Week 11

Course. Introduction to Machine Learning

Theory 11. Recommender Systems

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2. Recommendation techniques
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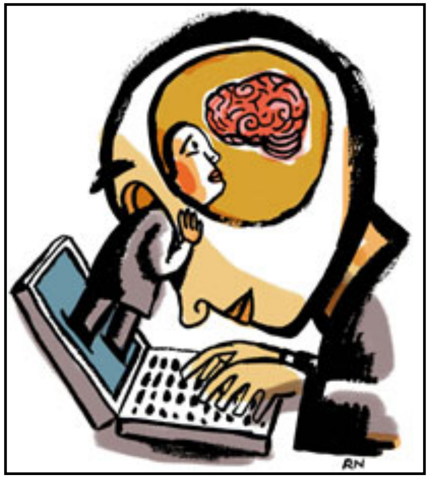
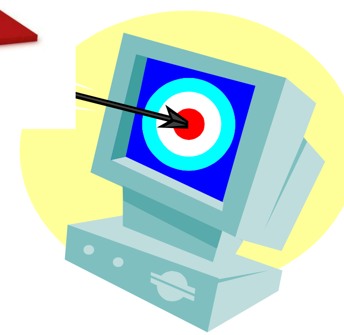
Introduction to Recommender Systems



Introduction to Recommender systems

Which one should I read?



Recommendations from friends

Recommendations from Online Systems

Which one should I choose?

Make a decision

For example, IMDb contained in January 2018
IMDb: 4.734.693 titles and 8.702.002 users

What travel should I do ?

I would like to escape from this ugly an tedious work life and relax for two weeks in a sunny place. I am fed up with these crowded and noisy places … just the sand and the sea … and some "adventure".

I would like to bring my wife and my children on a holiday … it should not be too expensive. I prefer mountainous places… not to far from home. Children parks, easy paths and good cuisine are a must.

I want to experience the contact with a completely different culture. I would like to be fascinated by the people and learn to look at my life in a totally different way.

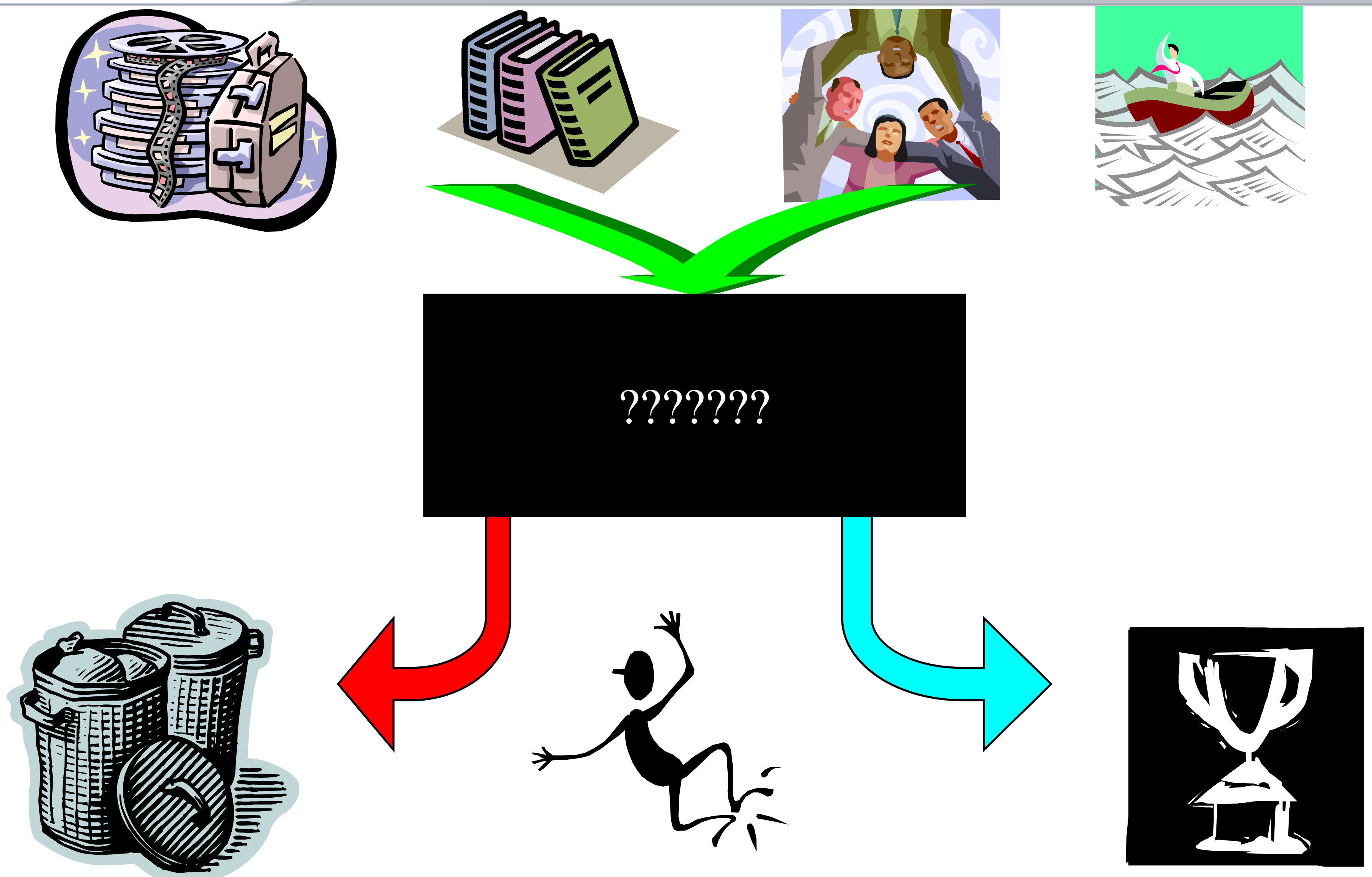
Information Overload

- **Internet = information overload**
  - the state of having too much information to make a decision or remain informed about a topic
- Information retrieval technologies can assist a user to **locate content** if the user knows exactly what he is looking for (with some difficulties!)
  - The user must be able to say “yes this is what I need” when presented with the right result
- But in many information search task, e.g., product selection, the user is…
  - not aware of the range of available options
  - may not know what to search
  - if presented with some results he may not be able to choose.

Decisions

- When there are 100 million of options it is obvious we need tools for searching, filtering, ranking options
- *But even when there are a few dozen of options we need support*
- **Examples**
  - Where to go for dinner tonight?
  - What flight for going to London?
  - What Digital camera should I buy?

A Solution



Recommender Systems

- A **recommender system** helps to make choices without sufficient personal experience of the alternatives
  - To **suggest products** to their customers
  - To provide consumers with **information to help them decide** which products to purchase

- They are based on a number of **technologies**: information filtering, machine learning, adaptive and personalized system, user modeling, intelligent user interfaces, …

Motivation for Recommenders

- Automates quotes like:
  - "I like this book; you might be interested in it"
  - "I saw this movie, you'll like it"
  - "Don't go see that movie!"

- Many of the top commerce sites use recommender systems to improve sales
- Users may find new books, music, or movies that were previously unknown to them
  - Also can find the opposite for e.g., movies or music that will definitely not be enjoyed

Use of RecSys

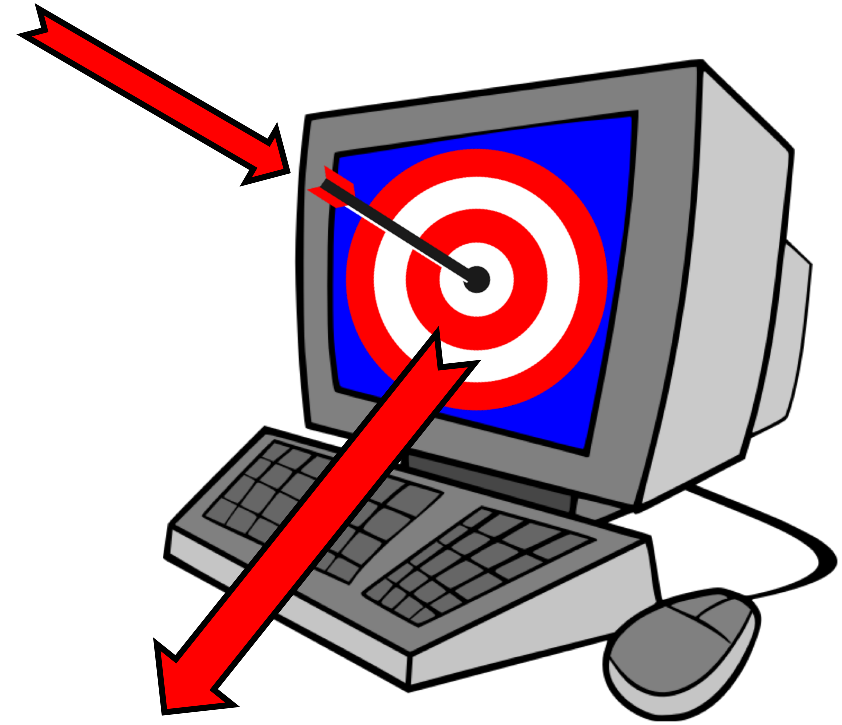
ebay
amazon.com
facebook
match.com
LinkedIn
YAHOO!
Google
NETFLIX
Spotify
iTunes

Recommender Systems

Basic interaction paradigm of RecSys

Input (ratings of movies):

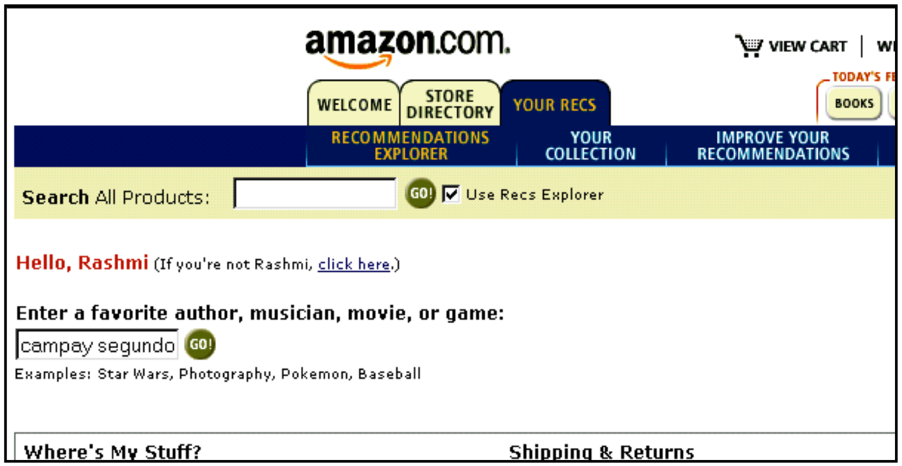
"I recently enjoyed *Harry Potter*, Star Wars, Joker"



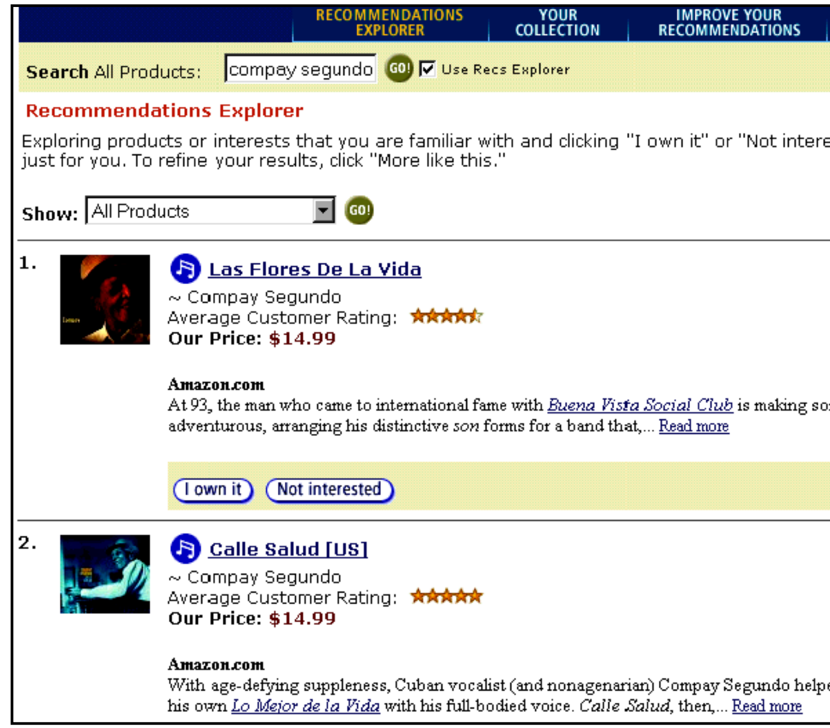
Output (Recommendations):

"Movies you might enjoy are… "

Amazon's Recommendation Process



Input: One artist/author name



Output: List of recommendations

Opportunity to Explore / Refine

The recommendation process from the user's perspective

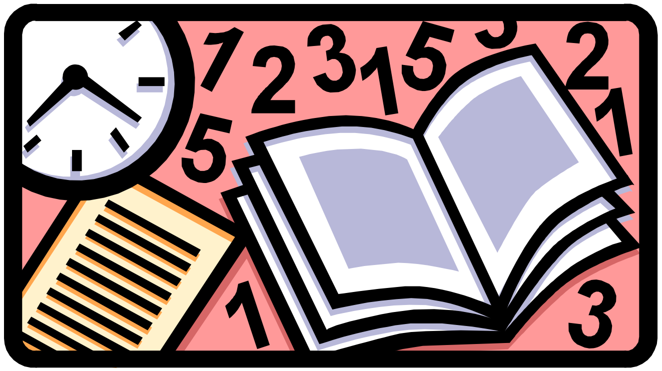
User inputs preferences…

Time and effort to input

Privacy concerns

…receives recommendations…

Time and effort to review recs



…and decides if he/she will sample recommendation

In the end, a user benefits only if recommendations turn out to be good ones.

