

Week 9

Course. Introduction to Machine Learning **Theory 9. Model Evaluation**

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Introduction to Machine Learning

Unsupervised Learning

Cluster Analysis

Factor Analysis

Visualization

K-Means,
Fuzzy C-means,
EM

PCA, ICA

Self Organized Maps (SOM) ,
Multi-Dimensional Scaling

Lazy Learning
(K-NN, IBL,
CBR)

**Overfitting,
model evaluation
and feature selection**

Kernel Learning

Ensemble Learning
(Trees, Adaboost)

Perceptron,
SVM

Supervised Learning

Non Linear Decision

Linear Decision

Decision Learning Theory

Basic concepts of
Decision Learning Theory

Bias/Variance,
VC dimension,
Practical advice of how
to use learning algorithms

Contents

1. Introduction to model evaluation
2. Evaluation metrics
3. Evaluation techniques
4. Overfitting and underfitting



Introduction to model evaluation

1. To know how well it works

- Reporting results
- Making scientific/business decisions
- Can we use it in real-life?



2. To compare models

- For choosing which models are best
- For tuning a given model's parameters



Definitions

- **Model Evaluation** is the process of assessing a property or properties of a model
 - Evaluation metrics
 - Evaluation techniques
- **Model Selection** is the process of choosing among many candidate models for a predictive modeling problem
 - Probabilistic measures
 - Resampling methods

How do we evaluate a model's performance?

- **Model Evaluation** is the process of assessing a property or properties of a model
 - It is often valuable to assess the efficacy of a model that has been learned
 - Such assessment is frequently relative— an evaluation of which of several alternative models is best suited to a specific application
 - There are many **evaluation metrics** by which a model may be assessed
 - Measures relating to *predictive efficacy*
 - Metrics based on *ROC analysis*

How do we evaluate a model's performance?

- **Model Evaluation** is the process of assessing a property or properties of a model
 - A learning algorithm must interpolate appropriate predictions of regions of the instance space that are not included in the training data
 - Algorithm **evaluation techniques** are designed to provide more reliable estimates of the accuracy of the models learned by an algorithm than would be obtained by assessing them on the training data



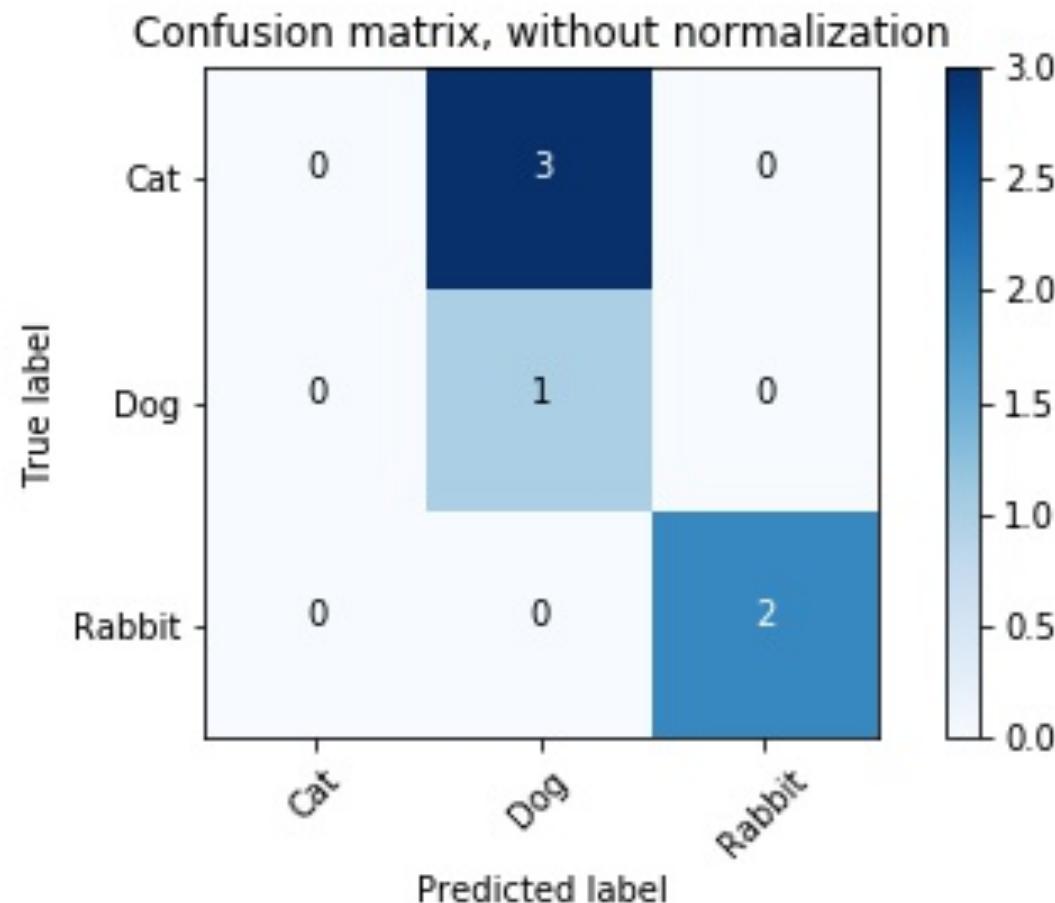
Evaluation metrics

Performance metrics

- Precision & Recall (and f1 score)
 - **Precision:** What percent of our predictions are accurate?
 - **Recall:** How many of the accurate predictions did we capture
 - **F1 score:** A single number that combines the two values above.
Good for ranking/sorting...
- $$F1 = 2 * \frac{\text{precision} * \text{recall}}{\text{precision} + \text{recall}}$$
- **Specificity/sensitivity** – very similar to precision/recall. Often used in medicine.
- Precision at N
 - How many accurate examples did we capture in our top N ranked examples. This is often used in information (document) retrieval

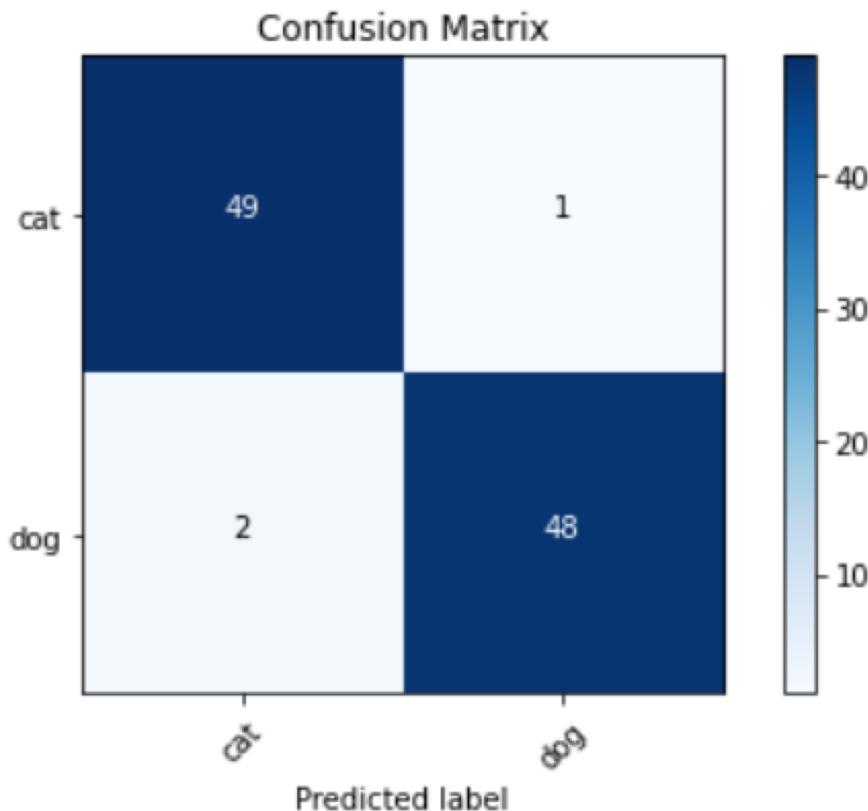
The confusion matrix

- How can we understand what types of mistakes a learned model makes?



Confusion matrix for 2-class problems

Confusion Matrix



	Actually Positive (1)	Actually Negative (0)
Predicted Positive (1)	True Positives (TPs)	False Positives (FPs)
Predicted Negative (0)	False Negatives (FNs)	True Negatives (TNs)

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$ErrorRate = 1 - Accuracy$$

Is accuracy an adequate measure?

- Accuracy may not be useful measure in cases where there is a large class skew
 - Is 98% accuracy good if 97% of the instances are negative?
- There are differential misclassification costs (i.e., getting a positive wrong costs more than getting a negative wrong)
 - For example, in a medical domain in which a false positive results in an extraneous test but a false negative results in a failure to treat a disease
- We are more interested in a subset of high-confidence predictions

Other accuracy metrics

- Purpose: Evaluating binary classification
- Many performance terms: Precision, recall, sensitivity, specific, true negative rate, true positive rate, PPV, NPV, type 1 error, type 2 error

		Condition (as determined by "Gold standard")		
		Condition Positive	Condition Negative	
Test Outcome	Test Outcome Positive	True Positive	False Positive (Type I error)	Positive predictive value = $\frac{\sum \text{True Positive}}{\sum \text{Test Outcome Positive}}$
	Test Outcome Negative	False Negative (Type II error)	True Negative	Negative predictive value = $\frac{\sum \text{True Negative}}{\sum \text{Test Outcome Negative}}$
		Sensitivity = $\frac{\sum \text{True Positive}}{\sum \text{Condition Positive}}$	Specificity = $\frac{\sum \text{True Negative}}{\sum \text{Condition Negative}}$	

*Even this picture is not complete (precision/recall on next slide)

