Computational Intelligence

Master AI

2019-2020

Àngela Nebot

Introduction to Fuzzy Computation

Class Index

- Introduction to CI - Lluís
- Examples - Lluís
- Introduction to Fuzzy Computation – Àngela
- Fuzzy LAB – Àngela + Enrique
- Introduction to Neural Computation I (Single and Multi-layer Perceptrons, Back-propagation) - Enrique
- Introduction to Neural Computation II (Convolutional Neural Networks) - Enrique
- Introduction to Neural Computation III (Deep Learning, Radial Basis Function) – Enrique + René
- Neural Computation LAB - Àngela + Enrique
- Experimental Issues - Enrique

Class Index

- Introduction to Evolutionary Computation I (Genetic Algorithms) - René
- Introduction to Evolutionary Computation II (Genetic Algorithms) - René
- Introduction to Evolutionary Computation III (Evolutionary Strategies) - René
- Genetic Algorithms and Evo. Str. LAB - Àngela +Enrique

Exam: 9/1/2020

FINAL = 35% Exam + 50% Pract + 5% LabExerFuzzy + 5% LabExerNN + 5% LabExerGA

Introduction to Fuzzy Computation

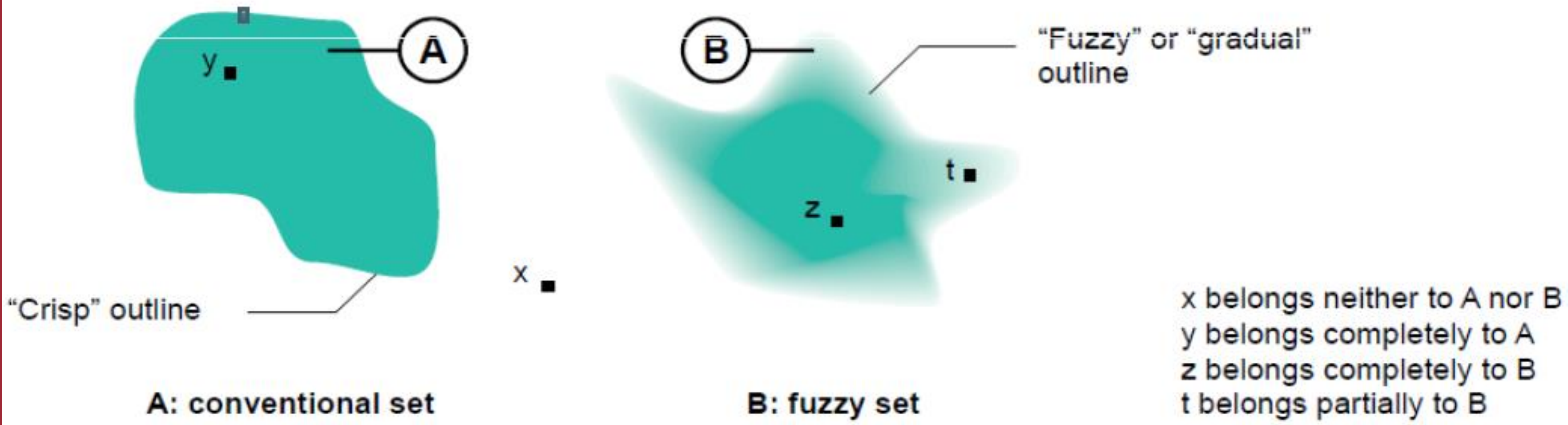
- Fuzzy logic
- Fuzzy sets
- Fuzzy rules
- Fuzzy rule-based systems (Mamdani and Sugeno)
- Adaptive Neuro-Fuzzy Inference System (ANFIS)
- Classification and Regression Trees (CART)
- Extracting Fuzzy Models from Data

Fuzzy Logic vs. Fuzzy Arithmetic

- *Fuzzy logic* is the generalization of ordinary logic and allows truth values other than absolute truth and utter falsity.
- *Fuzzy arithmetic* is a formalism for making calculations with numerical quantities that are imprecisely known. These quantities are known as fuzzy numbers and are defined as fuzzy sets on the real numbers.
- Fuzzy logic is comparable to fuzzy arithmetic in the same way that logic is comparable to arithmetic. They don’t have much in common in terms of the kinds of problems they’re used to address.

Fuzzy Logic

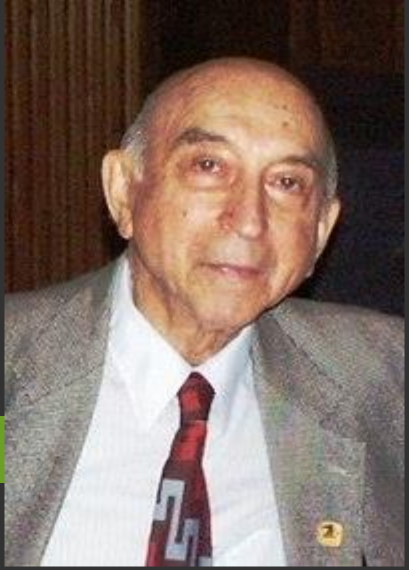
- Unlike Boolean logic (bivalent/Aristotelian), fuzzy logic is *multi-valued*
  - Fuzzy logic represents degrees of membership and degrees of truth
  - Things can be *part true* and *part false* at the same time


A
y
"Crisp" outline
x
B
"Fuzzy" or "gradual" outline
t
z
A: conventional set
B: fuzzy set
x belongs neither to A nor B
y belongs completely to A
z belongs completely to B
t belongs partially to B

Fuzzy Logic

- ***Heraclitus*** (535-475 BC): things can be simultaneously *True* and *False*
- ***Plato*** (428-348 BC): there is a third region beyond *True* and *False*
- ***Łukasiewicz*** (1878-1956): many valued logic. First described three-valued logic, it was later generalized to *n*-valued (for all finite *n*)
- ***Knuth*** (1938): three-valued logic similar to *Łukasiewicz*. His insight, apparently missed by *Łukasiewicz*, was to use the integral range [-1, 0, +1] rather than [0, 1, 2]

Fuzzy Logic



Lotfi A. Zadeh

- was a mathematician, computer scientist, electrical engineer, artificial intelligence researcher and professor emeritus at the University of California, Berkeley
- in his paper fuzzy sets (1965), proposed using a membership function operating on the domain of all possible values and new operations for the calculus of logic and showed that fuzzy logic was a generalization of Boolean logic.
- he also proposed fuzzy numbers as a special case of fuzzy sets, as well as the corresponding rules for consistent mathematical operations (fuzzy arithmetic).

Fuzzy Logic Characteristics

- Knowledge is interpreted as a collection of elastic/fuzzy constraints on a set of variables.
- Inference is viewed as a process of propagation of elastic constraints
- Fuzzy logic allows decision making with estimated values under incomplete or uncertain information
- Exact reasoning is viewed as a limiting case of approximate reasoning

Fuzzy Set Definition

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function

$$\mu_A : X \longrightarrow [0, 1]$$

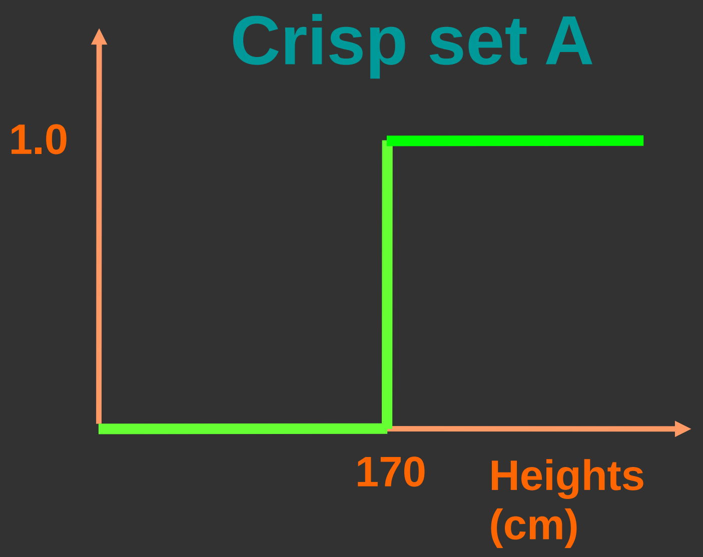
and $\mu_A(x)$ is interpreted as the degree of membership of element x in the fuzzy set A for each $x \in X$.

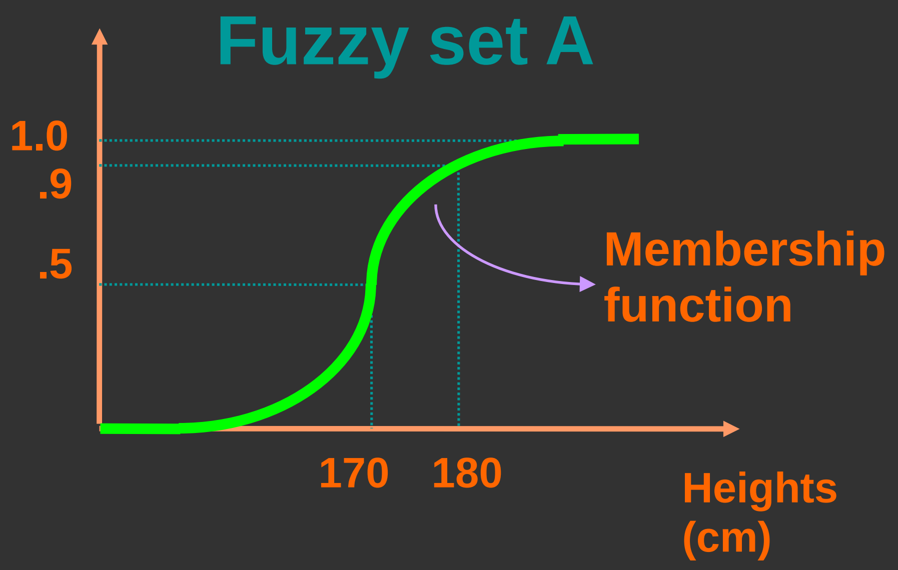
It is clear that *A* is completely determined by the set of ordered pairs:

$$A = \{(x, \mu_A(x)) | x \in X\}$$

Fuzzy Set Definition

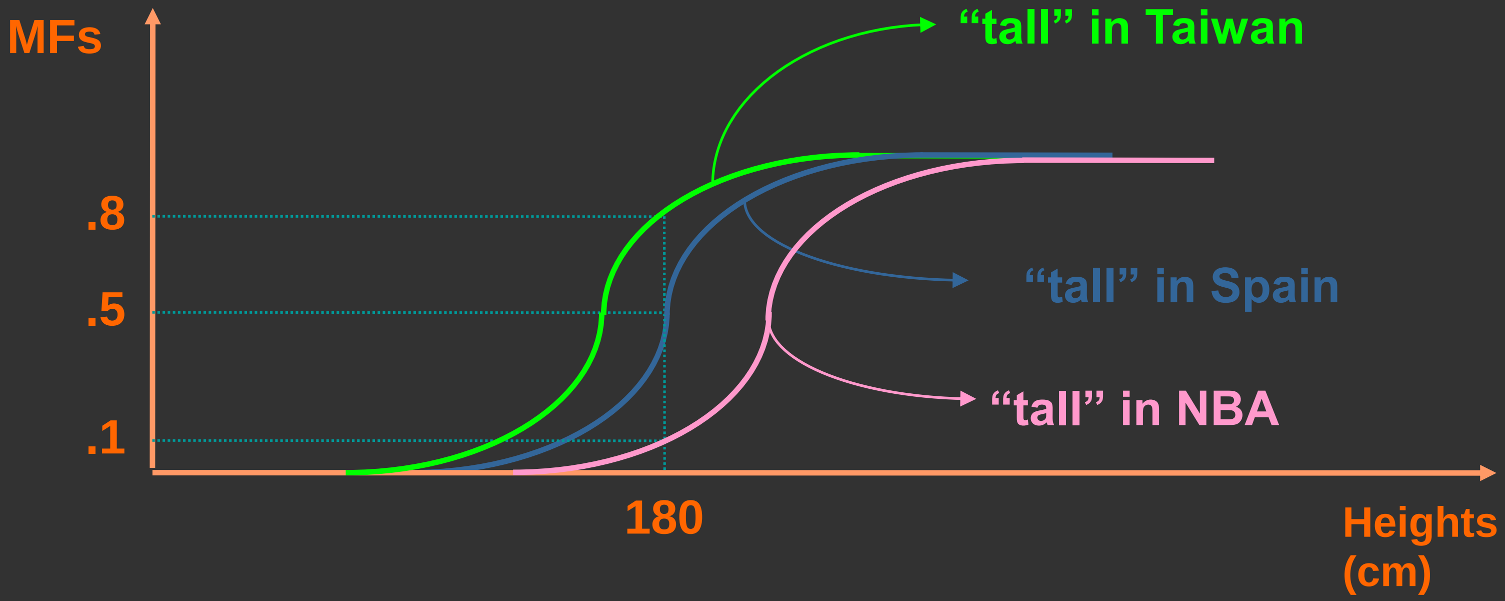
A = Set of tall people





Fuzzy Set Definition

- About MFs
  - Subjective measures
  - Not probability functions



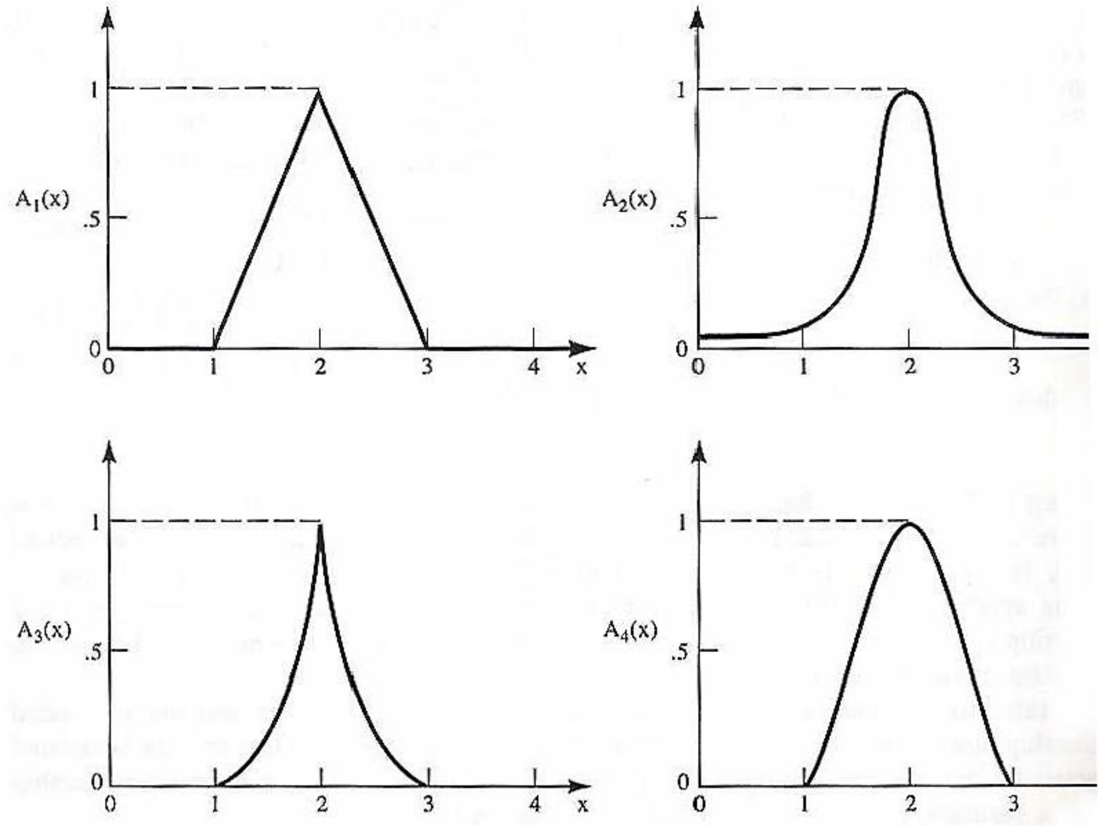
Fuzzy Sets

Fuzzy sets representing the same concept may vary considerably.

Class of real numbers that are close to 2

The four fuzzy sets should possess the properties:

- *A_i* (2)=1 and *A_i* (x)<1, for all x≠2;
- *A_i* is symmetric with respect to x=2;
- *A_i* decreases monotonically from 1 to 0



Examples of membership functions that may be used in different contexts for characterizing fuzzy sets of real numbers close to 2.

Fuzzy Sets: Types

Thus far we introduced only one type of fuzzy set!

Given a relevant universal set *X*, any arbitrary fuzzy set of this type (set *A*) is defined by a function of the form:

$$A : X \rightarrow [0, 1]$$

Ordinary Fuzzy Sets

- By far the most common in the literature and applications
- Their membership functions are often overly precise, i.e. they require that each element of the universal set be assigned a particular real number
- p. e. we may only be able to identify meaningful lower and upper bounds of membership grades for each element of the universal set. With OFS we should suppress the identification uncertainty by choosing for example the middle value or…

Fuzzy Sets: Types

Interval-valued fuzzy sets can further be generalized by allowing their intervals to be fuzzy.

