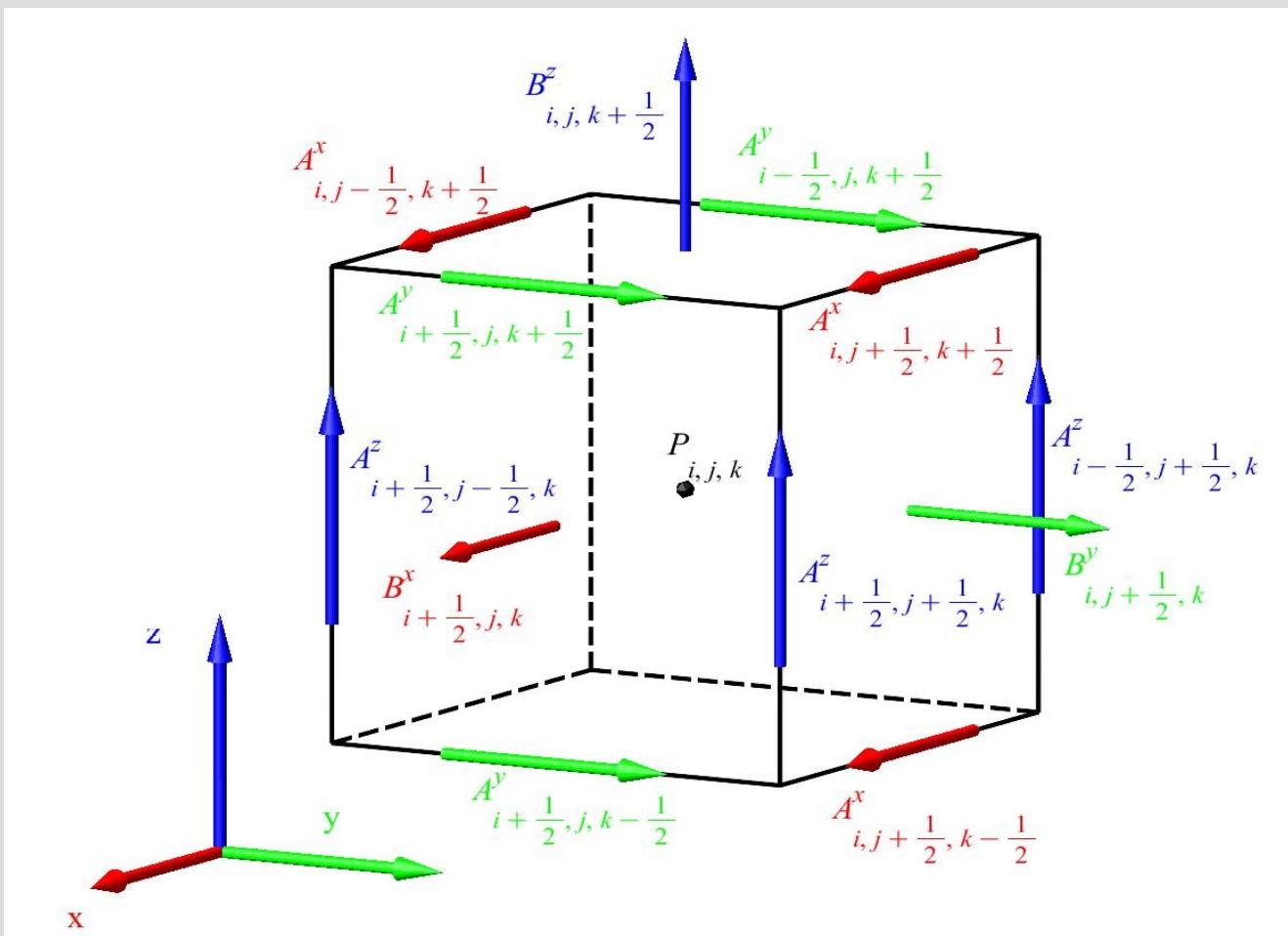
- **2nd- order convergence**

- BALASARA 1 ShockTube
- PPM + HLL
- Postshock oscillations avoided



STAGGERED VECTOR POTENTIAL

Each grid-cell



Variables' location

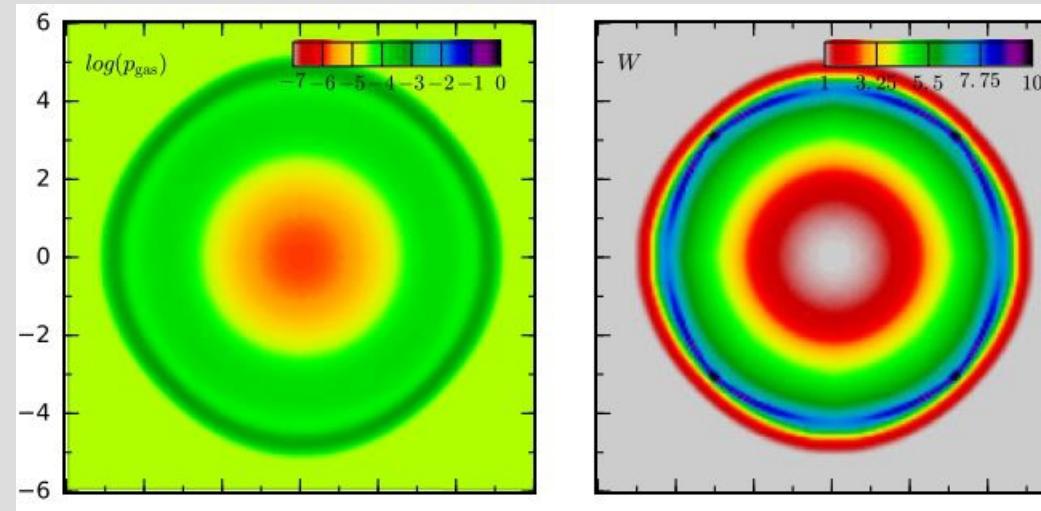
Symbol	Definition	Location
α	Lapse	(i, j, k)
β^m	m -component of the shift vector	(i, j, k)
γ^{mn}	mn -component of the spatial metric	(i, j, k)
γ	Determinant of the spatial metric	(i, j, k)
ρ	Rest-mass density	(i, j, k)
p_{gas}	Pressure	(i, j, k)
ε	Energy density	(i, j, k)
v_m	m -component of fluid velocity	(i, j, k)
B^1	x -component of magnetic field	$(i + \frac{1}{2}, j, k)$
B^2	y -component of magnetic field	$(i, j + \frac{1}{2}, k)$
B^3	z -component of magnetic field	$(i, j, k + \frac{1}{2})$
A_1	x -component of vector potential	$(i, j + \frac{1}{2}, k + \frac{1}{2})$
A_2	y -component of vector potential	$(i + \frac{1}{2}, j, k + \frac{1}{2})$
A_3	z -component of vector potential	$(i + \frac{1}{2}, j + \frac{1}{2}, k)$
Ψ_{mhd}	Scalar potential	$(i + \frac{1}{2}, j + \frac{1}{2}, k + \frac{1}{2})$

$$\vec{B} = \nabla \times \vec{A} \implies \nabla \cdot \vec{B} = 0$$

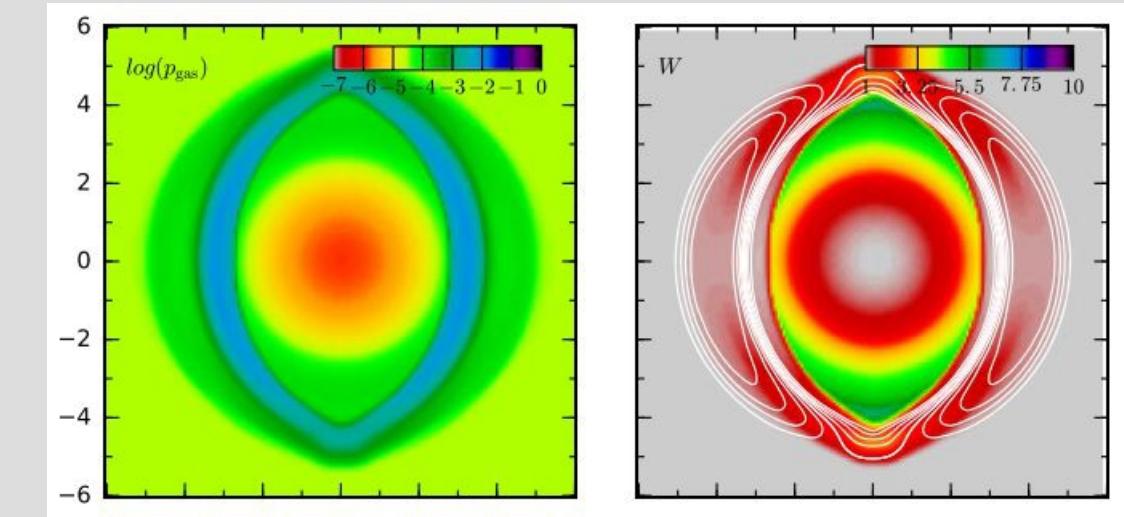
The divergence-free condition of the magnetic field should be satisfied at machine precision

Why several reconstruction methods of different order?

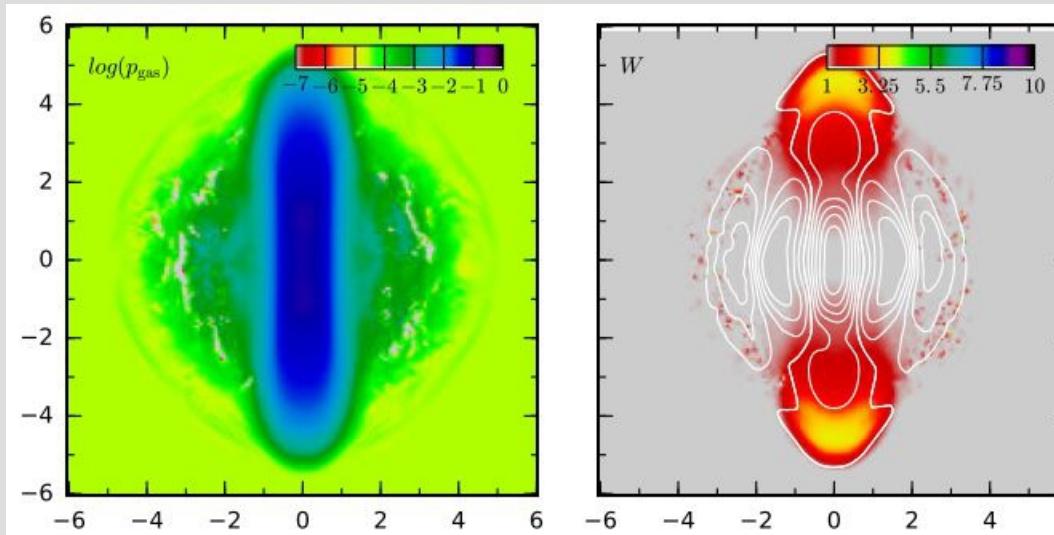
$$B^z = 0$$



$$B^z = 0.1$$



$$B^z = 1.0$$



Using lower-order reconstruction (e.g. MINMOD) might provide some dissipation for difficult problems

Analytical EOSs

■ Ideal Fluid EOS

$$p_{\text{gas}} = (\Gamma - 1) \rho \varepsilon$$

■ Polytropic EOS

$$\begin{aligned} p_{\text{gas}} &= K \rho^{\Gamma}, \\ \varepsilon &= \frac{K \rho^{\Gamma-1}}{(\Gamma-1)} \end{aligned}$$