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What Users Want

Fast

Engaging

New to me

PROCESS

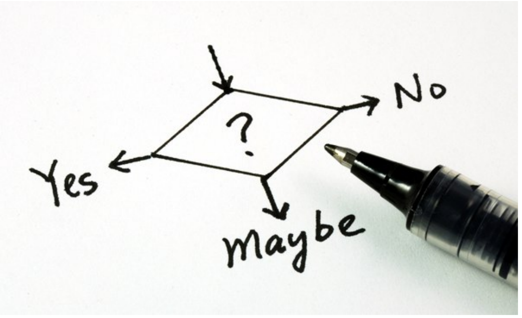
RECOMMENDATIONS

Easy

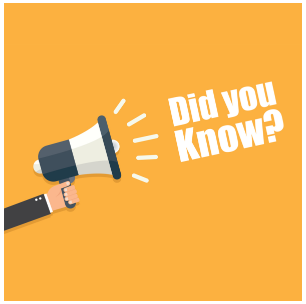
Good

To succeed, collaborative filtering recommender systems need a LOT of motivated regular users

Objective of RecSys



- Help people make decisions
  - Where to spend attention
  - Where to spend money



- Help maintain awareness
  - New products
  - New information

In both cases: Many options, limited resources

Definition of Recommender

- Recommender systems are a specific type of **information filtering** technique that attempt to present to the user information **items** (movies, *music*, books, news, web pages) the **user is interested in**. To do this the **user's profile** is compared to some reference characteristics

from:

http://en.wikipedia.org/wiki/Recommendation_system

Recommender definitions

- Recommender systems are personalized **information systems** that provide recommendations: suggestions for items likely to be of use to a user (Burke, 2007)
- **Recommendation** as a **prediction problem** (Celma & Lamere, 2007)
  - Attempt to predict items that a user might be interested in
  - compute *similarity* between objects
    - user-user
    - item-item
  - form *predictions* based on the computed similarities

Recommendation techniques

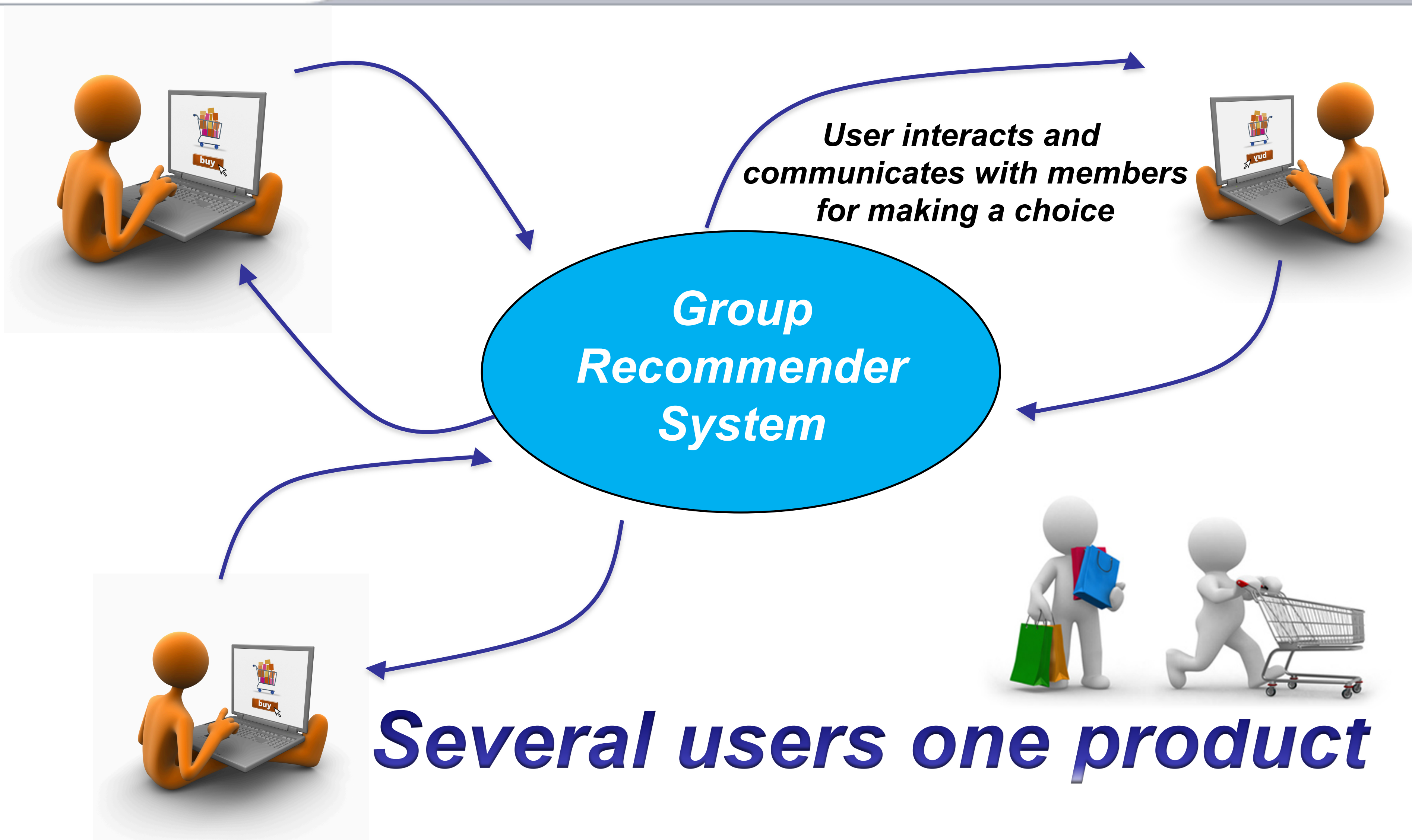
Single Recommenders

Single Recommender System

User interacts for making a choice

One user one product

Group Recommenders

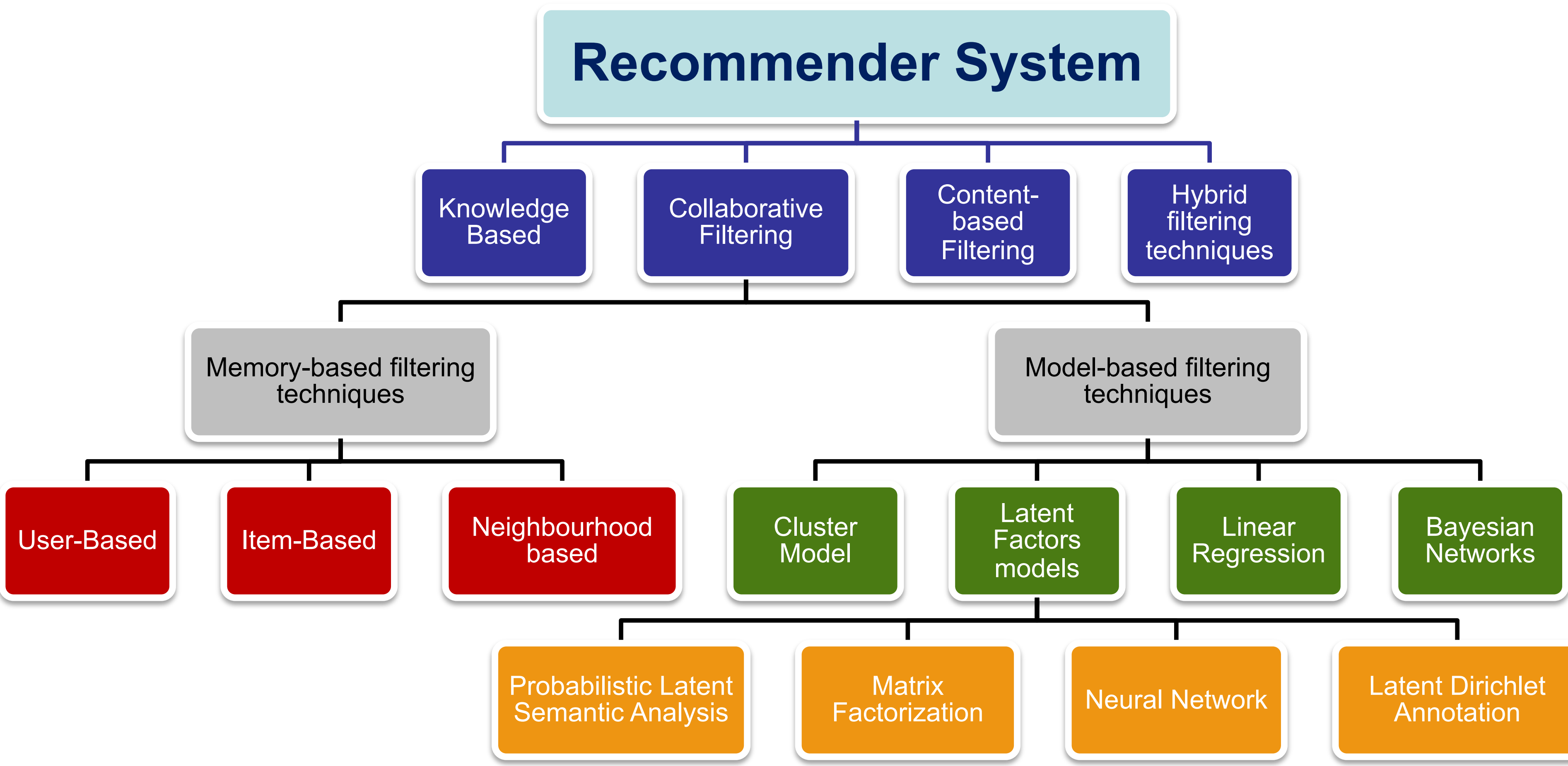


Several users one product

Objective

- Address recommendations to groups of users
- First we have to **identify user's individual preferences**
- Second, we try to find a **compromise that is fair** and **acceptable** to all the users

Taxonomy RecSys



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Recommendation techniques

- **Knowledge-based (KB)** – knowledge about users and products used to reason what meets the user's requirements, using discrimination tree, decision support tools, case-based reasoning (CBR)
- **Collaborative Filtering (CF)** – aggregation of consumers' preferences and recommendations to other users based on similarity in behavioral patterns
- **Content-based (CN)** – supervised machine learning used to induce a classifier to discriminate between interesting and uninteresting items for the user
- **Hybrid filtering techniques** – combine one or more of the techniques mentioned above

Knowledge sources in RecSys

Classification based on knowledge source

- **Knowledge-based (KB)**
  - Domain knowledge, product features, user’s need/query
  - Inferences about user’s needs and preferences
- **Collaborative (CF)**
  - User’s ratings “only”
- **Content-based (CN)**
  - Product features, user’s ratings
  - Classifications of user’s likes/dislikes

Additional knowledge source

- Context
  - What is context? Additional information, besides information on *Users*, that is *relevant* to recommendations
  - Relevant in:
    - Identifying pertinent subsets of data when computing recommendations
    - Building richer rating estimation models
    - Providing various types of constraints on recommendations outcomes

Context-aware recommender systems

Obtaining context

- Explicitly specified by the user
  - E.g., I want to watch a movie at home with my parents
- Observed or deduced by the system
  - Time (from system clock)
  - Location (from GPS)
  - Deduced from user's behaviour
- How to obtain the context is a separate problem that lies beyond the scope of this course
  - Context-aware recommender systems are very important for mobile computing

Weakness in Recs

- **Cold-start problem** – concerns the issue that the system cannot draw any inferences for users or items about which it has not yet gathered sufficient information

- **Latency problem** – new items incorporated into a Rec cannot be used in CF recommendations before a substantial amount of users have evaluated it

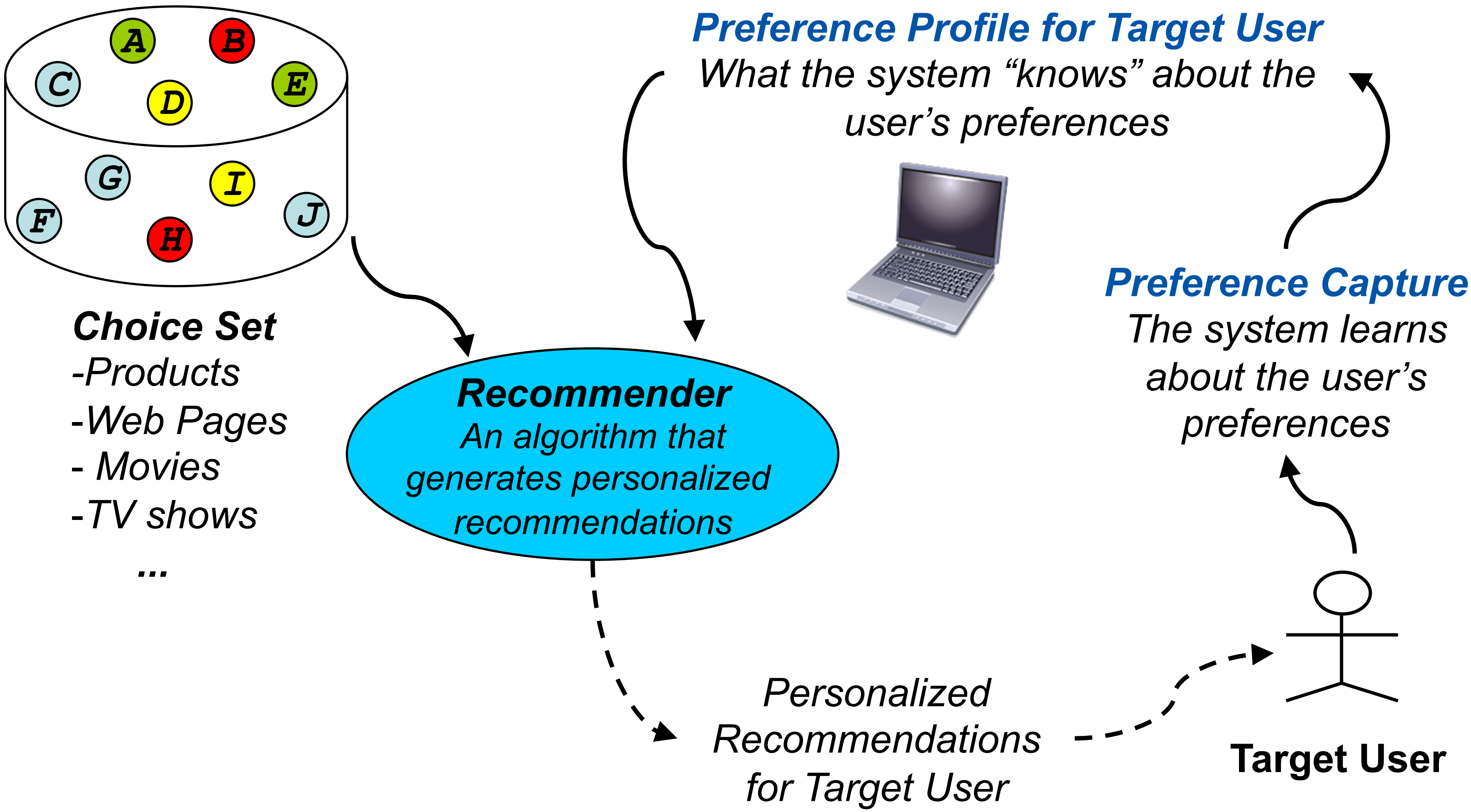
Weakness in Recs

- **Sparsity problem** – few users have rated the same items
- **Gray-sheep problem** – refers to a user that fall on a border between existing cliques of users

Summary of the techniques

- **Knowledge-based** methods go to the root of the decision problem
- **Collaborative-based** methods are the most popular
  - Suffers from bootstrapping problems
- **Content-based** methods are well rooted in information retrieval
- **Hybrid methods** are the most powerful and popular right now – there are plenty of options for hybridization

