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Computational intelligence:

Fuzzy Systems

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Master's degree in Artificial Intelligence
Barcelona, 2021

1 Description of membership functions & rules

In this laboratory, we design a Mamdani FIS and implement a controller in a compound pendulum to stabilize it at the angle indicated.

To understand this system it is necessary to describe its inputs, outputs, how these interact with each other and how they influence the behavior of the fuzzy controller. This system has two inputs and one output. One of the inputs is the error and the other is the derivative of the error. On the other hand, the output is the thrust that is applied at the joint as a response to the measured inputs.

At a certain time t, the error and its derivative are defined as follows:

$$\mathbf{e(t) = \theta_{ref} - \theta(t)}$$
$$\mathbf{\frac{de}{dt}(t) = \lim_{h \to 0} \frac{e(t+h) - e(t)}{h}}$$

But because the system measures the angle in discrete time, the derivative is approximated by the finite difference, so that:

$$\mathbf{\frac{de}{dt} \approx \frac{\Delta e}{\Delta t} = \frac{e^k - e^{k-1}}{t^k - t^{k-1}}}$$

where e^k is the error at certain sampling time t^k and e^{k-1} is the error at the previous sampling time t^{k-1}.

1.1 Membership functions

We used the triangular membership functions for the inputs and output, and the Mamdani method.

1.1.1 Inputs

For each input, three levels are defined and the membership functions are symmetrical about zero:

Error: We get a *Negative* result when we are at an angle greater than the desired value of the angle, *Positive* if it is less than the desired angle and *Zero* when it is considered "equal" to the desired angle. Its range is between [-80º, 80º] and the membership functions are of type *trimf* with the following parameters:

Negative	Zero	Positive
[-80 -40 0]	[-40 0 40]	[0 40 80]

Derivative of error: We get a decreasing result when the derivative is negative, increasing for the positive derivative and stationary when there is no variation (around the zero value). Its range is between [-5 5] and for each membership function:

Decreasing	Stationary	Increasing
[-5 -2 0]	[-2.5 0 2.5]	[0 2 5]

1.1.2 Output

Thrust: We get a negative result when a small force is applied, it is positive when a bigger force is applied and the center is between the range [-12.5 12.5]. To apply more or less force to the pendulum can be interpreted as thrust (torque), which makes it describe a rotational movement. Its range is between [-25 25] and for each membership function:

Negative	Stable	Positive
[-40 -20 0]	[-20 0 20]	[0 20 40]

1.2 Rules base

To understand how the rules are working, we need to describe how this system is working. In the following Figure 1 it can be seen how the inputs and outputs of the system are interpreted.

To make a good inference of the result, we consider all the statements and we will analyze the possible input values:

The error will be **negative** when $\theta > \theta_{ref}$ $=> e < 0$:

First rule: The pendulum moves away from the desired angle so the $de/dt < 0$. That is, it decreases. Therefore, we must apply a *thrust* < 0 to move it back.

Second rule: The pendulum approaches the desired angle and then the $de/dt > 0$. That is, it grows. Therefore, we should apply a *thrust* < 0 or a neutral value to be able to move it back faster or slower.

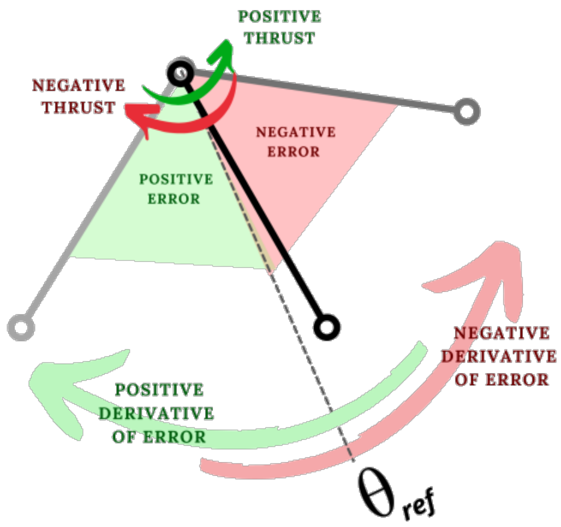

Figure 1 – System description, source: own construction

Third rule: However, when it reaches $\theta = \theta_{ref} \Rightarrow e = 0$ then we want to apply the opposite *thrust* > 0 (or a neutral value) in order to stop its tendency to move to the area where $\theta < \theta_{ref}$.

The error will be **positive** when $\theta < \theta_{ref} \Rightarrow e > 0$:

Fourth rule: The pendulum moves away from the desired angle so $de/dt > 0$. That is, it grows. Therefore, we must apply a *thrust* > 0 in order to move it back.

Fifth rule: The pendulum approaches the desired angle and then $de/dt < 0$. That is, it decreases. Therefore, we should apply a *thrust* > 0 or a neutral value to be able to move it back faster or slower.

Sixth rule: However, when it reaches $\theta = \theta_{ref} \Rightarrow e = 0$ then we want to apply the opposite *thrust* < 0 (or a neutral value) in order to stop its tendency to move to the area where $\theta > \theta_{ref}$.

When de/dt is stationary:
Seventh rule: the pendulum could be at the same position,so we assume thrust =<neutral value>.