Fuzzy sets of **type 2**

$$A : X \to \mathcal{F}([0, 1])$$

Fuzzy power set: Set of all ordinary fuzzy sets that can be defined within the universal set [0, 1]

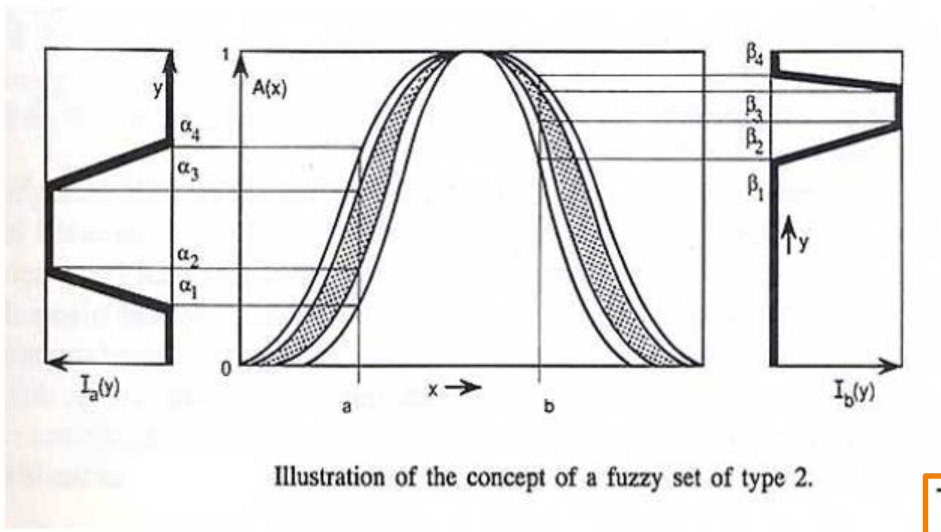
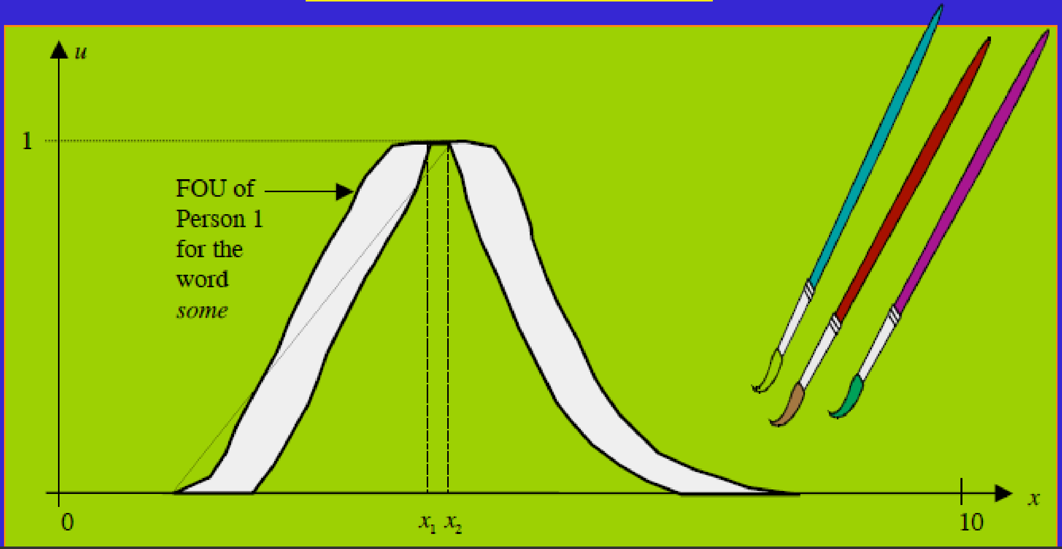


Illustration of the concept of a fuzzy set of type 2.

Types 3, 4, etc...

Examples Type-2 Fuzzy Set

- ***Intra-uncertainty*** **about a word can be modeled using a type-2 *person fuzzy set*** $\tilde{A}(p_j), j = 1, \ldots, n_{\tilde{A}}$

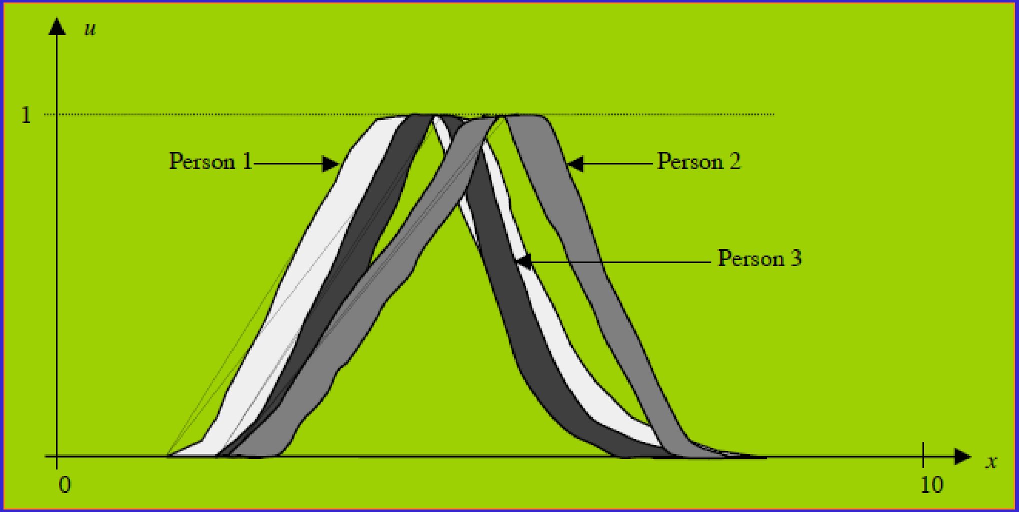


FOU: footprint of uncertainty

From J.M. Mendel IEEE-FUZZ

Examples Type-2 Fuzzy Set

- Collect person MFs from a group of people



From J.M. Mendel
IEEE-FUZZ

Linguistic Variables

- A *linguistic variable* is a fuzzy variable
  - e.g. the fact “*NBA players are tall*” implies linguistic variable “*NBA players*” takes the linguistic value “*tall*”
  - Use linguistic variables to form *fuzzy rules*:

IF	‘project duration’ is *long*
THEN	‘risk’ is *high*

IF	‘risk’ is *very high*
THEN	‘project funding’ is *very low*

Fuzzy Rules

- A *fuzzy rule* is a conditional statement in the familiar form:

  IF x is A

  THEN y is B

  – x and y are linguistic variables
  – A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

Fuzzy System

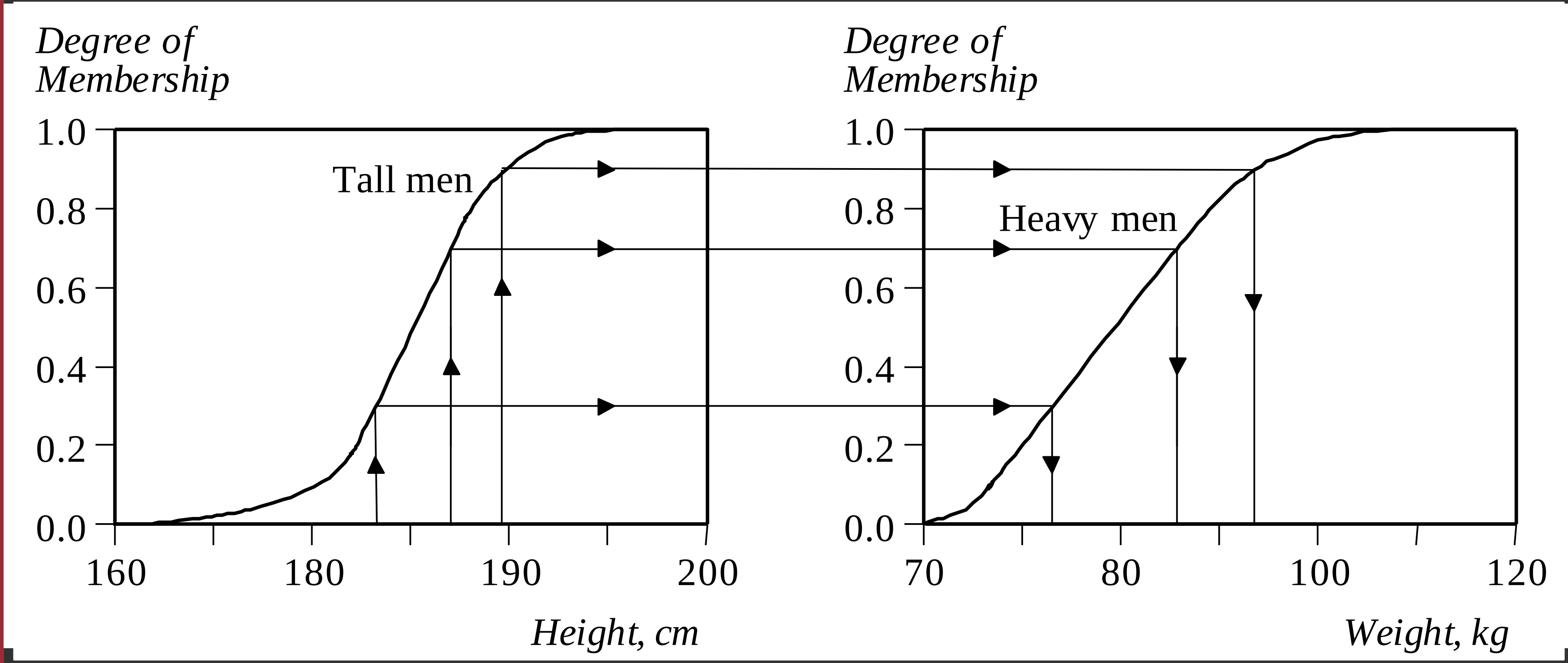
- A *fuzzy logic system* is a system that uses fuzzy rules, fuzzy logic, and fuzzy sets

- Many rules in a fuzzy logic system will fire to some extent
  - If the antecedent is true to some *degree of membership*, then the consequent is true to the *same degree* (Zadeh Extension Principle)

Fuzzy Rules

IF *height* is *tall*

THEN *weight* is *heavy*



Fuzzy Rules

- Other examples (multiple antecedents):
  - e.g. IF ‘*project duration*’ is *long*
    AND ‘*project staffing*’ is *large*
    AND ‘*project funding*’ is *inadequate*
    THEN *risk* is *high*
  - e.g. IF *service* is *excellent*
    OR *food* is *delicious*
    THEN *tip* is *generous*

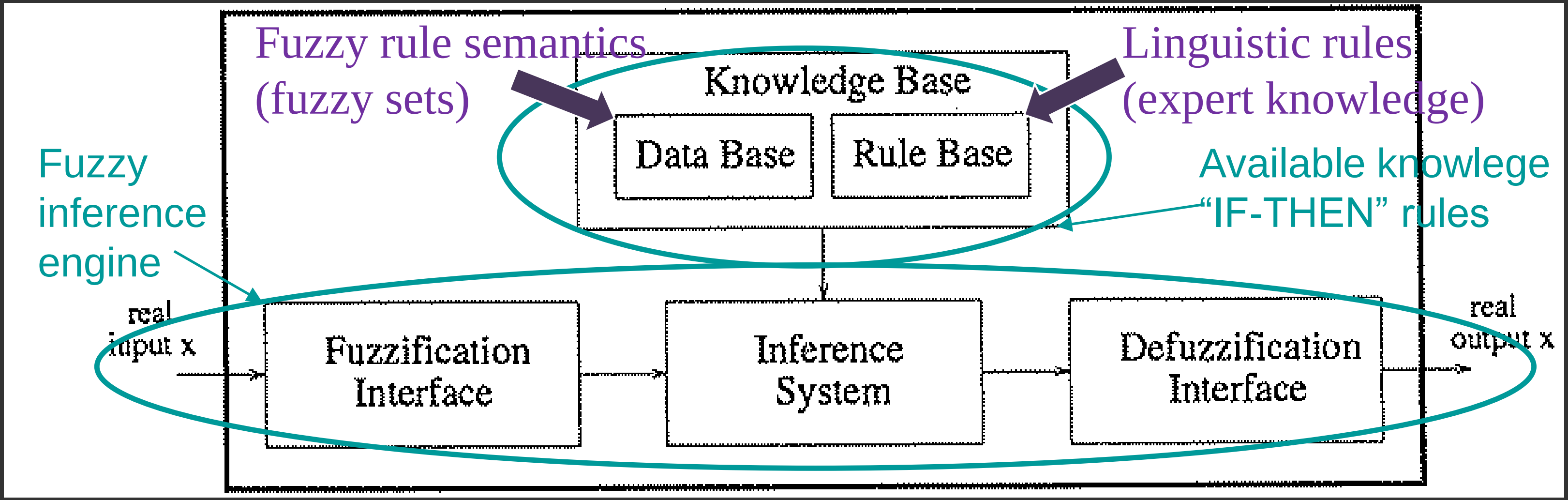
Fuzzy Rule-Based Systems

Principal FRBS for engineering problems (i.e., dealing with real-valued inputs and outputs):

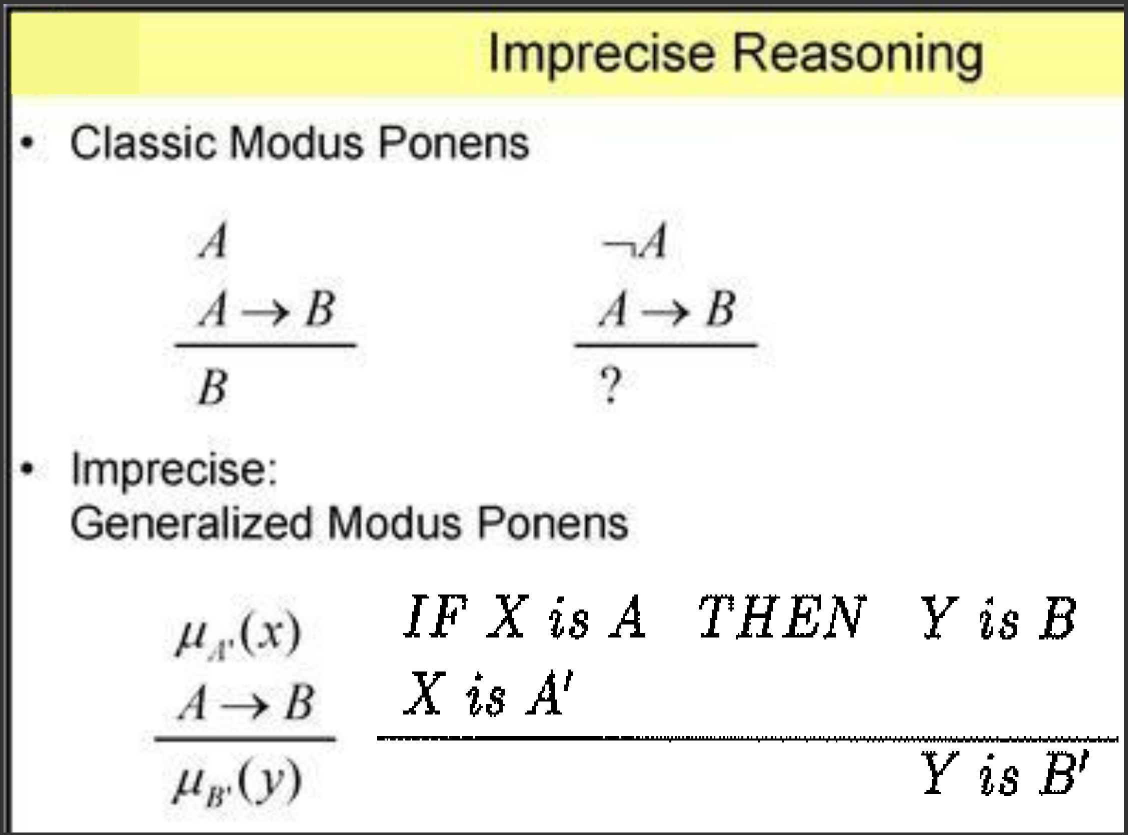
- *Mamdani*
- *Takagi-Sugeno-Kang*

Fuzzy Inference: Mamdani

- *Ebrahim Mamdani* (1974), the Mamdani method for fuzzy inference is:
  1. Fuzzify the input variables; 2.Evaluate the rules
  3. Aggregate the rule outputs; 4. Defuzzify



Fuzzy Inference: Mamdani



Imprecise Reasoning

- Classic Modus Ponens

$$\frac{\begin{array}{l} A \\ A \to B \end{array}}{B} \qquad \frac{\begin{array}{l} \neg A \\ A \to B \end{array}}{?}$$

- Imprecise:
  Generalized Modus Ponens

$$\frac{\begin{array}{l} \mu_{A'}(x) \\ A \to B \end{array}}{\mu_{B'}(y)} \qquad \frac{\begin{array}{l} IF\ X\ is\ A \quad THEN \quad Y\ is\ B \\ X\ is\ A' \end{array}}{Y\ is\ B'}$$

Fuzzy Inference: Mamdani

The membership function of the fuzzy set B' is obtained by applying the *compositional rule of inference:*

$$B' = A' \circ R$$

When applied to the ith rule:

$$R_i : IF\ X_{i1}\ is\ A_{i1}\ and\ \ldots\ and\ X_{in}\ is\ A_{in}\ THEN\ Y\ is\ B_i,$$