The EinsteinToolkit's `EOS_Omni` thorn provides support for those kinds EOS simply by setting some parameters

Tabulated EOSs and neutrino leakage

F Cipolletta et al 2021 Class. Quantum Grav. 38 085021

- Tabulated EOS: <https://compose.obspm.fr/home/> → $P = P(\rho, T, Y_e)$
- The `EOS_Omni` thorn provides support for that kind of EOS, but:
 - Need to select EOS's “slices” for ID: const. T or S slices → Table is reduced to “2D”
 - Code for producing and reading ID: **Lorene** - <https://lorene.obspm.fr/>
 - Code for setting Beta equilibrium
 - C2P which support “evolving” T and S: **Palenzuela1D** – *Siegel et al, ApJ (2018)*
 - Code for neutrino leakage: **ZelmaniLeak** - *Ott et al, PRD (2012)*

ZelmaniLeak - Neutrino Leakage (1)

Dominant processes:

- A) $e^- + n \rightarrow p + \nu_e$ ← Electron Capture
- B) $e^+ + n \rightarrow p + \bar{\nu}_e$ ← Positron Capture
- C) $e^- + e^+ \rightarrow \nu_i + \bar{\nu}_i$ ← Pair Annihilation
- D) $\gamma \rightarrow \nu_i + \bar{\nu}_i$ ← Plasmon Decay

Optical Depth: isotropic neutrino radiation → $\tau(x) = \min_{\gamma \in \Gamma} \int_{\gamma} k \rho \sqrt{\gamma_{ij} dx^i dx^j}$

1. Diffusive Regime (Absorption) - $\rho > 10^{12} \text{ g cm}^{-3}$

Sources of opacity:

$$\begin{aligned}
 \nu_i + \{n, p\} &\rightarrow \nu_i + \{n, p\} & \Rightarrow \sigma_{\nu_i, \{n, p\}} & \quad R_{\nu_i}^{\text{diff}} \propto T \\
 \nu_i + \{A, Z\} &\rightarrow \nu_i + \{A, Z\} & \Rightarrow \sigma_{\nu_i, \{A, Z\}} & \quad Q_{\nu_i}^{\text{diff}} \propto T^2 \\
 \nu_e + n &\rightarrow p + e^- & \Rightarrow \sigma_{\nu_e, n} & \quad E_{\nu_i}^{\text{diff}} \propto T \\
 \bar{\nu}_e + n &\rightarrow p + e^+ & \Rightarrow \sigma_{\bar{\nu}_e, p} &
 \end{aligned}$$

2. Free-streaming Regime (Emission) - $\rho < 10^{12} \text{ g cm}^{-3}$

A), B), C), D)

$N + N \rightarrow N + N + \nu + \bar{\nu}$

$$R_{\nu_i}^{\text{proc}}, Q_{\nu_i}^{\text{proc}} \propto T^{\frac{n}{q}}, \text{ with } \frac{n}{q} \gg 1$$

Constraining T
→ can avoid issues
at the NS surface

ZelmaniLeak - Neutrino Leakage (2)

3. Neutrino Re-absorption – Heating:

$$\begin{aligned} \sum_{\text{proc}} R_{\nu_i}^{\text{proc}} &= R_{\nu_i}^{\text{loc}} \\ \sum_{\text{proc}} Q_{\nu_i}^{\text{proc}} &= Q_{\nu_i}^{\text{loc}} \end{aligned} \Rightarrow E_{\nu_i}^{\text{ef}} = \frac{Q_{\nu_i}^{\text{ef}}(Q_{\nu_i}^{\text{diff}}, Q_{\nu_i}^{\text{loc}})}{R_{\nu_i}^{\text{ef}}(R_{\nu_i}^{\text{diff}}, R_{\nu_i}^{\text{loc}})}$$

$$Q_{\{\nu_e, \bar{\nu}_e\}}^{\text{heat}}(r) \equiv Q_{\{\nu_e, \bar{\nu}_e\}}^{\text{heat}} \left(L_{\{\nu_e, \bar{\nu}_e\}}^{\text{FRF}}, \sigma_{(\{\nu_e, n\}, \{\bar{\nu}_e, p\})}^{\text{heat}}, r, \rho, X_{\{n, p\}} \right)$$

where

$$L_{\nu_i}^{\text{FRF}}(r) \equiv L_{\nu_i}^{\text{FRF}}(r, \alpha, W, v, Q_{\nu_i}^{\text{ef}}) \quad \textbf{NOTE: } Q_{\nu_i}^{\text{ef, heat}} = Q_{\nu_i}^{\text{ef}} - Q_{\nu_i}^{\text{heat}} \implies Q_{\nu_i}^{\text{heat}} \text{ alters } L_{\nu_i}$$

4. Neutrino Pressure Handling: $\rho > 10^{12} \frac{\text{g}}{\text{cm}^3} \Rightarrow P_\nu$ added to $T^{\alpha\beta}$ source terms

5. Ray-by-ray approach: $(x, y, z) \rightarrow (r, \theta, \phi) \rightarrow (x, y, z)$

6. Operator-split: Y_e and ϵ should be updated at each time-step via P2C

TOV Tests

ID	GRMHD	symmetry	Beta-equilibrium	T-Evolution	Max B-Field	Neutrino-Leakage
00	Spritz	Octant	T-slice	X	-	Disabled
01	Spritz	Full 3D	S-slice	V	-	Disabled
02	GRHydro	Octant	S-slice	V	-	Disabled
03	Spritz	Octant	S-slice	V	-	Disabled
04	Spritz	Full 3D	S-slice	V	-	Enabled
05	GRHydro	Octant	S-slice	V	-	Enabled
06	Spritz	Octant	S-slice	V	-	Enabled
07	GRHydro	Octant	T-slice	V	-	Disabled
08	Spritz	Octant	T-slice	V	-	Disabled
09	Spritz	Octant	T-slice	V after 2 ms	-	Disabled
10	GRHydro	Octant	T-slice	V	-	Enabled
11	Spritz	Octant	T-slice	V	-	Enabled
12	Spritz	Octant	T-slice	V after 2 ms	-	Enabled after t = 3 ms
13	Spritz	Full 3D	S-slice	V	10^{16} G	Disabled
14	Spritz	Full 3D	S-slice	V	10^{16} G	Enabled
15	Spritz	Full 3D	T-slice	V	10^{16} G	Disabled
16	Spritz	Full 3D	T-slice	V	10^{16} G	Enabled after t = 3 ms

RESULTS

- LS220 EOS
- 5 refinement levels
- $dx_{\min} = 0.12 \rightarrow 60$ pts per r_{NS}
resolution of 180m for NS interior
- Evolutions for 6 ms (limited resources)
- Consider or not the heating

