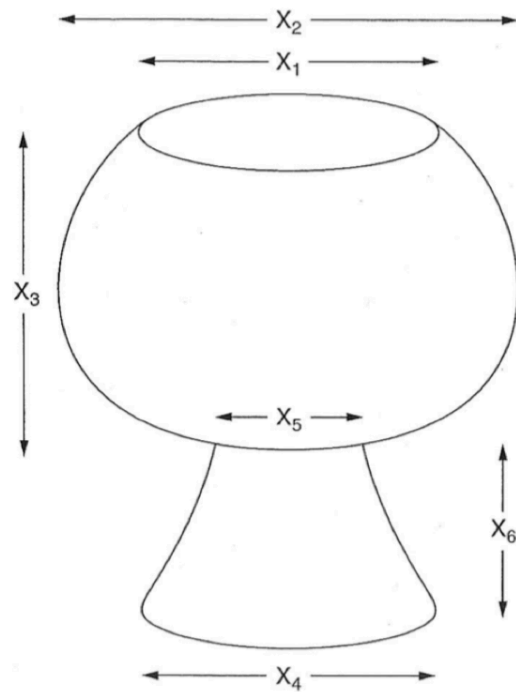


Example: (archeological measurements of goblets from Thailand)

	X1	X2	X3	X4	X5	X6
1	13	21	23	14	7	8
2	14	14	24	19	5	9
3	19	23	24	20	6	12
4	17	18	16	16	11	8
5	19	20	16	16	10	7
6	12	20	24	17	6	9
7	12	19	22	16	6	10
8	12	22	25	15	7	7
9	11	15	17	11	6	5
10	11	13	14	11	7	4
11	12	20	25	18	5	12
12	13	21	23	15	9	8
13	12	15	19	12	5	6
14	13	22	26	17	7	10
15	14	22	26	15	7	9
16	14	19	20	17	5	10
17	15	16	15	15	9	7
18	19	21	20	16	9	10
19	12	20	26	16	7	10
20	17	20	27	18	6	14
21	13	20	27	17	6	9
22	9	9	10	7	4	3
23	8	8	7	5	2	2
24	9	9	8	4	2	2
25	12	19	27	18	5	12

Measurements on archeological goblets (cm)



Descriptive statistics (covariances and correlations):

	X1	X2	X3	X4	X5	X6
X1	9.04	8.13	6.33	8.41	4.48	5.55
X2	8.13	18.81	22.11	14.89	5.43	10.85
X3	6.33	22.11	36.92	21.23	3.29	16.36
X4	8.41	14.89	21.23	17.17	4.36	11.84
X5	4.48	5.43	3.29	4.36	4.66	1.96
X6	5.55	10.85	16.36	11.84	1.96	9.86

	X1	X2	X3	X4	X5	X6
X1	1.00	0.62	0.35	0.67	0.69	0.59
X2	0.62	1.00	0.84	0.83	0.58	0.80
X3	0.35	0.84	1.00	0.84	0.25	0.86
X4	0.67	0.83	0.84	1.00	0.49	0.91
X5	0.69	0.58	0.25	0.49	1.00	0.29
X6	0.59	0.80	0.86	0.91	0.29	1.00

The correlations between variables are positive (direct relations)

PCA of the covariance matrix:

	PC1	PC2	PC3	PC4	PC5	PC6
X1	0.20	0.67	-0.23	-0.27	0.61	-0.11
X2	0.46	0.19	0.62	-0.44	-0.37	-0.22
X3	0.66	-0.54	0.11	0.16	0.49	0.06
X4	0.44	0.18	-0.45	0.46	-0.39	-0.45
X5	0.11	0.44	0.39	0.60	-0.01	0.52
X6	0.33	0.00	-0.45	-0.36	-0.31	0.68

	PC1	PC2	PC3	PC4	PC5	PC6
eigenvalue	77.56	11.48	4.17	1.71	0.93	0.61
fraction	0.80	0.12	0.04	0.02	0.01	0.01
cumulative	0.80	0.92	0.97	0.98	0.99	1.00

- **PC1:** we can see that all the variables are positively correlated. The first Principal Component is the size of the goblet. Big goblets have a large and positive PC1 and tiny goblets have a small and negative PC1 (*Size-effect*). Usually PC1 contributes to the size effect.



- **PC2:** the second Principal Component is not correlated to X6. X2 and X4 hardly contribute to PC2. PC2 contrasts variables: when X1 is large, X3 is small. And the other way around. This component represents the shape of the goblet; if PC2 is high, then the goblet is wide and short, and if PC2 is small, then the goblet is narrow and tall. It captures the contrast between both variables.



- **PC3:** we do not need it because it explains a small fraction of variability. Starting from PC3, they explain almost nothing – their contribution is so small, that we rather leave them out.