Other accuracy metrics

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$ErrorRate = 1 - Accuracy$$

$$TP\ Rate = \frac{TP}{TP + FN}$$

$$FP\ Rate = \frac{FP}{FP + TN}$$

$$Precision = \frac{TP}{TP + FP}$$

$$Recall = \frac{TP}{TP + FN}$$

$$F-measure = \frac{(1 + \beta)^2 \cdot TP}{\beta^2 \cdot TP + FP + FN}$$

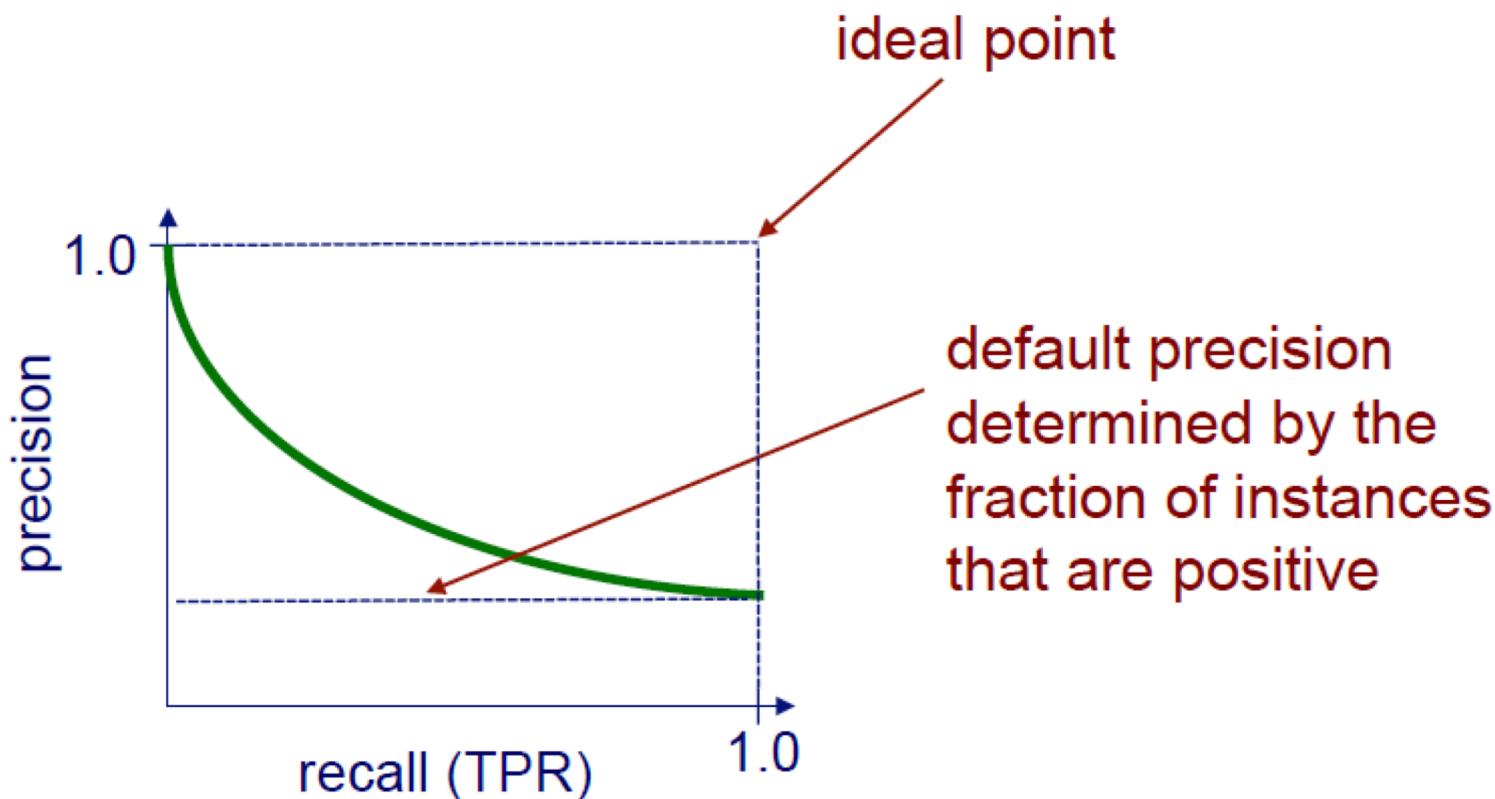
$\beta=1$, usually

Confusion Matrix

	Actually Positive (1)	Actually Negative (0)
Predicted Positive (1)	True Positives (TPs)	False Positives (FPs)
Predicted Negative (0)	False Negatives (FNs)	True Negatives (TNs)

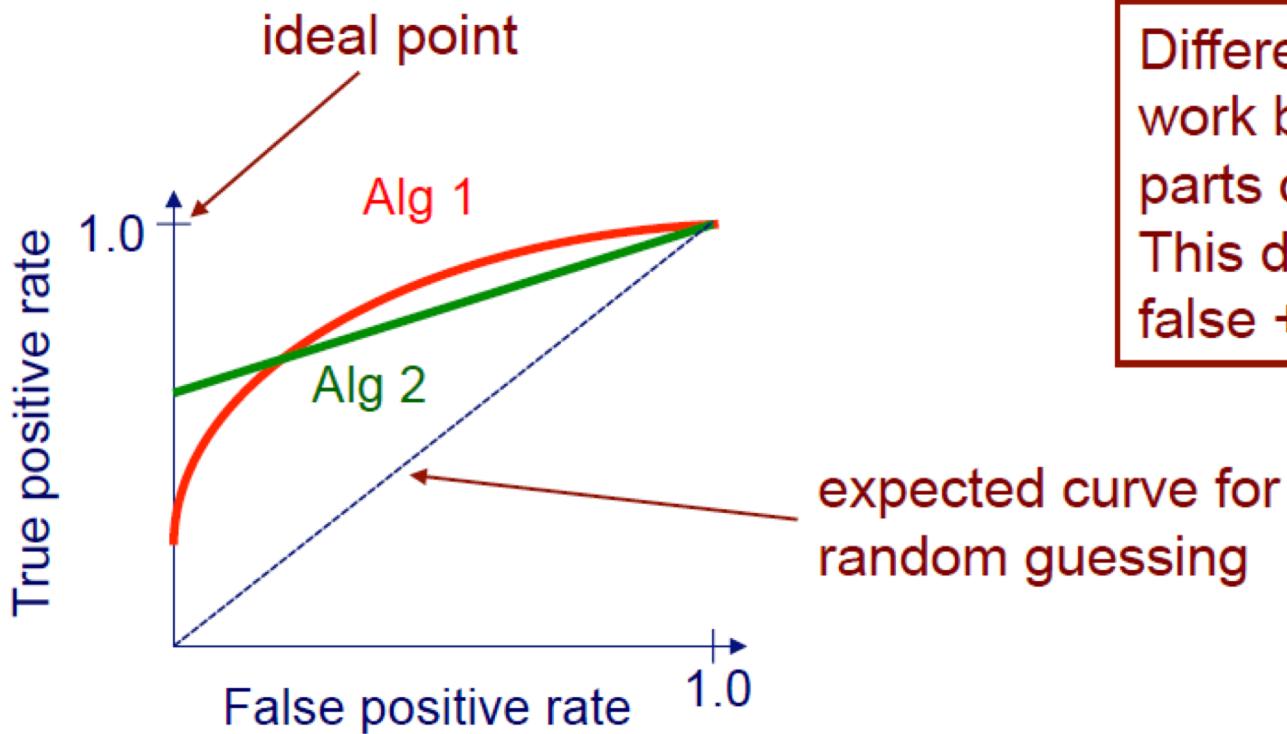
Precision/recall curves

A *precision/recall* (PR) curve plots the precision vs. recall (TP-rate) as a threshold on the confidence of an instance being positive is varied



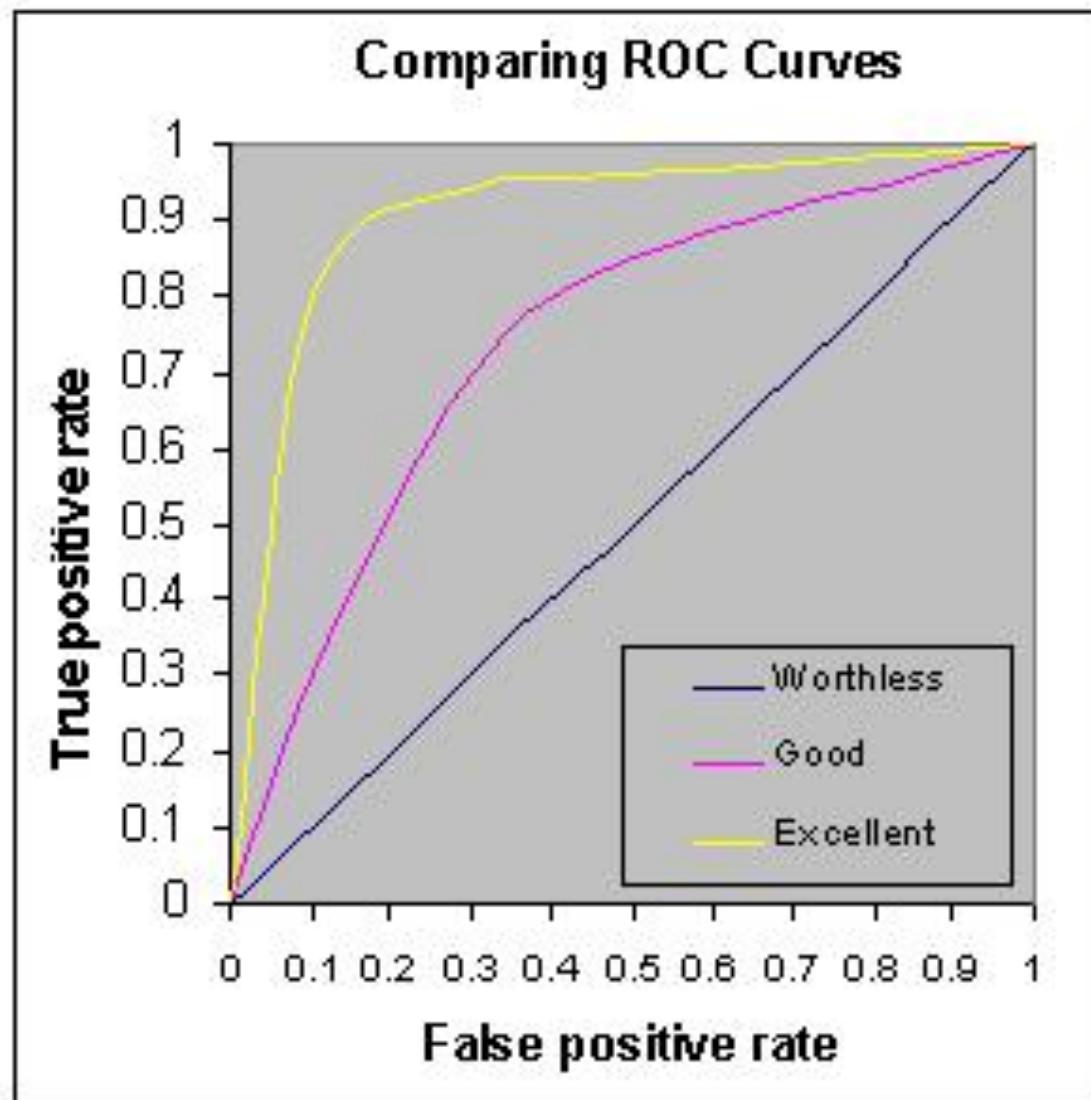
ROC Curves

A *Receiver Operating Characteristic* (ROC) curve plots the TP-rate vs. the FP-rate as a threshold on the confidence of an instance being positive is varied