Maximum Rest mass Density

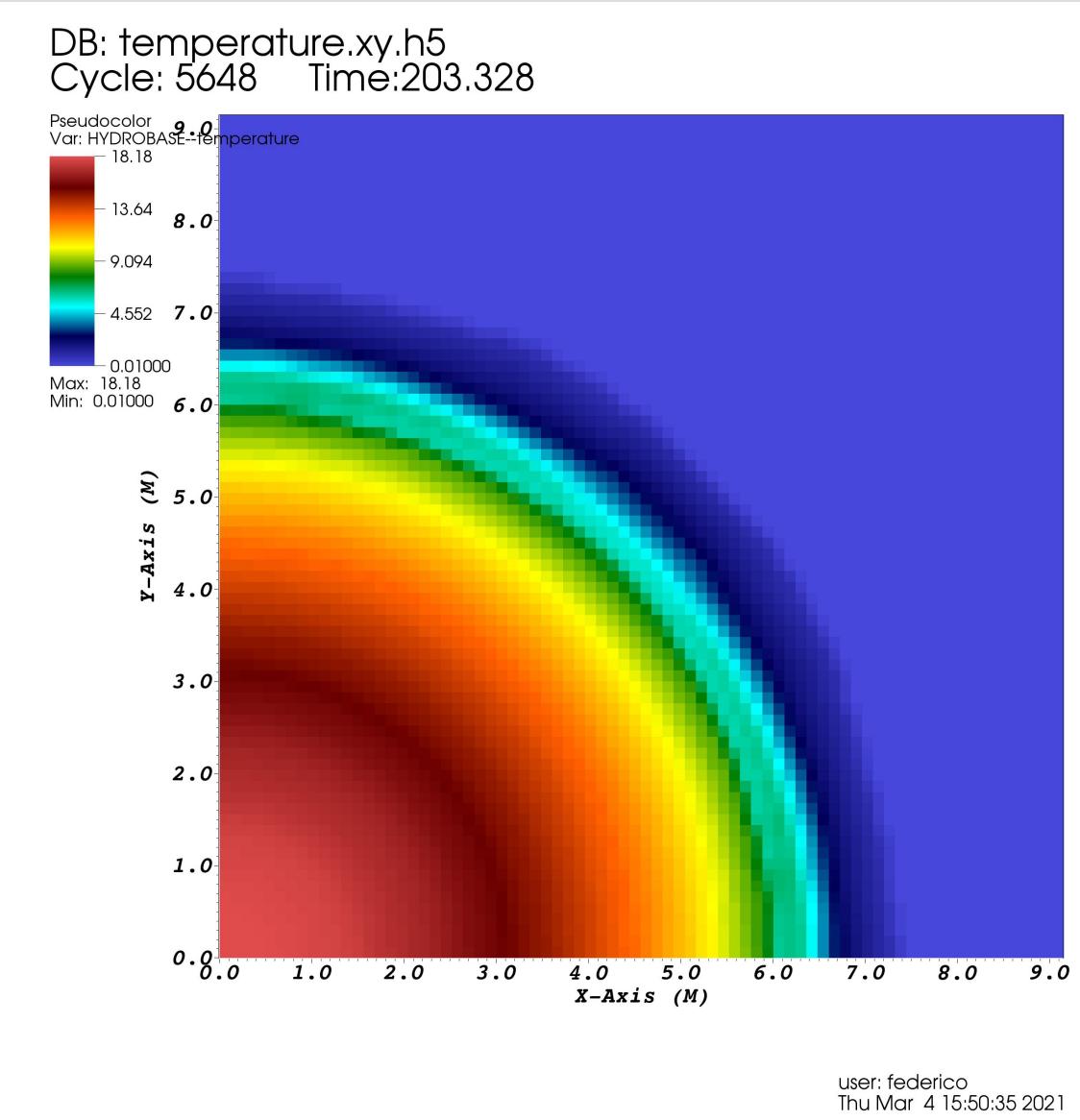
Maximum Temperature

Maximum norm of B

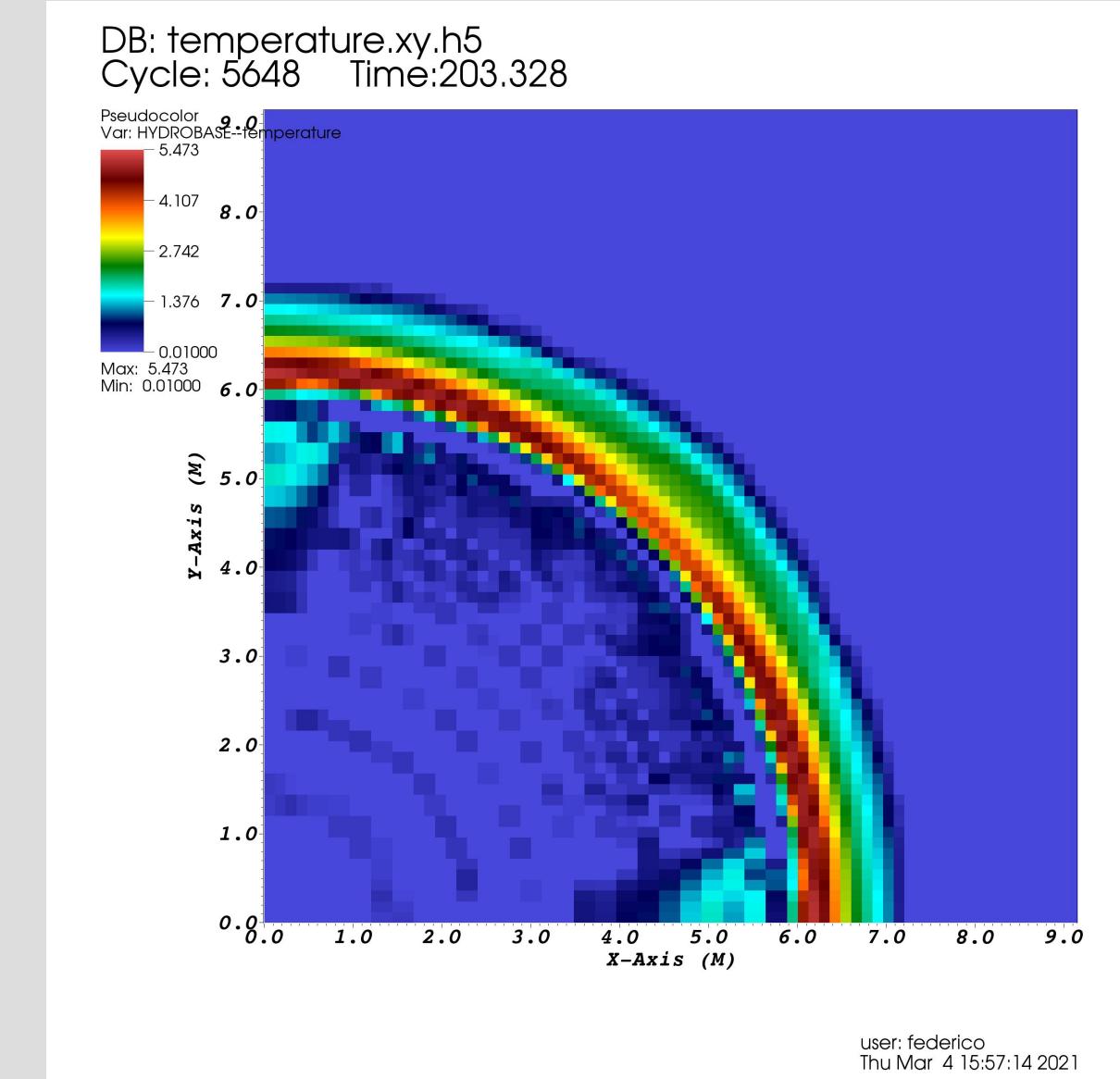
Luminosity of neutrinos

Note on the maximum temperature

Sim 03 – octant symmetry – const S ID

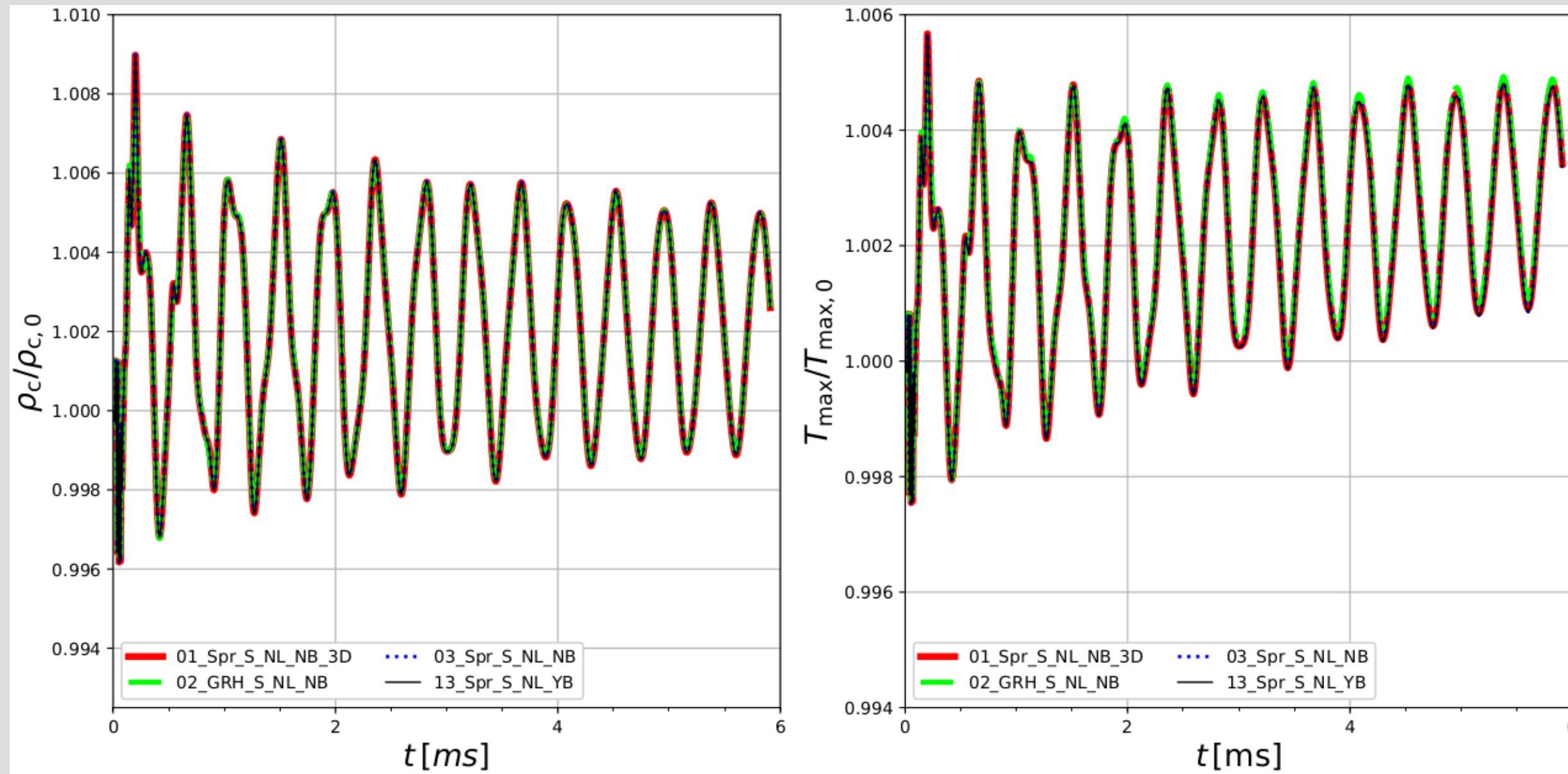


Sim 08 – octant symmetry – const T ID



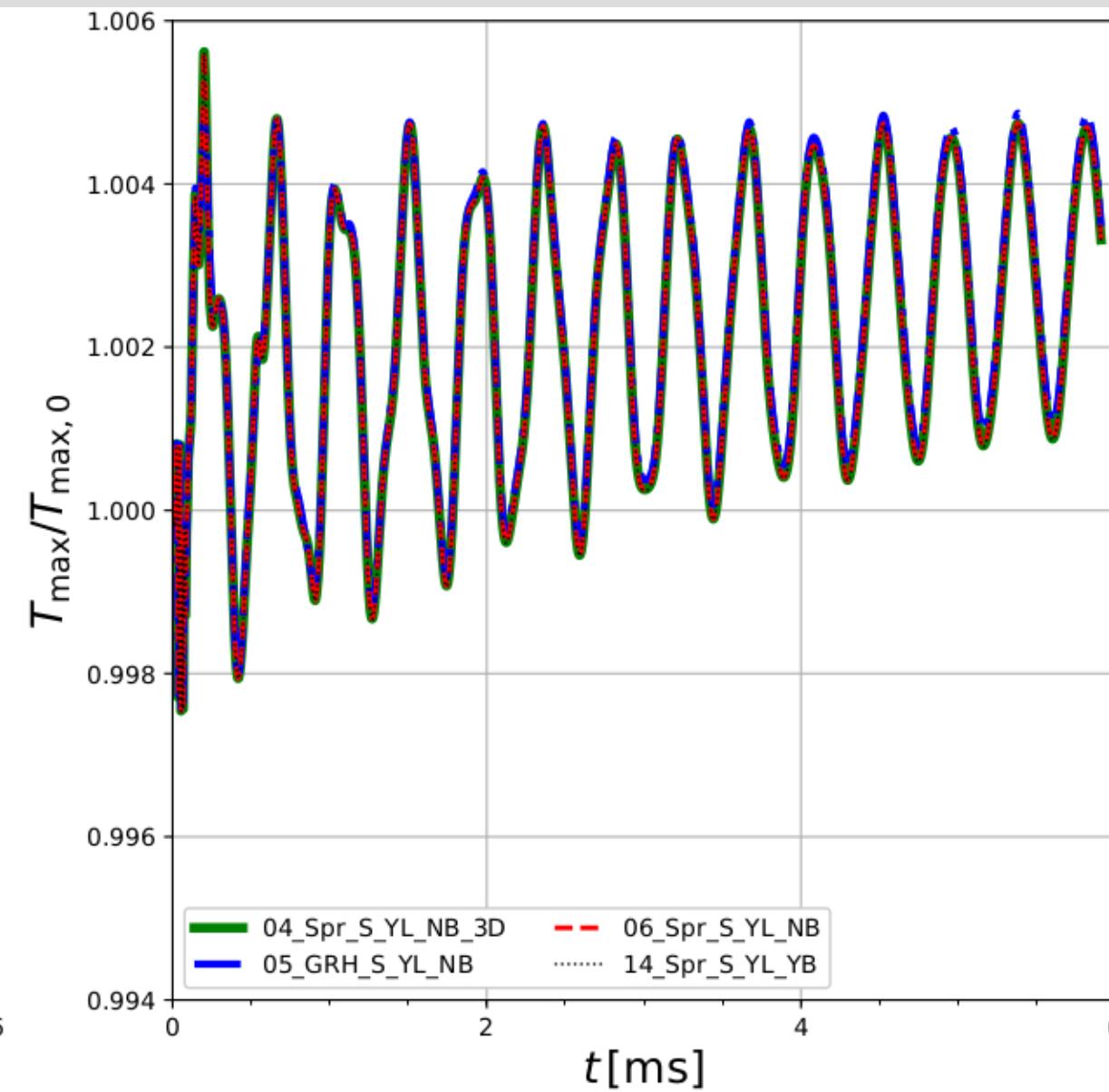
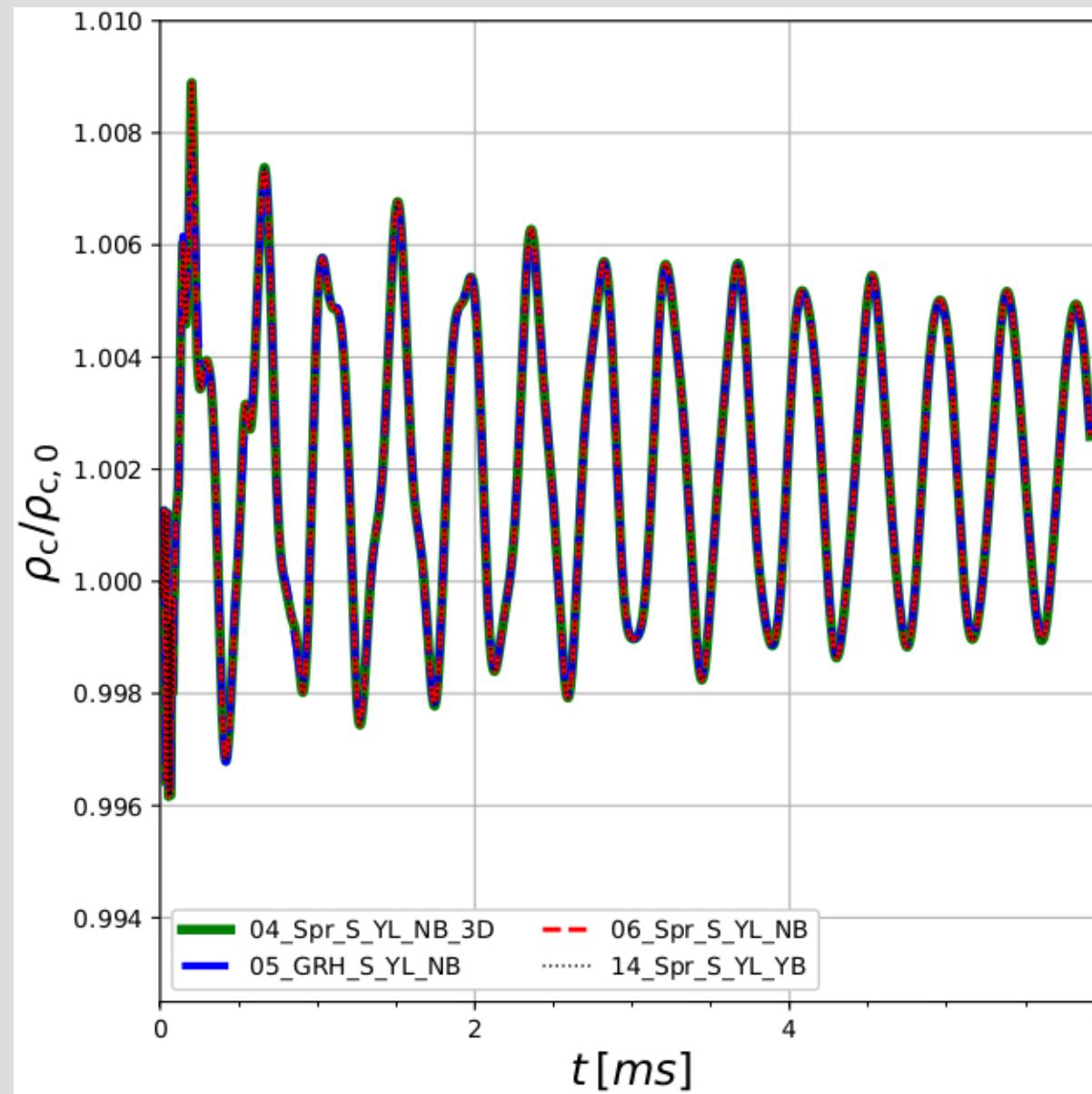
The simulations with const T ID are the most delicate ones

