

# ABSTRACT DATATYPES

ADSI, S2023

# ABSTRACT DATATYPES (ADTs)

are objects with a set of operations,  
but we don't care about how it's  
actually implemented.

ADTs

List  
Stack

Queues

← vs →

DS → data structure

Linked List  
Array List

## WE DISTINGUISH

ADT

Data objects  
for a  
problem

from

DS

Their  
representation  
in memory

# ABSTRACT DATATYPES IN JAVA

ADT  
↓

```
public interface ADListADT<T> {  
    .public void add(T elm);  
    .public void insert(int index, T elm);  
    .public boolean remove(T elm);  
    .public int indexOf(T elm);  
    .public int size();  
    .public boolean contains(T elm);  
    .public T remove(int index);  
    .public T set(int index, T elm);  
}
```

```
public class ADSLinkedList<T> implements  
ADListADT<T> {  
    private Node<T> first;  
    private int size;  
  
    public ADSLinkedList() {  
        first = null;  
        size = 0;  
    }  
  
    @Override  
    public void add(T elm) {  
        if(elm == null){  
            return;  
        }  
        Node<T> newNode = new Node(elm, null);  
        if(size == 0){  
            first = newNode;  
            ... blah blah blah ...  
        }  
    }  
}
```

# SETS, MAPS AND HASHING

ADSI, S2023

## SETS

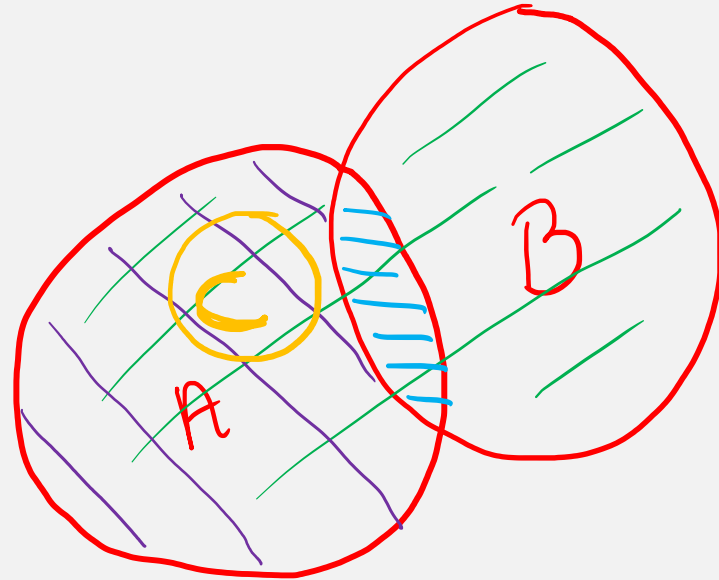
A collection of elements with  
no duplicates and no order

$S = \{ \text{"apple"}, \text{"pear"}, \text{"banana"} \}$

add "apple" to  $S \rightarrow$  nothing happens

# OPERATIONS ON A SET INCLUDE ...

- ... testing for membership
- ... adding elements
- ... removing elements
- ... union  $A \cup B$
- ... intersection  $A \cap B$
- ... set difference  $A - B$
- ... subset  $C \subseteq A$



~

## A SMALL EXERCISE

$$A = \{1, 3, 5, 7\}$$

$$B = \{2, 3, 4, 5\}$$

Find

$$A \cup B = \{1, 2, 3, 4, 5, 7\}$$

$$A \cap B = \{3, 5\}$$

$$A - B = \{1, 7\}$$

$$B - A = \{2, 4\}$$



## LISTS VS SETS

Sets ...

have no duplicates

have no positions

⇒ can give <sup>average</sup> constant time  
complexity for lookup,  
insert,  
delete

# MAPS aka DICTIONARIES

↳ sets of ordered pairs  
(key, value)