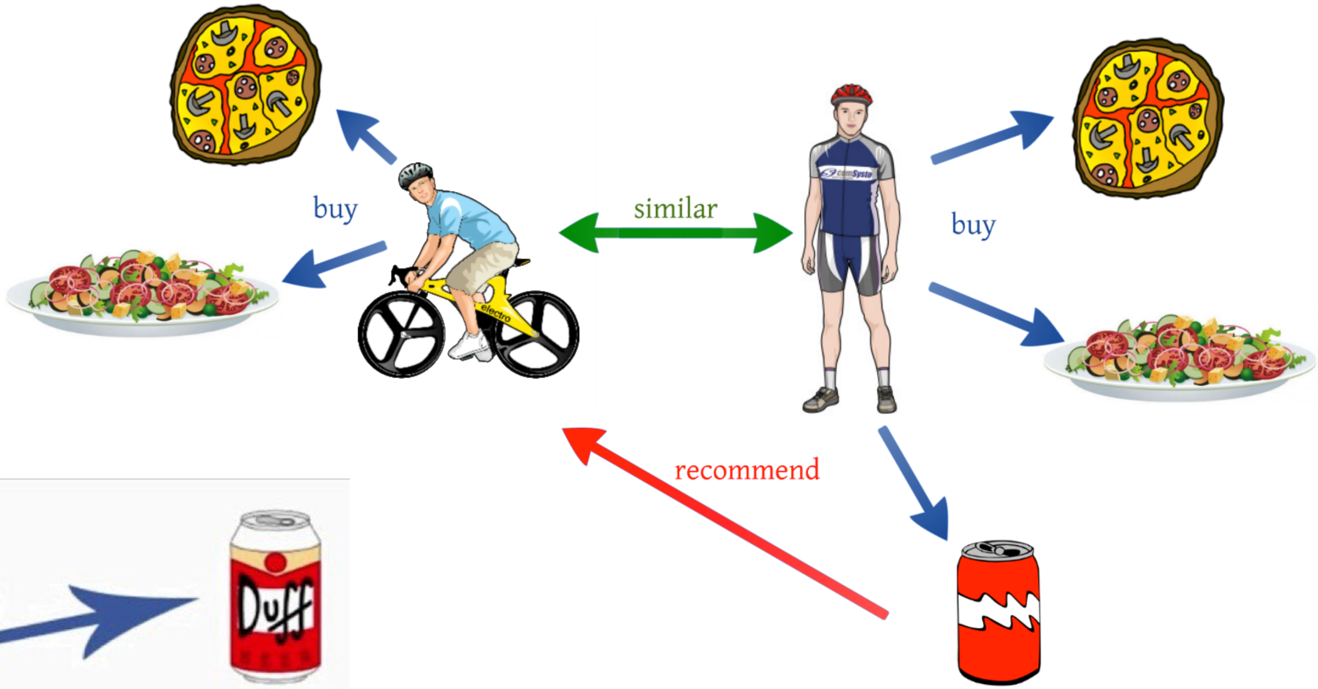
Components of a recommender

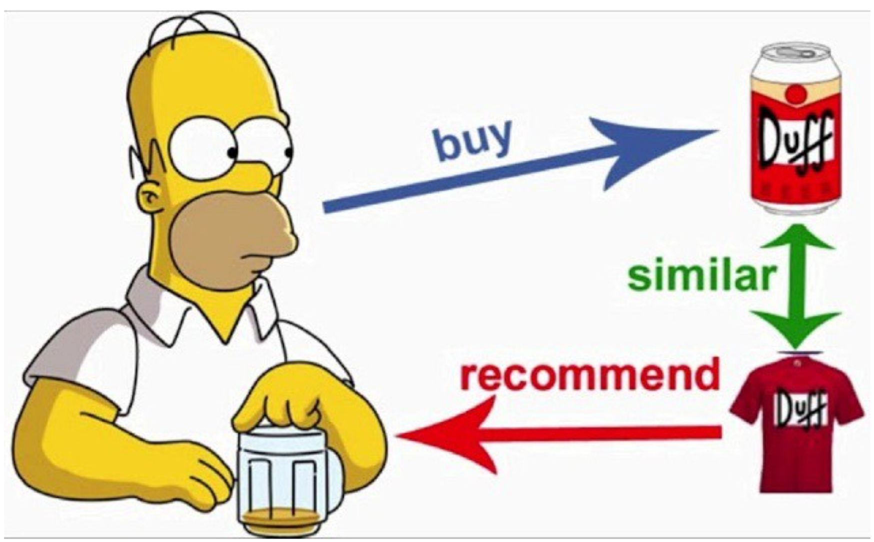


(Cantador, 2008)

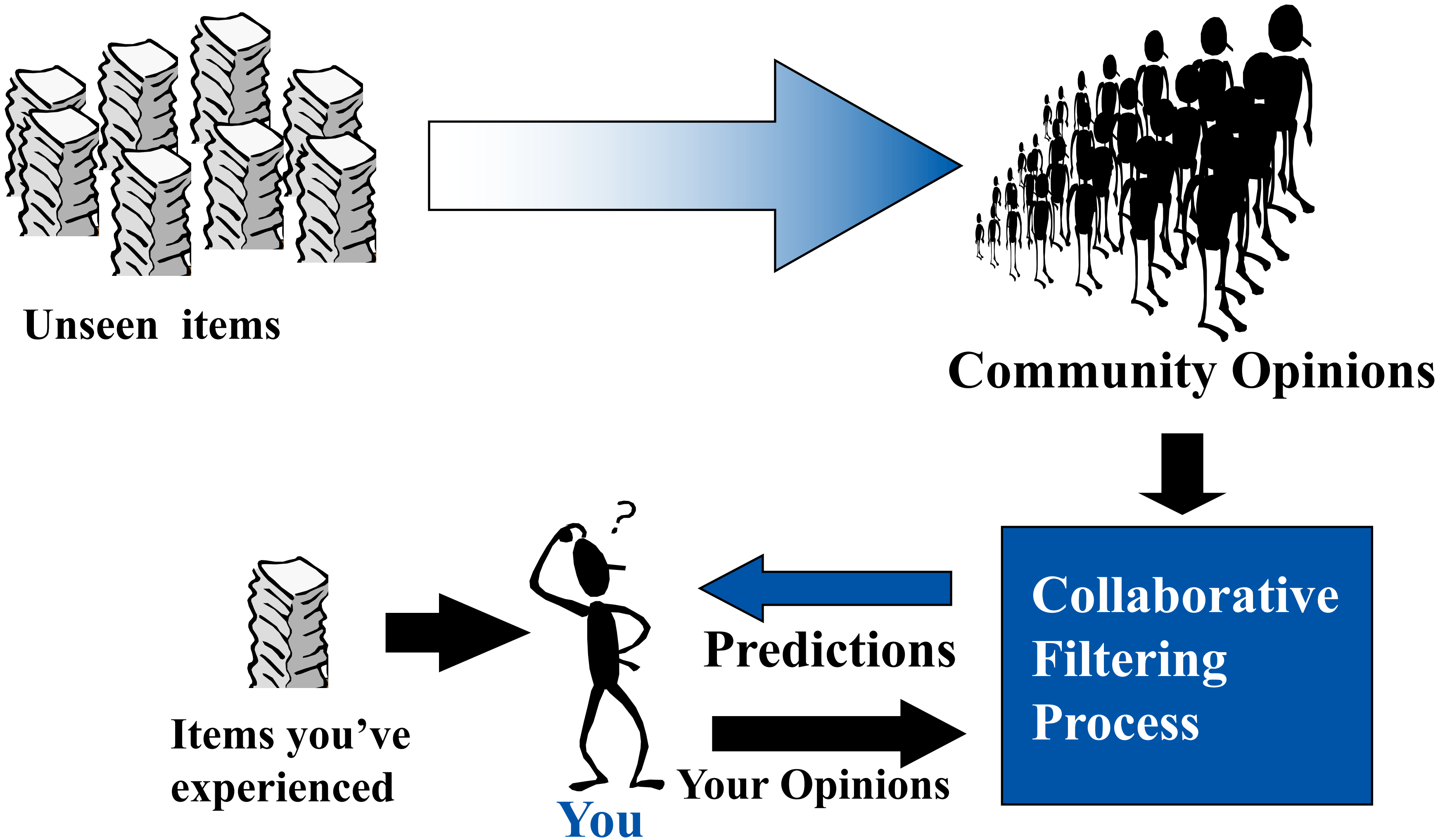
An overview of Collaborative Filtering Recommenders