Hydro Results – const S id without leakage



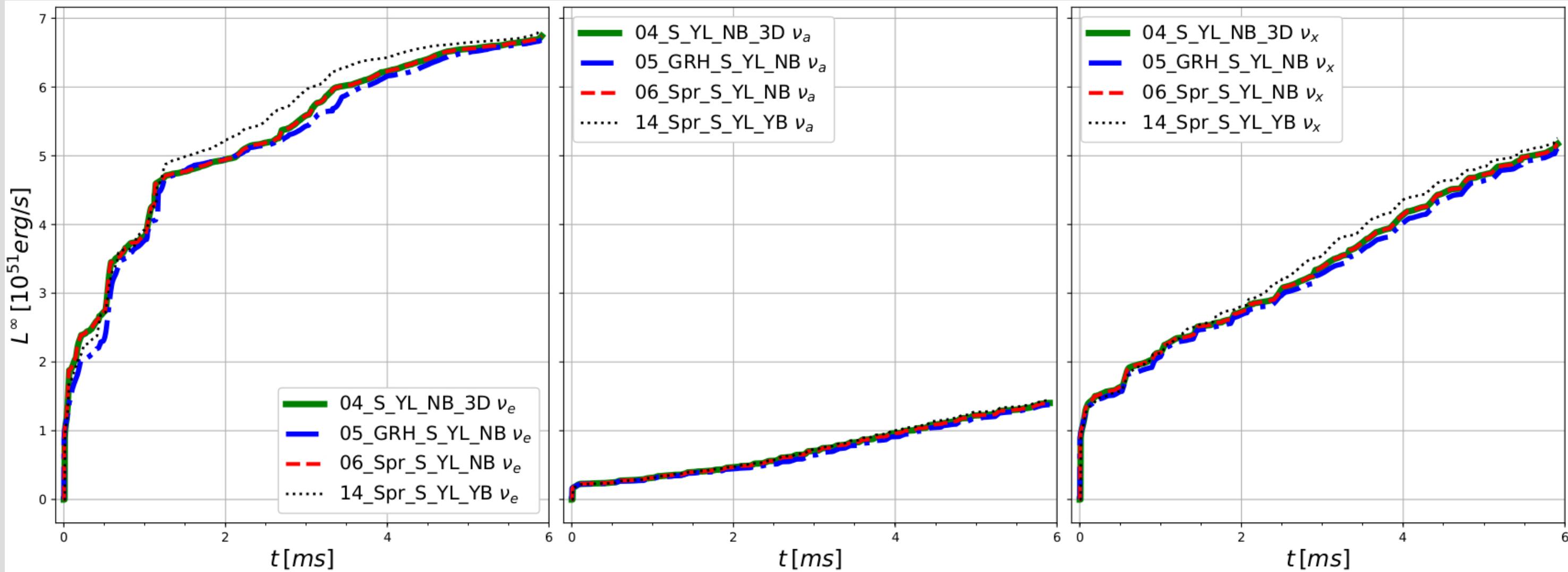
Maximum of T at NS center

Hydro Results – const S id with leakage



Maximum of T at NS center

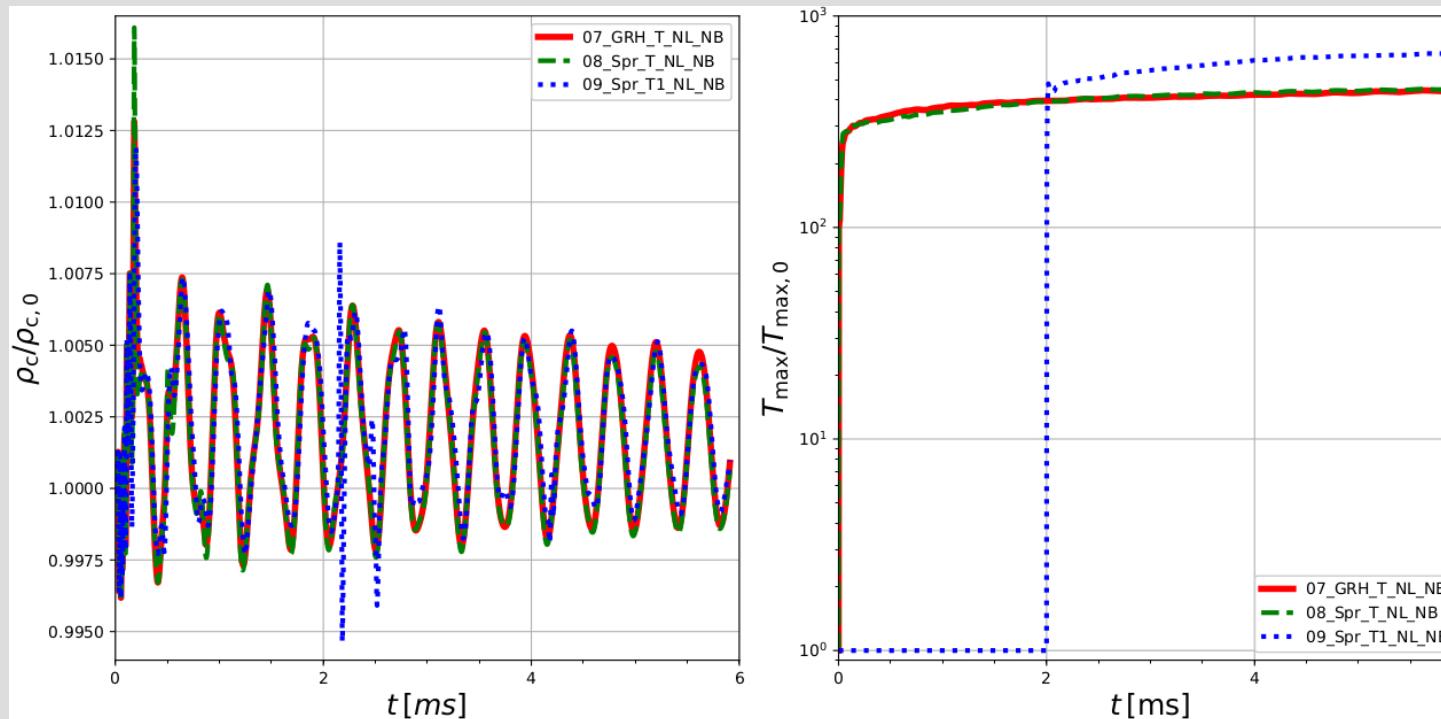
Luminosity Results – const S id with leakage

 L_{ν_e} L_{ν_a} L_{ν_x} 

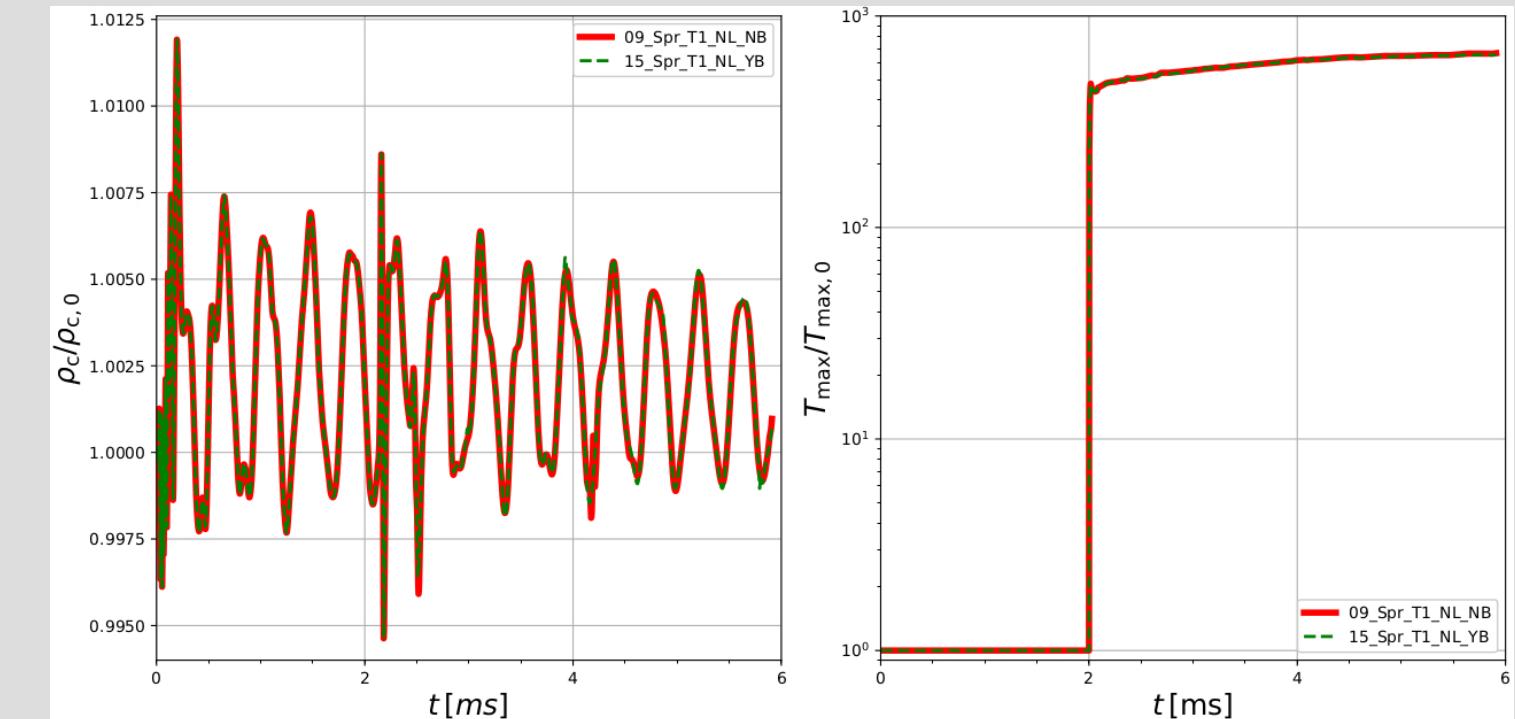
Electron capture seems the dominant process