efficient way  
of storing  
information

no duplicate  
keys

may appear  
several times

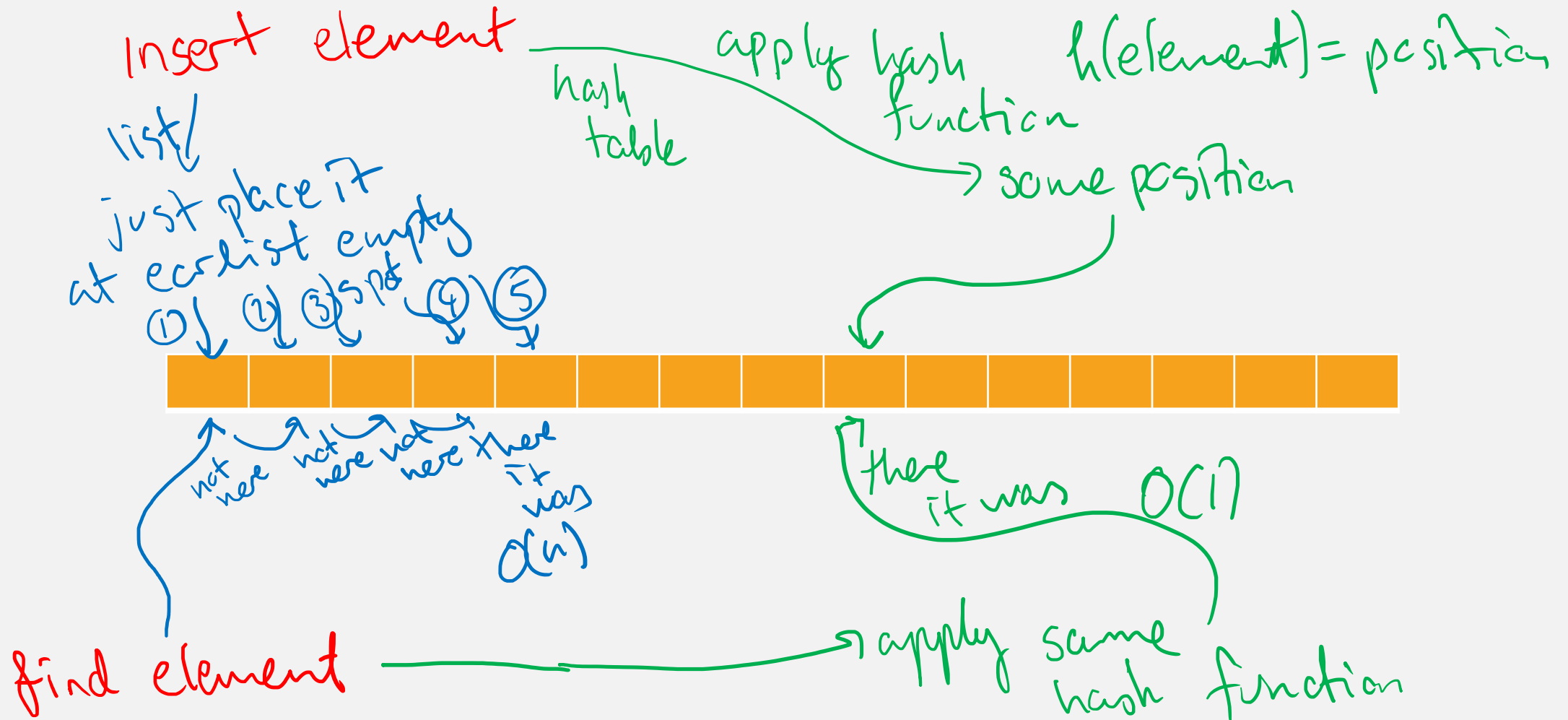
```
planet_radii = { ('mercury', 2440), ('venus', 6052), ('earth', 6378), ... }
```

## OPERATIONS ON A MAP INCLUDE ...

- ... getting a value given a key (*get*)
- ... adding a pair (key, value) (*put*)

how to implement  
sets & maps?

# HASH TABLES



## A SET OF PLANETS

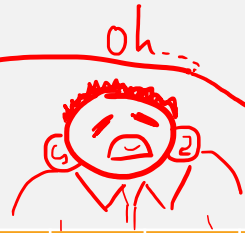
hash fct = position in alphabet of first letter in planet name

$h(\text{"Mercury"}) = 12$

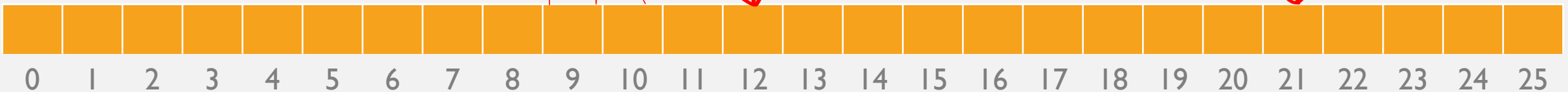
$h(\text{"Venus"}) = 21$

$h(\text{"Earth"}) = 4$

$h(\text{"Mars"}) = 12$



COLLISION



## THE JAVA STRING HASH FUNCTION

unicode char of last letter  $\times 31^0$   
+ \_\_\_\_\_ 2nd last \_\_\_\_\_  $\times 31^1$   
+ \_\_\_\_\_ 3rd last \_\_\_\_\_  $\times 31^2$   
+ - - - - -

$$\begin{aligned} h(\text{"Earth"}) &= \text{"h"} \times 31^0 + \text{"t"} \times 31^1 + \text{"r"} \times 31^2 + \text{"a"} \times 31^3 + \text{"E"} \times 31^4 \\ &= 104 \times 1 + 116 \times 31 + 114 \times 31^2 + 97 \times 31^3 + 69 \times 31^4 \\ &= 66725930 \end{aligned}$$