Basic idea of RecSys

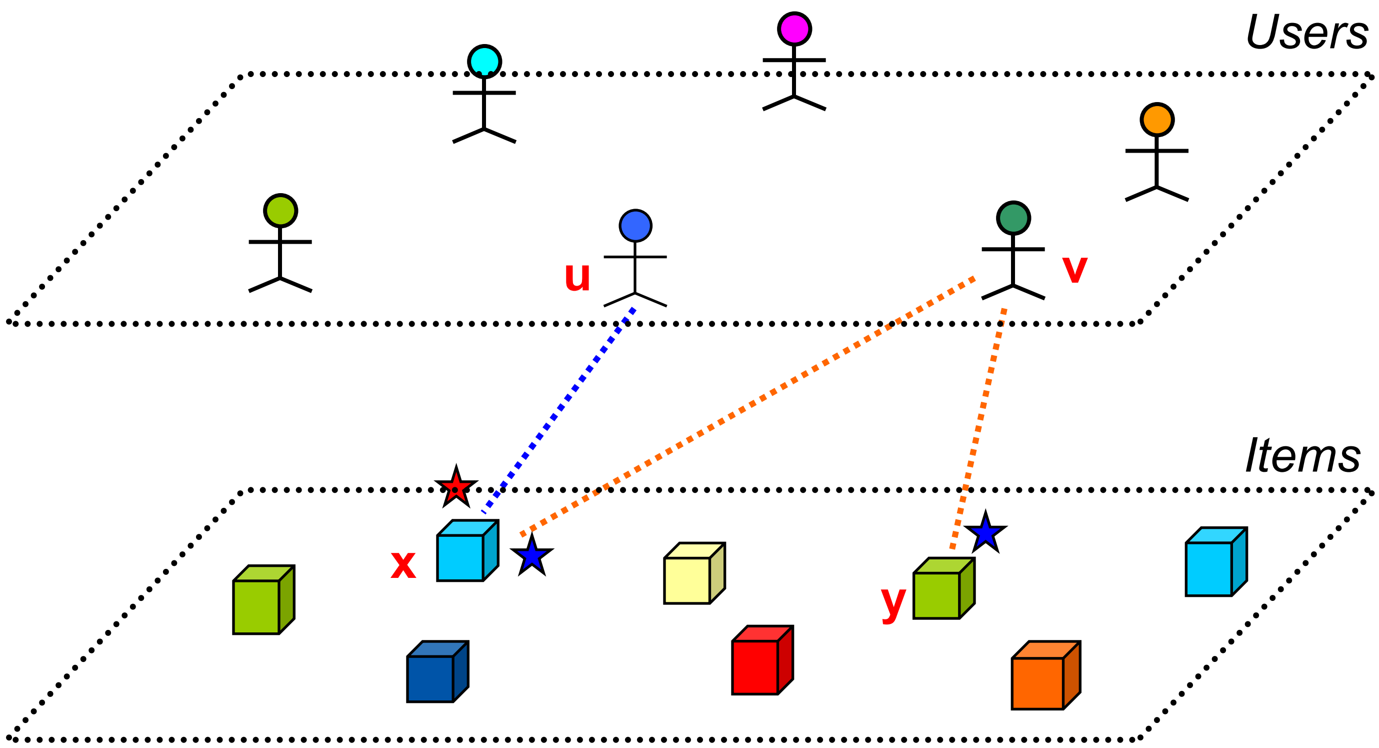




Collaborative Filtering



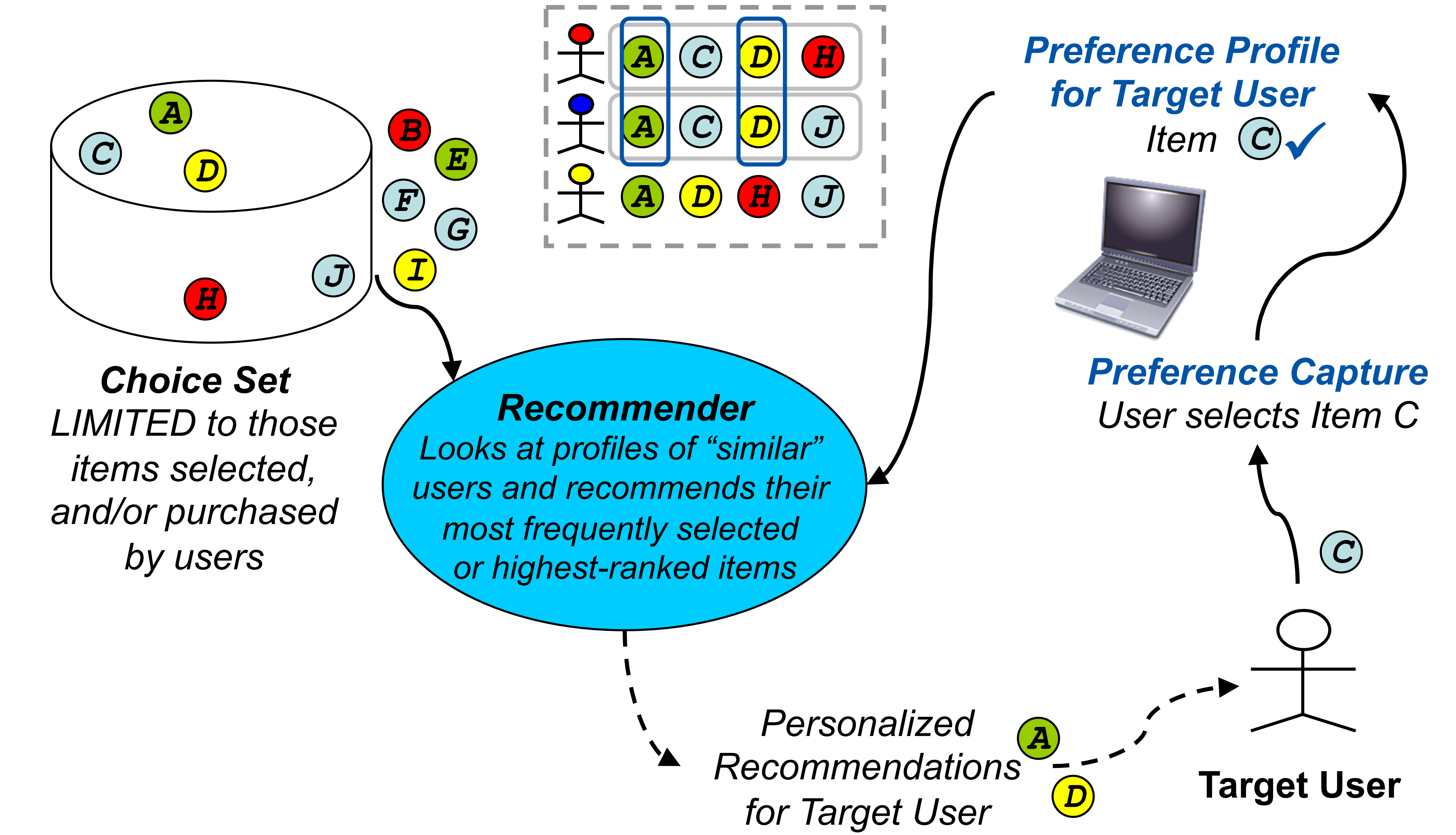
Collaborative Filtering



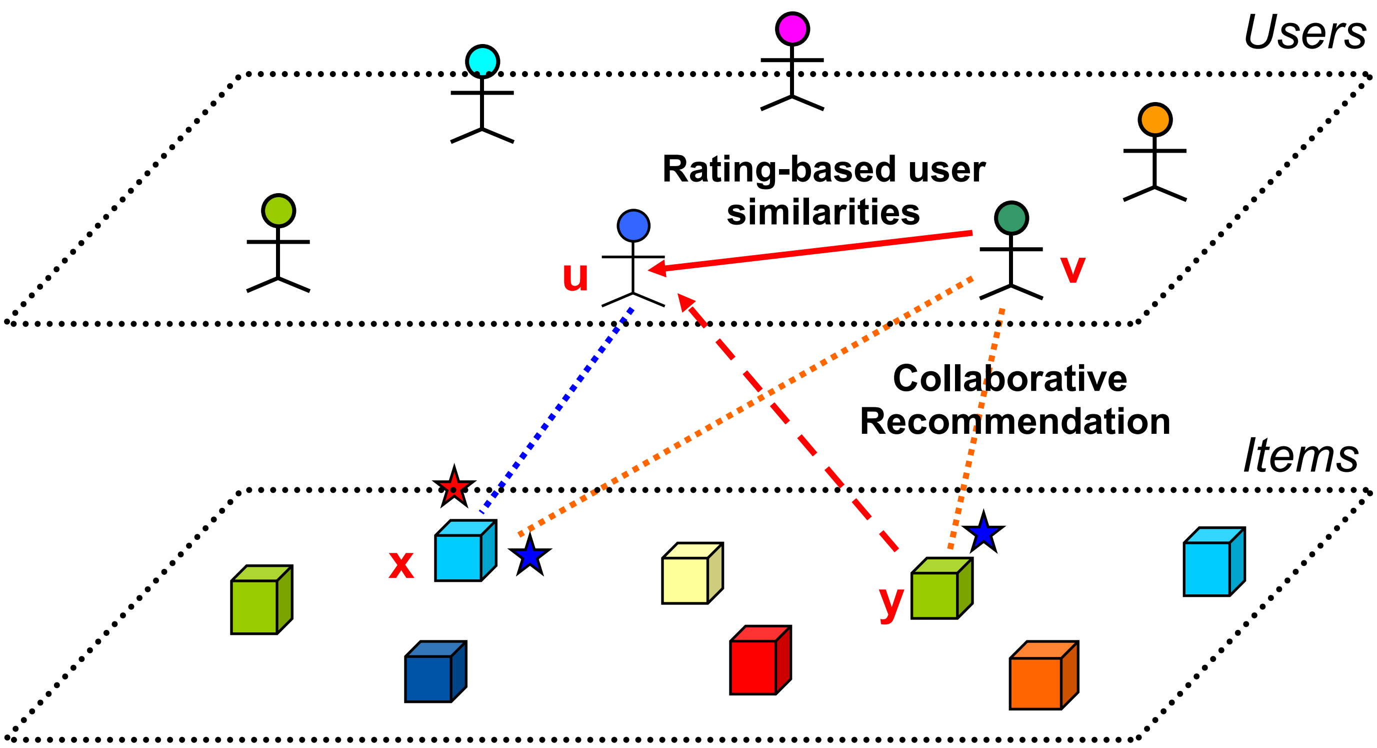
Definition

- **Collaborative Filtering (CF)** is the process of filtering or evaluating items through the opinions of other people
  - Maintain a database of many users' ratings of a variety of items
  - For a given user, find other similar users whose **ratings** strongly correlate with the current user
  - Recommend items rated highly by these similar users, but not rated by the current user
  - Almost all existing commercial recommenders use this approach (e.g., Amazon)

User-based


A C D H
A C D J
A D H J
Preference Profile
for Target User
Item C
A C D J H
B E F G I
Choice Set
LIMITED to those
items selected,
and/or purchased
by users
Recommender
Looks at profiles of "similar"
users and recommends their
most frequently selected
or highest-ranked items
Preference Capture
User selects Item C
C
Personalized
Recommendations
for Target User
A
D
Target User

Collaborative Filtering



User-based

Rating predictions

- Recommend to ***u*** those items ranked by ***v*** "*similar*" to ***u***

$$(a) \qquad r(u,x) = c \sum_{v \in \eta(u)} sim(u,v)\, r(v,x)$$

Similarity between users

$$(b) \qquad r(u,x) = \bar{r}(u) + c \sum_{v \in \eta(u)} \overbrace{sim(u,v)}\big(r(v,x) - \bar{r}(v)\big)$$

Predicted vote for "*active user*" ***u*** is a weighted sum

$$\bar{r}(u) = \frac{1}{|S(u)|} \sum_{x \in S(u)} r(u,x) \qquad S(u) = \{x \in \mathcal{I} \mid r(u,x) \neq \varnothing\}$$

$$c = \frac{1}{\sum_{v \in \eta(u)} |sim(u,v)|}$$

Normalizing factor

Similarity between users

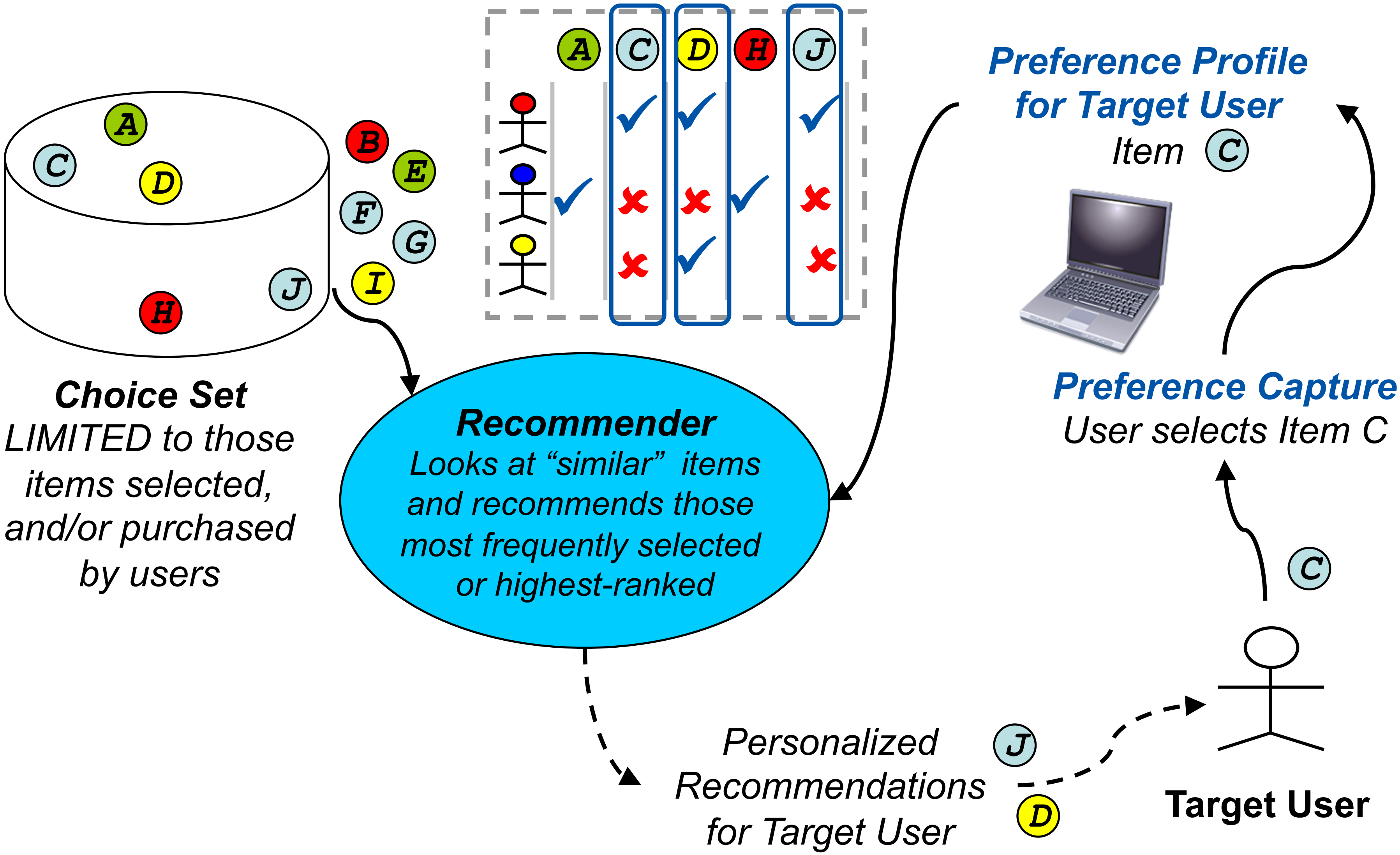
- The most popular approaches are:
  - **Cosine-based** user similarity (from IR)

$$sim(u,v)=\cos\left(r(u),r(v)\right)=\frac{\sum_{x\in\mathcal{I}} r(u,x)\,r(v,x)}{\sqrt{\sum_{x\in\mathcal{I}} r(u,x)^2 \sum_{x\in\mathcal{I}} r(v,x)^2}}$$

  - **Correlation-based** user similarity (from GroupLens)

$$sim(u,v)=\frac{\sum_{x\in\mathcal{I}}\left(r(u,x)-\bar{r}(u)\right)\left(r(v,x)-\bar{r}(v)\right)}{\sqrt{\sum_{x\in\mathcal{I}}\left(r(u,x)-\bar{r}(u)\right)^2 \sum_{x\in\mathcal{I}}\left(r(v,x)-\bar{r}(v)\right)^2}}$$

Item-based



A
C
D
H
J
Preference Profile
for Target User
Item C
A
C
D
J
H
B
E
F
G
I
Choice Set
LIMITED to those
items selected,
and/or purchased
by users
Recommender
Looks at "similar" items
and recommends those
most frequently selected
or highest-ranked
Preference Capture
User selects Item C
C
Personalized
Recommendations
for Target User
J
D
Target User

Rating predictions

- Recommend to ***u*** items "similar" to those best ranked by ***u***

Weighted sum

$$r(u,x) = c \sum_{y \in S(u)} sim(x,y)\, r(u,y)$$

$$c = \frac{1}{\sum_{y \in S(u)} \left| sim(x,y) \right|}$$

Similarity between items

Cosine

$$sim(x,y)=\cos(r(x),r(y))=\frac{\sum_{u\in\mathcal{U}} r(u,x)r(u,y)}{\sqrt{\sum_{u\in\mathcal{U}} r(u,x)^2 \sum_{u\in\mathcal{U}} r(u,y)^2}}$$

Pearson

$$sim(x,y)=\frac{\sum_{u\in\mathcal{U}}\left(r(u,x)-\overline{r}(x)\right)\left(r(u,y)-\overline{r}(y)\right)}{\sqrt{\sum_{u\in\mathcal{U}}\left(r(u,x)-\overline{r}(x)\right)^2 \sum_{u\in\mathcal{U}}\left(r(u,y)-\overline{r}(y)\right)^2}}$$

Adjusted Cosine

$$sim(x,y)=\frac{\sum_{u\in\mathcal{U}}\left(r(u,x)-\overline{r}(u)\right)\left(r(u,y)-\overline{r}(u)\right)}{\sqrt{\sum_{u\in\mathcal{U}}\left(r(u,x)-\overline{r}(u)\right)^2 \sum_{u\in\mathcal{U}}\left(r(u,y)-\overline{r}(u)\right)^2}}$$

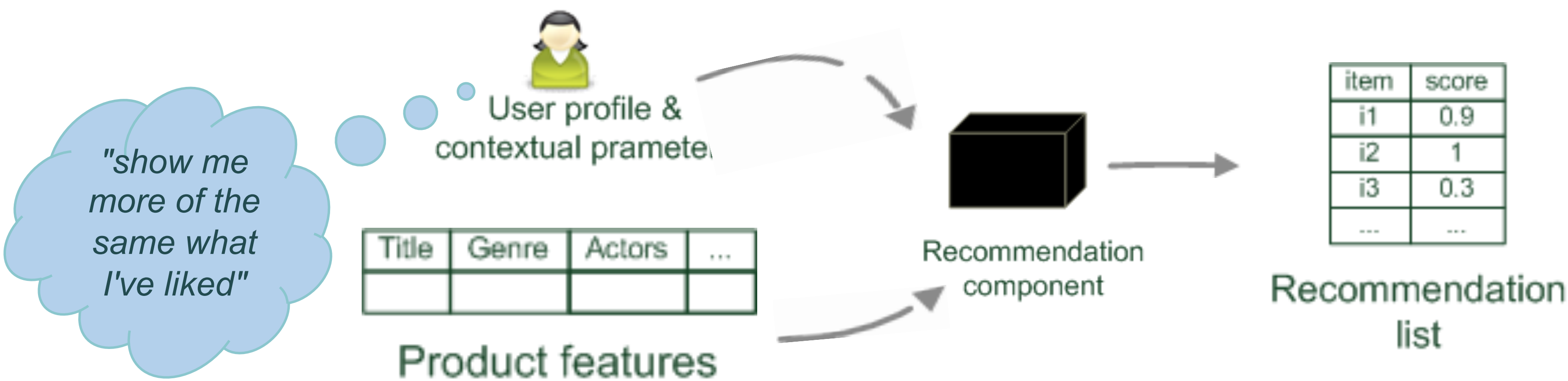
An overview of Content-based Recommenders

Introduction to content-based recs

- Based on the conjecture that a person likes items with features similar to those of other items she liked in the past
- Closely related to **Information Retrieval**
- Items are suggested according to:
  - A comparison between their content and the user profiles
- A **user profile** contains information about the user' tastes, interests and needs