Hydro Results – const T id without leakage

Pure Hydro

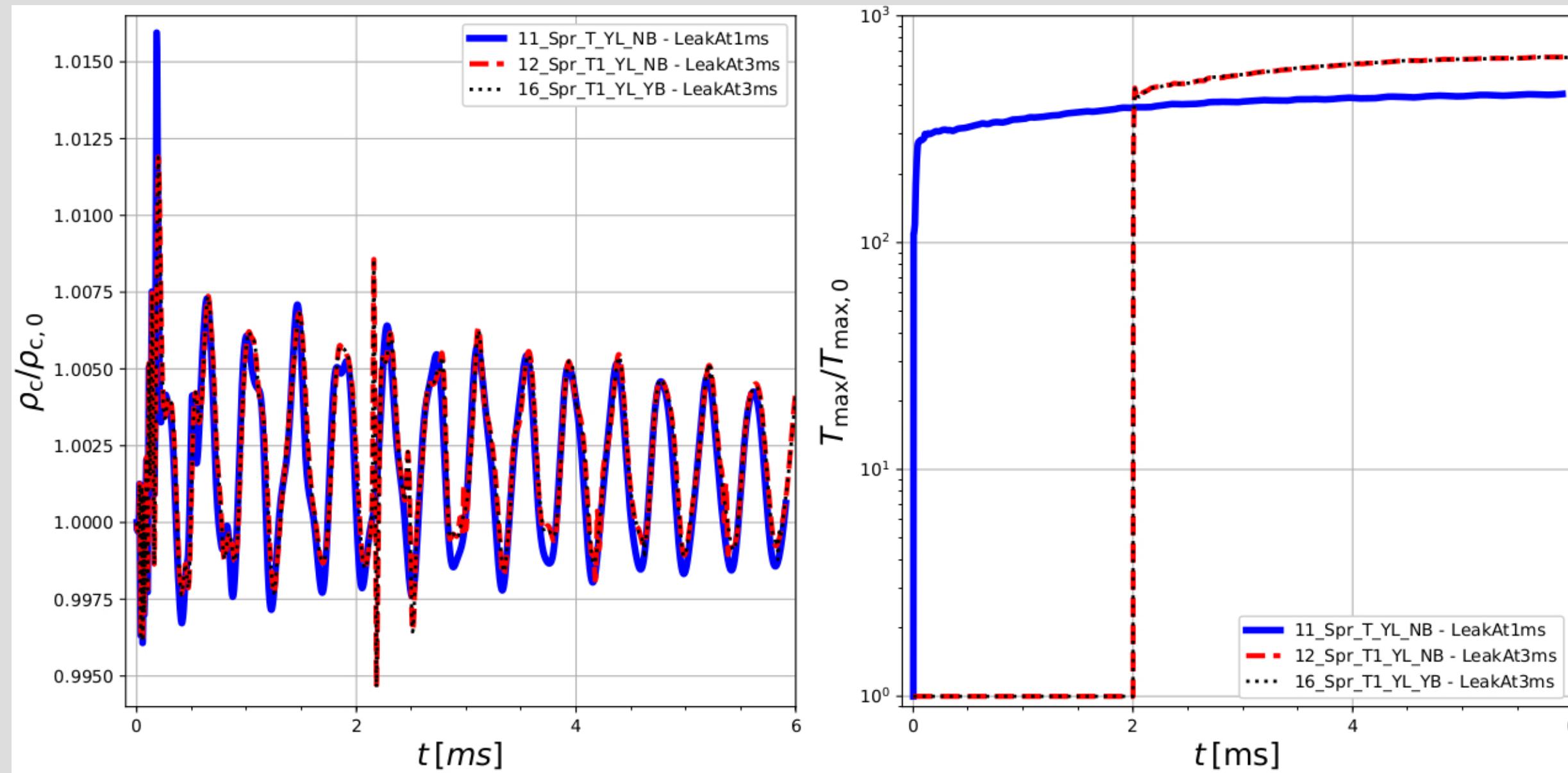


Pure Hydro VS Magnetized



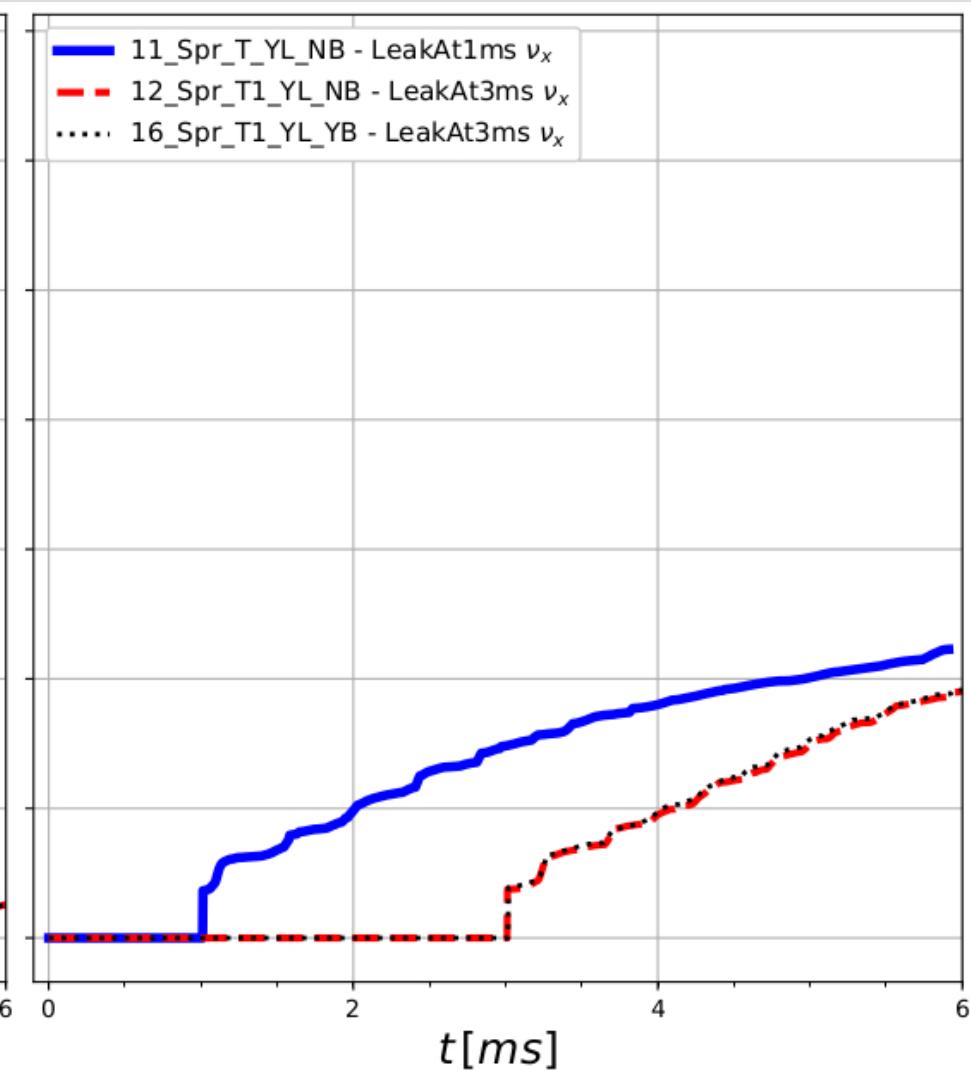
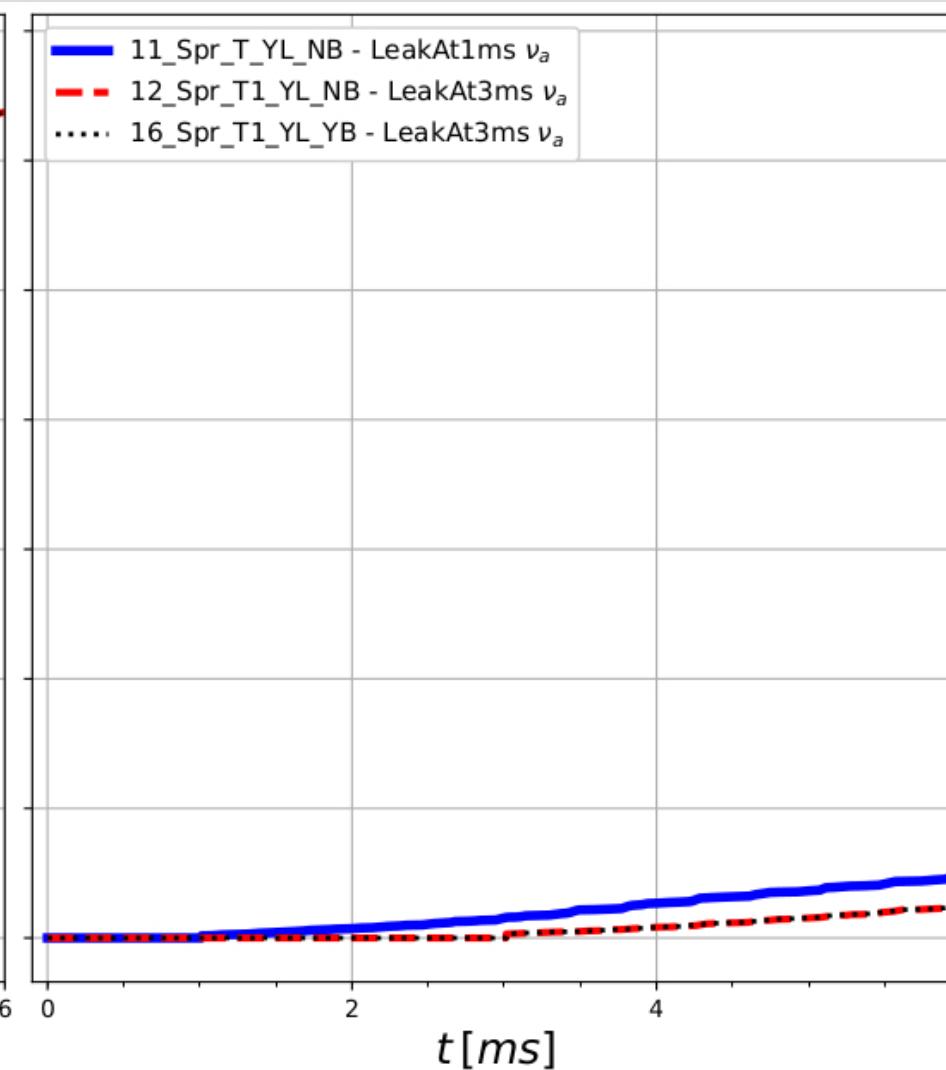
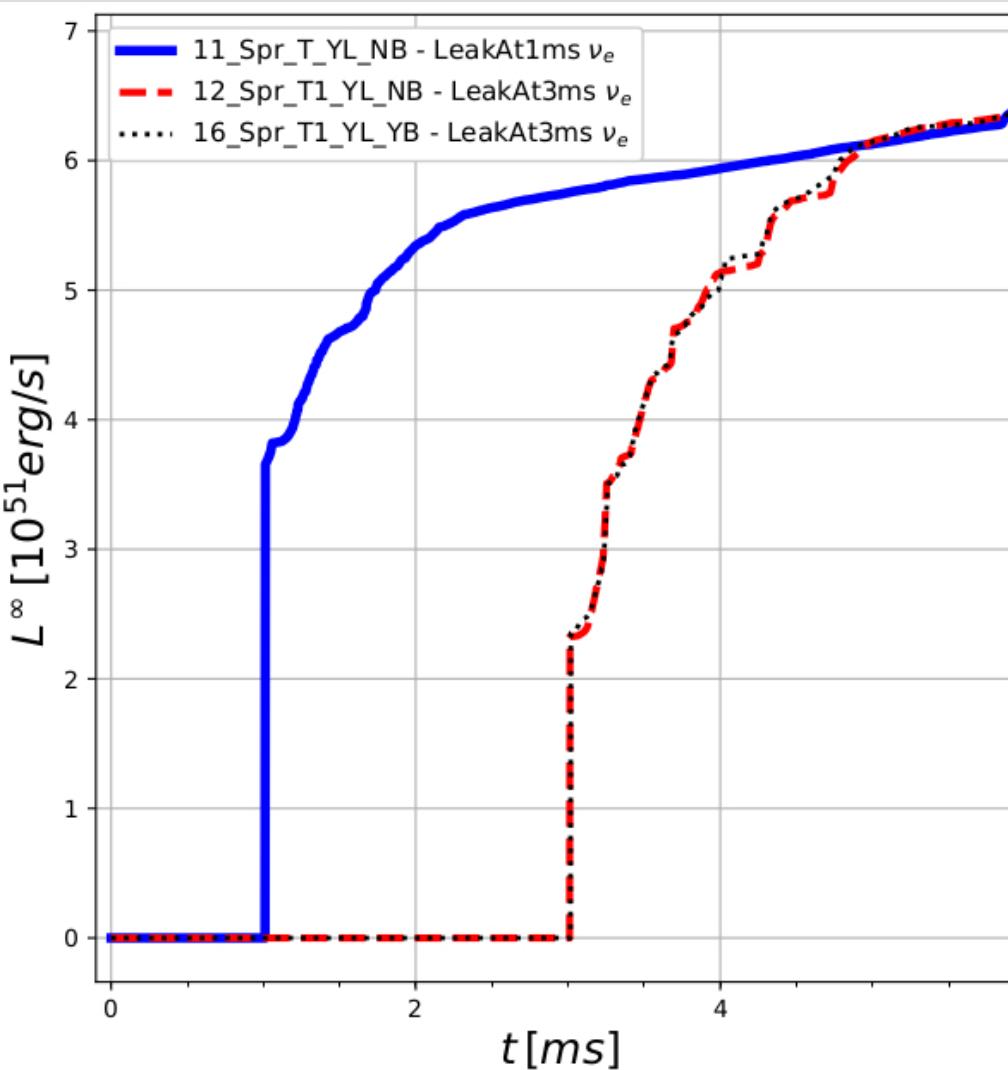
Maximum of T at NS surface (!!)