Different methods can work better in different parts of ROC space. This depends on cost of false + vs. false -

ROC curve example



Comments on ROC and PR curves

- Both
 - Allow predictive performance to be assessed at various levels of confidence
 - Assume binary classification tasks
 - Sometimes summarized by calculating area under the curve
- ROC curves
 - Insensitive to changes in class distribution (ROC curve does not change if the proportion of positive and negative instances in the test set are varied)
 - Can identify optimal classification thresholds for tasks with differential misclassification costs
- Precision/recall curves
 - Show the fraction of predictions that are false positives
 - Well suited for tasks with lots of negative instances



Evaluation techniques



**Don't test your model on data
it's been trained with!**



Testing on ‘realistic data’

- Can you evaluate the learned model on real-life/real use case data? → **Ideal**
- Sometimes you can only evaluate on simulated data
- Testing on biased datasets can lead to inaccurate estimation of performance



I thought our results were better

- **Unbiased data**

- Data that represents the diversity in our target distribution of data on which we want to make predictions (i.e., it is a random, representative sample. This is a good scenario)

- **Biased data**

- Data that does not accurately reflect our target population
 - Example – we only asked people with glasses who they will vote for, instead of asking a random sample of the population
 - Take home message – training on biased data can lead to inaccuracies
 - There are methods to correct bias

Holdout test set – considerations

- Can the data be split randomly?
 - If not, it may lead to underfitting (remember me)
 - Test set and training set should be interchangeable, except for data size

- Do we have enough data?
 - Example: Guess if a car is American or Japanese based on 50 examples?
Will the test set be big enough to accurately report our model performance?



Holdout test set: The naïve approach

The simplest evaluation strategy: Simulate exploitation data.

We are given a dataset \mathcal{D} and it is divided in two sets

$$\mathcal{D} = \mathcal{D}_{train} \cup \mathcal{D}_{test}$$

Training

Use \mathcal{D}_{train} to learn the model.

Evaluation

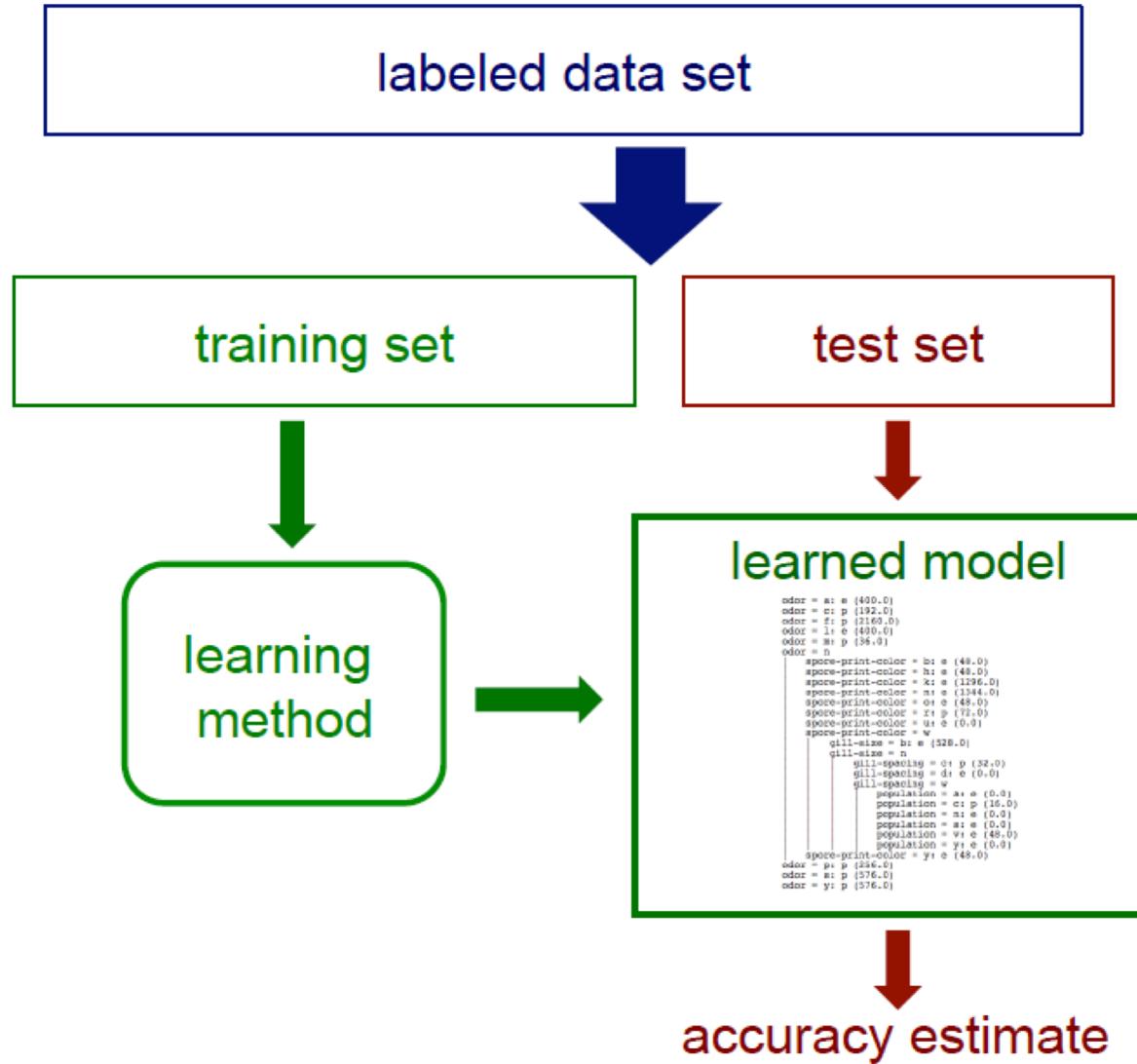
Use \mathcal{D}_{test} to compute the performance of the method.

Exploitation

Apply the model to new data and **hope** it predicts correctly.



Holdout test set: The naïve approach





The three-way split

Simulate exploitation data.

We are given a dataset \mathcal{D} and it is divided in three sets

$$\mathcal{D} = \mathcal{D}_{train} \cup \mathcal{D}_{validation} \cup \mathcal{D}_{test}$$

Training and model selection

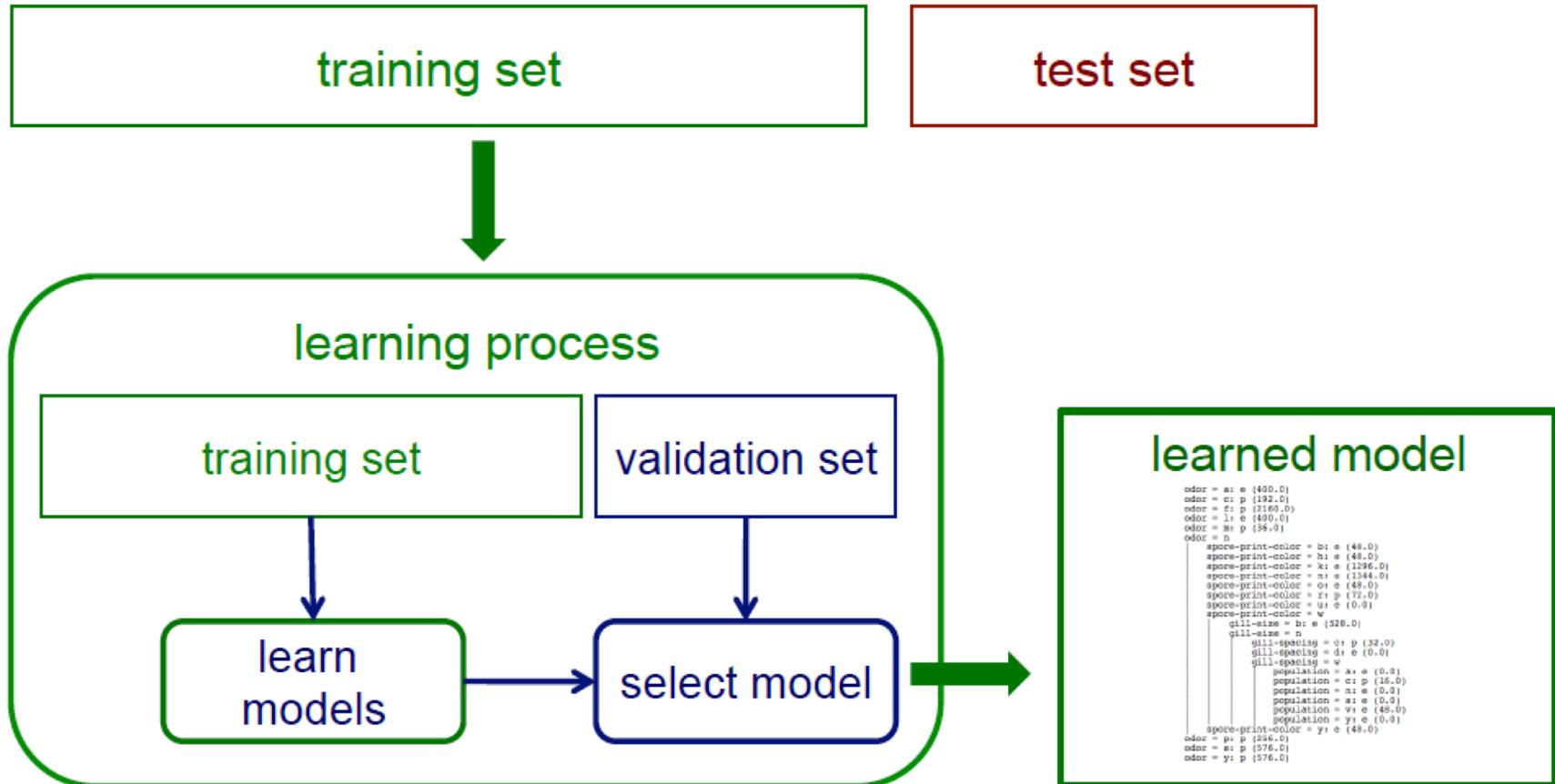
Use \mathcal{D}_{train} to learn both models. Use $\mathcal{D}_{validation}$ to decide which of the two models better adapts to our problem.

Evaluation

Use \mathcal{D}_{test} to compute the performance of the selected method.

Apply the selected model to new data and **hope** it predicts correctly.