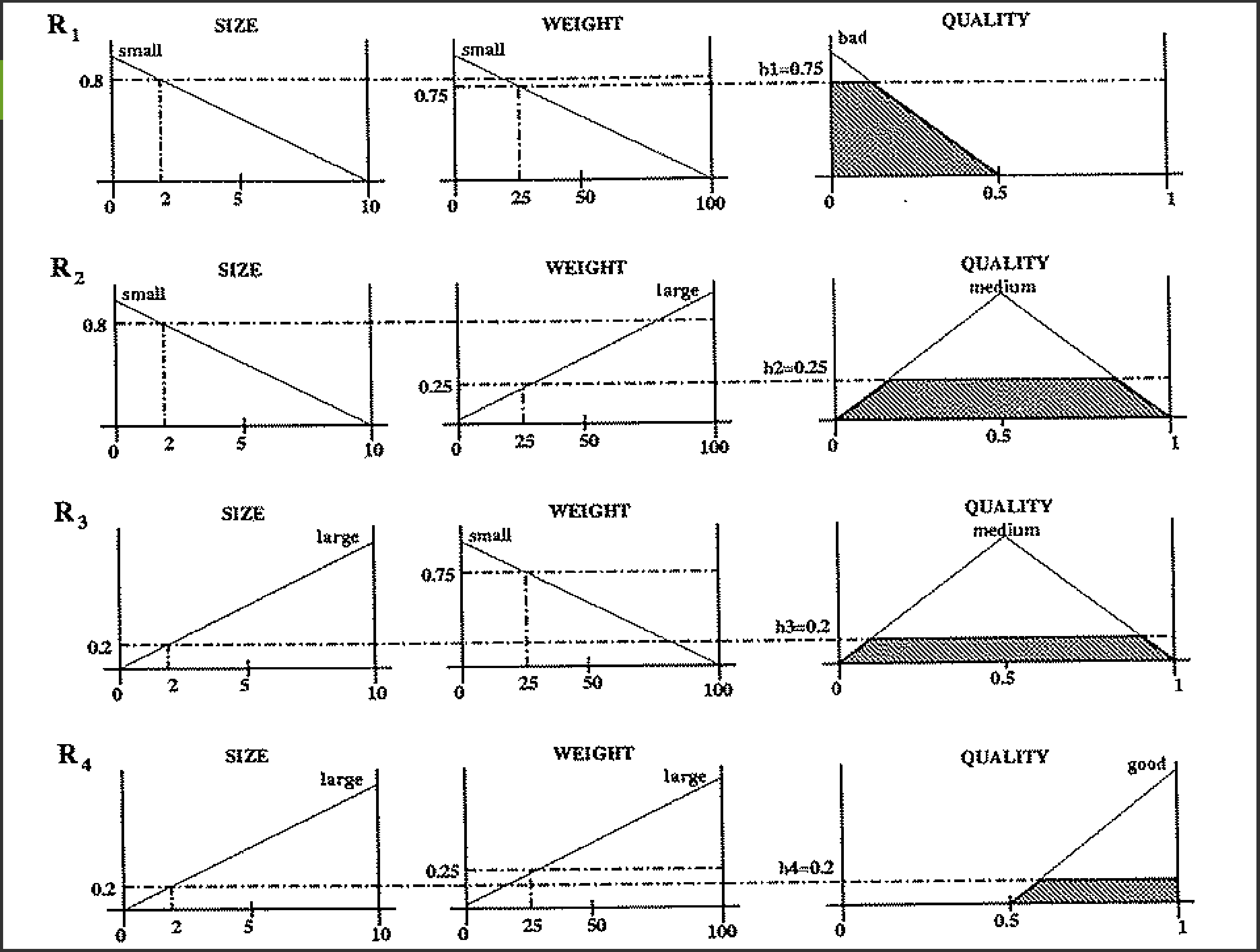
Simplest expression CRI: $\mu_{B'_i}(y) = I(\mu_{A_i}(x_0), \mu_{B_i}(y))$

where $\mu_{A_i}(x_0) = T(\mu_{A_{i1}}(x_1), \ldots, \mu_{A_{in}}(x_n)),\ x_0 = (x_1, \ldots, x_n)$
is the current system input, T is a fuzzy conjunctive operator (t-norm) and I a fuzzy implication operator (common choice *minimum t-norm*)

Fuzzy Inference: Mamdani

Current system input = (2, 25)

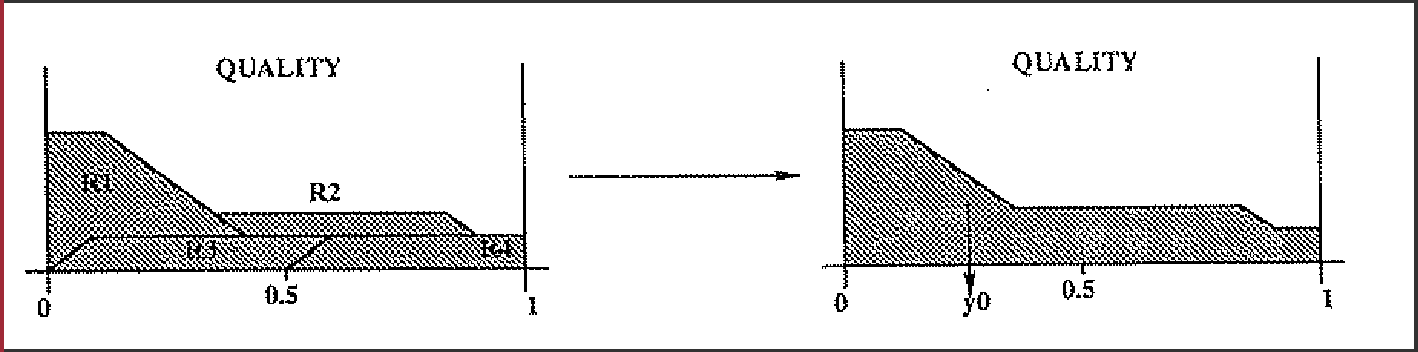
Degree of firing or firing strength between the antecedent part of the i-th rule and the current inputs to the system.
T-norm = conjunctive operators (minimum, algebraic product)



Mamdani – Defuzzification

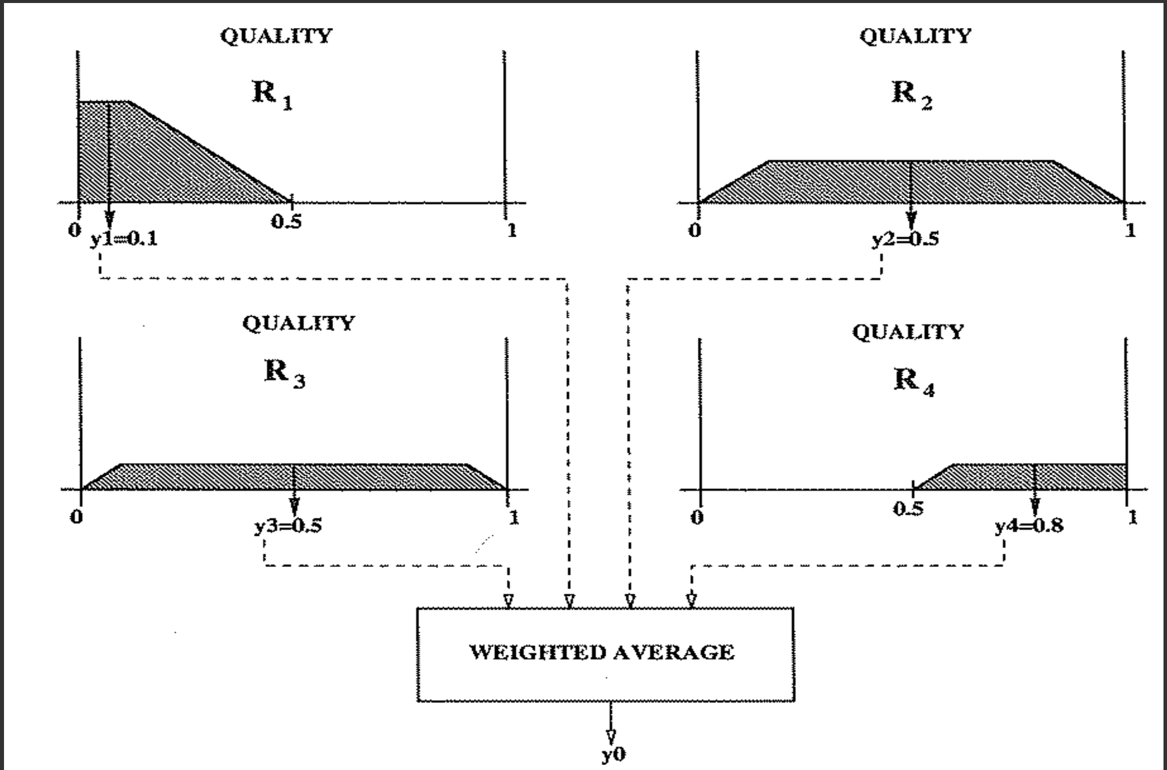
First Aggregation then Inference - FATI

(center of maximums, center of gravity, etc.)



First Inference then Aggregation - FITA

(maximum value)



Mamdani – Example

4. Defuzzification

– Calculate the *center of gravity* (*cog*):

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

- An easier approximation (with some imprecision...):

$$y = \frac{\sum_{j=1}^{r} \mu_j \cdot s_j}{\sum_{j=1}^{r} \mu_j}$$

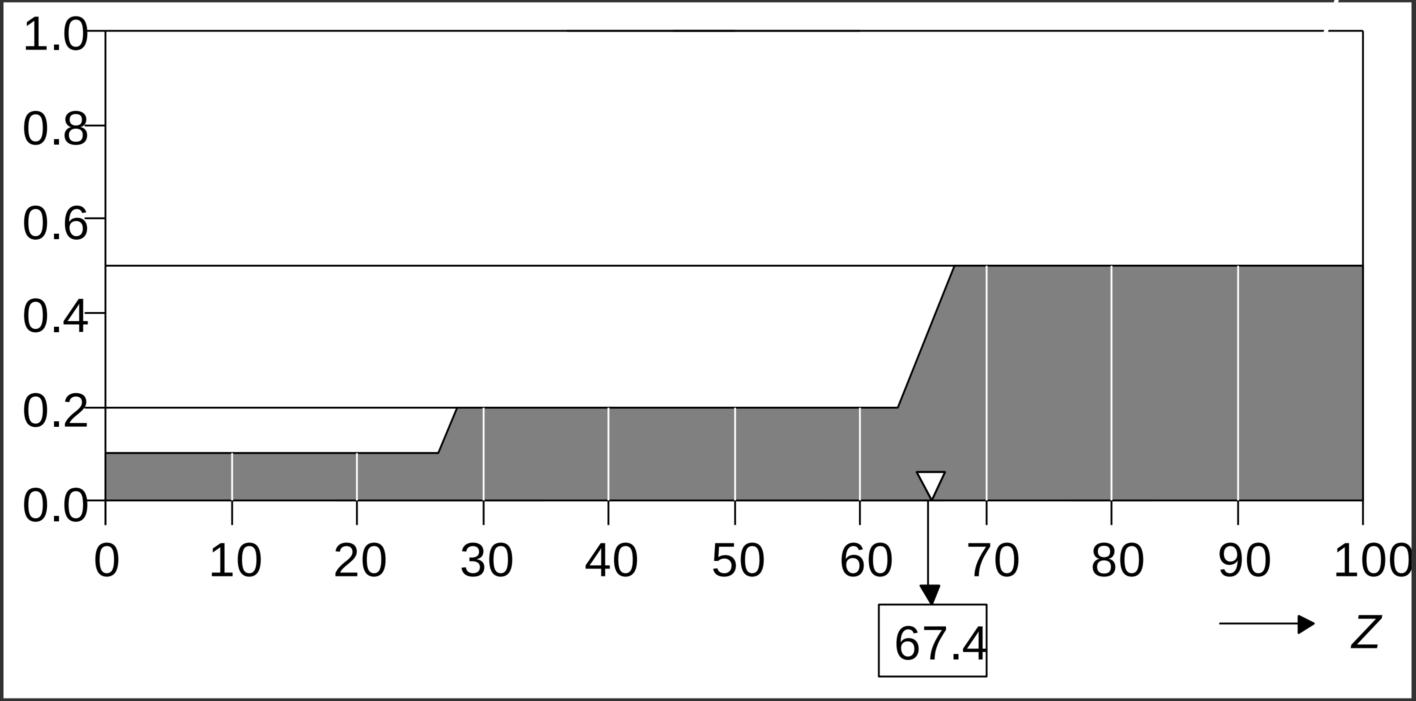
S_j is the center of gravity of set *j*

$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

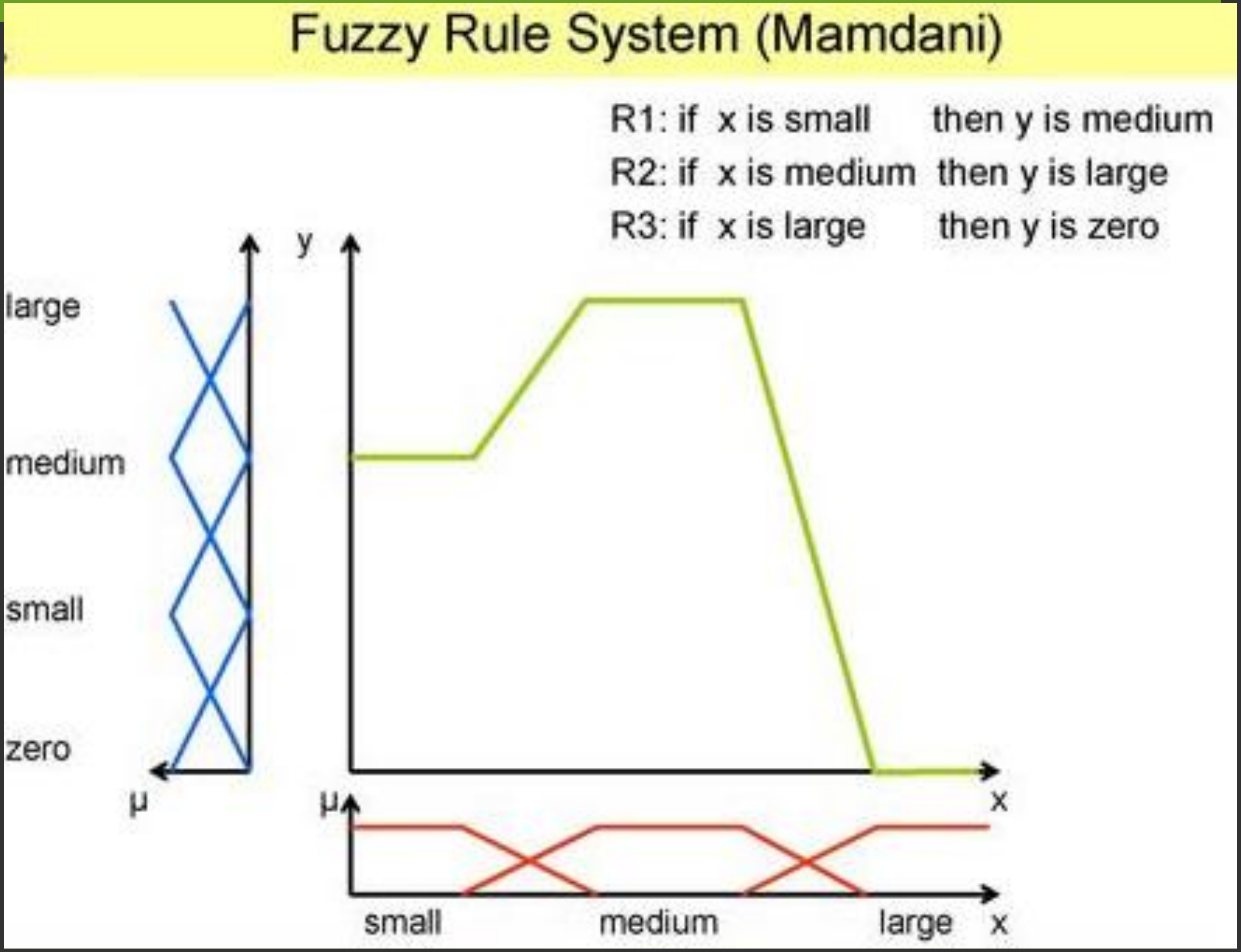
Mamdani – Example

4. Defuzzification

– Use a reasonable sampling of points



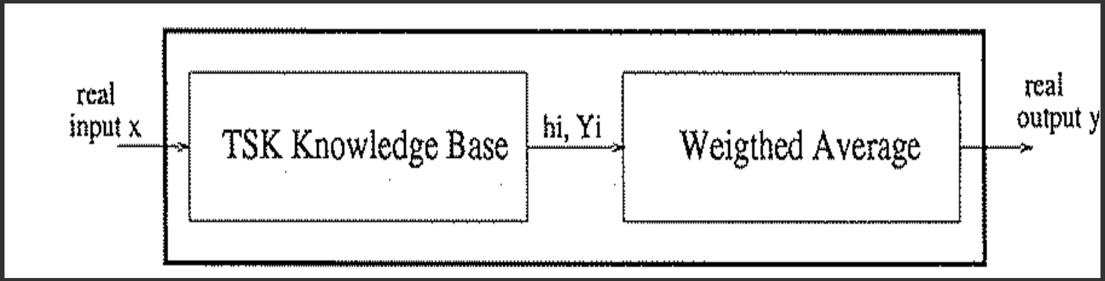
Mamdani: Visual example


Fuzzy Rule System (Mamdani)

R1: if x is small then y is medium
R2: if x is medium then y is large
R3: if x is large then y is zero

Fuzzy Inference: TSK

- Takagi-Sugeno-Kang (1985), the TSK method for fuzzy inference is:



$$\begin{aligned}&IF\ X_1\ is\ A_1\ and\ \ldots\ and\ X_n\ is\ A_n\\&THEN\ Y = p_1 \cdot X_1 + \cdots + p_n \cdot X_n + p_0,\end{aligned}$$

Polynomial function

Output TSK (m rules): $\dfrac{\sum_{i=1}^{m} h_i \cdot Y_i}{\sum_{i=1}^{m} h_i}$

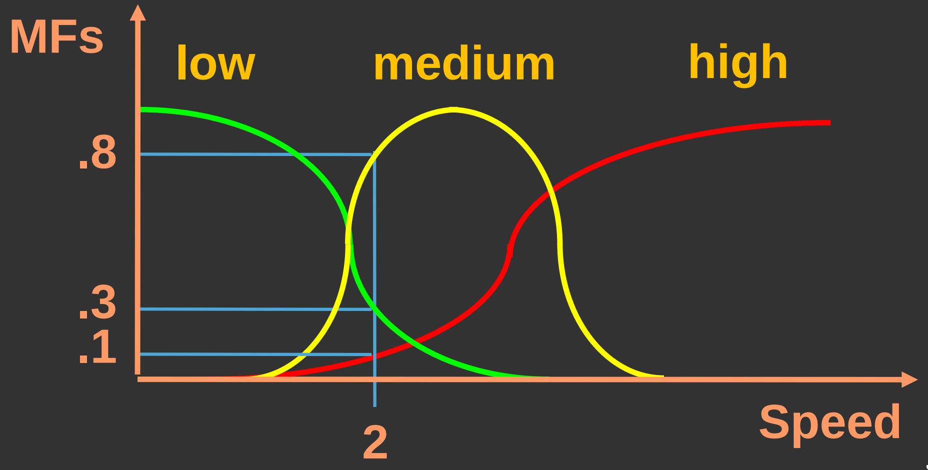
$$h_i = T(\mu_{A_{i1}}(x_1), \ldots, \mu_{A_{in}}(x_n))$$