Hydro Results – const T id with leakage



Maximum of T at NS surface (!!!)

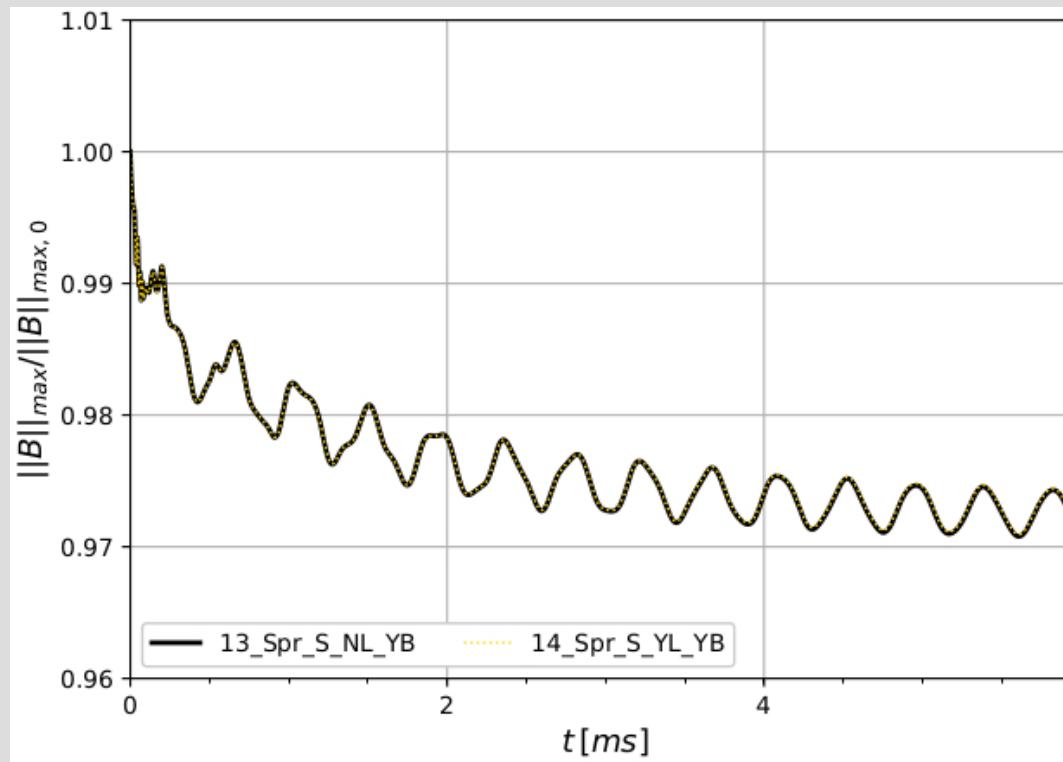
Luminosity Results – const T id with leakage

 L_{ν_e} L_{ν_a} L_{ν_x} 

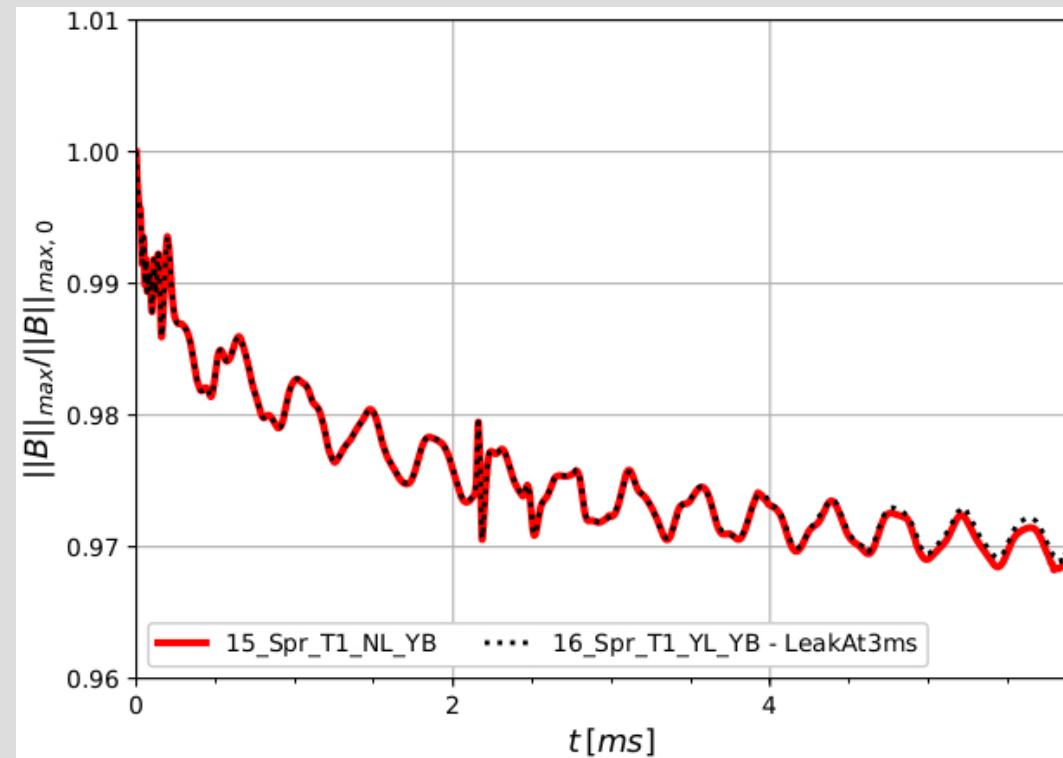
Electron capture seems the dominant process
NOTE: the delayed leakage activation is evident