Content-based recommendation

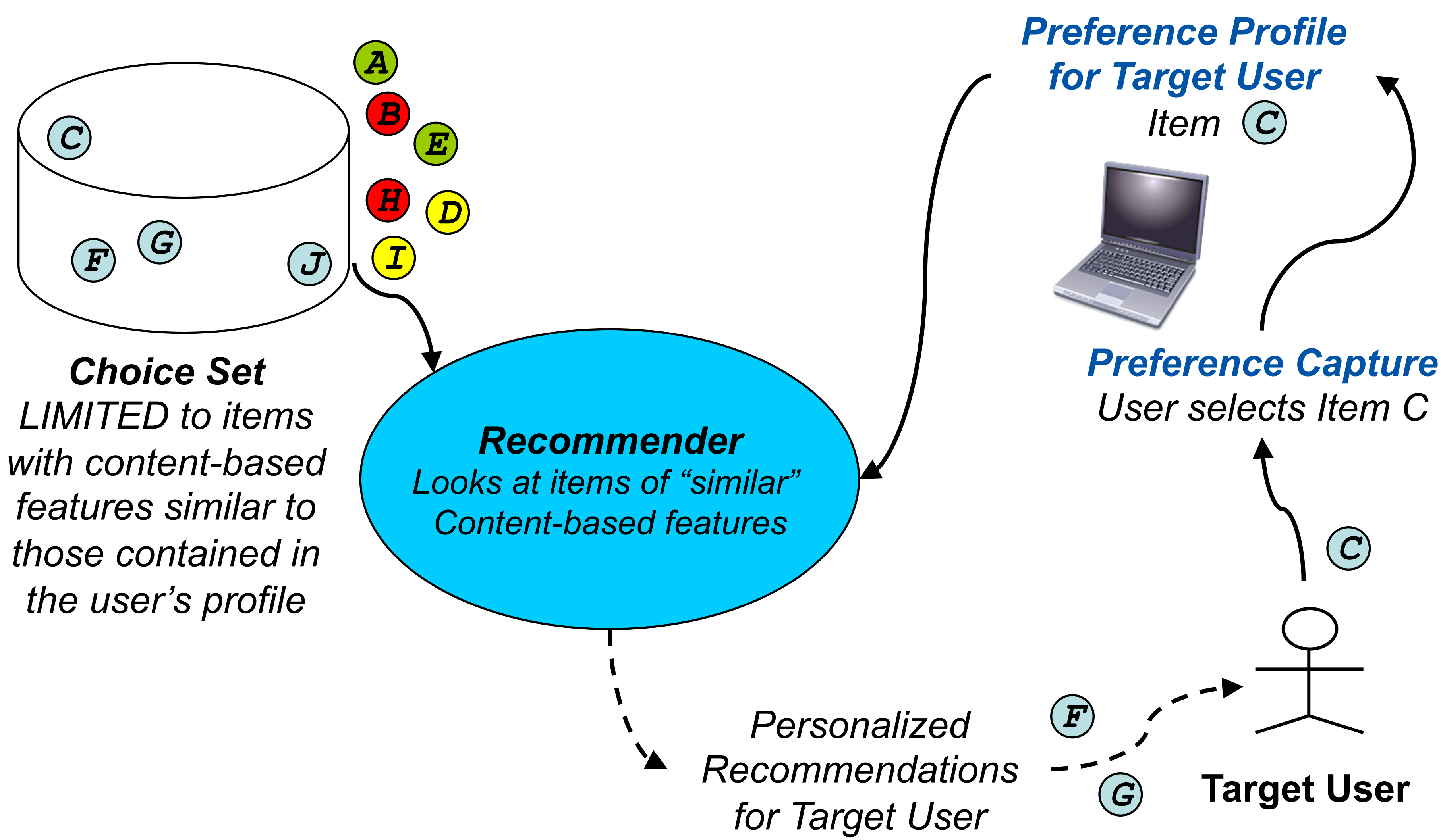
- ***While CF – methods do not require any information about the items,***
  - *it might be reasonable to exploit such information; and*
  - *recommend fantasy novels to people who liked fantasy novels in the past*
- ***What do we need:***
  - *some information about the available items such as the genre ("content")*
  - *some sort of user profile describing what the user likes (the preferences)*
- ***The task:***
  - *learn user preferences*
  - *locate/recommend items that are "similar" to the user preferences*



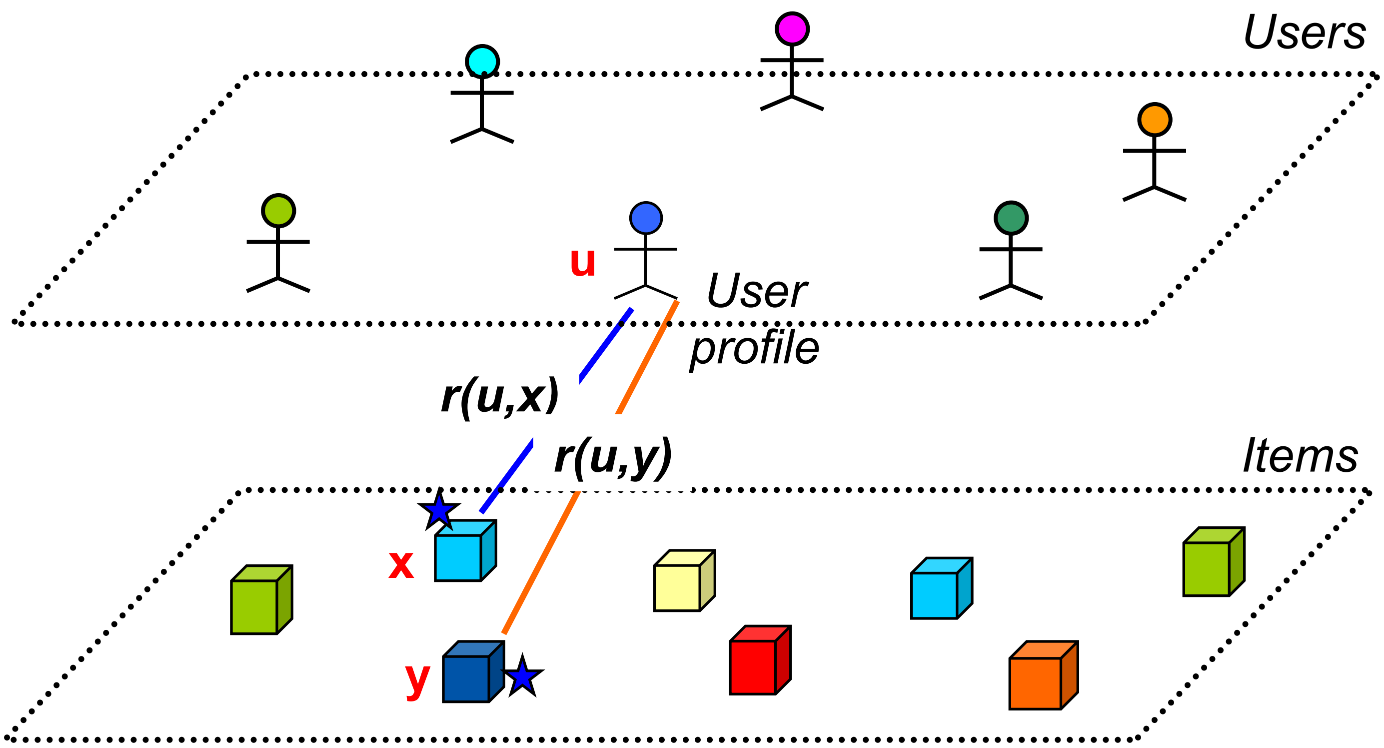
Content-based recommenders

- **Content-based recommendation** systems share in common a means for:
  - Describing the items that may be recommended
  - Creating a profile of the user that describes the types of items the user likes
  - Comparing items to the user profile to determine what to recommend
- The profile is often created and updated automatically in response to feedback on the desirability of items that have been presented to her

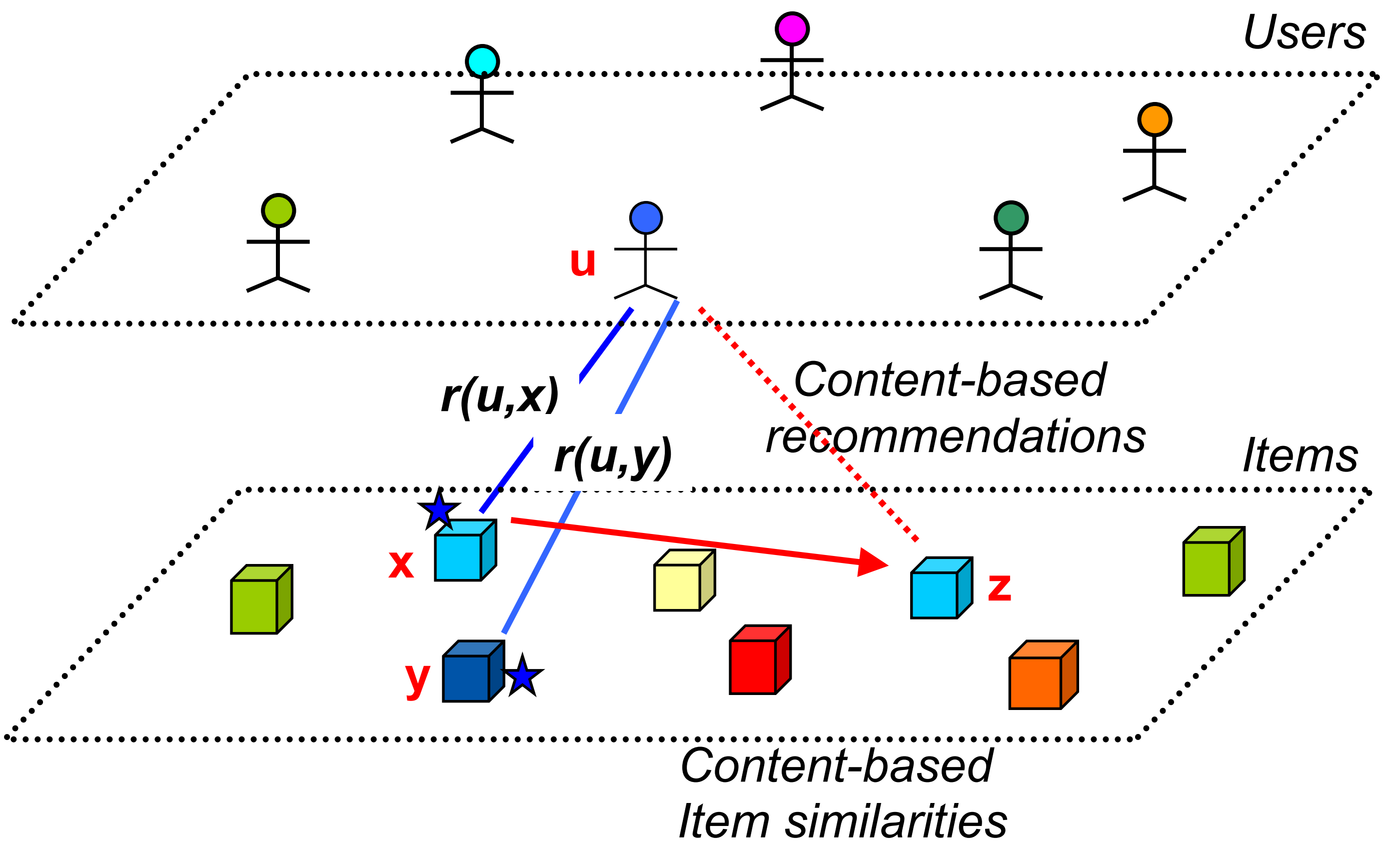
Content-based



Content-based



Content-based



Definition

- **Content-based recommendation** systems analyze item descriptions to identify items that are of particular interest to the user
- Items are suggested according to a comparison between their content and user profiles, which contain information about the users' tastes, interests and needs
- Most systems differ in the representation of items and how they get the feedback from users

Representation of items

Called attributes, features, characteristics, fields or variables

- **Database table**

Id	Name	Cuisine	Service	Cost
10001	Bella Italia	Italian	Counter	Low
10002	Amour	French	Table	Medium
10003	Jacques Bistro	French	Table	High

  – Attributes names with well-defined values
- **Unstructured data**, unrestricted text such as news, articles
  – There are no attribute names with well-defined values
- **Semi-structured data**
  – Some attributes with a set of restricted values and
  – Some free-text fields

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User profiles

- A profile of the user's interest is used by most recommendation systems
- **Types of information in a profile**:
  - A model of the user's preferences, a description of the types of items that interest the user
  - A history of the user's interactions with the recommendation system. This includes:
    - Storing the items that the user has viewed and
    - Other information about the user's interaction

Learn a user model

Learning a user model

- Creating a user model of the user's preference from the user history is a form of classification learning
- **Explicit feedback**, the user rates items via some interface for collecting feedback
  - Few data but an accurate set of preferences
- **Implicit feedback**, the system observes the user's interactions with items
  - Can collect a large amount of data with some uncertainty as to whether the user actually likes the item

Advantages of Content-Based Recs

- User independence, no need for data on other users
  - No cold-start or sparsity problems
- No grey sheep problem, it is able to recommend to users with unique tastes
- New item, it is able to recommend new and unpopular items
  - No first-rater problem
- Transparency, can provide explanations of recommended items by listing content-features that caused an item to be recommended

Disadvantages of Content-Based Recs

- **Requires content analysis**, content that can be encoded as meaningful features
  - The effectiveness of the Rec depends on the available descriptive data
- Users' tastes must be represented as a learnable function of these content features
- **Content over-specialization**, it only retrieves items that score highly against a specific user profile
- **Portfolio-effect**, non diversity problem, the user should be presented with a diverse range of options, not with a homogeneous set of alternatives
- Unable to exploit quality judgments of other users
  - Unless these are somehow included in the content features

Conversational Recommender Systems

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Conversational recommenders

Conversational recommenders

(Jannach et al., 2020) guide the user through a complex problem space by alternatively making **suggestions** and using **user feedback** to influence future suggestions.

One way to provide **feedback** is based on **critiquing elicitation** *(Chen and Pu, 2012)*

Case-based recommender systems

- **Items are retrieved using similarity measures**
- Distance similarity

$$similarity(p, REQ) = \frac{\sum_{r \in REQ} w_r * sim(p, r)}{\sum_{r \in REQ} w_r}$$

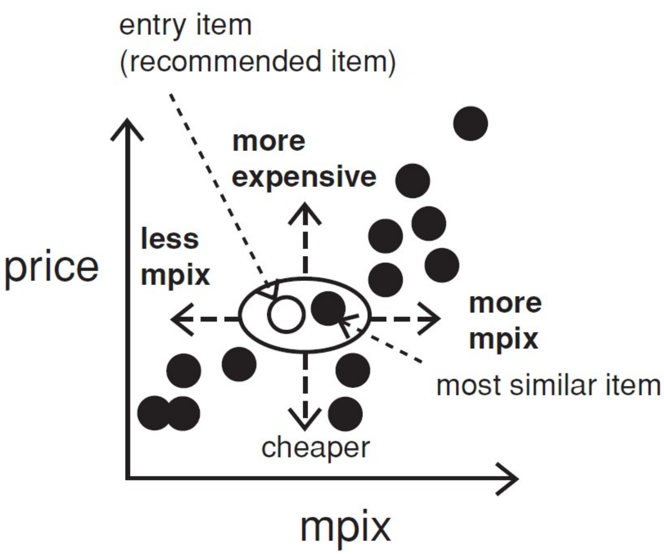
  - similarity (p, r) expresses for each item attribute value φr (p) its distance to the customer requirement r ∈ REQ.
  - w_r is the importance weight for requirement r
- In real world, customer would like to:
  - maximize certain properties. i.e. resolution of a camera, "more is better" (MIB)
  - minimize certain properties. i.e. price of a camera, "less is better" (LIB)

Forms of feedback

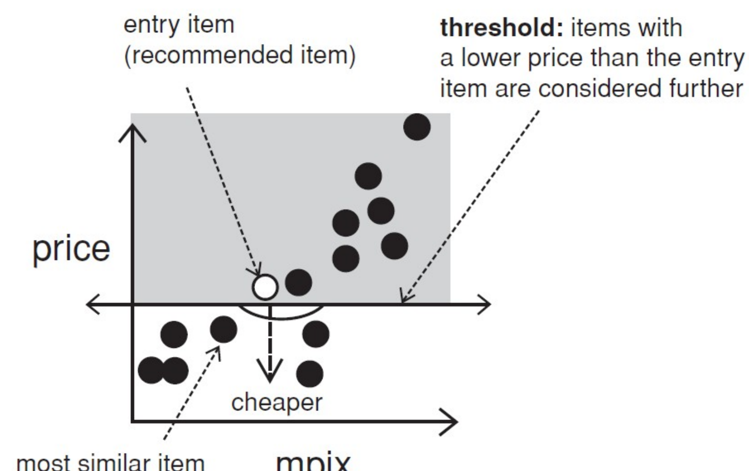
- **Value elicitation**, approaches ask the user to provide details relating to specific features
- **Preference-based feedback and Rating-based**,
  - ask the user to indicate which product they prefer when presented with a small set of alternatives
  - to provide ratings for these alternatives
- **Critiquing or tweaking**, the user expresses a directional preference over the value-space for a particular product feature

Interacting with case-based recommenders

- Customers maybe not know what they are seeking
- Critiquing is an effective way to support such navigations
- Customers specify their change requests (*price or mpix*) that are not satisfied by the current item (*entry item)*



Critique on price



The Critiquing Approach

- **"FindMe Systems"**
  - Robin Burke [DePaul University], Kris Hammond [Northwestern University]
  - Previously of Computer Science Department of the University of Chicago.
  - Use CBR as a way of recommending products in e-commerce catalogs and provide critique-based navigation as a primary user interface.
- **Basic Assumption**
  - User has some product requirement.
  - Requirement is sufficiently vague (user cannot simply list all of the features).
  - User provides some initial features.
  - User is helped to locate a good product by critiquing additional features from intermediate recommendations.
  - Recommendations help influence the user's elaborate their initial query.
- **Practical Applications**
  - Restaurants, cars, cameras, property etc.