Matching degree between the antecedent part of the i-th rule and the current inputs to the system. T-norm = conjunctive operators (minimum, algebraic product)

Zero-Order TSK FRBS

Takagi-Sugeno-Kang

If speed is low then resistance = 2
If speed is medium then resistance = 4
If speed is high then resistance = 8



Rule 1: h1 = .3; Y1 = 2
Rule 2: h2 = .8; Y2 = 4
Rule 3: h3 = .1; Y3 = 8

Resistance = Σ(hi*Yi) / Σhi
= (0.3*2+0.8*4+0.1*8)/(0.3+0.8+0.1)
= 3.83

First-Order TSK FRBS

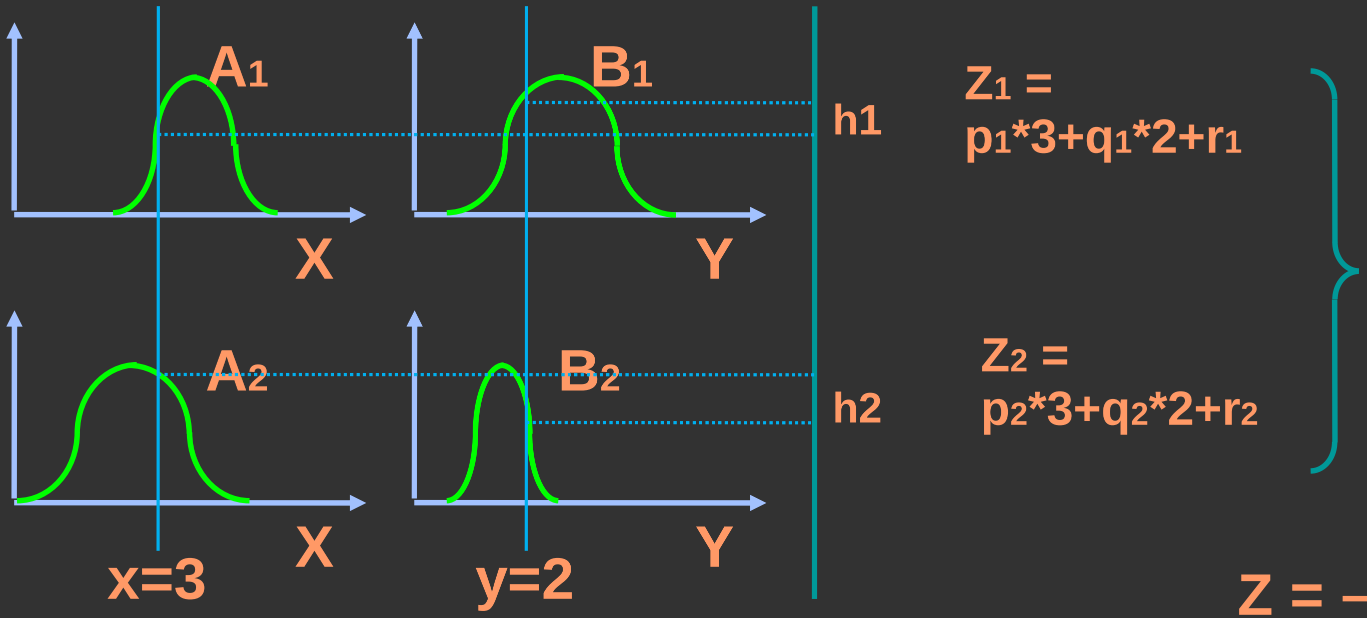
Takagi-Sugeno-Kang

Rule base

If X is A1 and Y is B1 then Z = p1*x + q1*y + r1

If X is A2 and Y is B2 then Z = p2*x + q2*y + r2

Fuzzy reasoning



$Z_1 = p_1*3+q_1*2+r_1$

$Z_2 = p_2*3+q_2*2+r_2$

$$Z = \frac{h_1*Z_1+h_2*Z_2}{h_1+h_2}$$

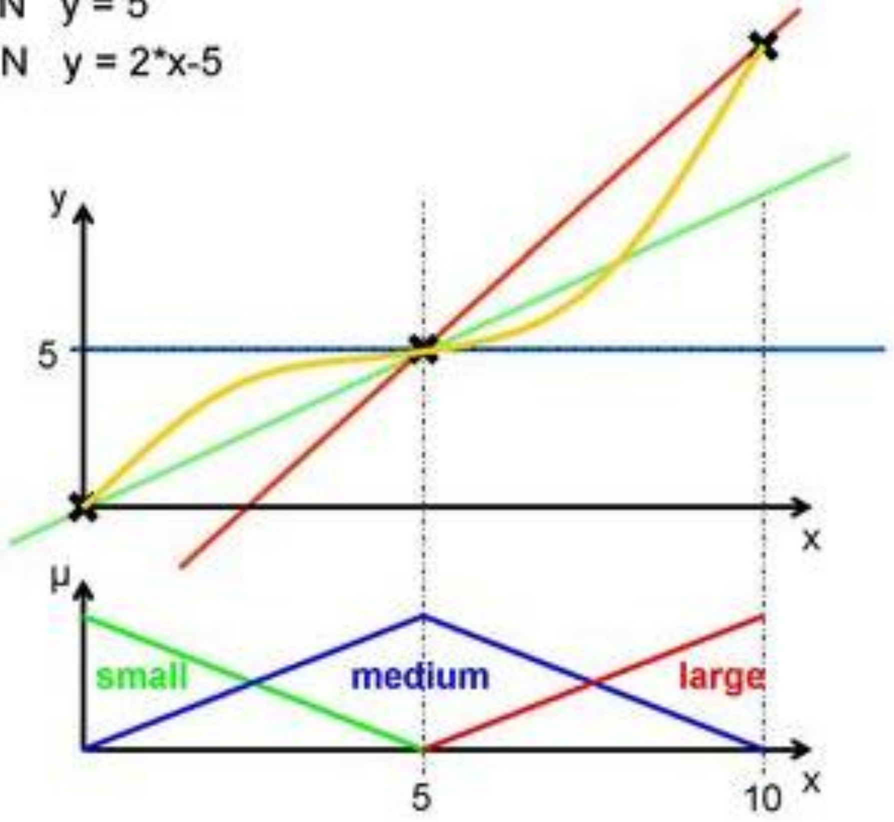
TSK: Visual example

R1: IF x IS small THEN y = x

R2: IF x IS medium THEN y = 5

R3: IF x IS large THEN y = 2*x-5

$$y = \frac{\sum_{i=1}^{r} \mu_{R_i} \cdot y_i(\vec{x})}{\sum_{i=1}^{r} \mu_{R_i}}$$



Demo TSK

Fuzzy Juggling ball system

Matlab: juggler

Mamdani vs. TSK

Advantages of the TSK Method

- It is computationally efficient.
- It works well with linear techniques (e.g., PID control).
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

Advantages of the Mamdani Method

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.

Some Misconceptions on Fuzzy Logic

Fuzzy logic is the same as imprecise logic

Fuzzy logic is not any less precise than any other form of logic: it is an organized and mathematical method of handling *inherently* imprecise concepts.

Some Misconceptions on Fuzzy Logic

Fuzzy logic is a new way of expressing probability

Fuzzy logic and probability refer to different kinds of uncertainty. Fuzzy logic is specifically designed to deal with **imprecision** of facts (fuzzy logic statements), while probability deals with **chances** of that happening *(but still considering the result to be precise)*

Example:

- ➢ P(A) = 0.5 means that A may be true or may be false
- ➢ A logical value of 0.5 means both true and false at the same time

➡ However, still is a point of controversy

Applications of Fuzzy Logic

- Why use *fuzzy systems*?
  - Apply fuzziness (*and therefore accuracy*) to linguistically defined terms and rules
  - Lack of crisp or concrete mathematical models exist

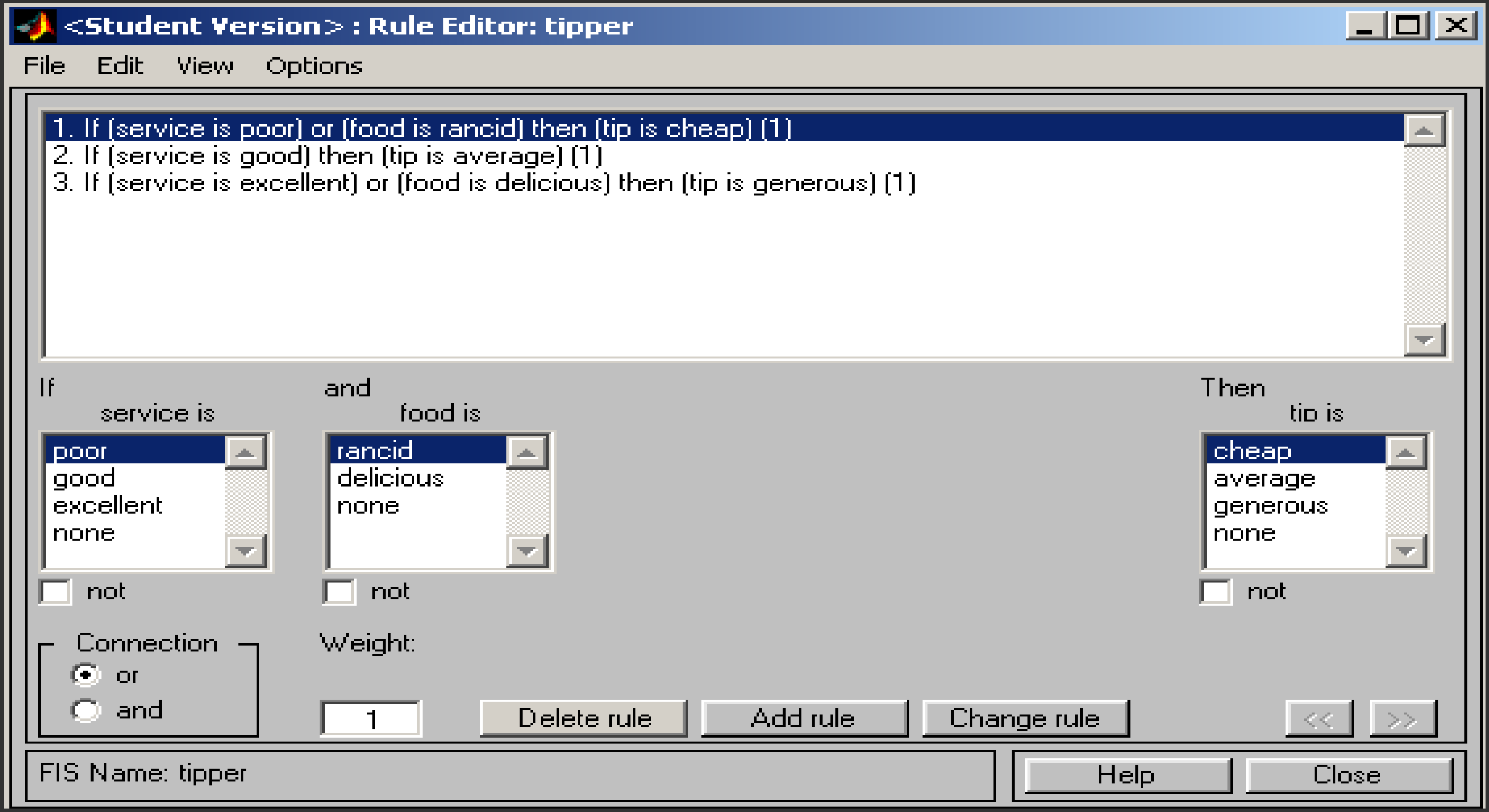
- When do you avoid *fuzzy systems*?
  - Traditional approaches produce acceptable results
  - Crisp or concrete mathematical models exist and are easily implemented

Applications of Fuzzy Logic

- Real-world applications include:
  - Control of robots, engines, automobiles, elevators, etc.
  - Cruise-control in automobiles
  - Temperature control
  - Reduce vibrations in airplanes, cameras, etc.
  - Handwriting recognition
  - Predictive and diagnostic systems (e.g. cancer)
  - Etc.

Example: Dinner for two (Mamdani)

Rule Editor



<Student Version> : Rule Editor: tipper

File Edit View Options

1. If (service is poor) or (food is rancid) then (tip is cheap) (1)
2. If (service is good) then (tip is average) (1)
3. If (service is excellent) or (food is delicious) then (tip is generous) (1)

If service is: [✓ poor] good excellent none — not

and food is: [✓ rancid] delicious none — not

Then tip is: [✓ cheap] average generous none — not

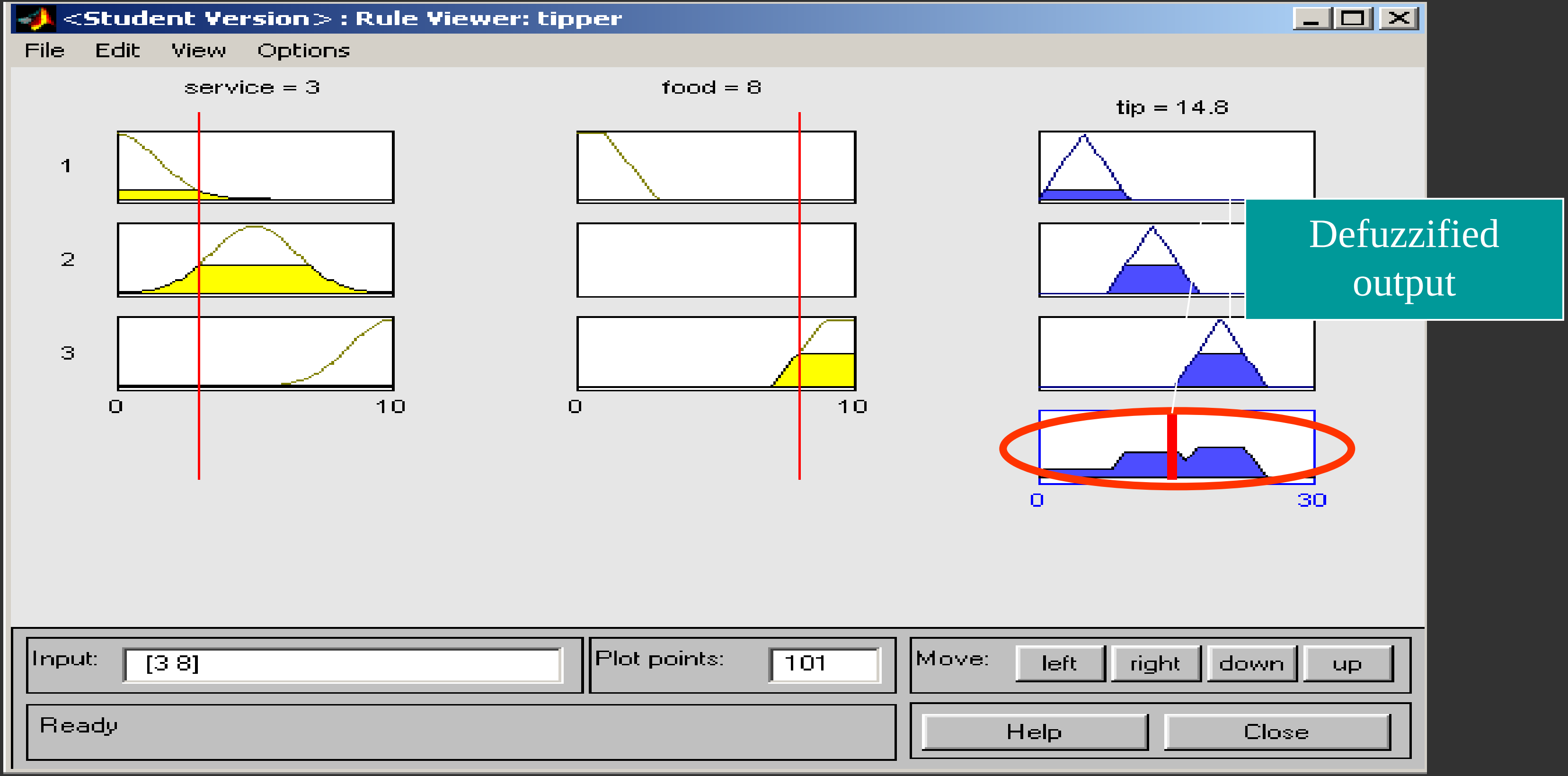
Connection: [✓ or] and

Weight: 1

FIS Name: tipper

Example: Dinner for two (Mamdani)