RESULTS – B norm max

Const S id



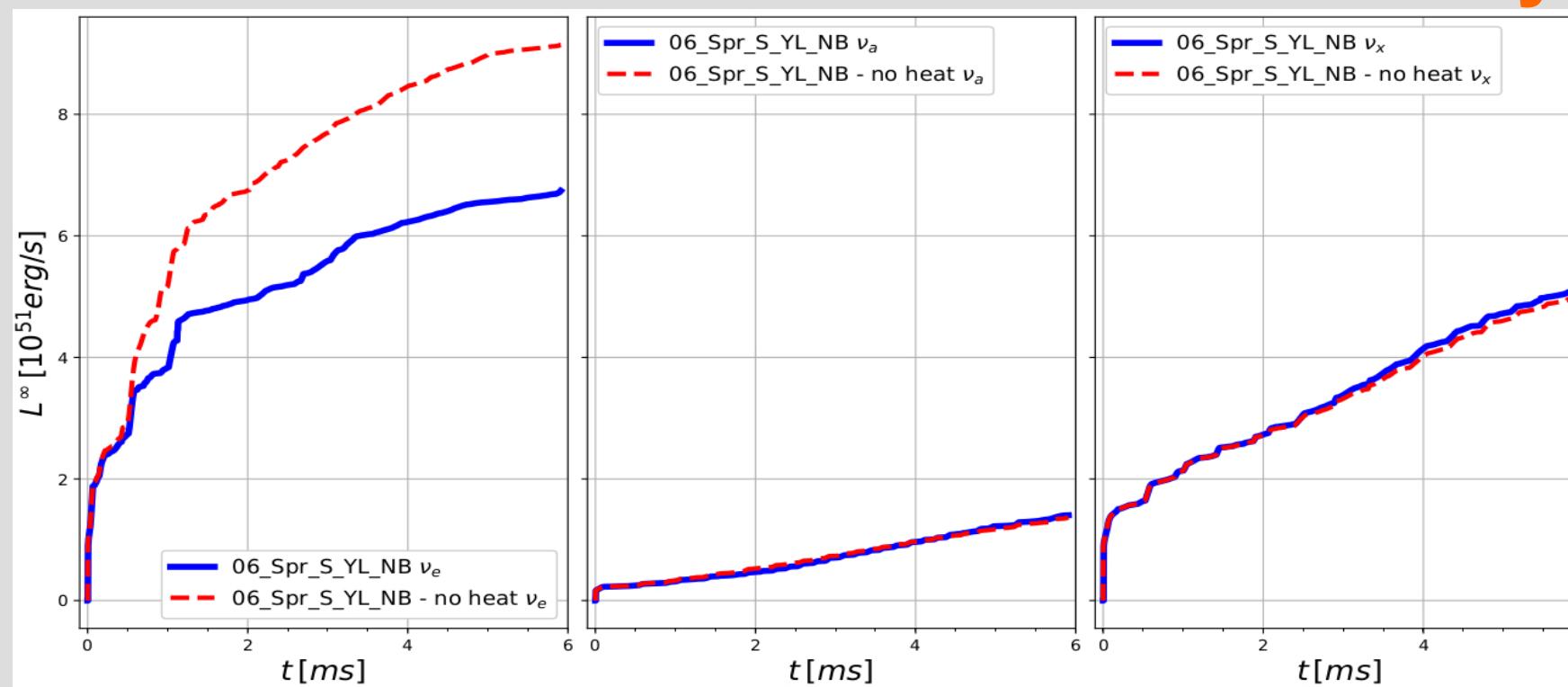
Const T id



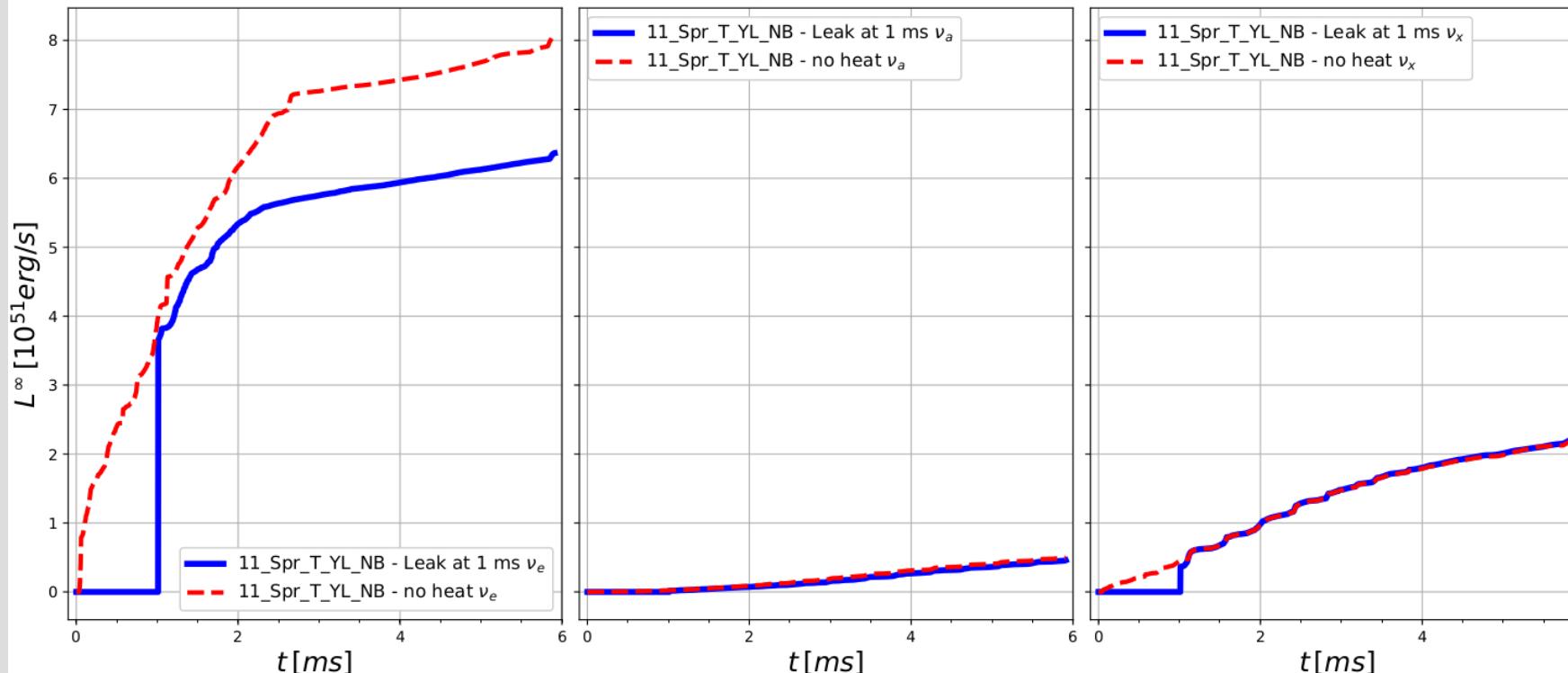
Lekage activation does not alter the maximum B-field evolution

RESULTS – neutrino Luminosity

Const S id



Const T id



Heating may considerably affect the neutrino luminosity observed

Heating effects need to be handled with care

Spritz code in the TCAN collaboration

Theory and Computational Network on Neutron Star Mergers

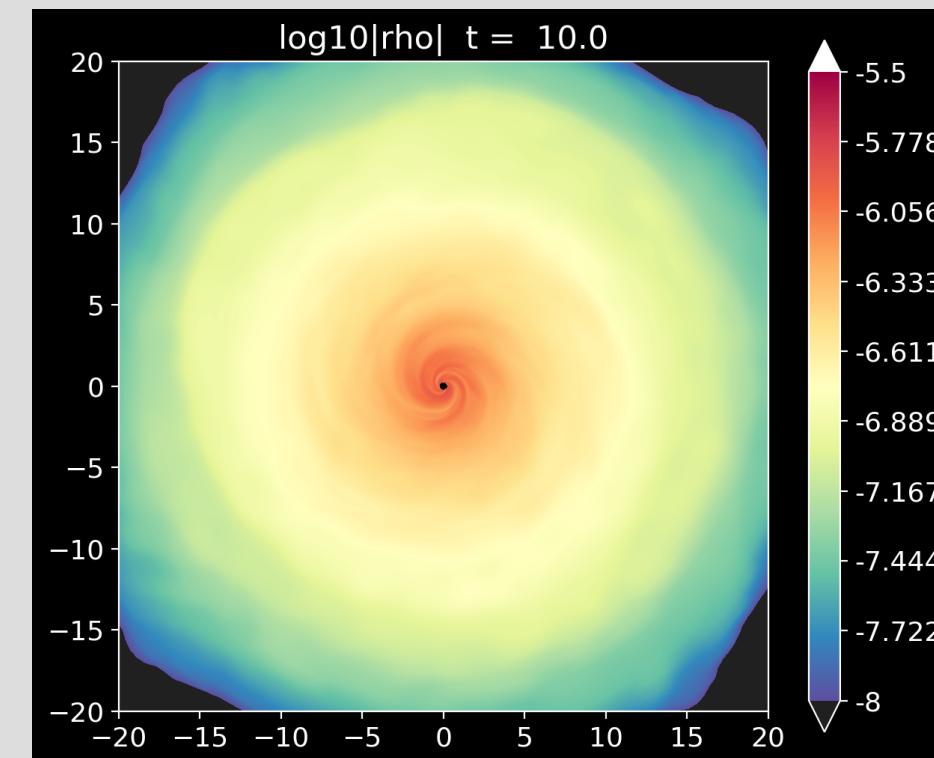


Advancing Computational Methods to Understand the Dynamics of Ejections, Accretion, Winds and Jets in Neutron Star Mergers



Take advantage of the strength of each code:

- absence of symmetry → **CARTESIAN** coordinates
- axial symmetry → **SPHERICAL** coordinates



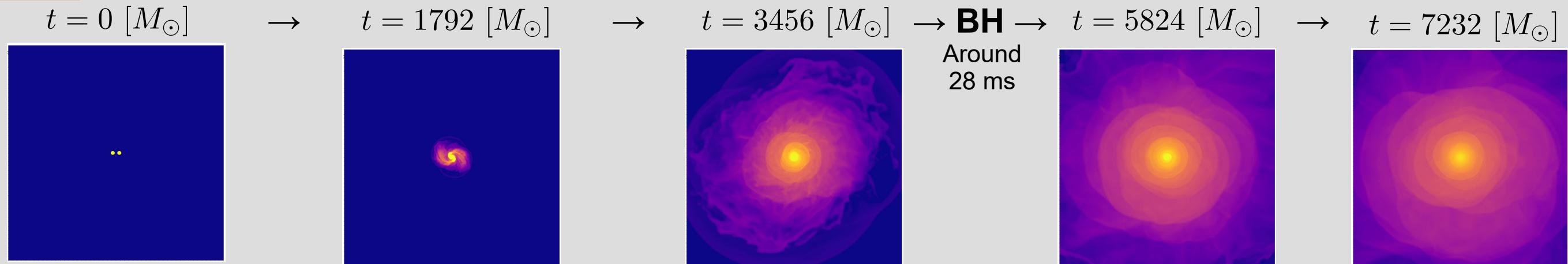
*Work in progress:
Armengol-Lopez et al.*

Hand-off from BNS simulation in Cartesian coordinates (**IGM**) to postmerger simulations in spherical coords (**HARM3D**)

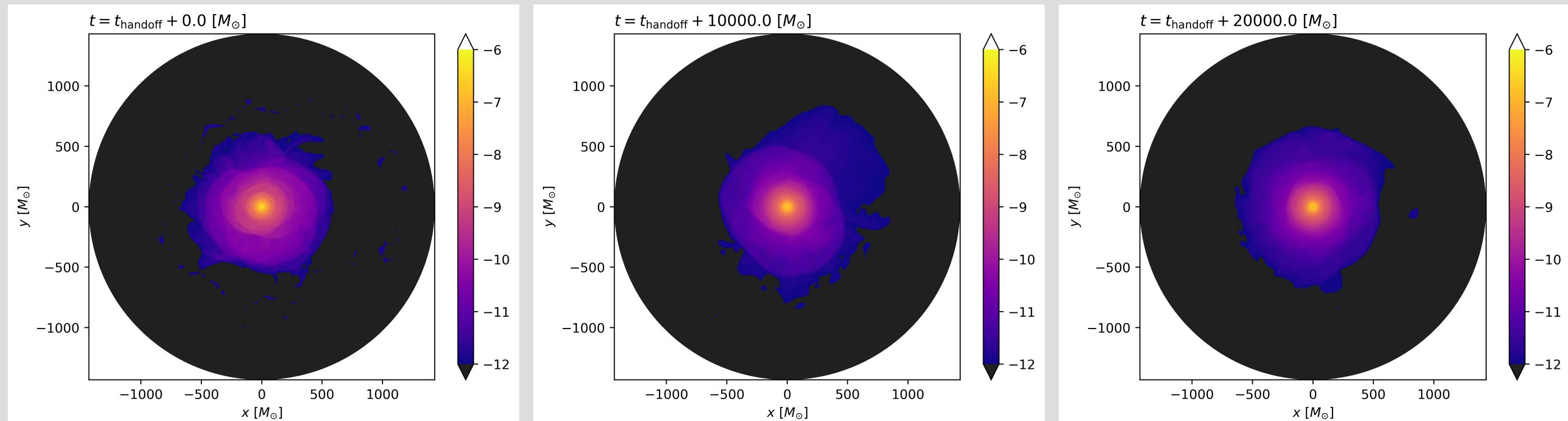
Goal: long-term BNS simulations with:

- Dynamical GR-MHD
- Nuclear and Neutrino Physics, EOS
- Neutrino/photon transport
- R-processes/nucleosynthesis