An Example – “Find Me” Systems

Figure 1: Entry point for the Entree system

- **Entrée – A Restaurant Recommender System**
  - Users start off by indicating what kind of a restaurant they are looking for.
  - The system retrieves restaurants that are considered to be **similar** to the user's current requirements.

Retrieval and User Feedback in Entrée

- **User Feedback**
  - The user indicates a feature critique or tweak to constrain the value range of that feature. E.g., Show me “more like this but cheaper”

Critique-based feedback

- **Unit critiquing**
  - Express preference over a single feature
  - [price, < , 1500]
- **Compound critiquing**
  - Critique operates over multiple features
  - [price, < , 1500] and [memory, >, 2]

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Critique-based feedback

Critique-based feedback

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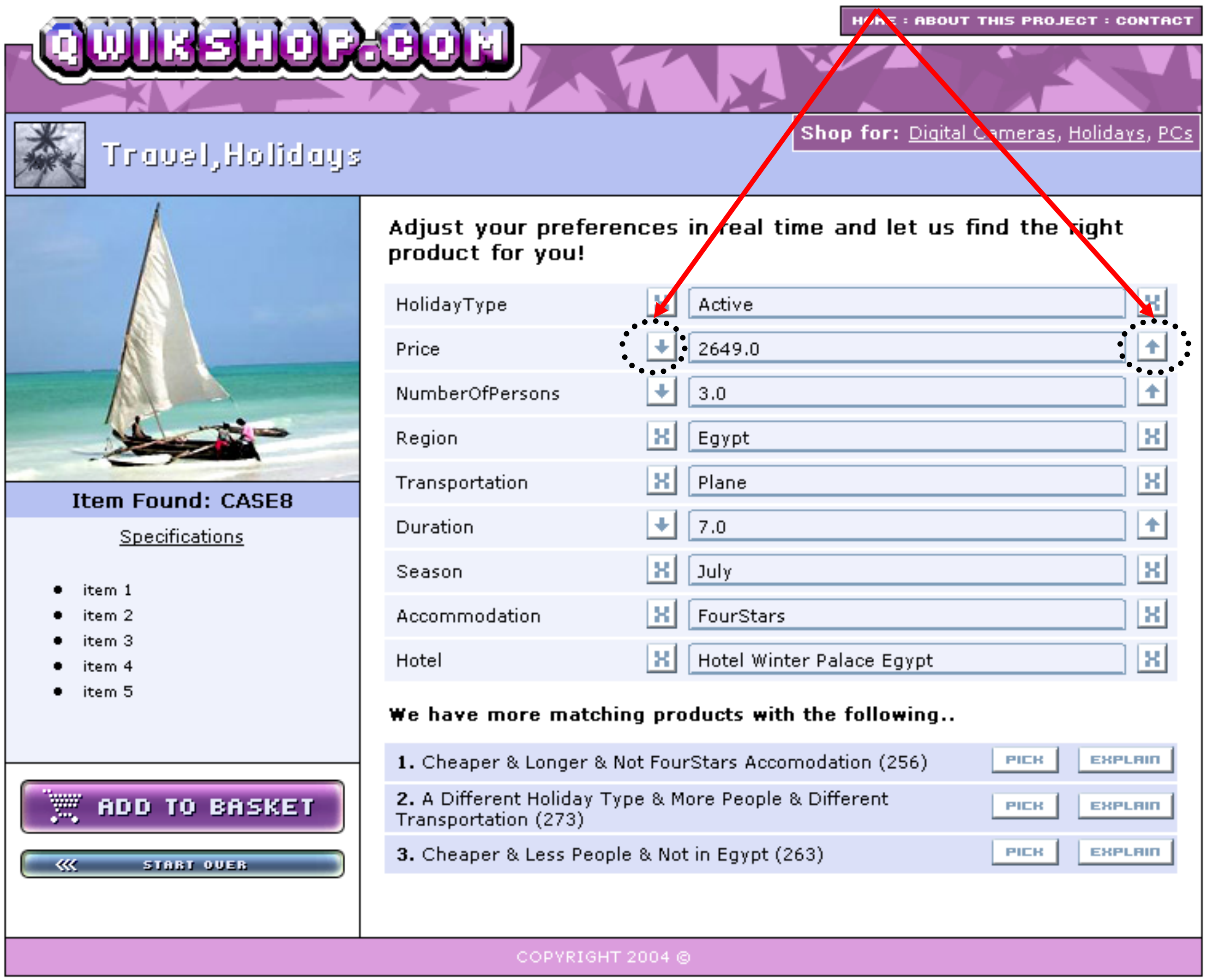
Unit Critiques



The user indicates a **directional feature preference** in relation to a presented recommendation

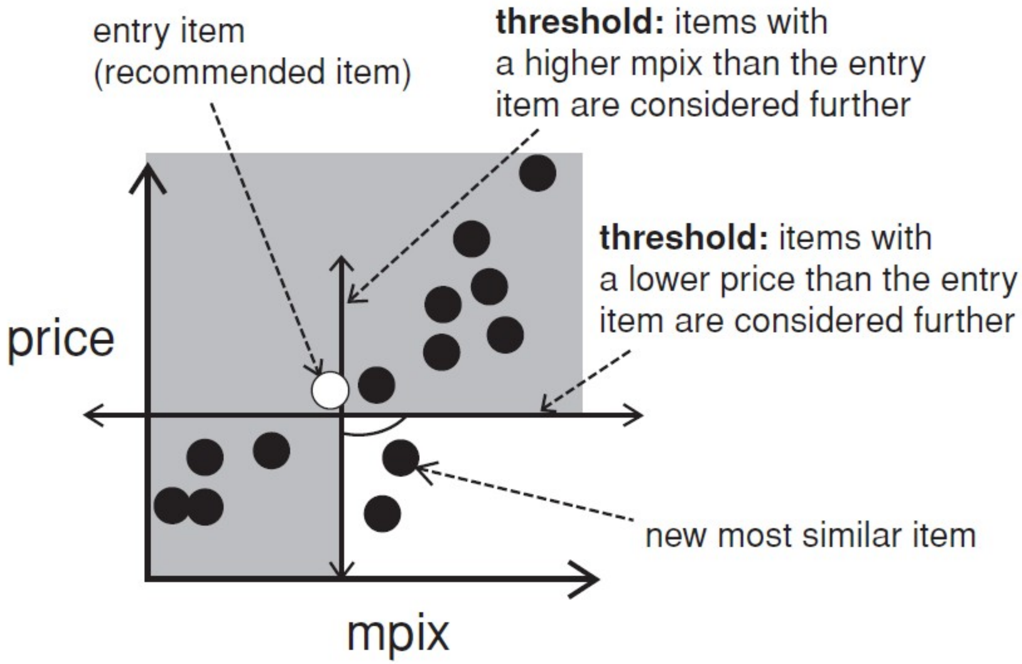
Critique-based feedback

Unit Critiques

The traditional implementation of critiquing can lead to **protracted recommendation sessions** as there is a tendency towards **relatively minor changes** in the values of critiqued features from **cycle to cycle**

Compound critiques

- Operate over multiple properties can improve the efficiency of recommendation dialogs

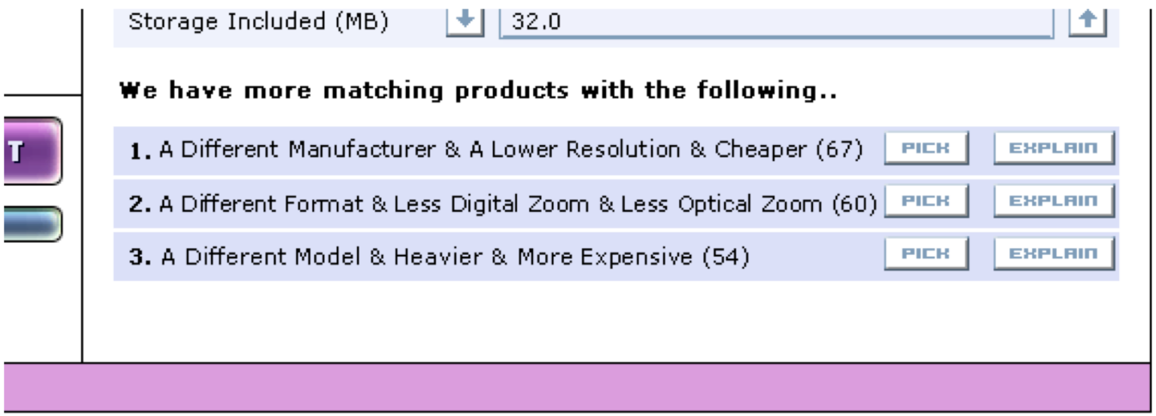


Dynamic Critiquing

Unit Critiques

Compound Critiques

Explanatory Benefits



Diversity Enhancement

1. A Different Manufacturer & Less Pixels & Cheaper (72) PICK
2. Less Pixels & Less Memory & Cheaper (84) PICK
3. A Different Type of Memory & Different Software & Cheaper (80) PICK

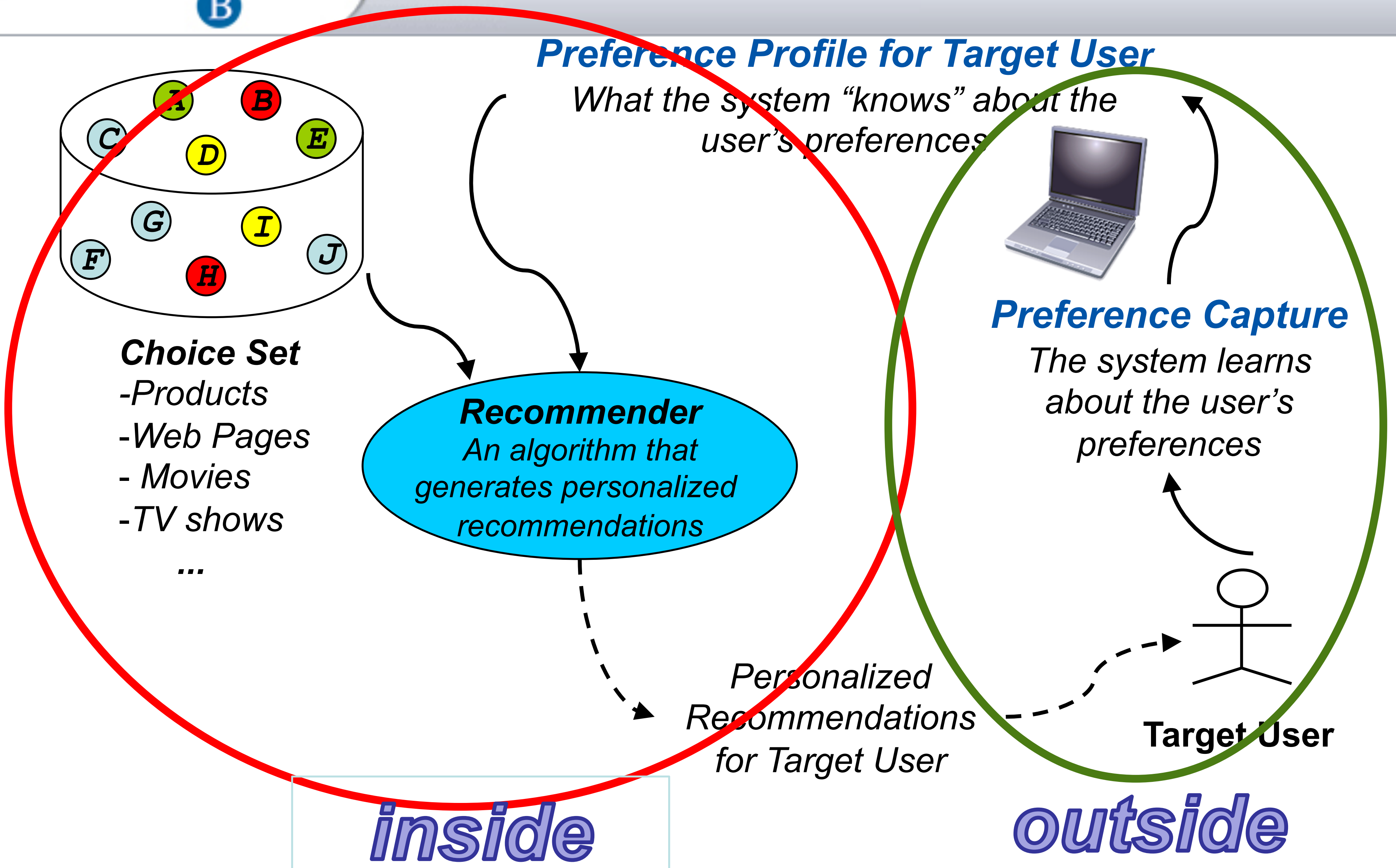
1. A Different Manufacturer & Less Pixels & Cheaper (72) PICK
2. A Different Model & More Memory & More Expensive (79) PICK
3. A Different Type of Memory & Different Cable & Less Memory (83) PICK

Conversational Recommenders

- **Advantages**
  - No ratings are necessary
  - Avoid cold start problem
- **Limitations**
  - cost of knowledge acquisition
    - from domain experts
    - from users
    - from web resources
  - accuracy of preference models
    - very fine granular preference models require many interaction cycles