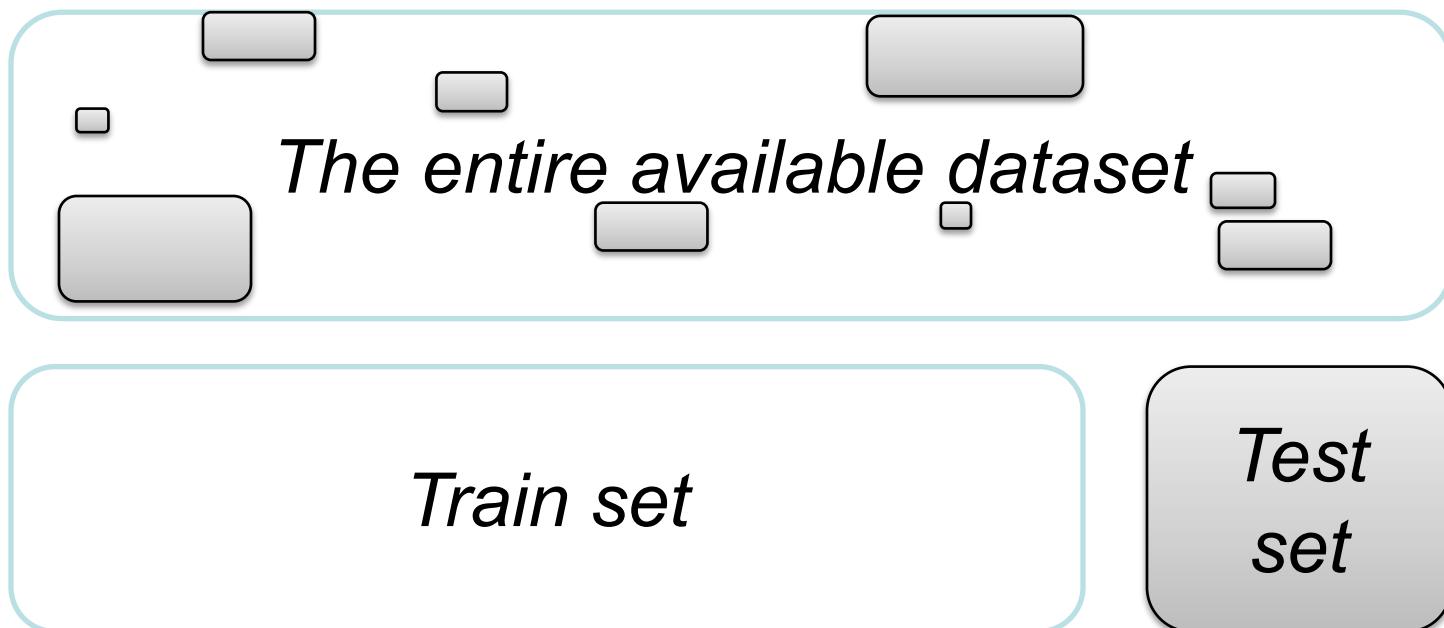
The three-way split



Partition training data into separate training/validation sets

How to perform the split?

- How many examples in each data set?
 - Training: Typically 60-80% of data
 - Test set: Typically 20-30% of your trained set
 - Validation set: Often 20% of data
- Examples
 - 3 way: Training: 60%, CV: 20%, Test: 20%
 - 2 ways: Training 70%, Test: 30%



How to perform the split?

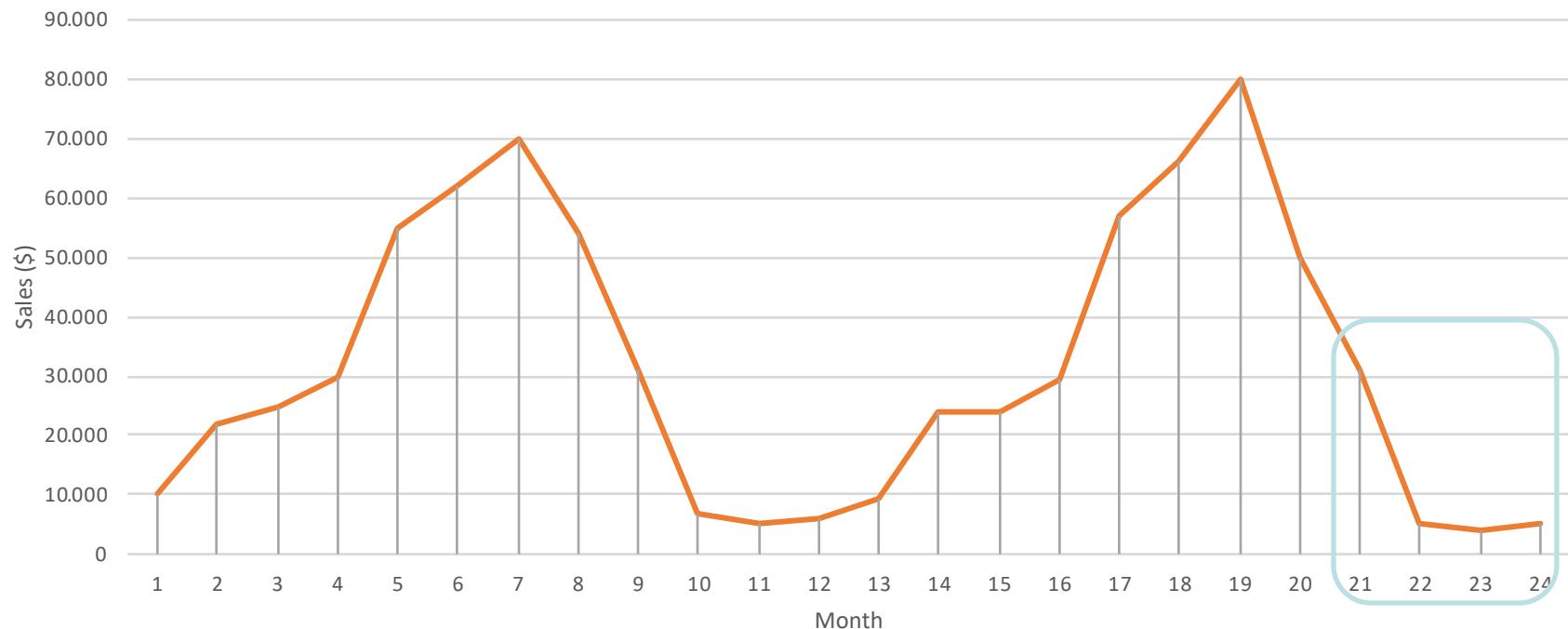
- Time based – If time is important



Can you use Jul 2011 stock price to try and predict Apr 2011 stock price?

How to perform the split?

- Some datasets are effected by seasonality



Is it a good test set?

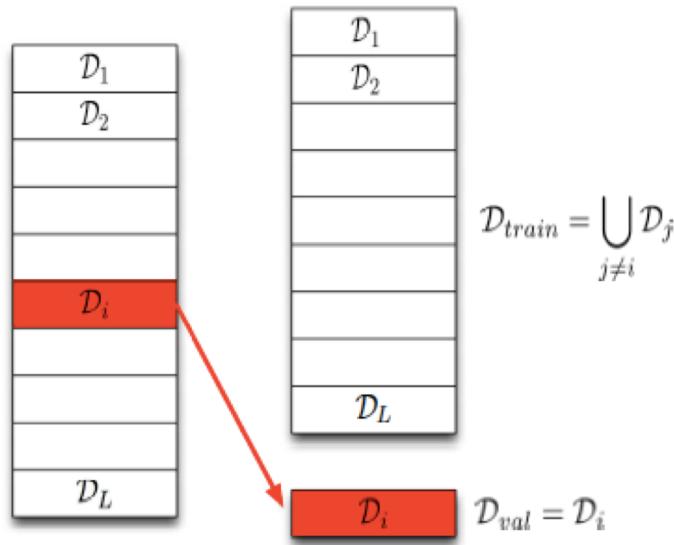
Cross-validation

- Popular method for validating models
- Cases when it is often used
 - When a hold-out plan is difficult to design or was not designed at the beginning
 - When the dataset is small, and you want to use all the data for good model training



Cross-validation

① for $i = 1..L$:



①

② Train the model on \mathcal{D}_{train} and evaluate $e_{val}(\mathcal{D}_i)$

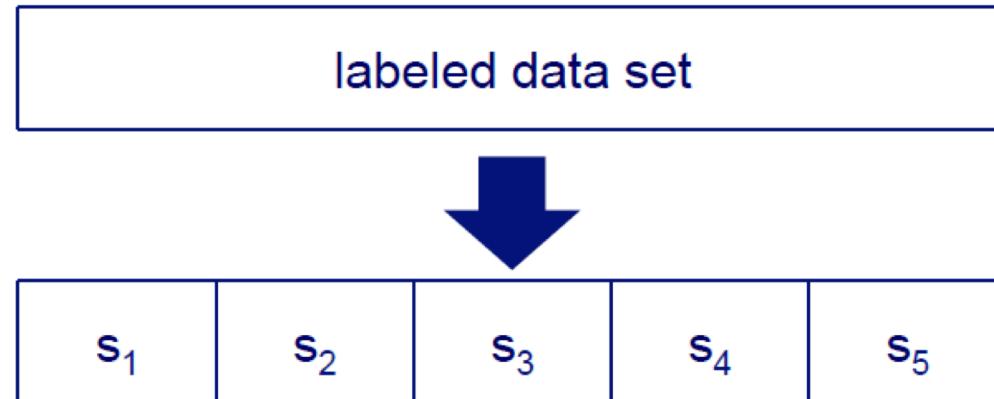
endfor

② $E_{val} = \sum_{i=1}^L e_{val}(\mathcal{D}_i)$

if $L = N \implies K = 1$ the process is called **Leave-one-out**. Otherwise, the method is called **L -fold cross-validation**.