PureHydro Ideal Fluid BNS MERGER (the “Missing Link”) + HANDOFF SPRITZ



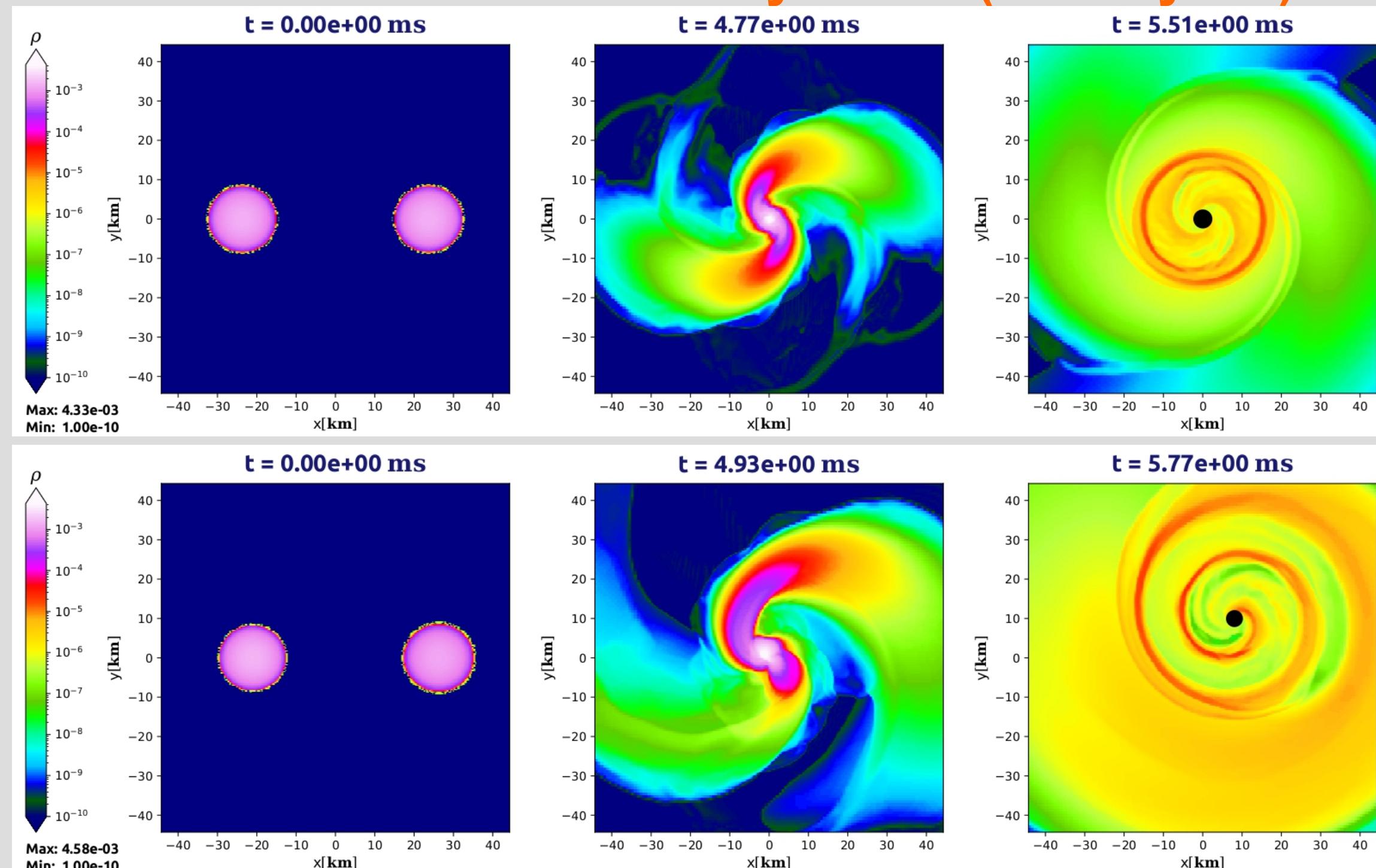
HARM3D



Images for HARM3D: credits to Federico G. L. Armengol @RIT

PRELIMINARY TESTS FOR BNS WITH SLy4 EOS (PureHydro)

- SLy4 EOS
- 6 refinement levels
- $\Delta x_{\min} = 0.24 \sim 354$ m
- Cold initial data

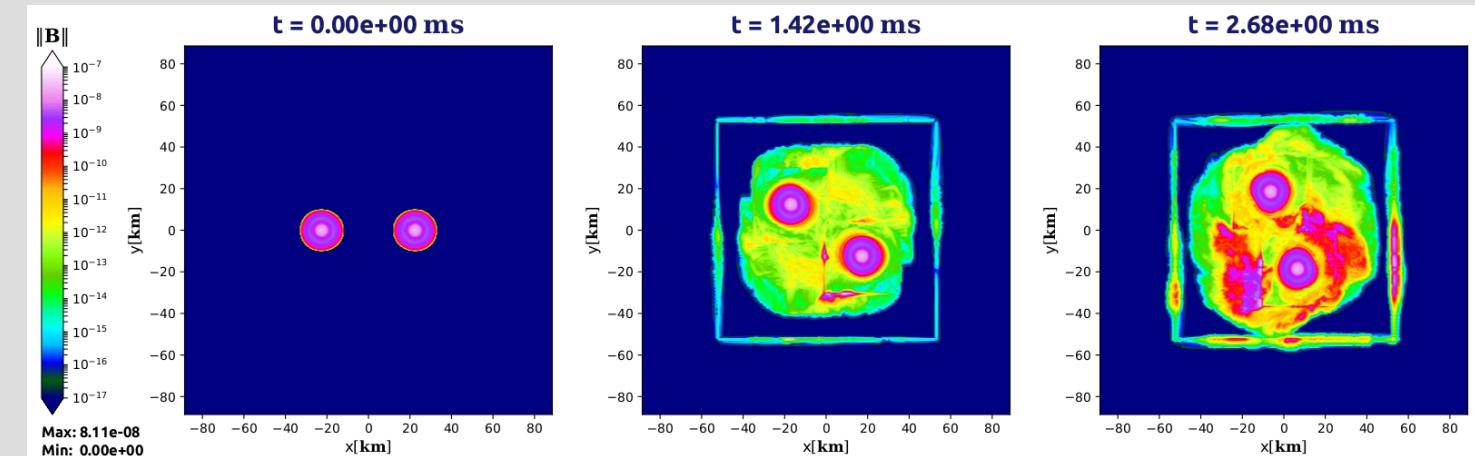


Images: courtesy of Lorenzo Ennoggi @UniMIB

Improving the Interpolation Scheme for the Generalized LG

Previous Interpolation Scheme for A and Ψ_{mhd}

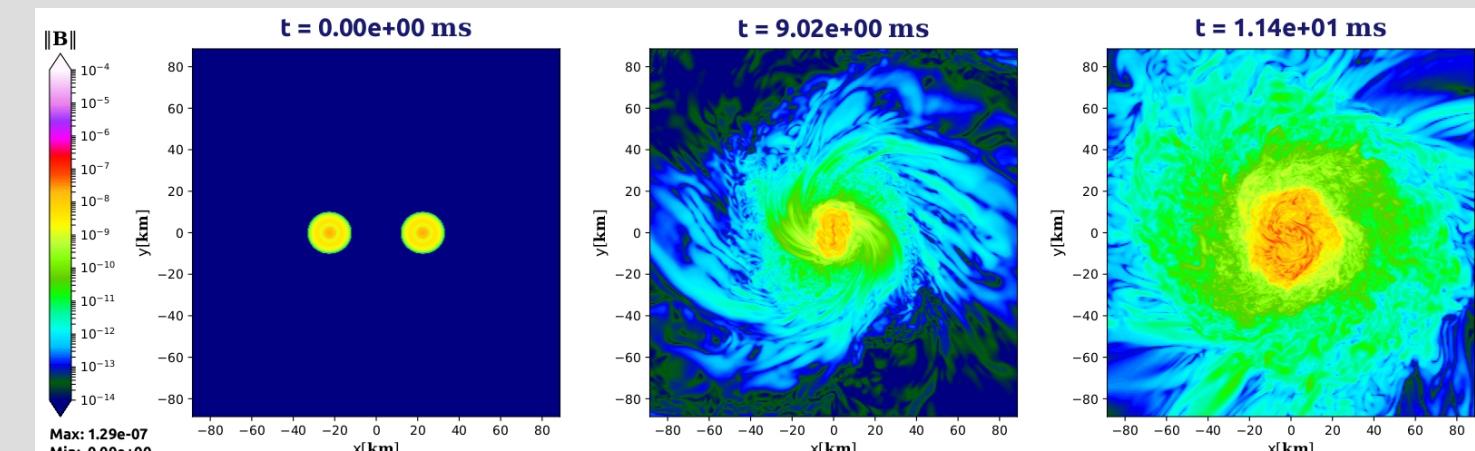
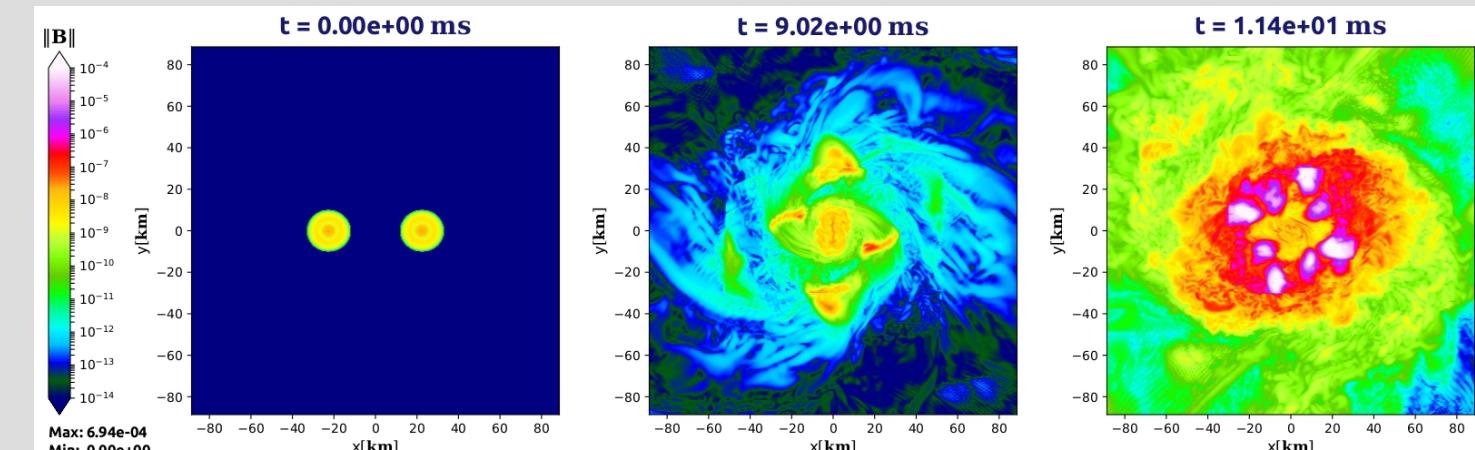
Instabilities developing during BNS inspiral
in the
Magnetized “Missing Link” simulation



Current Interpolation Scheme for A and Ψ_{mhd}

No
Dissipation
After
Merger

Dissipation
After
Merger



NOTE: Work ongoing at CCRG of RIT

Images: courtesy of Lorenzo Ennoggi @RIT

FURTHER AND FUTURE DEVELOPMENTS

- See talk by Jay Kalinani on July 28th at 12:35 pm CT ([Improving C2P](#))
- Testing the handling of B-field outside the NS
- Improving the neutrino treatment