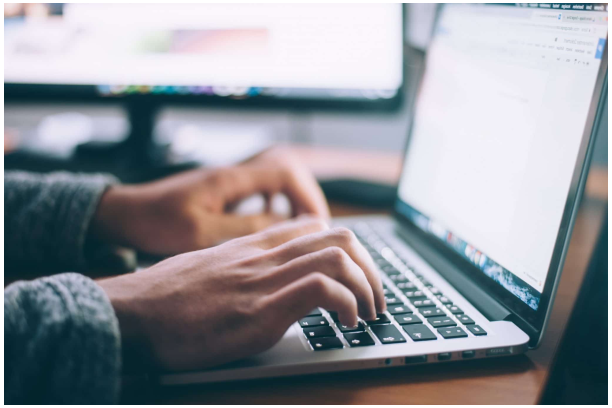
Outside Recommender Systems

Components of a recommender



Outside Recommender Systems

- **Recommenders need an interface for capturing user's interactions**
- **The interface is the key component** for obtaining user's preferences and it is designed based on:
  - The type of recommender
  - The type of preferences that should be collected for the user profiles
- Common taxonomy of interfaces:
  - 2D interfaces
  - Mobile interfaces
  - 3D Interfaces



Conversational Recommender Systems

- Recommenders and its interaction to 2D (web-based) and 3D interfaces (Virtual Worlds)
- Recommenders and natural language interaction with assistants
- Single Recommenders
- Group Recommenders

Outside Conversational Recommender Systems

1. 3D Virtual Worlds
2. Cognitive Assistants
3. gCoach: Collaborative Group Recommender

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3D Virtual Worlds and Recommenders

- *Contreras, D., Salamó, M., Rodríguez, I., Puig, A.* ***Shopping Decisions Made in a Virtual World: Defining a State-Based Model of Collaborative and Conversational User-Recommender Interactions****, Journal IEEE Consumer Electronics Magazine, 7(4), pp. 26-35 (2018), doi: 10.1109/MCE.2017.2728819*
- *Contreras, D., Salamó, M., Rodríguez, I., Puig, A.* ***A 3D Visual Interface for Critiquing-based Recommenders: Architecture and Interaction****. Journal IJIMAI 3 (3): pp. 7-15 (2015)*

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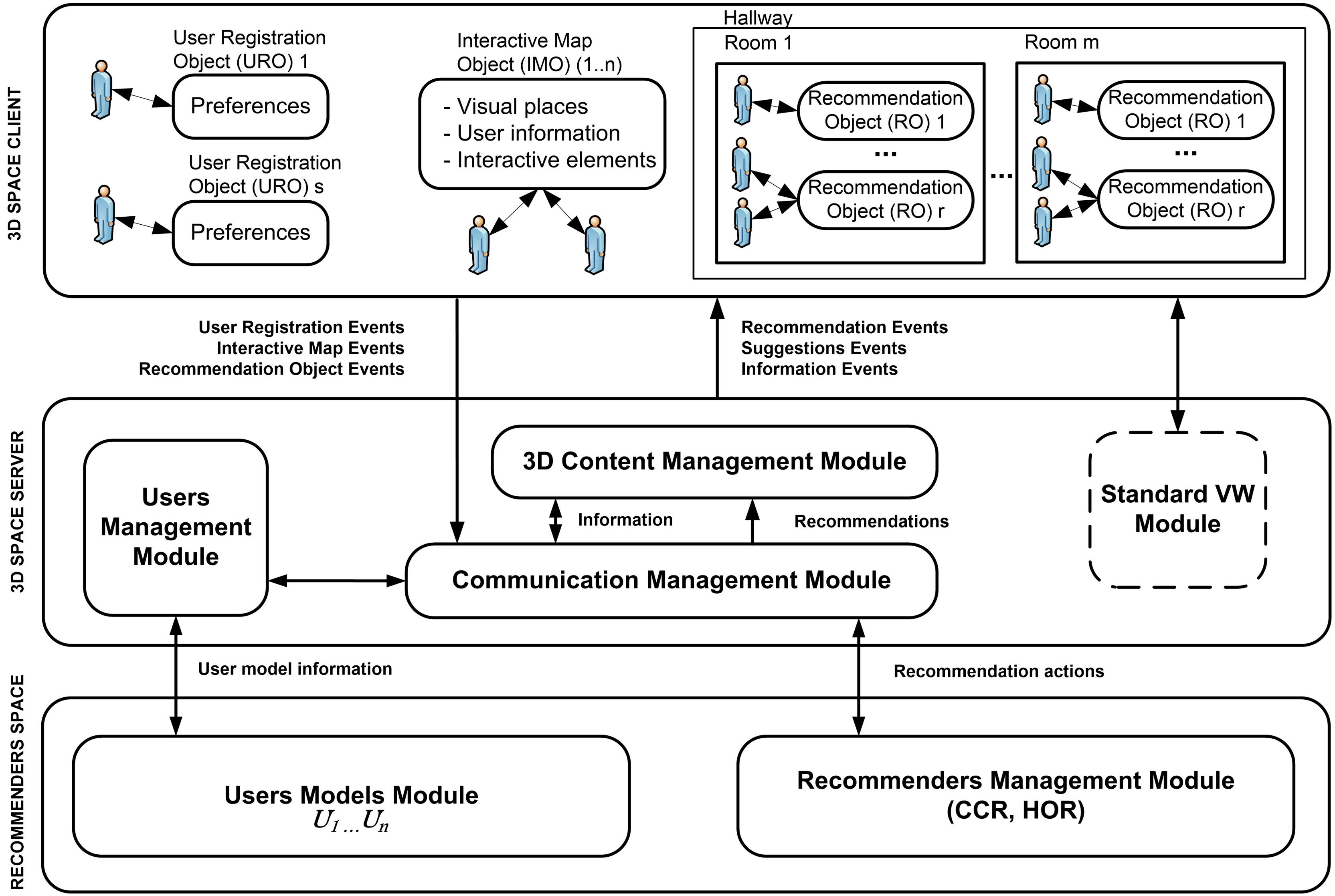
Goal in 3D Virtual Conv. Rec

Design a SCALABLE framework to enhance and facilitate user buying experience in a large 3D eCommerce space

(OPTIONAL) *https://youtu.be/jX91kSnmQn8*

Framework 3D VW and Conv. Rec



Interaction Objects (I)

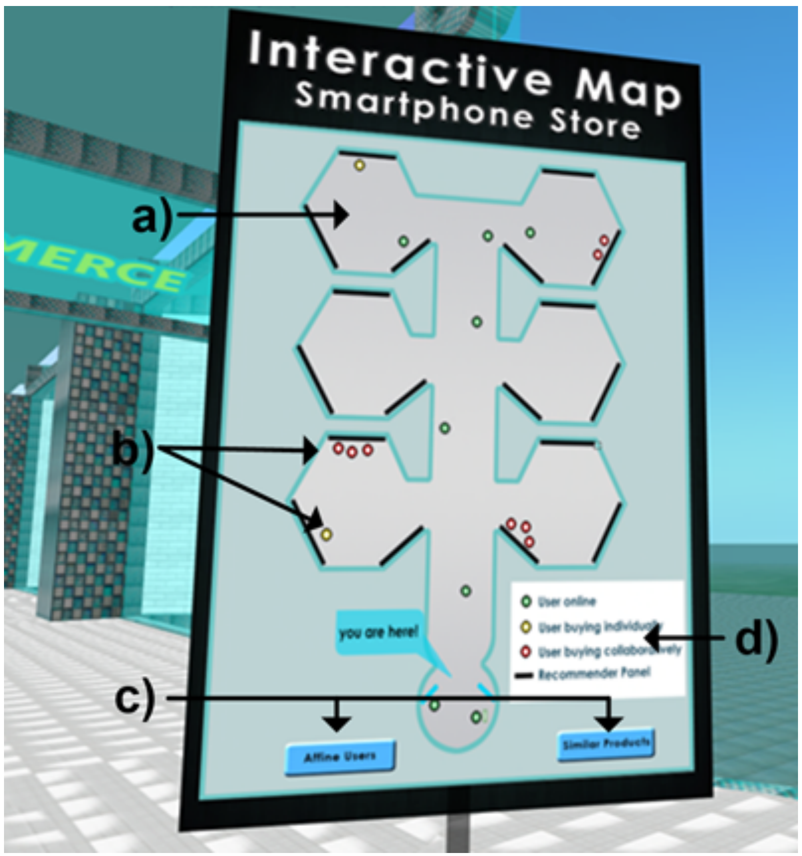
User Registration Object (URO)

- URO allows users register their initial preferences when they arrive to 3D eCommerce Space.
- Later, the system provides the users with recommendations based on these preferences.
- It complements the conversational recommendation process.

In the figure are shown the following preferences:

- mobile phone operating system and price, in the Smartphone domain

Interactive Map Object (IMO)



The user can:

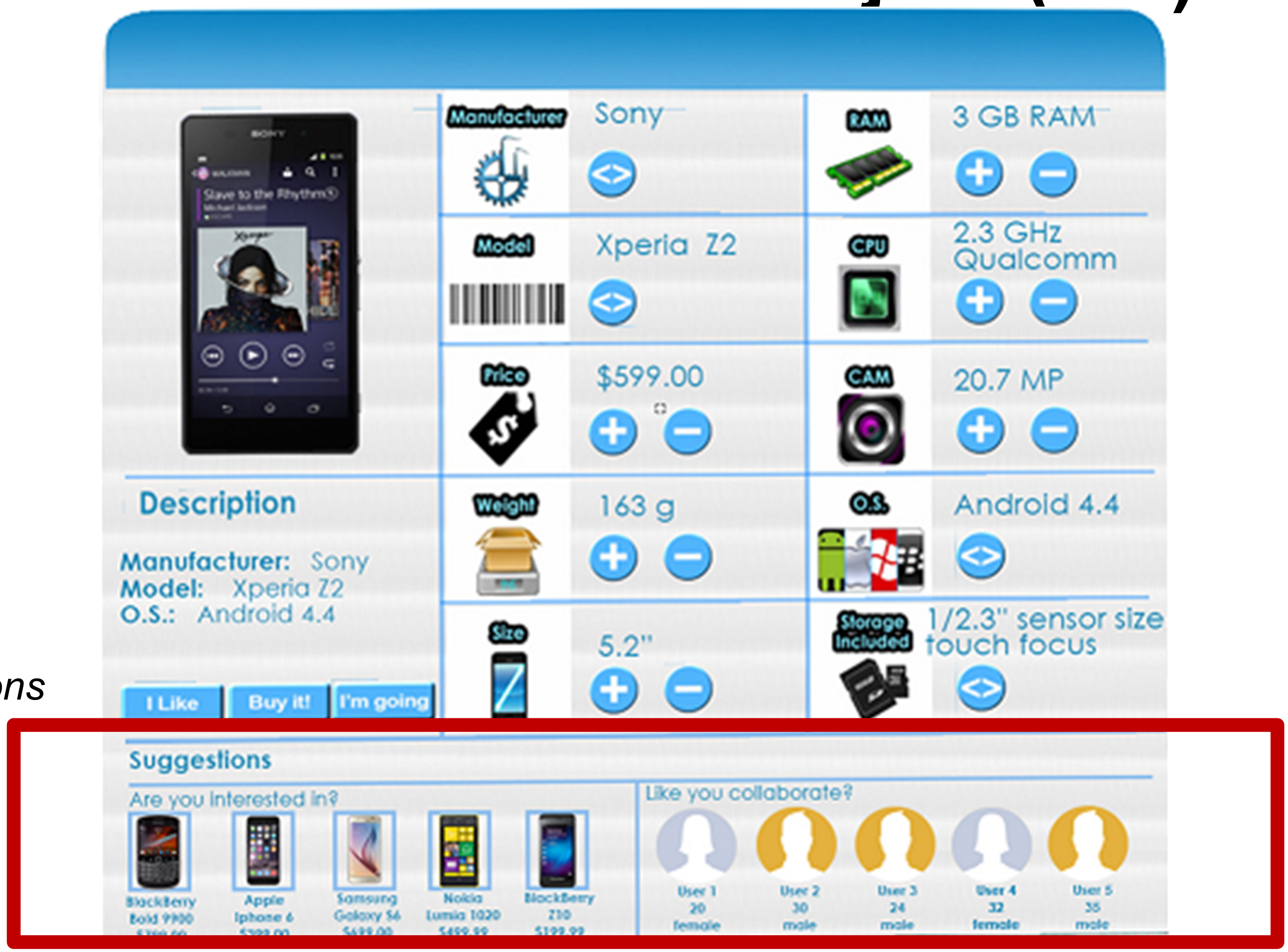
a) See all rooms in the eCommerce space,

b) Know which other users are interacting with Recommendation Objects (RO).

c) Ask for suggestions about rooms where they should go to find the users or products that matches their preferences.

Interaction Objects (III)

Recommendation Object (RO)



Based on User Preferences and History recommendations

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Cognitive Assistant

Güell, M., Salamó, M., Contreras, D., Boratto, L. ***Integrating a cognitive assistant within a critique-based recommender system.*** *Journal Cognitive Systems Research, Volume 64, pp. 1-14, (2020), https://doi.org/10.1016/j.cogsys.2020.07.003*

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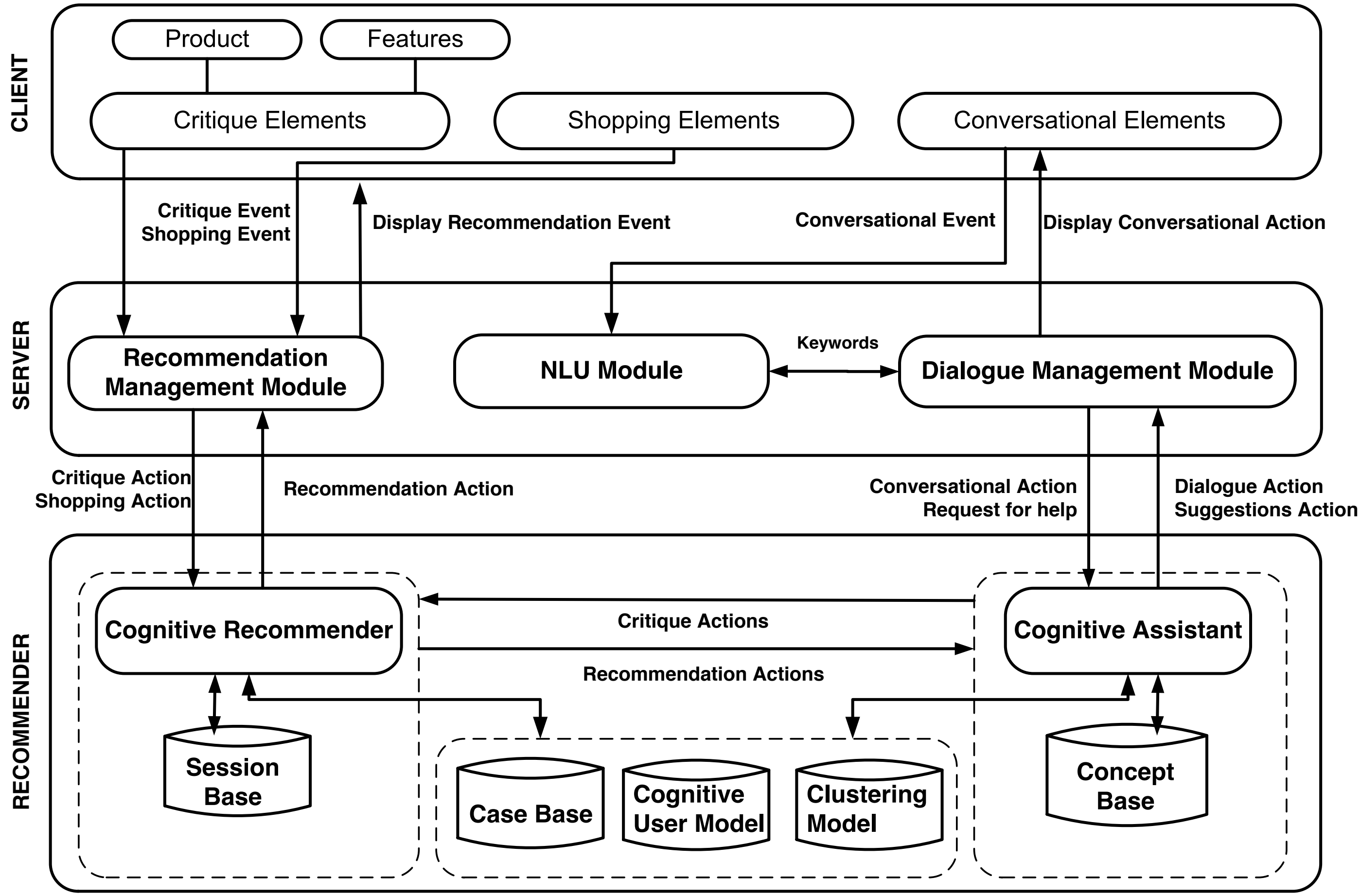
Conceptual Architecture