Cross-validation

partition data
into n subsamples



iteratively leave one
subsample out for
the test set, train on
the rest

iteration	train on	test on
1	$s_2 \ s_3 \ s_4 \ s_5$	s_1
2	$s_1 \ s_3 \ s_4 \ s_5$	s_2
3	$s_1 \ s_2 \ s_4 \ s_5$	s_3
4	$s_1 \ s_2 \ s_3 \ s_5$	s_4
5	$s_1 \ s_2 \ s_3 \ s_4$	s_5



Cross-validation example

Suppose we have 100 instances, and we want to estimate accuracy with cross validation

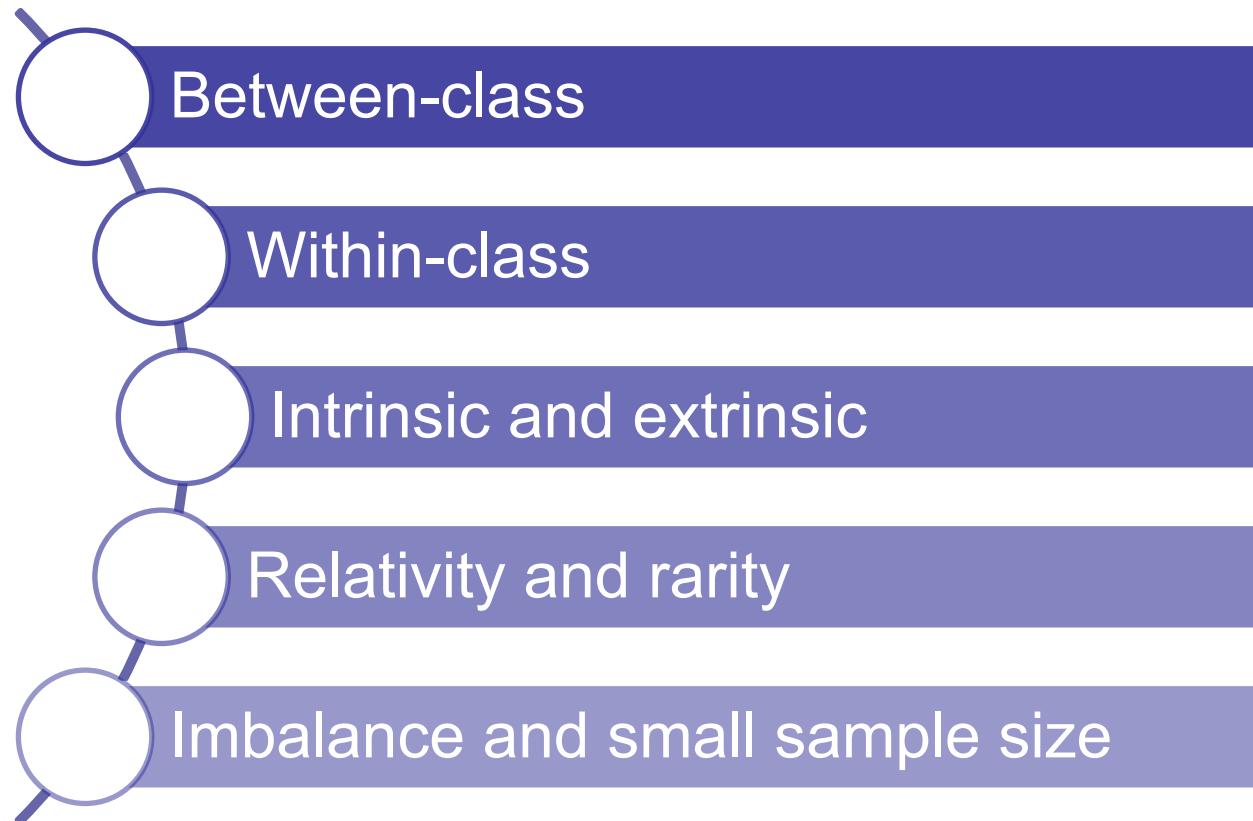
iteration	train on	test on	correct
1	$s_2 \ s_3 \ s_4 \ s_5$	s_1	11 / 20
2	$s_1 \ s_3 \ s_4 \ s_5$	s_2	17 / 20
3	$s_1 \ s_2 \ s_4 \ s_5$	s_3	16 / 20
4	$s_1 \ s_2 \ s_3 \ s_5$	s_4	13 / 20
5	$s_1 \ s_2 \ s_3 \ s_4$	s_5	16 / 20

$$\text{accuracy} = 73/100 = 73\%$$

How many folds are needed?

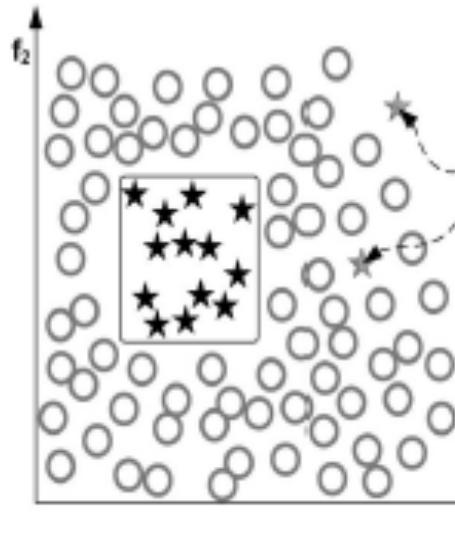
- **Common choice:** 5-fold or 10-fold cross validation (probably since these numbers sound nice)
- **Large datasets:** even a 3-Fold cross validation will be quite accurate
- **Smaller datasets:** we will prefer a bigger L
- **Leave-One-Out** approach ($L=N$). In this case the number of folds L is equal to the number of examples N . Used for smaller datasets

Imbalance is everywhere

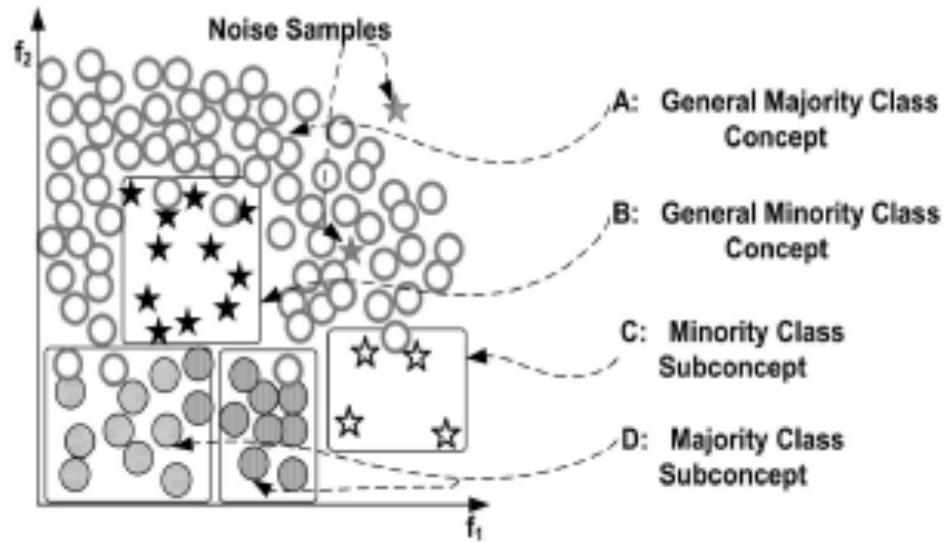


Source: H. He and E. A. Garcia, "Learning from Imbalanced Data," IEEE Trans. Knowledge and Data Engineering, vol. 21, issue 9, pp. 1263-1284, 2009

The Nature of Imbalance Learning



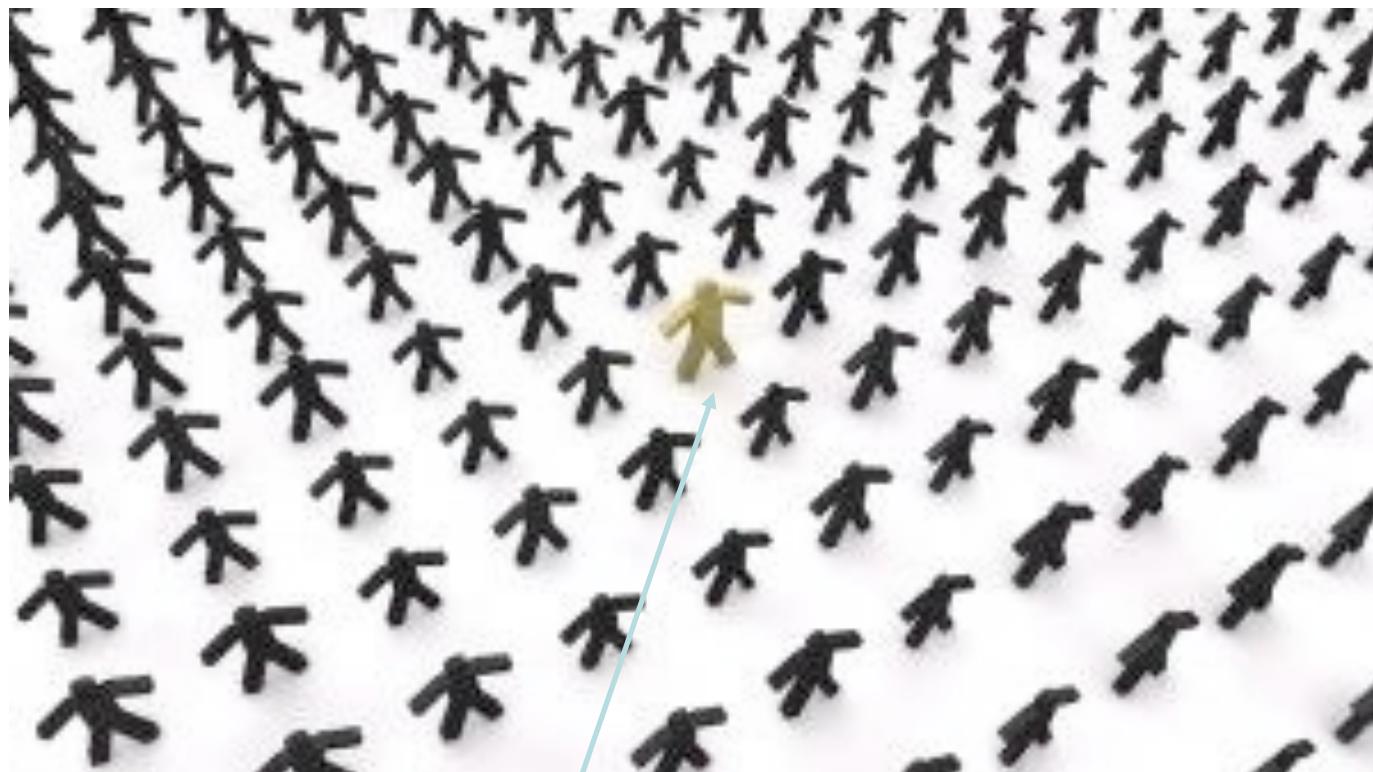
(a)



(b)

Fig. 2. (a) A data set with a between-class imbalance. (b) A high-complexity data set with both between-class and within-class imbalances, multiple concepts, overlapping, noise, and lack of representative data.

Dealing with imbalanced classes



Class 1: Short people

Class 2: Tall people

Can we build a classifier that can find the needles in the haystack?