Rule Viewer

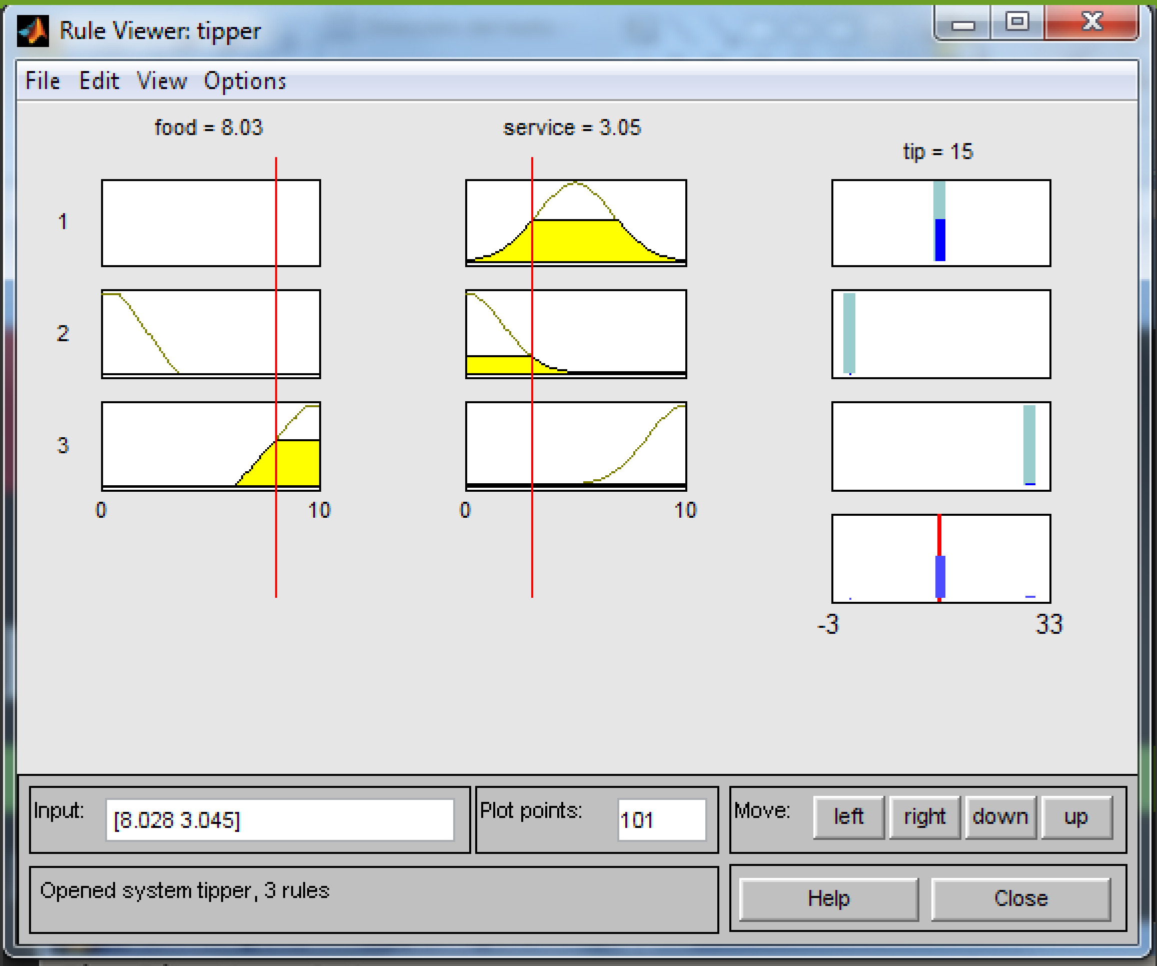


Input: [3 8]

Plot points: 101

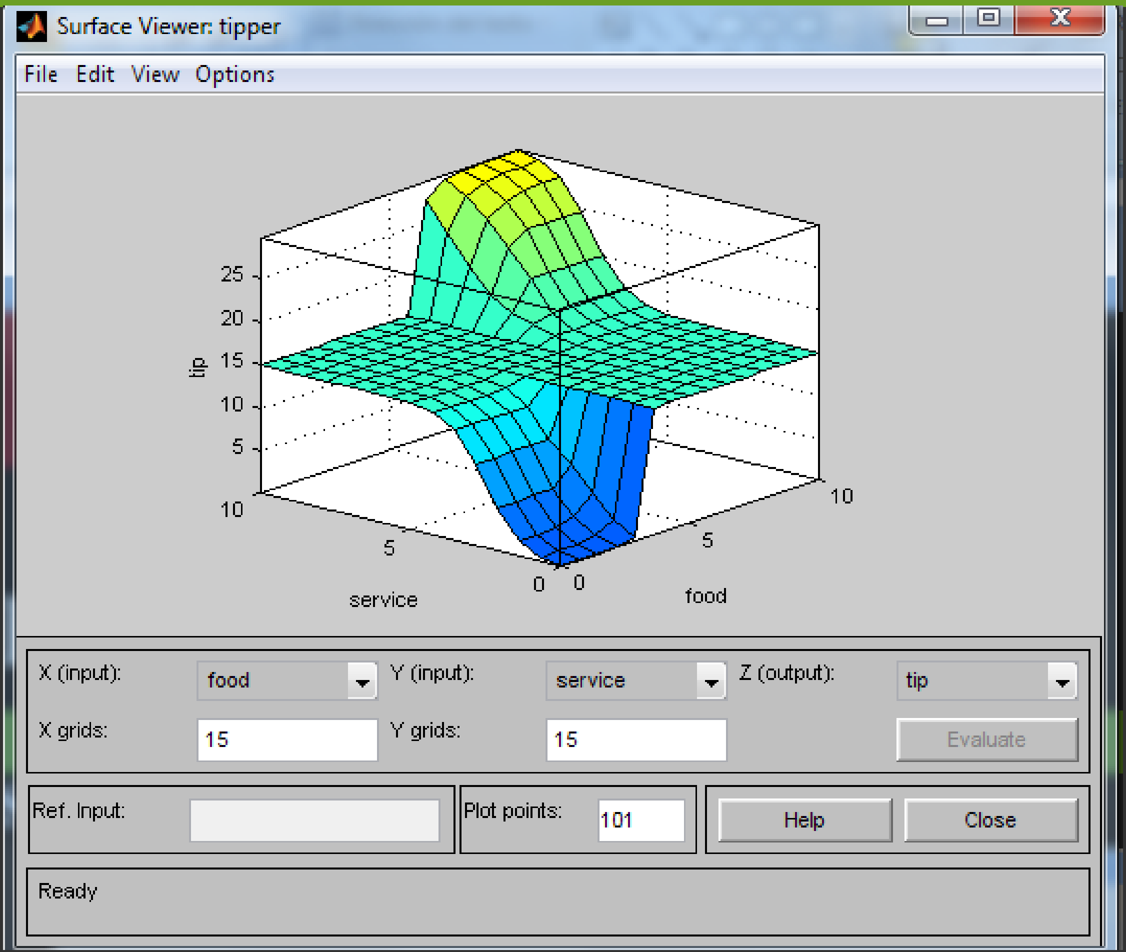
Ready

Example: Dinner for two (Sugeno)



Opened system tipper, 3 rules

Example: Dinner for two (Sugeno)

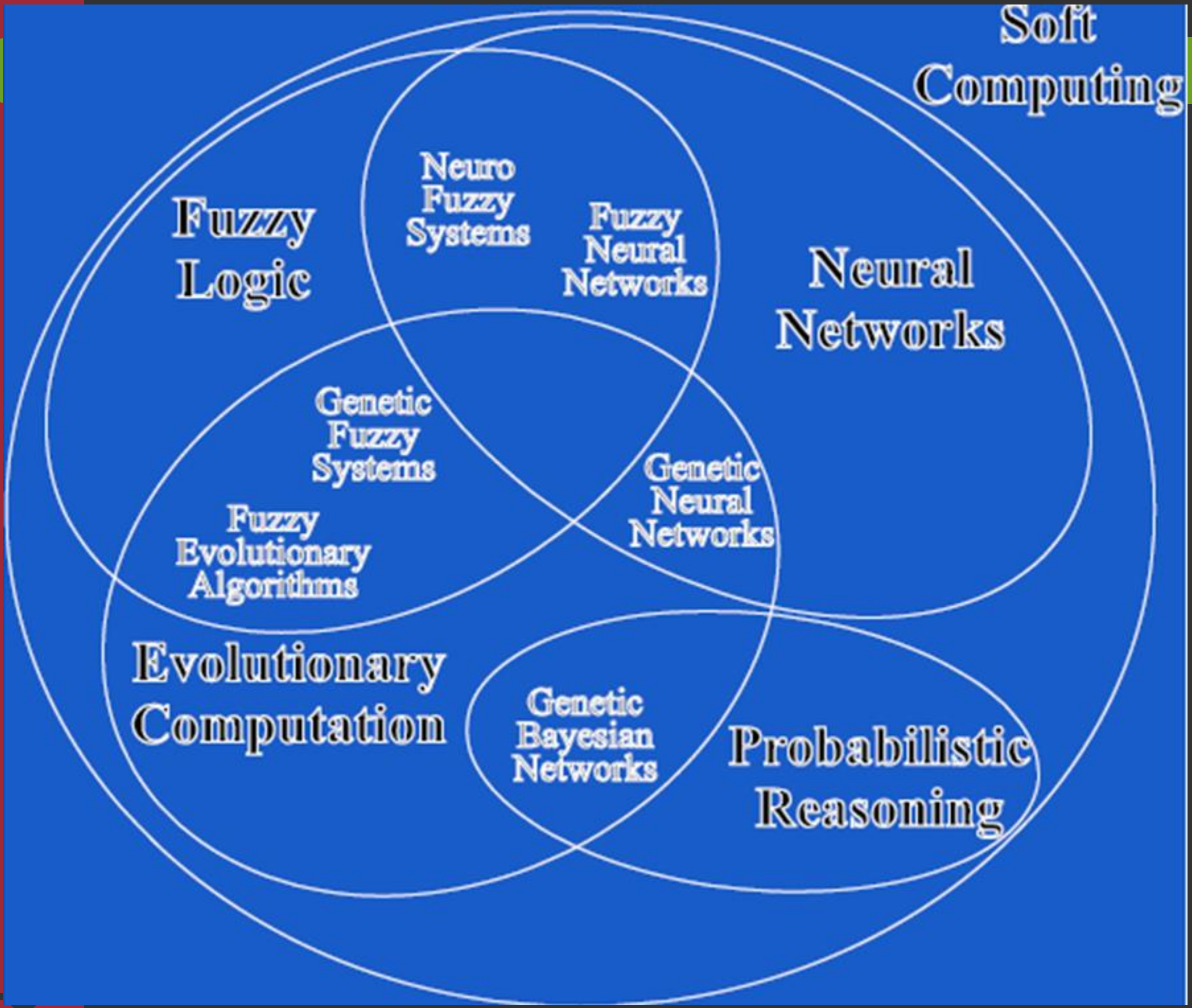


X (input): food
Y (input): service
Z (output): tip
X grids: 15
Y grids: 15

Ref. Input:
Plot points: 101

Ready

Hybridization

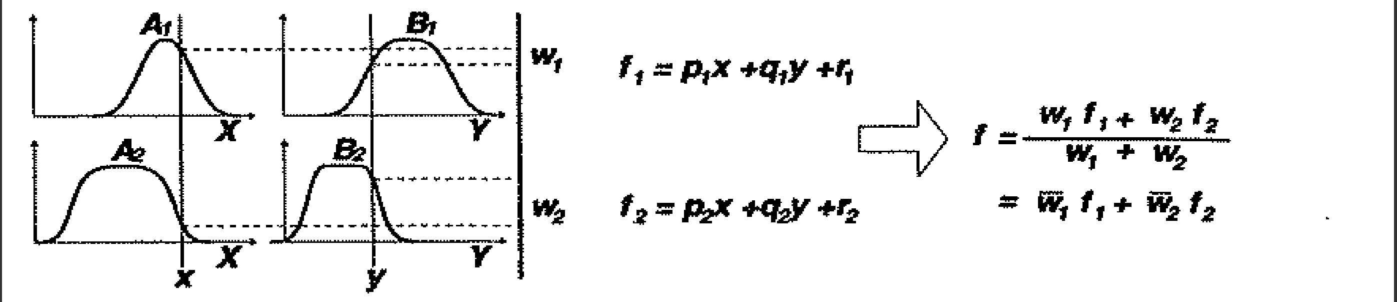


- FSs are neither capable of learning, adaptation or parallel computation, whereas these characteristics are clearly attributed to NNs.
- NNs lack flexibility, human interaction or knowledge representation, which lies at the core of FL.

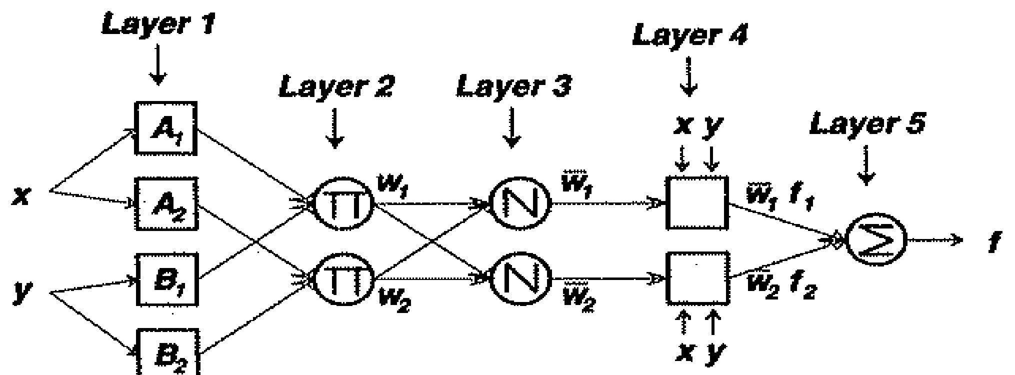
ANFIS: Adaptive Neuro-Fuzzy Inference System

Sugeno

Rule 1: If x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$,
Rule 2: If x is A_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$.

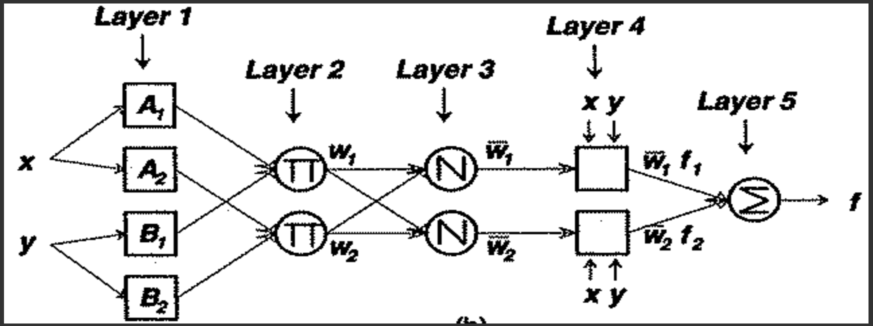

A_1
B_1
A_2
B_2
X
Y
x
y
w_1
w_2
$f_1 = p_1x + q_1y + r_1$
$f_2 = p_2x + q_2y + r_2$
$f = \frac{w_1 f_1 + w_2 f_2}{w_1 + w_2}$
$= \bar{w}_1 f_1 + \bar{w}_2 f_2$

(a)



(b)

ANFIS: Architecture



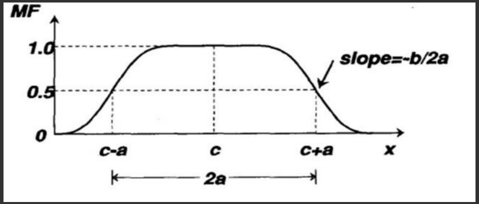
Layer 1: Every node *i* is an adaptive node with a node function:

$$O_{1,i} = \mu_{A_i}(x), \quad \text{for } i = 1, 2, \text{ or}$$
$$O_{1,i} = \mu_{B_{i-2}}(y), \quad \text{for } i = 3, 4,$$

where *x* (or *y*) is the input to node *i* and A_i (or B_{i-2}) is a linguistic label ("small", "large",…) associated with this node

The membership function for *A* or B (=A_1, A_2, B_1 or B_2) can be any appropriate parameterized membership function such as the generalized bell function:

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b}},$$

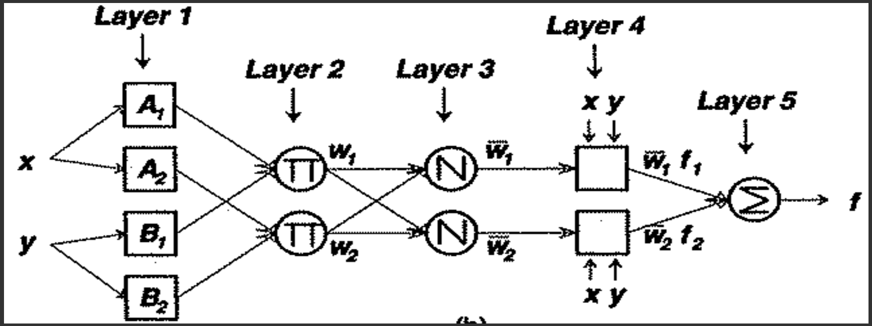


where $\{a_i, b_i, c_i\}$ is the parameter set (**premise parameters**)

Layer 2: Every node in this layer is a fixed node labeled Π, whose output is the result of applying any other T-norm operator that performs fuzzy AND

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \; i = 1, 2.$$

ANFIS: Architecture



Layer 3: Every node in this layer is a fixed node labeled N. The *i*th node calculates the ratio of the *i*th rule's firing strength to the sum of all rules' firing strengths

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2.$$

Outputs of this layer are called **normalized firing strengths**

Layer 4: Every node in this layer is an adaptive node with a node function

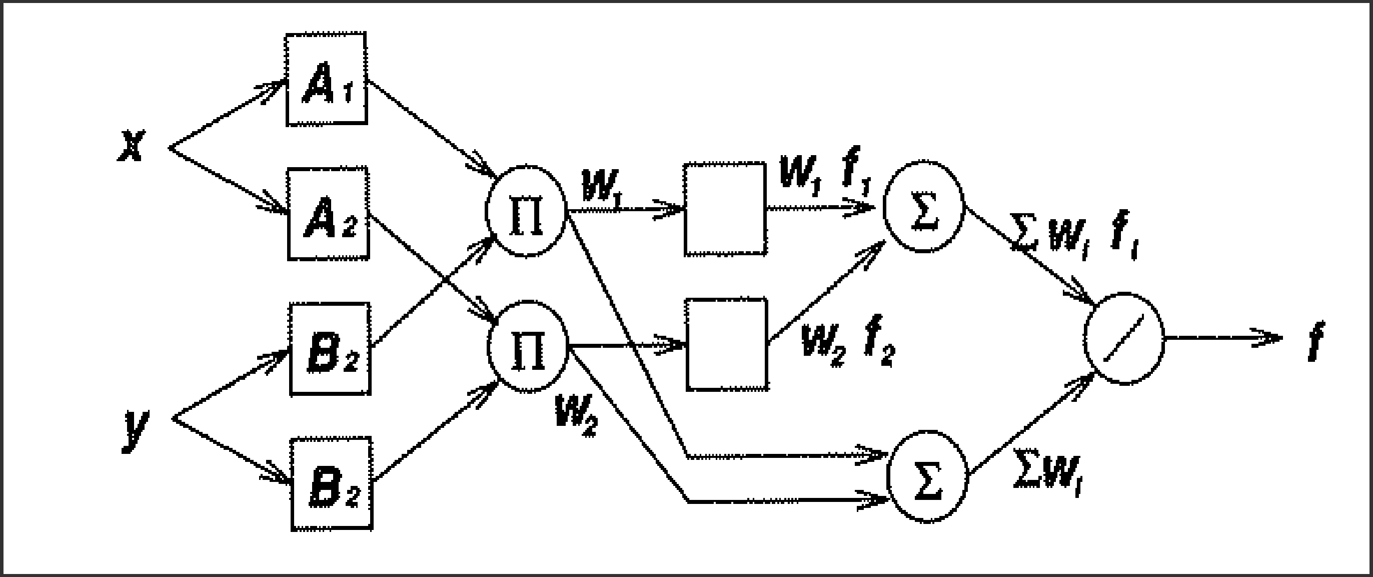
$$O_{4,i} = \overline{w}_i f_i = \overline{w}_i (p_i x + q_i y + r_i),$$

where $\overline{w}_i$ is a normalized firing strength from layer 3 and $\{p_i, q_i, r_i\}$ is the parameter set of this node (**consequent parameters**)