Güell, M., Salamó, M., Contreras, D., Boratto, L. ***Integrating a cognitive assistant within a critique-based recommender system.*** *Journal Cognitive Systems Research, Volume 64, pp. 1-14, (2020), https://doi.org/10.1016/j.cogsys.2020.07.003*

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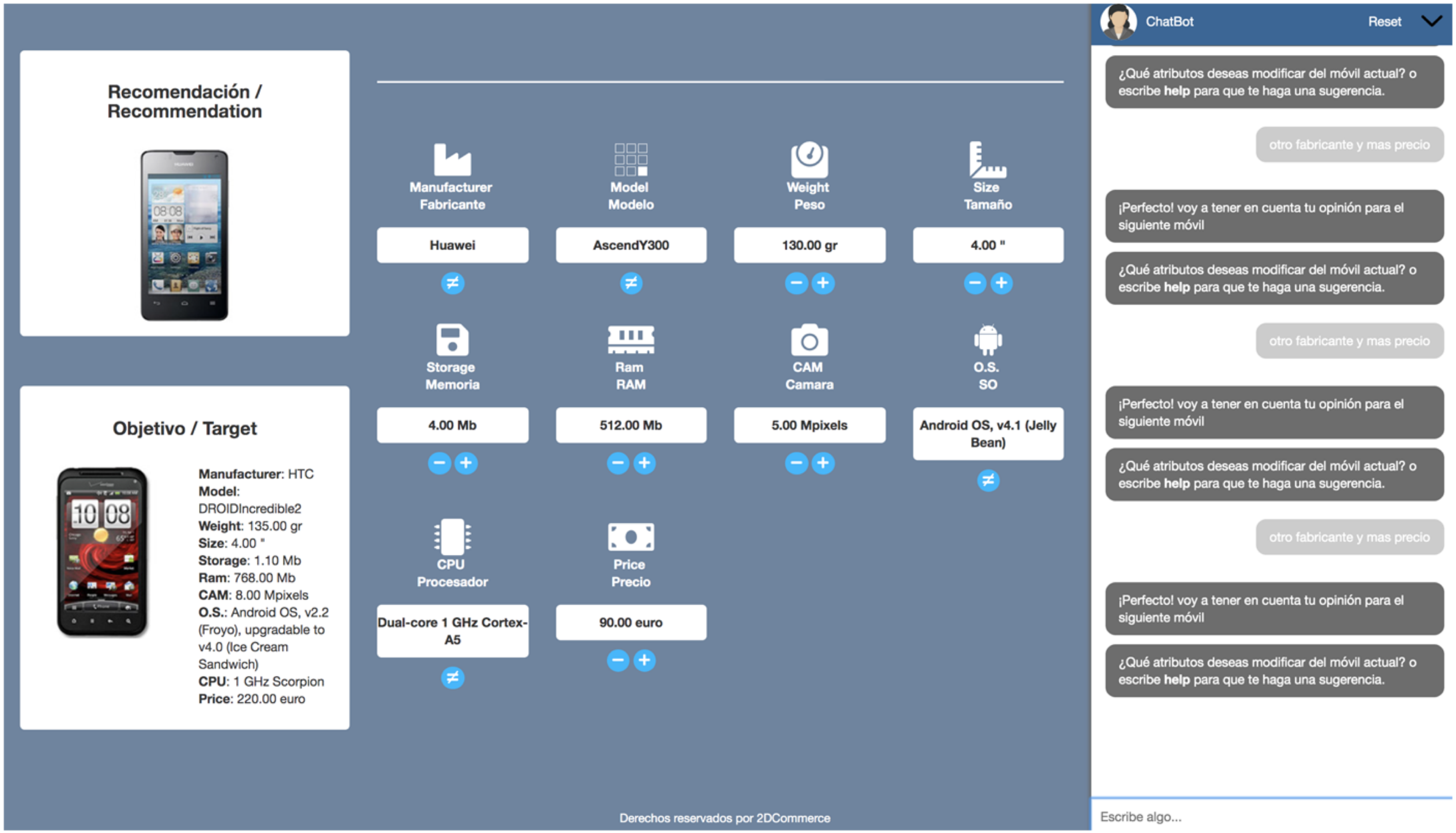
User Interface

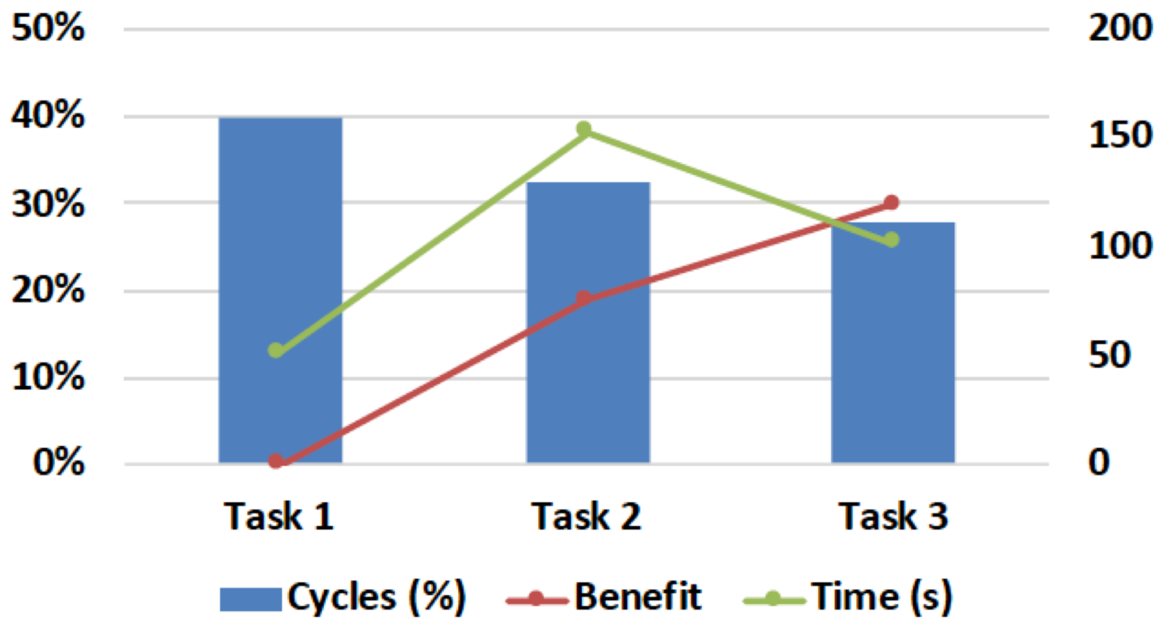


(OPTIONAL) https://youtu.be/JRuFRXku4Qw

Results: Efficiency

Task 1. Only buttons
Task 2. Only the assistant
Task 3. Both buttons and assistant

Between subject test
40 users



Smartphone dataset

gCoach for online Group Recommendation

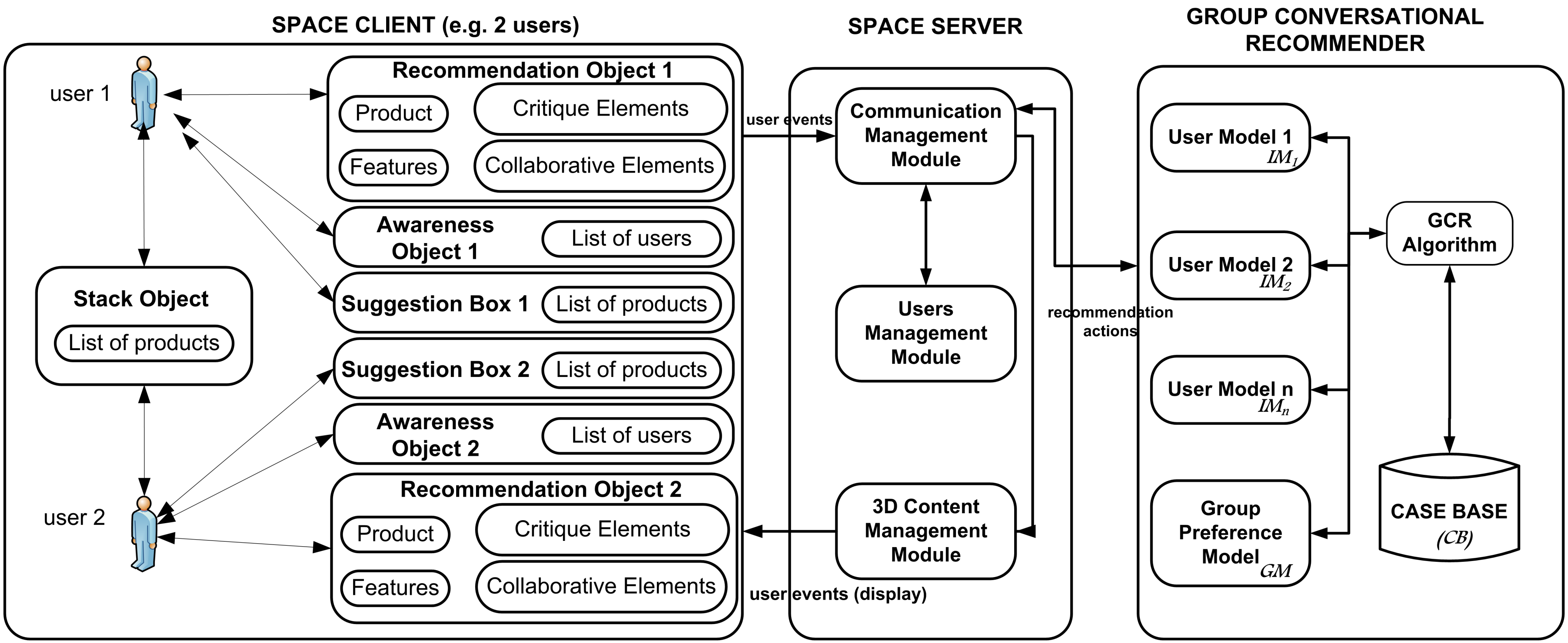
Contreras, D., Salamó, M., Pascual, J. ***A Web-Based Environment to Support Online and Collaborative Group Recommendation Scenarios****. Applied Artificial Intelligence. pp 480-499 (2015)*

gCOACH

- **gCOACH: Collaborative Advisory Channel for group recommendations**
- **Novel web-based environment** that supports on-line group recommendation
  - **Synchronizes all the actions** of users
  - Contains **multiple interaction modalities** to communicate, coordinate and persuade the group
- Conversational case-based group recommender

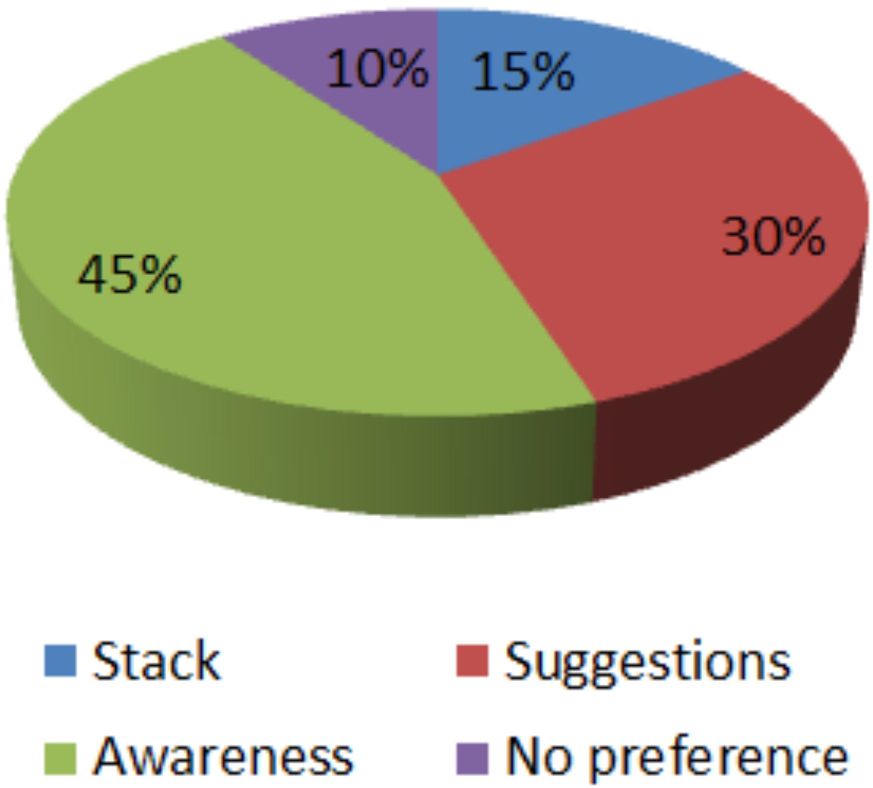
Contreras, D., Salamó, M., Pascual, J. ***A Web-Based Environment to Support Online and Collaborative Group Recommendation Scenarios****. Applied Artificial Intelligence. pp 480-499 (2015)*

gCOACH conceptual architecture



gCOACH interaction modalities

Results: Usability Evaluation



The majority of the users prefer the "Awareness Area"

What else?

TO READ:

An Introduction to Recommender Systems

Aggarwal, C.C.

Sections 1.1, 1.2, 1.3

Sections 1.4, 1.5, 1.6 and 1.7 are optional

Look at the video (OPTIONAL)

Recommender Systems: Beyond Machine Learning

Joseph A. Konstan

https://youtu.be/uupJmZG5xxA

From minute 5:30 to the end

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Week 11

Course. Introduction to Machine Learning

Theory 11. Recommender Systems

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