Imbalanced Class Sizes

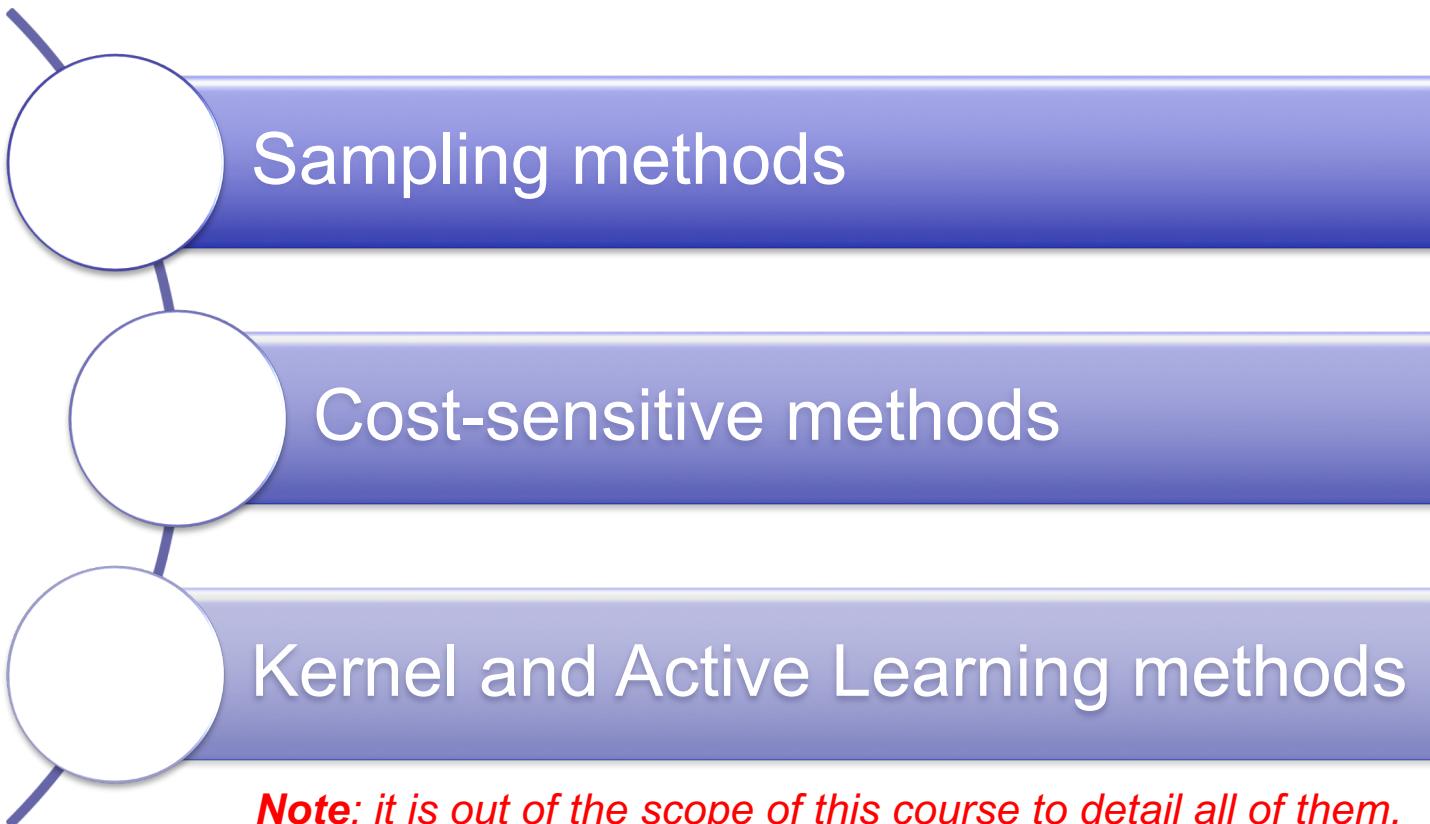
- Sometimes, classes have very unequal frequency
 - Cancer diagnosis: 99% healthy, 1% disease
 - eCommerce: 90% don't buy, 10% buy
 - Security: >99.99% of Americans are not terrorists
- Similar situation with multiple classes
- This creates problems for training and evaluating a model

Evaluating a model with imbalanced data



- Example: 99% of people do not have cancer
- If we simply create a ‘trivial classifier’ – predict that nobody has cancer, then 99% of our predictions are correct! Bad news! – we incorrectly predict nobody has cancer
- If we make only a 1% mistake on healthy patients, and accurately find all cancer patients
 - Then ~50% of people that we tell have cancer are actually healthy!

Solutions to imbalanced learning



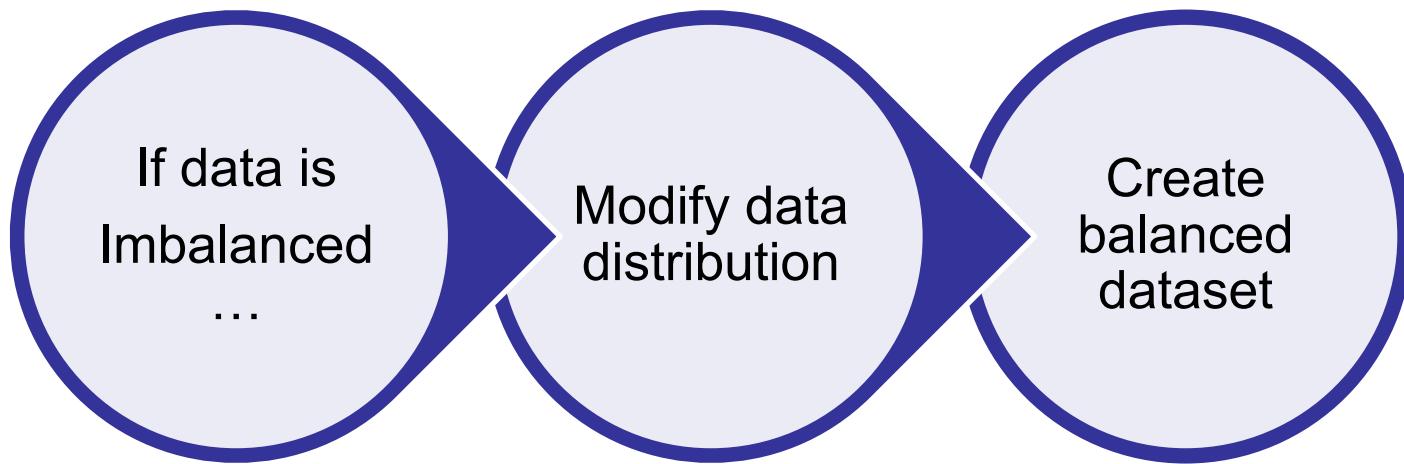
Note: it is out of the scope of this course to detail all of them.
In this course, we will concentrate on defining basic sampling methods



TO READ: OPTIONAL

H. He and E. A. Garcia, "Learning from Imbalanced Data," IEEE Trans. Knowledge and Data Engineering, vol. 21, issue 9, pp. 1263-1284, 2009

Sampling methods



Create balance through sampling

Balancing the classes

- Learning with imbalanced datasets is often done by balancing the classes
- **Important: Estimate the final results using an imbalanced held-out (test) set**
- How to create a “balanced” set
 - Treat the majority class → **undersampling**
 - Down-sample
 - Bootstrap (repeat downsampling with replacement)
 - Treat the small class → **oversampling**
 - Up-Sample the small class
 - Assign larger weights to the minority class samples

S : training data set; S_{min} : set of minority class samples, S_{maj} : set of majority class samples; E : generated samples

Random oversampling

- Expand the minority
- $|S'_{min}| \leftarrow |S_{min}| + |E|$
- $|S'| \leftarrow |S_{min}| + |S_{maj}| + |E|$
- Overfitting due to multiple “tied” instances

Random undersampling

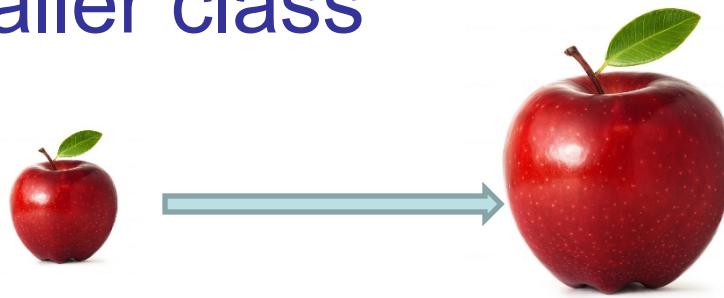
- Shrink the majority
- $|S'_{maj}| \leftarrow |S_{maj}| - |E|$
- $|S'| \leftarrow |S_{min}| + |S_{maj}| - |E|$
- Loss of important concepts

Down-sample and Up-sample

- Down-sample → undersampling
 - Sample (randomly choose) a **random subset of points** in the majority class
- Up-sample → oversampling:
 - Repeat minority points
 - Synthetic Minority Oversampling Technique (**SMOTE**)