Layer 5: The single node in this layer is a fixed node labeled Σ, which computes the overall output as the summation of all incoming signals:

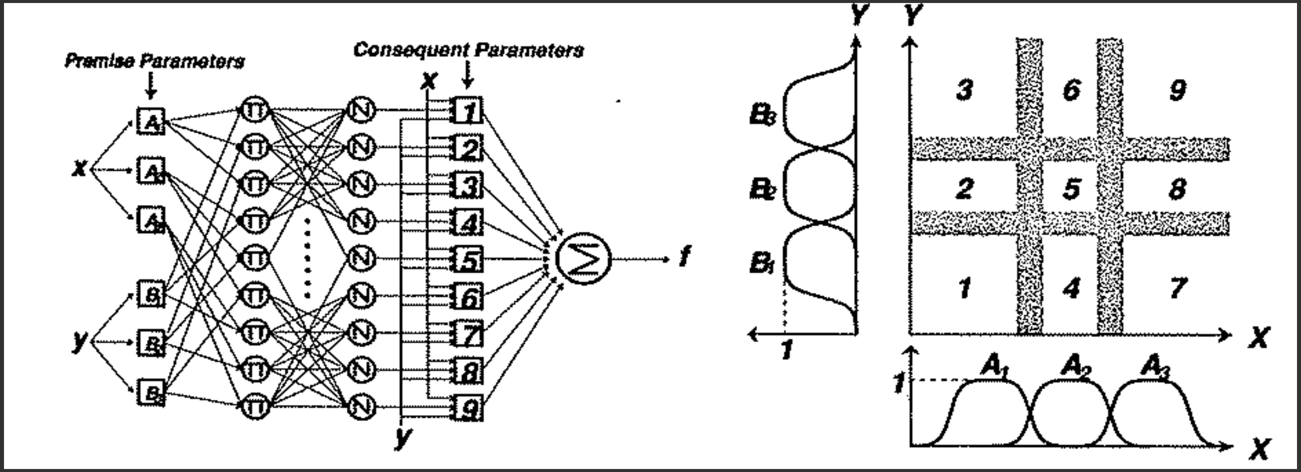
$$O_{5,1} = \sum_i \overline{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$

ANFIS: Another Architecture

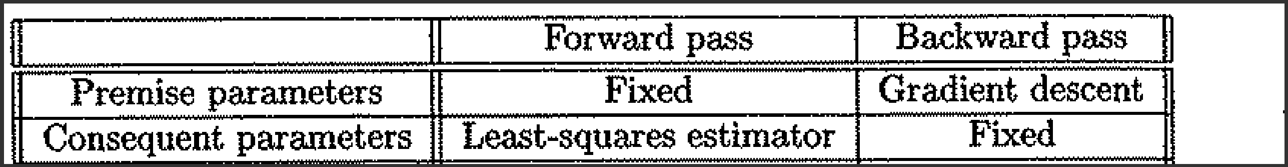


Weight normalization is performed at the very last layer

ANFIS: Hybrid Learning



The premise part of a rule defines a fuzzy region, while the consequent part specifies the output within the region



	Forward pass	Backward pass
Premise parameters	Fixed	Gradient descent
Consequent parameters	Least-squares estimator	Fixed

S1= Set of premise (nonlinear) parameters
S2= Set of consequent (linear) parameters

Example 1: Two-Input Sinc Function

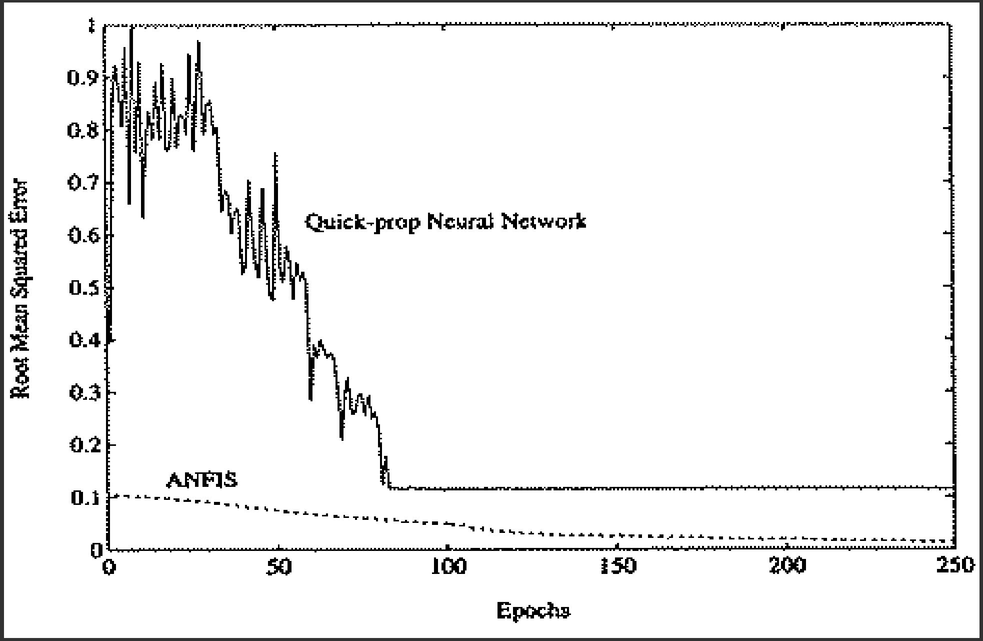
$$z = \text{sinc}(x, y) = \frac{\sin(x)\sin(y)}{xy}.$$

ANFIS

- 121 training data pairs
- 16 rules with 4 membership functions assigned to each input
- 72 total fitting parameters (24 premise and 48 consequent)

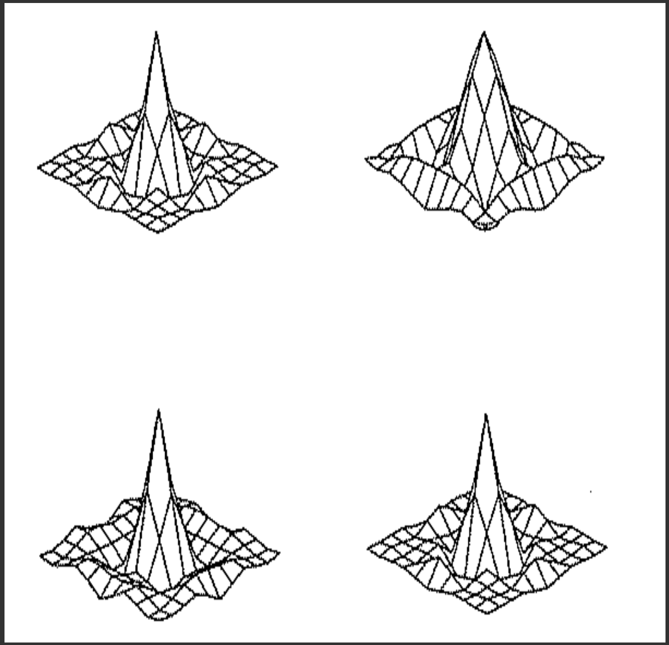
2-18-1 Backpropagation MLP

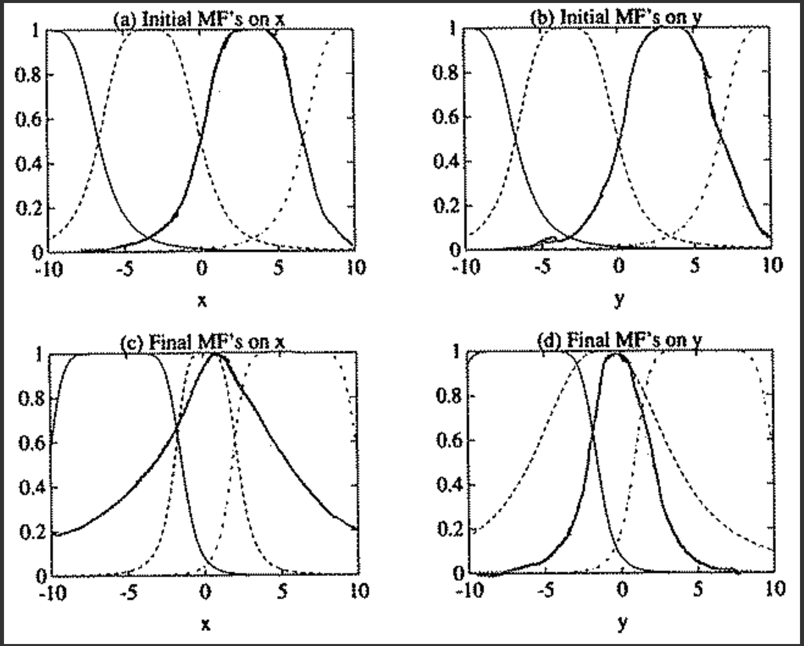
- 121 training data pairs
- Trained with quick propagation
- 73 total fitting parameters (connection weights and thresholds)



ANFIS approximates the highly nonlinear surface more effectively than the MLP

Example 1: Two-Input Sinc Function



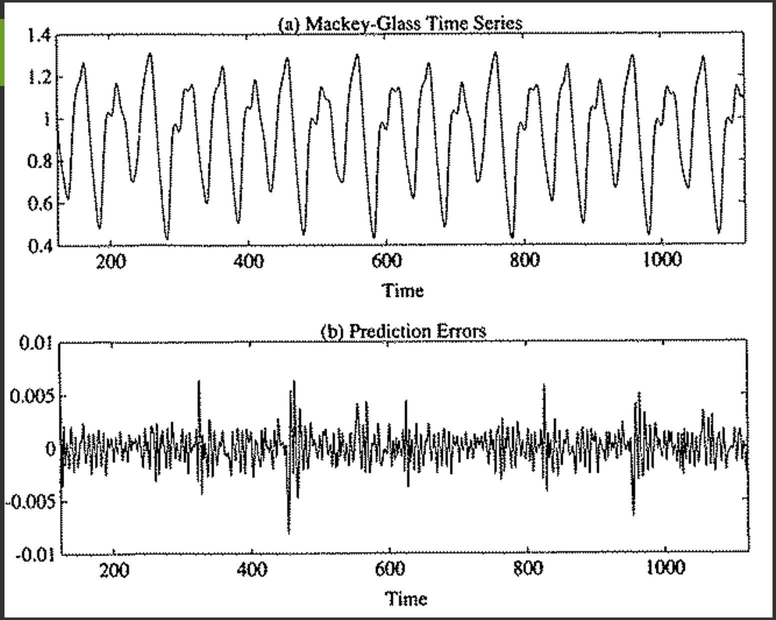


- Training data and reconstructed surfaces at different epochs: 0.5, 99.5 and 249.5

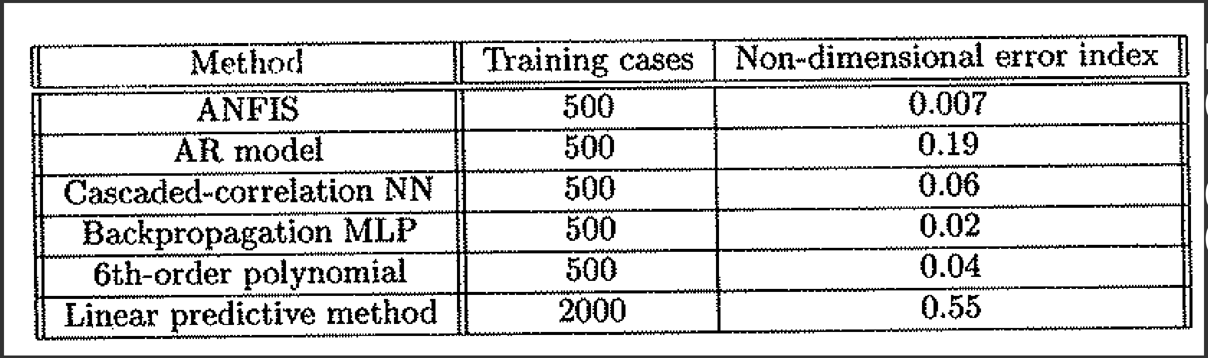
- Initial and final membership functions

Example 2: Predicting Mackey-Glass Chaotic Time Series

- Prediction from t=124 to 1123
- Desired and predicted values for both training and test data are essentially the same



NDEI: root mean square error divided by the standard deviation of the target series; Prediction: 500 values test data set



Method	Training cases	Non-dimensional error index
ANFIS	500	0.007
AR model	500	0.19
Cascaded-correlation NN	500	0.06
Backpropagation MLP	500	0.02
6th-order polynomial	500	0.04
Linear predictive method	2000	0.55

PARAM
(104)

(693)
(540)

Advantages of ANFIS

The remarkable generalization capability of ANFIS is due to:

1. ANFIS can achieve a highly nonlinear mapping. Therefore, it is superior to common linear methods in reproducing nonlinear time series.

2. The initial parameters are intuitively reasonable and all the input space is covered properly (fast convergence).

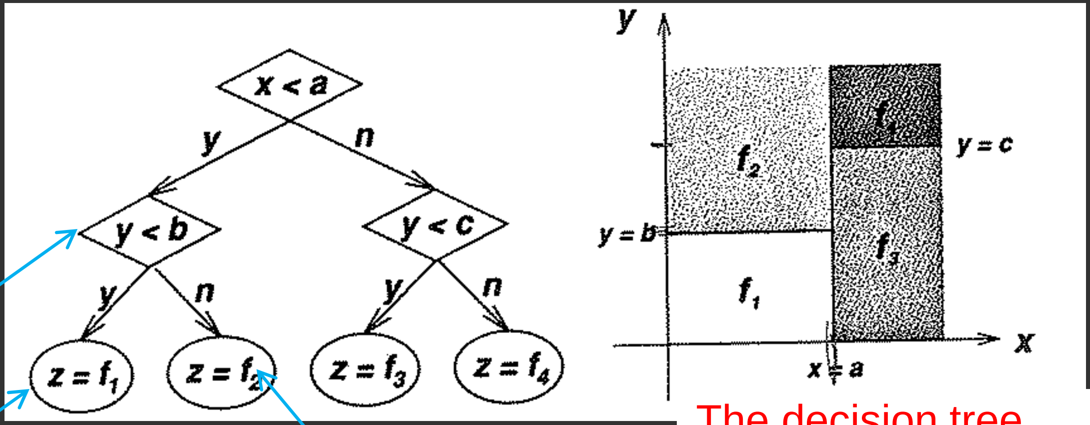
3. ANFIS consists of fuzzy rules that are local mappings instead of global ones, then facilitate the minimal disturbance principle (the adaptation should not only reduce the output error for the current training pattern but also minimize disturbance to response already learned).

CART: Classification and Regression Trees

- ANFIS only deals with **parameter identification**, we still need methods for **structure identification** to determine an initial ANFIS architecture.
- CART generates a tree partitioning of the input space, which relieves the "curse of dimensionality" problem (num. of rules increasing exponentially with num. of inputs)

CART: Classification and Regression Trees

Binary regression tree with two inputs and one output



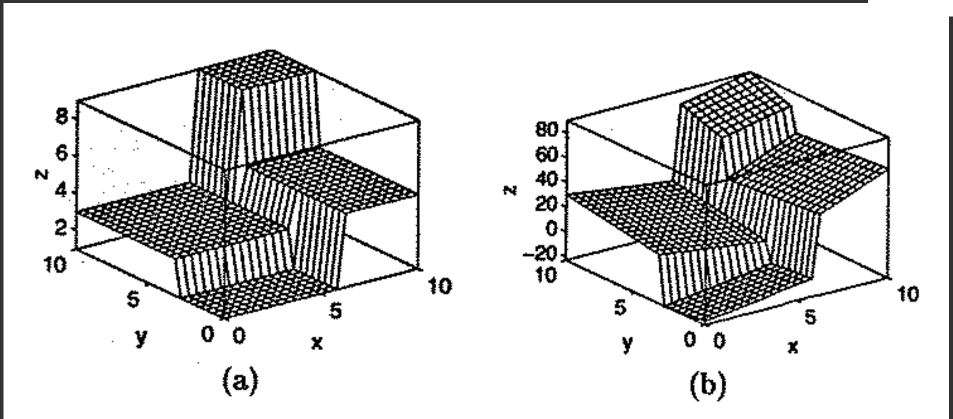
Internal node

Terminal node

Constants or equations

The decision tree partitions the input space into 4 regions

Constants: a=6, b=3, c=7
f1=1, f2=3, f3=5, f4=9



Linear functions:
f1= 2x-y-20,
f2= -2x+2y+10,
f3=6x-y+5,
f4=3x+4y+20

CART: Classification and Regression Trees

CART first **grows** the tree extensively based on a training data set, and then **prunes** the tree back based on a minimum cost-complexity principle

Tree Growing

- CART grows a decision tree by determining a succession of splits (decision boundaries) that partition the training data into disjoint subsets
- Starting from the root node (contains all the training data), an exhaustive search is performed to find the split that best reduces an error measure (cost function)
- The data set is partitioned into two subsets and the same splitting method is applied to both child nodes
- This recursive procedure terminates either when the error measure associated with a node falls below a certain tolerance level, or when the error reduction does not exceed a certain threshold value