2 Plots of the results

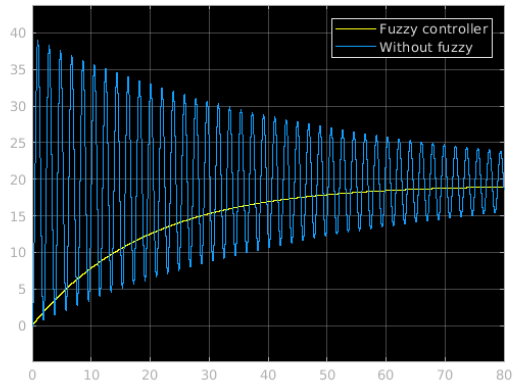

Figure 2 – $\theta_{ref} = 20^{\circ}$ (3 output MFs).

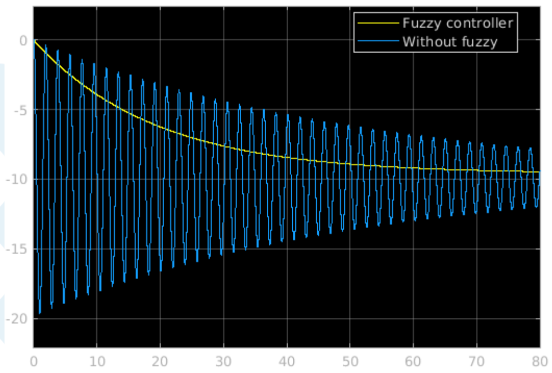

Figure 3 – $\theta_{ref} = -10^{\circ}$ (3 output MFs).

As we mentioned in the previous section, the rules were chosen for all the values of θ. From the Figure 2 and Figure 3 we can see that with the fuzzy controller, we have a smooth convergence and we don't surpass the desired angles since we are near the neutral area and we try to keep the pendulum at this exact position applying small positive and negative thrust continuously. The thrust values come from the defuzzification process which uses the centroids in this case. Because of that after convergence we don't use just zero thrust in the neutral area, but a thrust which has a membership degree in the positive or negative area as well.

3 Behavior increasing the number of membership functions

In order to test the influence of the number of output membership functions on the results, we added two new membership functions to our thrust output: small positive thrust (SP) and small negative thrust (SN). See figure 4.

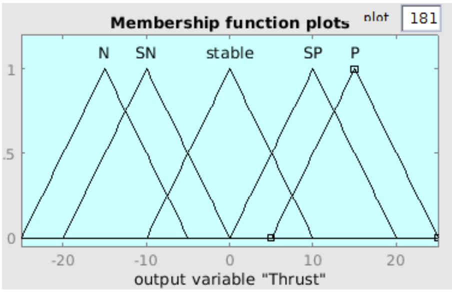

Figure 4 – Five output membership functions

As we can appreciate in figures 5 and 6, when we increase the number of membership functions in the output from 3 to 5, the result improves, obtaining a faster response from the system in both cases (desired angles 20º and -10º).

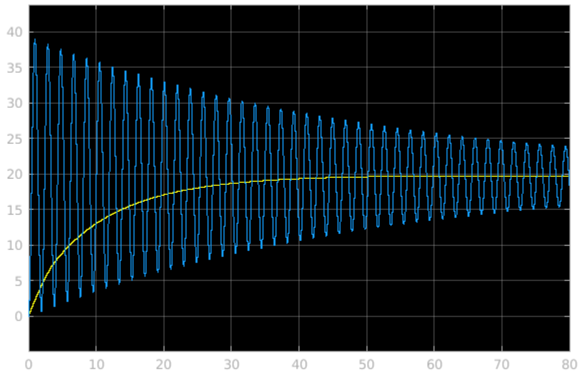

Figure 5 – $\theta_{ref} = 20^{º}$ (5 output MFs).

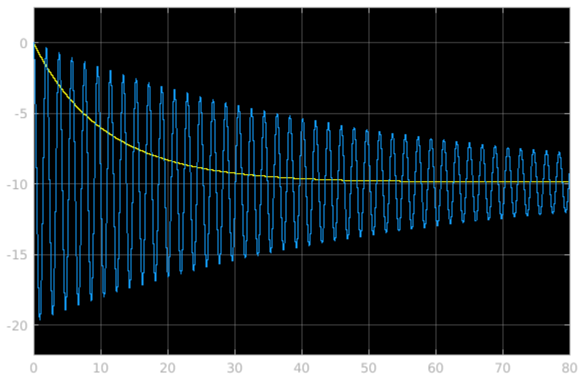

Figure 6 – $\theta_{ref} = -10^{º}$ (5 output MFs).

If we continue to increase the number of membership functions, we might improve the system more; however, we increase the risk of overfitting the control system.

On the other hand, we think there are two approaches trying to increase the number of membership functions, when we having an odd or an even total number of membership functions:

- In the even case, it would either end up having an asymmetric or a symmetric model in which there would not be a membership function centered in the value 0 thrust, this would give us as a result an unstable model that would never stop at the desired angle.

- In the odd case, we decided to test it and simulate it for different function definitions. When testing with 5 or more membership functions in the output and trying different kinds of rules, we came to the conclusion we can get faster convergences, these rule updates favor more abrupt changes. An example of such a system is one with equally distributed membership functions along the domain or a system similar to the initial case with a membership function of the same width inserted between every pair of previously existing functions (the former converges faster).

4 References

[1] Li-Xin Wang, "A course in fuzzy systems and control," 1997 The Hong Kong University of Science and Technology. 1ISBN:978-0-13-540882-7