Reweighting

- Idea: Assign larger weights to samples from the smaller class

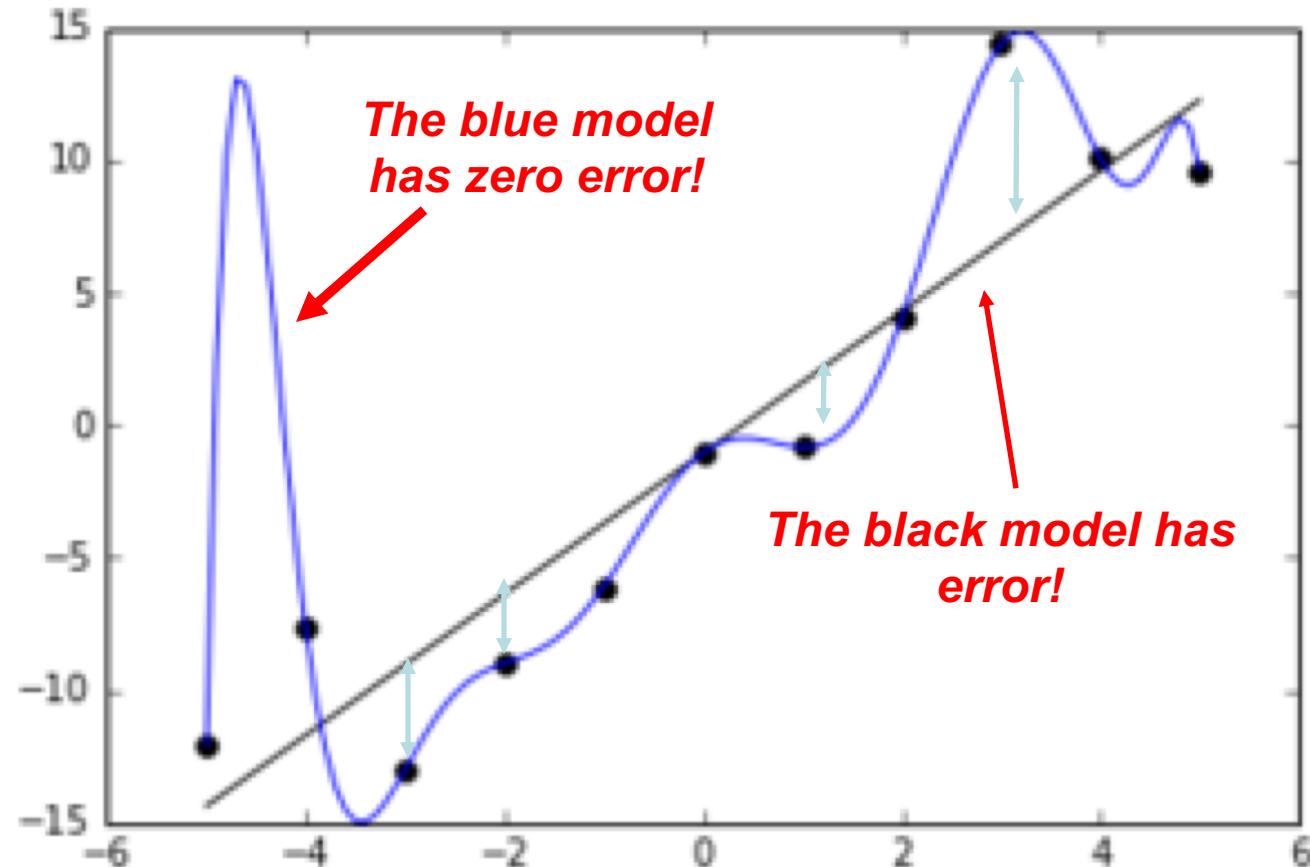


- A commonly used weighting scheme is linearly by class size: $w_c = \frac{n}{n_c}$
 - Where n_c is the size of the class c and $n = \sum_c n_c$ is the total sample size



Overfitting and Underfitting

Overfitting and underfitting



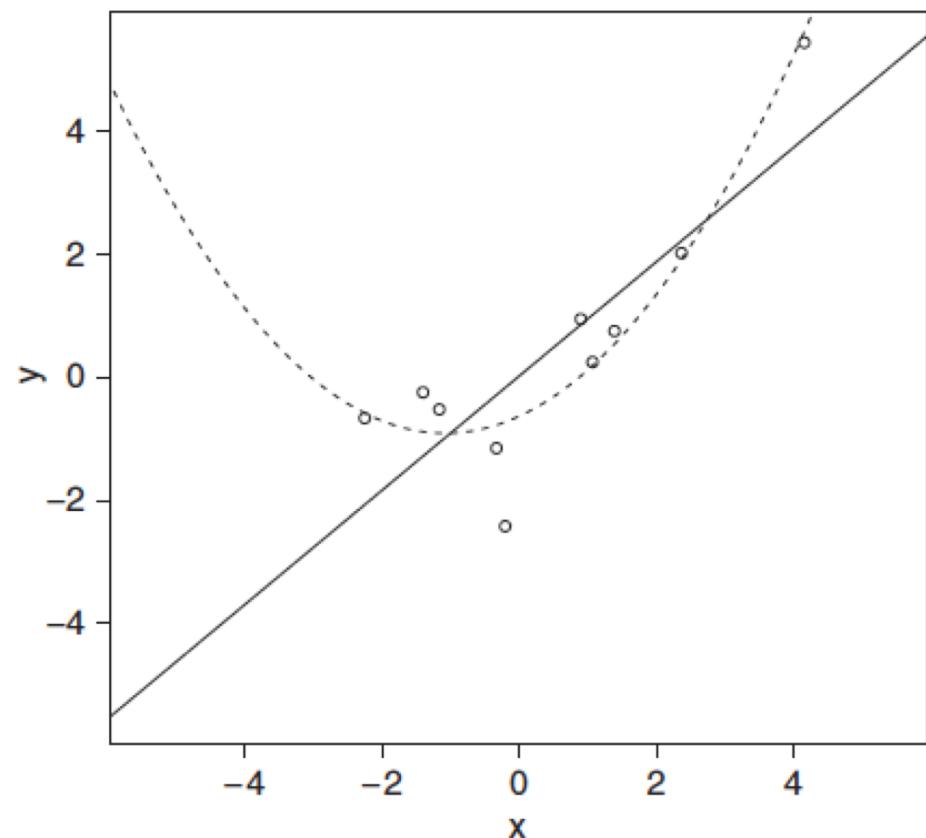
So which model is better : blue or black?

Overfitting

- A model overfits the training data when it describes features that arise from noise or variance in the data, rather than the underlying distribution from which the data were drawn
- Overfitting usually leads to loss of accuracy on out-of-sample data
- Approaches to avoid overfitting include:
 - Using low variance learners
 - Minimum description length technique
 - Pruning
 - Regularization
 - Stopping criteria

Overfitting example

- The figure illustrates overfitting
- Linear and polynomial models fitted to random data drawn from a distribution for which the linear model is a better fit



Underfitting (high Bias)

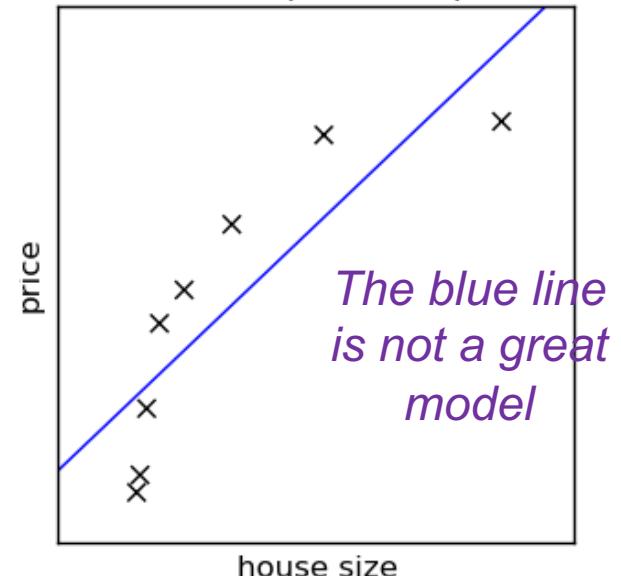
- In underfitting, the training error and test error are high

- What does too simple mean?

- Too few features
 - Use of features is not ‘complex’

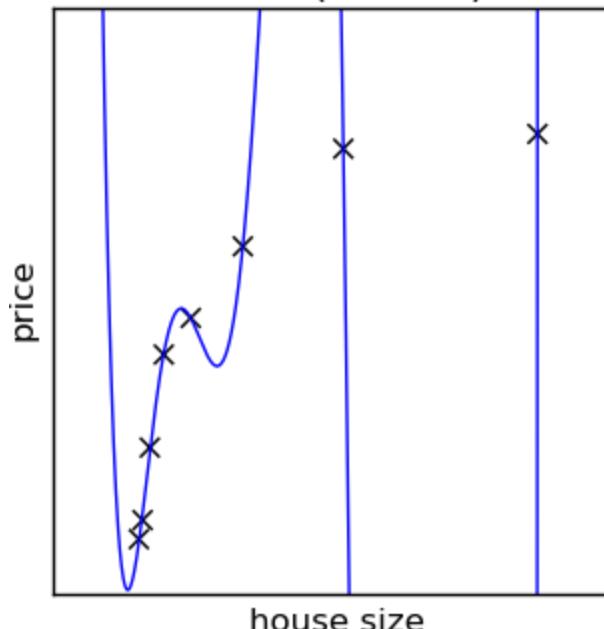
- Examples

- Can you predict well if an email is spam using only the word ‘free’?
Probably not. More features (words) are needed)
 - Can you predict well housing prices using only the year it was built?
Probably not - More features are needed: size, location, #windows, etc.



Overfitting (high Variance)

- If the model overfits
 - It cannot generalize well to new data
 - It memorizes the training data
 - It has a low training error, and high test error



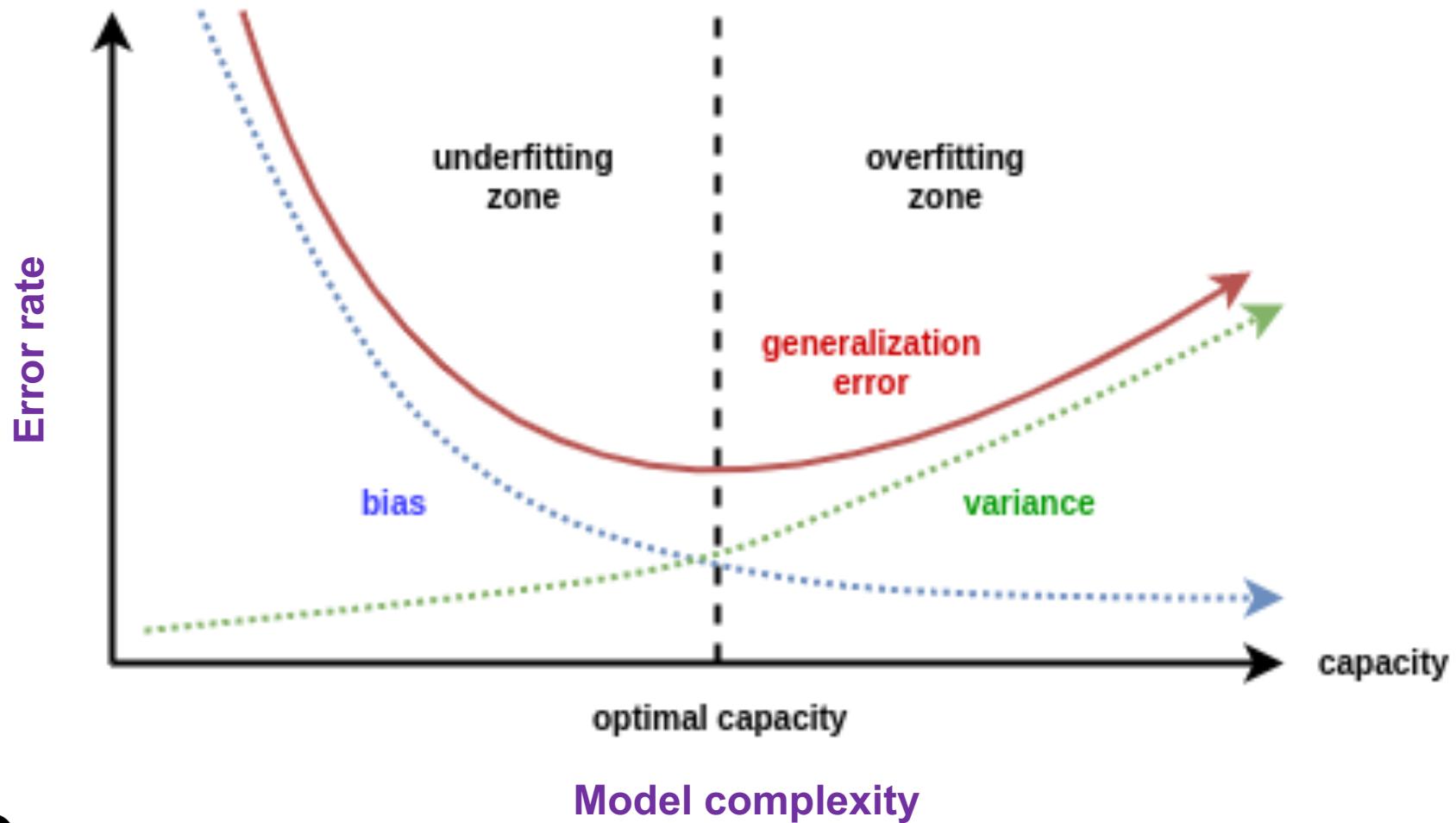
The model has no error, but it does not seem to represent the data well