CART: Classification and Regression Trees

For a regression tree, the error measure that quantifies the performance of a node *t* in separating data into disjoint subsets is usually taken as the square error (or residual) of a local model employed to fit the data set of the node:

$$E(t) = \min_{\boldsymbol{\theta}} \sum_{i=1}^{N(t)} (y_i - d_t(\mathbf{x}_i, \boldsymbol{\theta}))^2,$$

where {x_i,y_i} is a data point, $d_t(\mathbf{x}_i, \boldsymbol{\theta})$ is a local model for node *t* (with modifiable parameter θ) and E(*t*) is the mean-square error of fitting the local model to the data set in the node

Constant (d(x, θ) = θ): the minimizing of *E*(*t*) is the average value of the desired output

$$\theta^* = \frac{1}{N(t)} \sum_{i=1}^{N(t)} y_i.$$

CART: Classification and Regression Trees

Linear model ($d(x, \theta)$ = linear model with linear parameters): least-squares methods to identify the minimizing θ^*

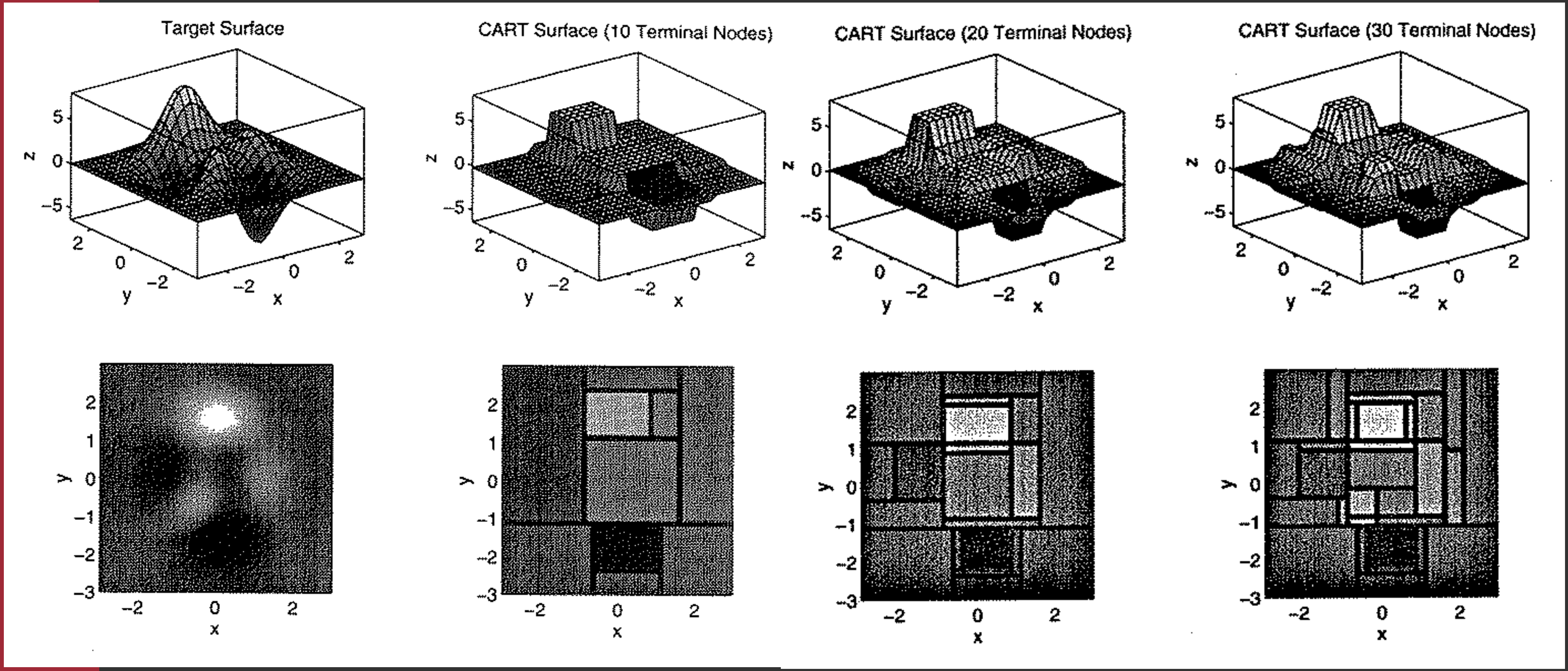
For any split *s* of node *t* into t_l and t_r, the change in the error measure is expressed as: $\Delta E(s,t) = E(t) - E(t_l) - E(t_r)$

the best split *s** is the one that maximizes the decrease in the error measure:

$$\Delta E(s^*,t) = \max_{s \in S} \Delta E(s,t).$$

The strategy for growing a regression tree is to split nodes (or data set) iteratively and thus maximize the decrease in $E(T) = \sum_{t \in \tilde{T}} E(t)$, the overall error measure (or cost) of the tree.

CART: Classification and Regression Trees



CART: Classification and Regression Trees

Tree Pruning

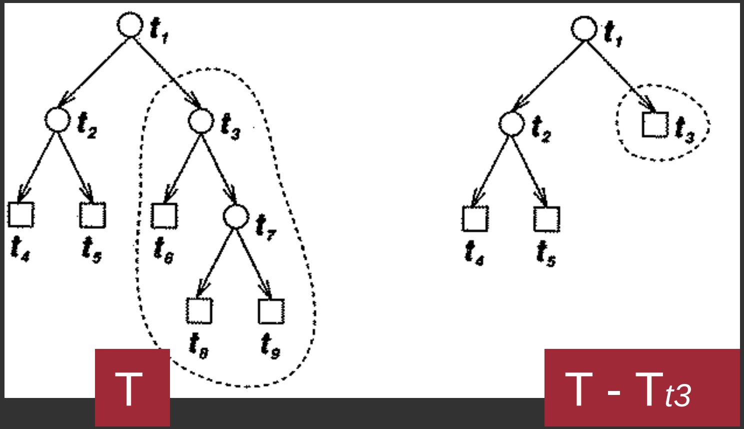
- The tree that the growing procedure yields is often too large and it is biased towards the training data set (overfitting)

Principle of minimum cost-complexity: prune the tree finding the weakest subtree in it, i.e. considering both the training error measure and the number of terminal nodes (measure of tree's complexity)

The internal node with the smallest α_t is chosen as the target node for shrinking

$$\alpha_t = \frac{E(t) - E(T_t)}{|\tilde{T}_t| - 1}.$$

number of terminal nodes in t



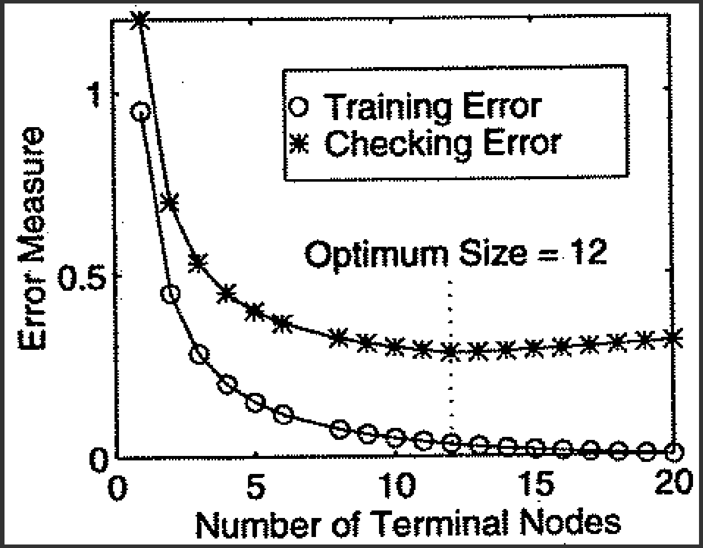
CART: Classification and Regression Trees

Therefore, a tree-pruning cycle consists of the following tasks:

1. Calculate α_t for each internal node *t* in T*i*
2. Find the minimal α_t and choose T - T_t as the next minimizing tree

Next step: select one of these candidate trees as the optimum-size tree:

• use an independent test (checking)
• cross-validation

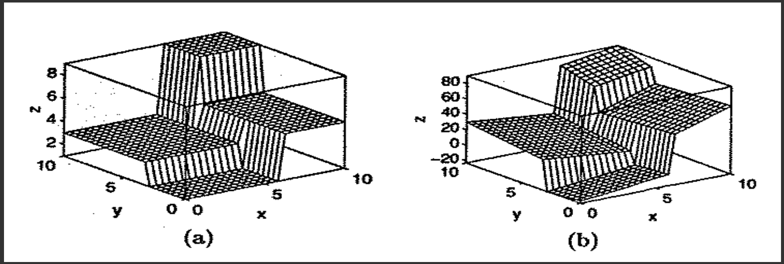


CART - ANFIS

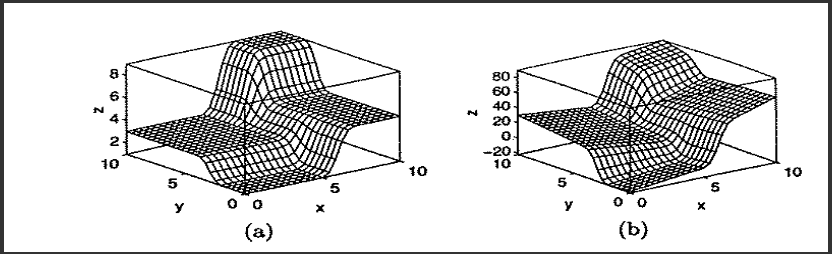
CART is used to find the number of ANFIS rules and the initial locations of membership functions before training

$$\begin{cases} \text{If } x < a \text{ and } y < b, \text{ then } z = f_1. \\ \text{If } x < a \text{ and } y \geq b, \text{ then } z = f_2. \\ \text{If } x \geq a \text{ and } y < c, \text{ then } z = f_3. \\ \text{If } x \geq a \text{ and } y \geq c, \text{ then } z = f_4. \end{cases}$$

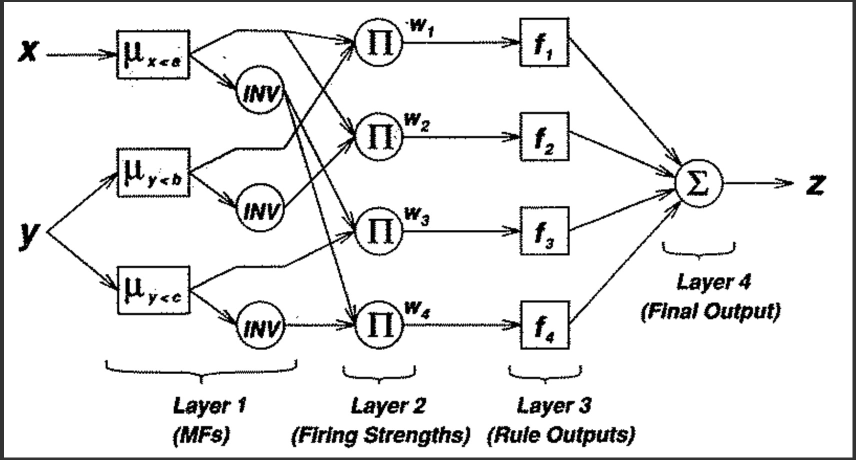
a=6; b=3; c=7



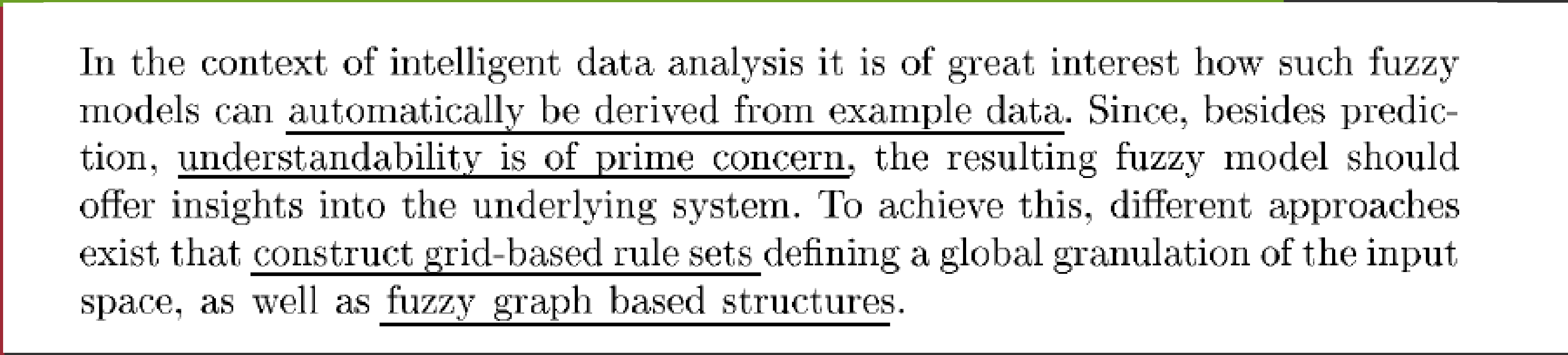
Crisp: discontinuous boundaries



Fuzzy: continuous boundaries



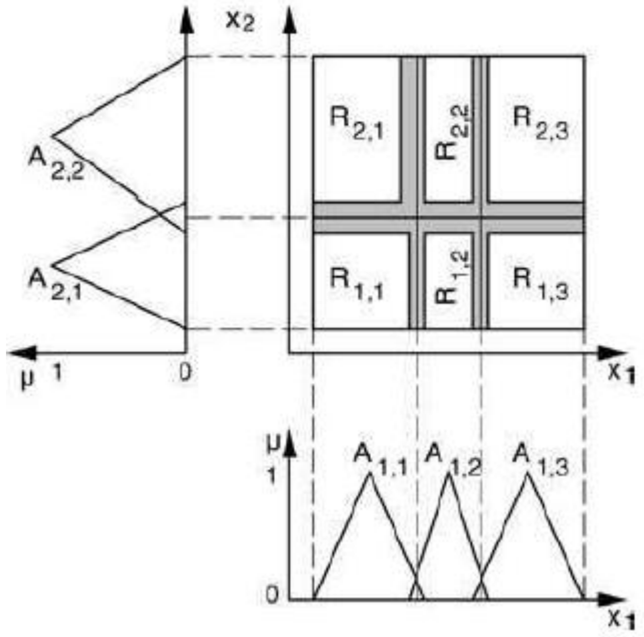
Extracting Fuzzy Models from Data



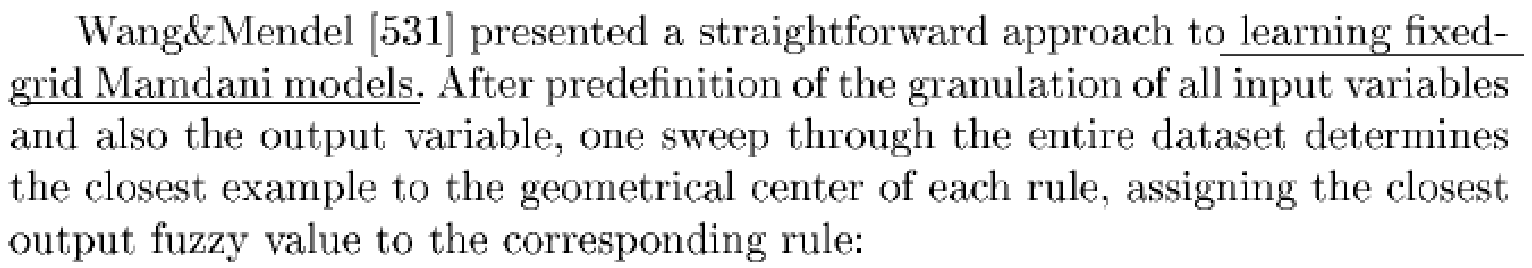
In the context of intelligent data analysis it is of great interest how such fuzzy models can automatically be derived from example data. Since, besides prediction, understandability is of prime concern, the resulting fuzzy model should offer insights into the underlying system. To achieve this, different approaches exist that construct grid-based rule sets defining a global granulation of the input space, as well as fuzzy graph based structures.

