```
In[1]:= Solve \left[x^2 + 3x == 5, x\right]
Out[1]:= \left\{\left\{x \to \frac{1}{2}\left(-3 - \sqrt{29}\right)\right\}, \left\{x \to \frac{1}{2}\left(-3 + \sqrt{29}\right)\right\}\right\}
In[2]:= NSolve \left[x^2 + 3x == 5, x\right]
Out[2]:= \left\{\left\{x \to -4.19258\right\}, \left\{x \to 1.19258\right\}\right\}
```

# Дифференцирование и интегрирование

```
In[3]:= Sin'[x]
Out[3]= Cos [x]
 In[4]:= Sin''[x]
Out[4]= - Sin[x]
 In[5]:= Sin''''[x]
Out[5]= \cos[x]
 In[6]:= D[f[x], x]
Out[6]= f'[x]
 In[7]:= Integrate[Sin[x], x]
Out[7]= -\cos[x]
 In[8]:= NIntegrate[Sin[x], {x, 0, 1}]
Out[8]= 0.459698
 In[9]:= \int \sin[x] dx
Out[9]= -\cos[x]
In[10]:= \int_{1}^{3} Sin[x] dx
Out[10]= 2 Sin [1] Sin [2]
```

-0.1 [-

## Дифференциальные уравнения

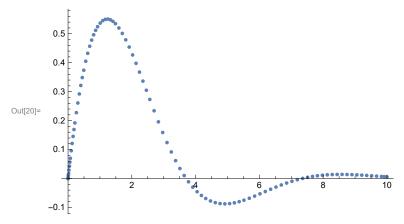
```
ln[11]:= sol = DSolve[x''[t] == -x[t], x, t]
\text{Out} [\texttt{11}] = \; \big\{ \big\{ x \rightarrow Function \big[ \big\{ t \big\} \text{, } C \big[ 1 \big] \text{ Cos} \big[ t \big] + C \big[ 2 \big] \text{ Sin} \big[ t \big] \big] \big\} \big\}
                   (Set[]) \neq = (Equal[])
 In[12]:= x /. sol
\label{eq:out_12} \text{Out}_{\text{[12]}} = \left\{ \text{Function}\left[\,\left\{\,t\,\right\}\,,\,\,C\,[\,1\,\right]\,\,\text{Cos}\left[\,t\,\right]\,+\,C\,[\,2\,\right]\,\,\text{Sin}\left[\,t\,\right]\,\right\}
 In[13]:= x[t] /. sol[[1]]
Out[13] = C[1] Cos[t] + C[2] Sin[t]
 ln[14]:= DSolve[x''[t] == -x'[t] - Sin[x[t]], x, t]
          .... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution
                 information
Out[14]= DSolve[x''[t] = -Sin[x[t]] - x'[t], x, t]
 In[15]:= sol = NDSolve[{
                 x''[t] = -x'[t] - Sin[x[t]],
                 x'[0] = 1, x[0] = 0
               x, {t, 0, 10}][1]
Out[15]= \{x \rightarrow InterpolatingFunction | 
                                                                         Output: scalar
 ln[16]:= Plot[x[t] /. sol, {t, 0, 10}]
          0.5
          0.4
          0.3
Out[16]=
          0.2
          0.1
```

```
In[17]:= Plot[x'[t] /. sol, {t, 0, 10}]
       0.4
       0.3
       0.2
       0.1
 Out[17]=
                                                        10
      -0.1
      -0.2
      -0.3 h
 In[18]:= sol[1, 2]
                                      Domain: {{0., 10.}}
 Out[18]= InterpolatingFunction
                                      Output: scalar
 In[19]:= sol[1, 2] // FullForm
Out[19]//FullForm=
      InterpolatingFunction[List[List[0.`, 10.`]],
        List[5, 7, 2, List[95], List[4], 0, 0, 0, 0, Automatic, List[], List[], False],
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```

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-0.007561309457221403`, 0.0016858501605269156`]], List[Automatic]]
```

#### $ln[20] := ListPlot[{sol[1, 2, 3, 1], sol[1, 2, 4, 3, sol[1, 2, 4, 2, ;; -2] + 1]]}^T]$



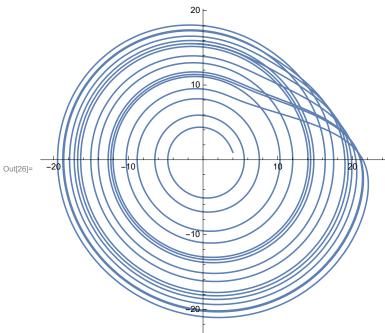
## Задачи

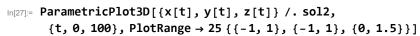
### 3.1

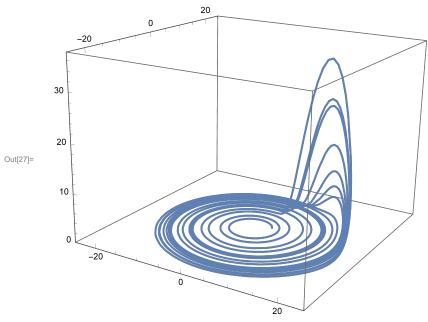
```
In[21]:= sol1 = NDSolve[{
          x''[t] = -x'[t] - Sin[x[t]],
          x'[0] = 1, x[0] = 0
         }, x, {t, 0, 10}];
      Plot[{x[t], x'[t]} /. sol1 // Evaluate, {t, 0, 10}]
      0.6
      0.4
      0.2
Out[22]=
      -0.2
```

## 3.2

```
ln[23]:= a = b = .2; c = 5.7;
     a = b = .1; c = 14;
     sol2 = NDSolve[{
          x'[t] = -y[t] - z[t],
          y'[t] = x[t] + a y[t],
          z'[t] = b + z[t] (x[t] - c),
          x[0] = 4, y[0] = 1, z[0] = 1
         \{x, y, z\}, \{t, 0, 100\}];
     ParametricPlot[{x[t], y[t]} /. sol2, {t, 0, 100}]
```







#### 3.3

In[28]:= 
$$F[r_] := -\frac{1}{r} + \frac{1}{r^2};$$

$$sol3 = NDSolve \Big[ \Big\{ x''[t] == F[r] * \frac{x[t]}{r}, x[\theta] == \theta, x'[\theta] == 1, y''[t] == F[r] * \frac{y[t]}{r}, \\ y[\theta] == 1, y'[\theta] == \emptyset \Big\} /. \Big\{ r \to \sqrt{x[t]^2 + y[t]^2} \Big\}, \{x, y\}, \{t, \theta, 100\} \Big];$$

ParametricPlot[ $\{x[t], y[t]\}$  /. sol3,  $\{t, 0, T\}$ , PlotRange  $\rightarrow$  3.5  $\{\{-1, 1\}, \{-1, 1\}\}\}$ ; Animate[Pic[T],  $\{T, 0.1, 100\}$ ]