Overfitting is caused by:

- Too much model complexity
 - Typically too many features
- Too little data or not enough data diversity
 - **Big data:** The more data we have, the more complex a model we can use
 - **Diversity:** Redundant data does not add information and thus does not improve the model



Bias –Variance tradeoff



TO READ:

<https://djsaunde.wordpress.com/2017/07/17/the-bias-variance-tradeoff/>



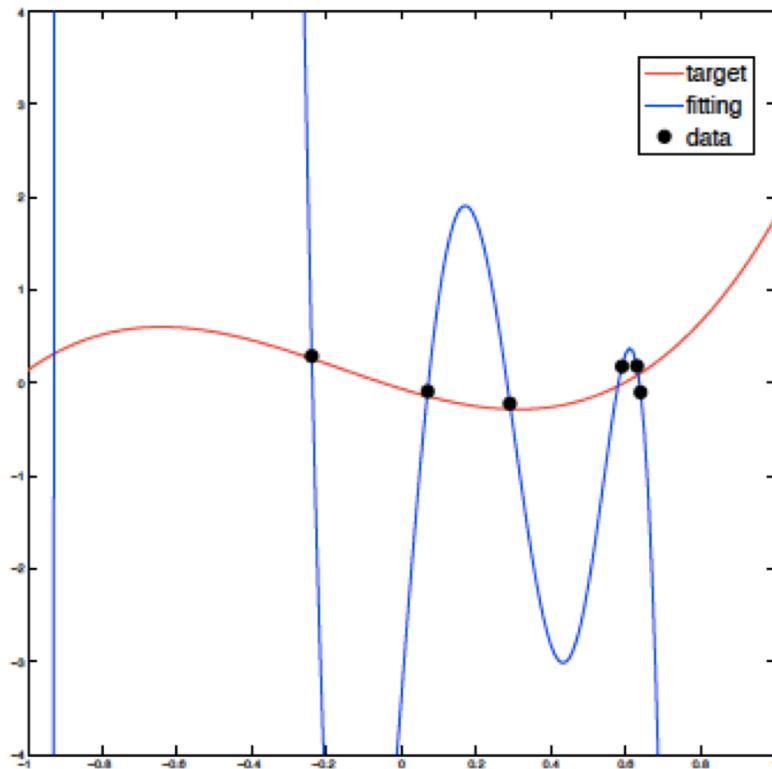
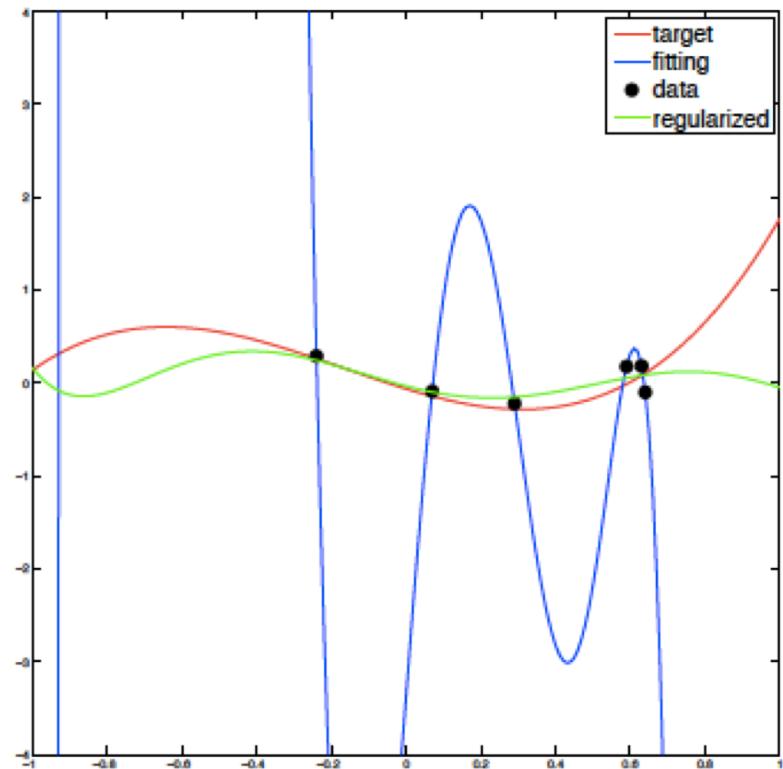
Addressing Overfitting

- More Data → a problem that comes up many time
- Reduce # of features or dimension
 - Select which features to keep
 - Reduce dimension
 - Select which model to use
- Regularization → Putting the brakes

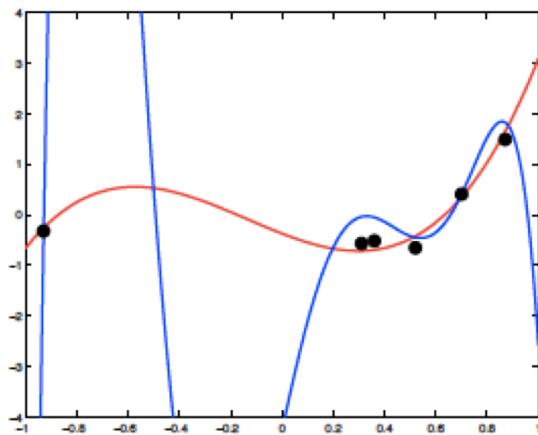
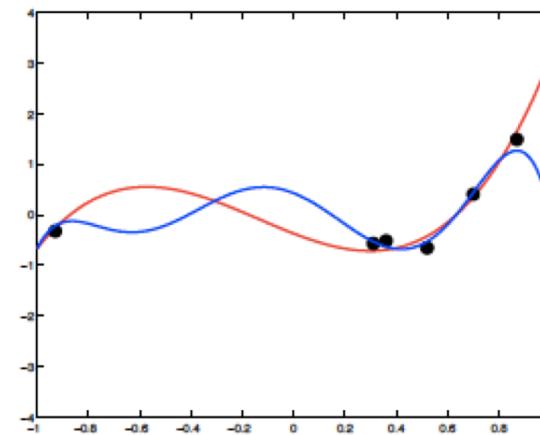
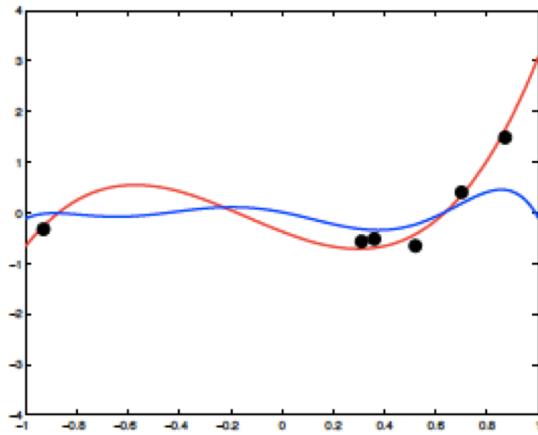
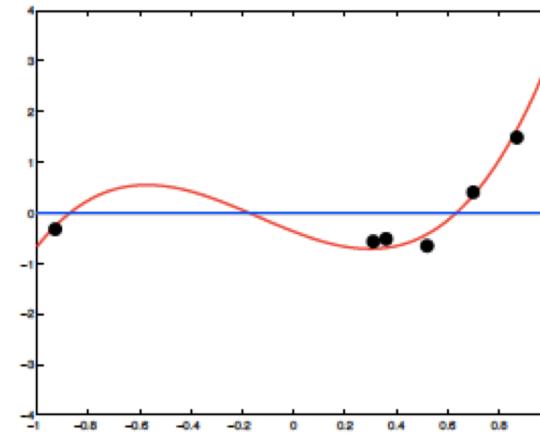
Regularization

- Regularization plays a key role in many machine learning algorithms
 - Exactly fitting a model to the training data is generally undesirable, because it will fit the noise in the training examples (overfitting), and is doomed to predict (generalize) poorly on unseen data
 - In contrast, a simple model that fits the training data well is more likely to capture the regularities in it and generalize well
- A regularizer is introduced to quantify the complexity of a model
 - There is a variety of regularizers, which yield different statistical and computational properties
 - In general, there is no universally best regularizer, and a regularization approach must be chosen depending on the dataset

Regularization

free fit**restrained fit**

Regularization

 $(\lambda = 0)$  $(\lambda = 0.5)$  $(\lambda = 10)$  $(\lambda = 1000)$ ⌂ ▶ ⌄ ⌁ ▶

Regularization formally

- In general:

Data: $(\mathbf{x}_1, y_1) \dots (\mathbf{x}_n, y_n) \in \mathbb{R}^N \times \mathbb{R}$
Estimate: $f : \mathbb{R}^N \rightarrow \mathbb{R}$

Hypothesis space \mathcal{H} .

$$f^* = \arg \min_{f \in \mathcal{H}} \frac{1}{n} \sum_i \mathcal{L}(f(\mathbf{x}_i), y_i) + \Omega(f)$$

fit to data + complexity penalty

- Occam's razor: “simpler” hypotheses are better
- Regularization encodes a notion of complexity to be minimized



What else?



Look at the video: **Diagnosing bias/variance**

<https://youtu.be/fDQkUN9yw44>



Look at the video: **Learning curves**

<https://youtu.be/ISBGFY-gBug>



TO READ:

Evaluating Machine Learning Models

A beginner's guide to key concepts and pitfalls

Alice Zheng

Chapter 2 and 3

The remaining of the chapters are optional



Week 9

Course. Introduction to Machine Learning

Theory 9. Model Evaluation

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