Extracting Grid-Based Fuzzy Models from Data

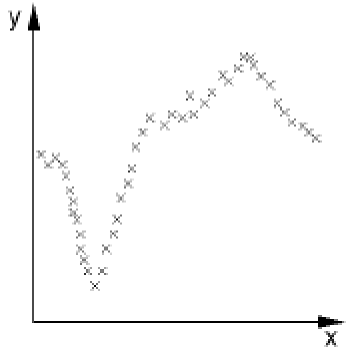
Grid-based rule sets model each input variable through a usually small set of linguistic values. The resulting rule base uses all or a subset of all possible combinations of these linguistic values for each variable, resulting in a global granulation of the feature space into "tiles":

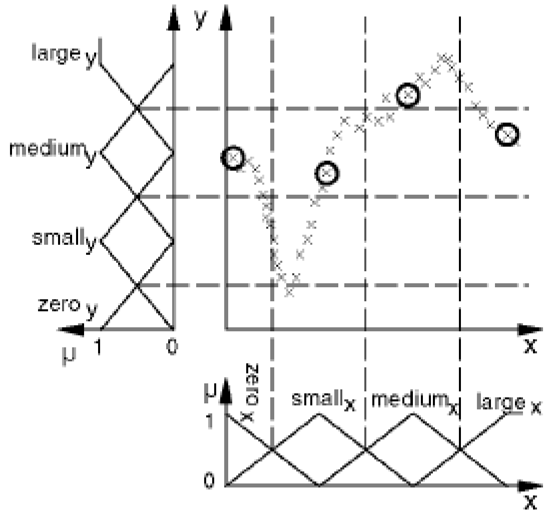


Extracting Grid-Based Fuzzy Models from Data: Wang&Mendel

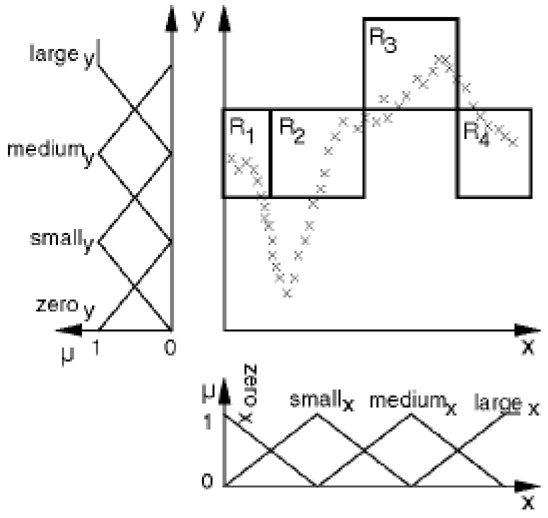

Wang&Mendel [531] presented a straightforward approach to learning fixed-grid Mamdani models. After predefinition of the granulation of all input variables and also the output variable, one sweep through the entire dataset determines the closest example to the geometrical center of each rule, assigning the closest output fuzzy value to the corresponding rule:

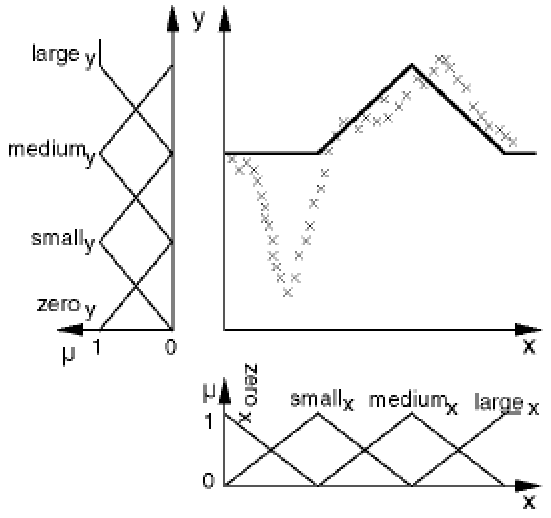
Extracting Grid-Based Fuzzy Models from Data: Wang&Mendel





R_1 : IF x IS $zero_x$ THEN y IS $medium_y$
R_2 : IF x IS $small_x$ THEN y IS $medium_y$
R_3 : IF x IS $medium_x$ THEN y IS $large_y$
R_4 : IF x IS $large_x$ THEN y IS $medium_y$



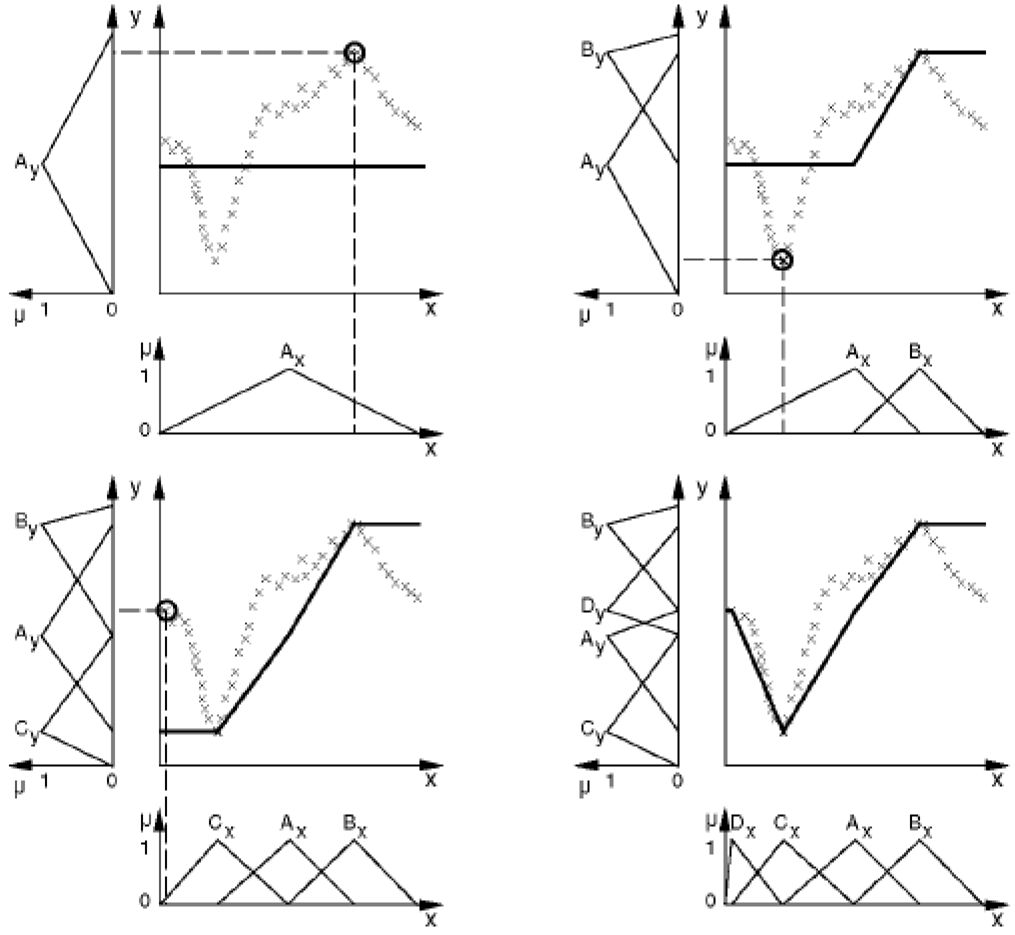


Extracting Grid-Based Fuzzy Models from Data: Higgins&Goodman

For practical applications it is obvious, however, that using such a predefined, fixed grid results in a fuzzy model that will either not fit the underlying functions very well or consist of a large number of rules. This is why approaches are of more interest that fine-tune or even automatically determine the granulations of both input and output variables.

Higgins&Goodman: Initially only one membership function is used to model each of the input variables as well as the output variable, resulting in one large rule covering the entire feature space. Subsequently new membership functions are introduced at points of maximum error. This is done until a maximum number of divisions is reached or the approximation error remains below a certain threshold.

Extracting Grid-Based Fuzzy Models from Data: Higgins&Goodman



IF x IS A_x THEN y IS A_y

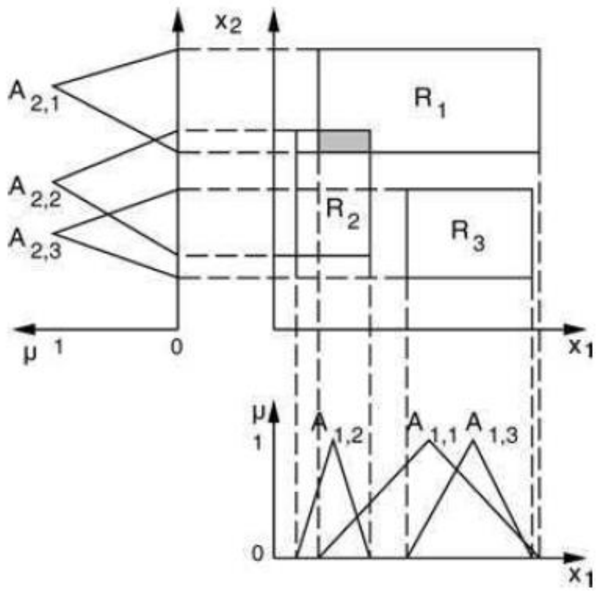
IF x IS B_x THEN y IS B_y

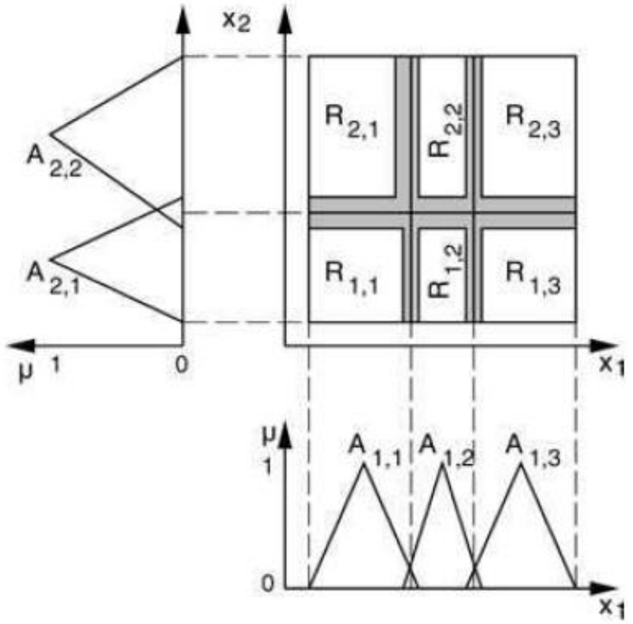
IF x IS C_x THEN y IS C_y

IF x IS A_x THEN y IS D_y
IF x IS B_x THEN y IS B_y
IF x IS C_x THEN y IS C_y
IF x IS D_x THEN y IS D_y

This approach is able to model extrema much better than Wang&Mendel algorithm, but has a strong tendency to concentrate on outliers

Extracting Fuzzy Graphs from Data





We mentioned before that especially in high-dimensional feature spaces a global granulation results in a large number of rules. For these tasks a fuzzy graph based approach is more suitable.

A possible disadvantage of the individual membership functions is the potential loss of interpretation.

Fuzzy Graphs

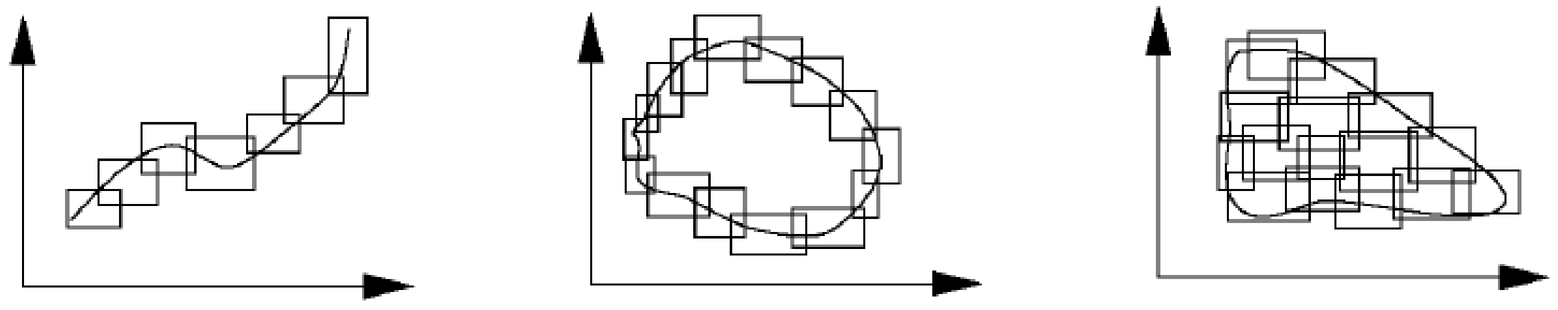
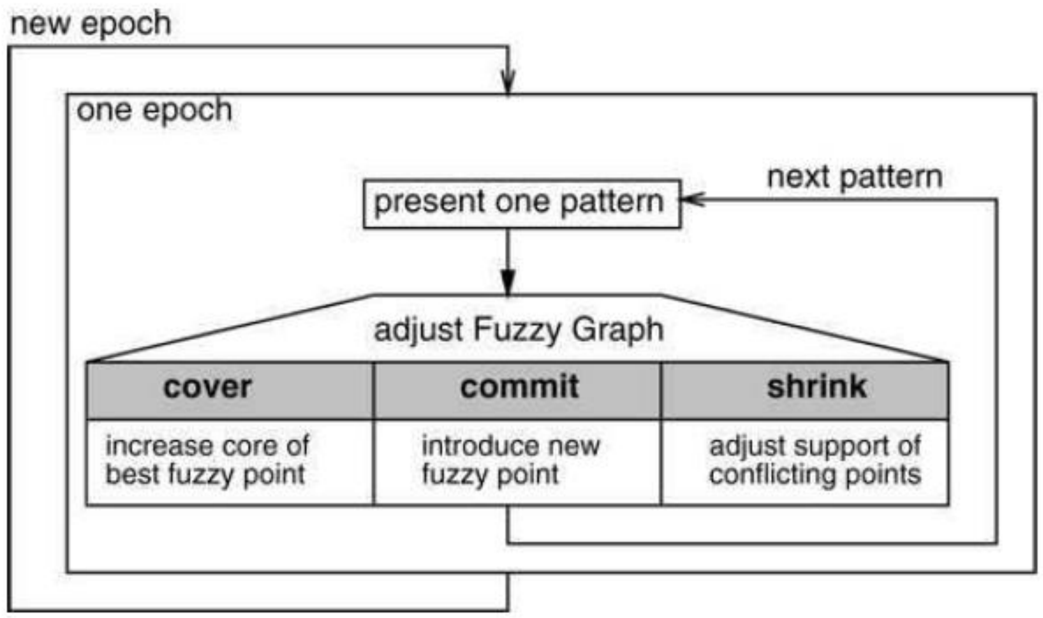


Fig. 9.9. Approximate representations of functions, contours, and relations (after Zadeh [559]).

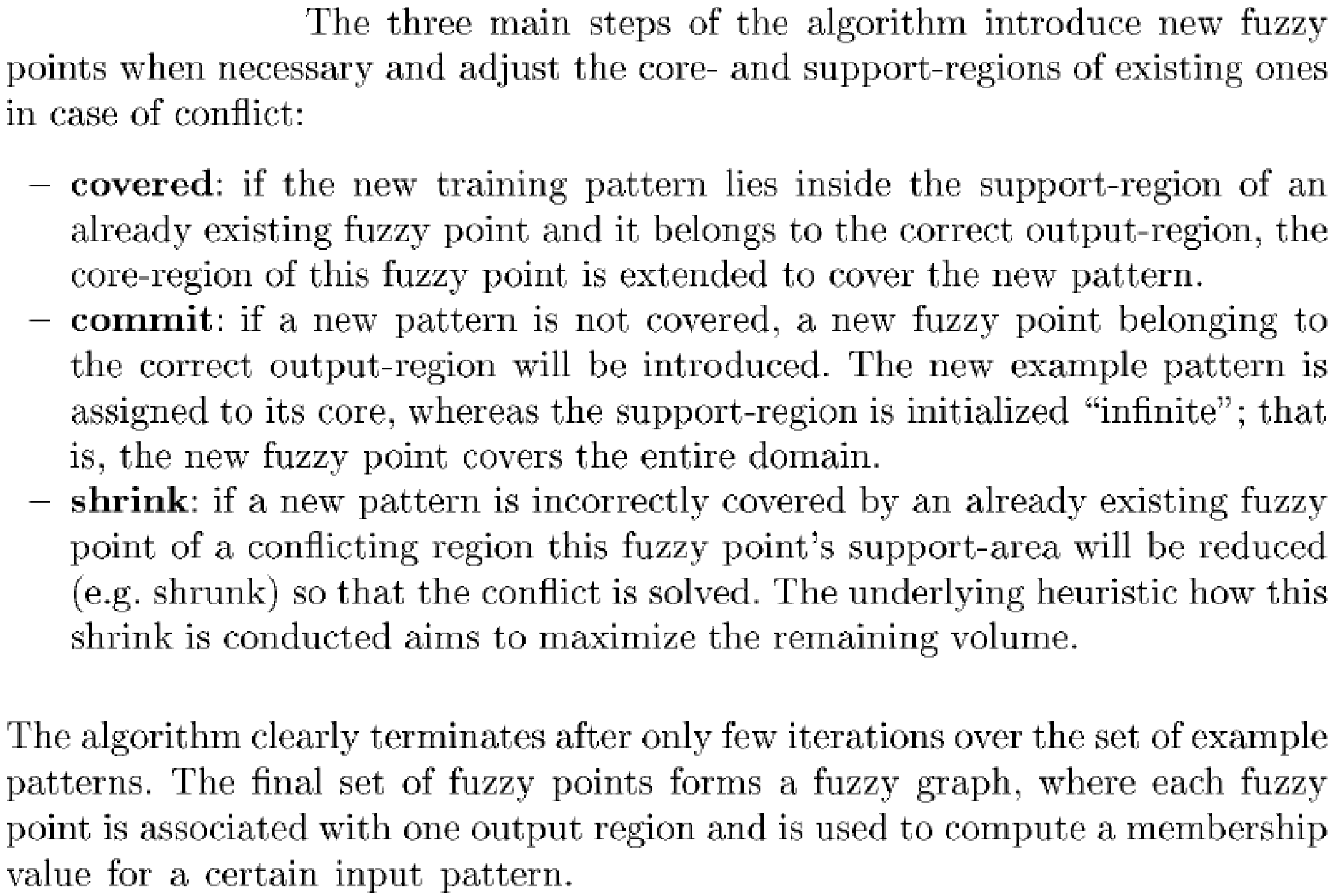
In contrast to a global grid, specified through linguistic variables, fuzzy graphs employ individual membership functions for each fuzzy rule; that is, the membership function of the constraint on the joint variable is defined individually for each rule.

Extracting Fuzzy Graphs from Data: Berthold and Huber

The only parameter specified by the user is the granulation of the output variable y; that is, the number and shape of the membership functions of y have to be determined manually . In most applications this is no disadvantage, because it enables the user to define foci of attention or areas of interest where a finer granulation is desired.


new epoch
one epoch
present one pattern
next pattern
adjust Fuzzy Graph
cover
commit
shrink
increase core of best fuzzy point
introduce new fuzzy point
adjust support of conflicting points

Extracting Fuzzy Graphs from Data: Berthold and Huber



The three main steps of the algorithm introduce new fuzzy points when necessary and adjust the core- and support-regions of existing ones in case of conflict:

- **covered**: if the new training pattern lies inside the support-region of an already existing fuzzy point and it belongs to the correct output-region, the core-region of this fuzzy point is extended to cover the new pattern.
- **commit**: if a new pattern is not covered, a new fuzzy point belonging to the correct output-region will be introduced. The new example pattern is assigned to its core, whereas the support-region is initialized "infinite"; that is, the new fuzzy point covers the entire domain.
- **shrink**: if a new pattern is incorrectly covered by an already existing fuzzy point of a conflicting region this fuzzy point's support-area will be reduced (e.g. shrunk) so that the conflict is solved. The underlying heuristic how this shrink is conducted aims to maximize the remaining volume.

The algorithm clearly terminates after only few iterations over the set of example patterns. The final set of fuzzy points forms a fuzzy graph, where each fuzzy point is associated with one output region and is used to compute a membership value for a certain input pattern.