## "GOOD" HASH FUNCTIONS

→ spreads values evenly

→ are cheap to compute

BUT WE STILL NEED TO HANDLE COLLISIONS

## COLLISIONS

OPEN ADDRESSING

If position taken,  
find another one

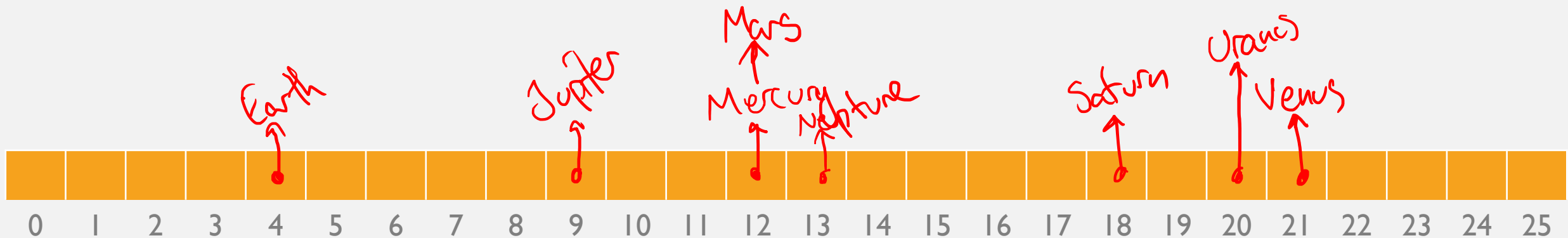
SEPARATE CHAINING

Accept multiple  
items at same  
position



# SEPARATE CHAINING

Each table slot references a *linked list*  
↓  
"bucket"



*danger: if linked lists long,  
no improvement at all*

# OPEN ADDRESSING

If the hashed position is occupied, *find a new one*

$i = 0$   
repeat:  
     $pos = h(key, i)$   
    if  $table[pos] == null$ :  
         $table[pos] = key$   
        return  
    else:  
         $i = i + 1$

*hash function*

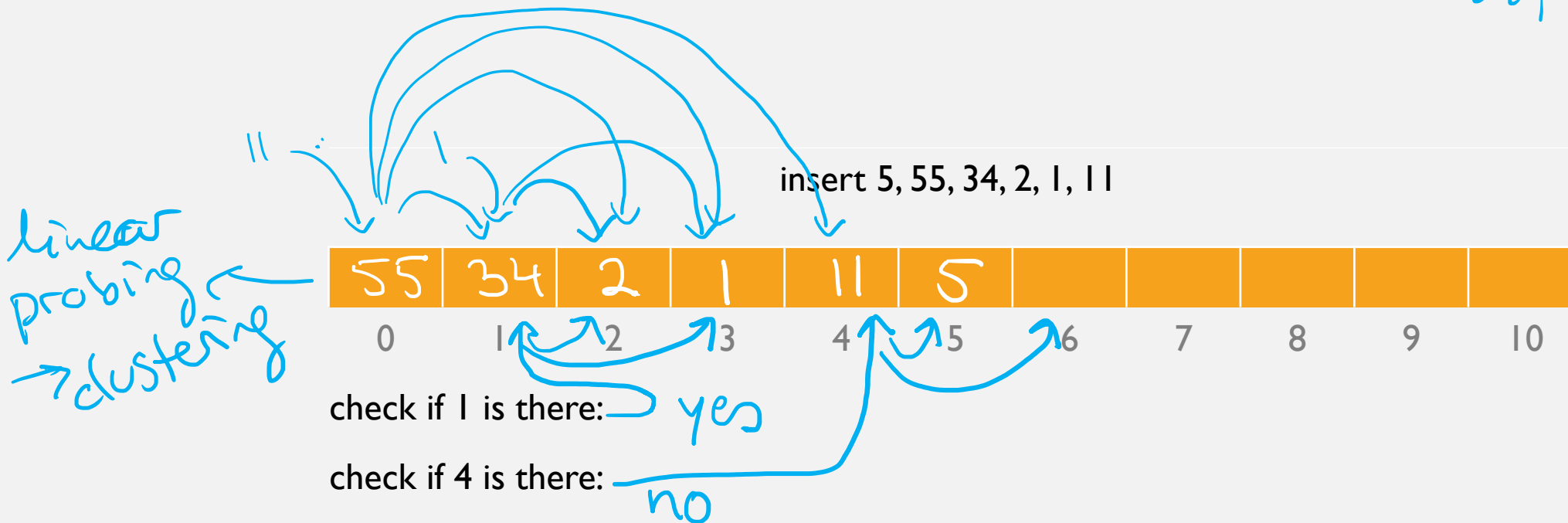
*this is where  
the magic happens*

## OPEN ADDRESSING I: LINEAR PROBING

$$h_1(k) = k \bmod 11$$

$$h(k, i) = h_1(k) + \text{linear term}$$

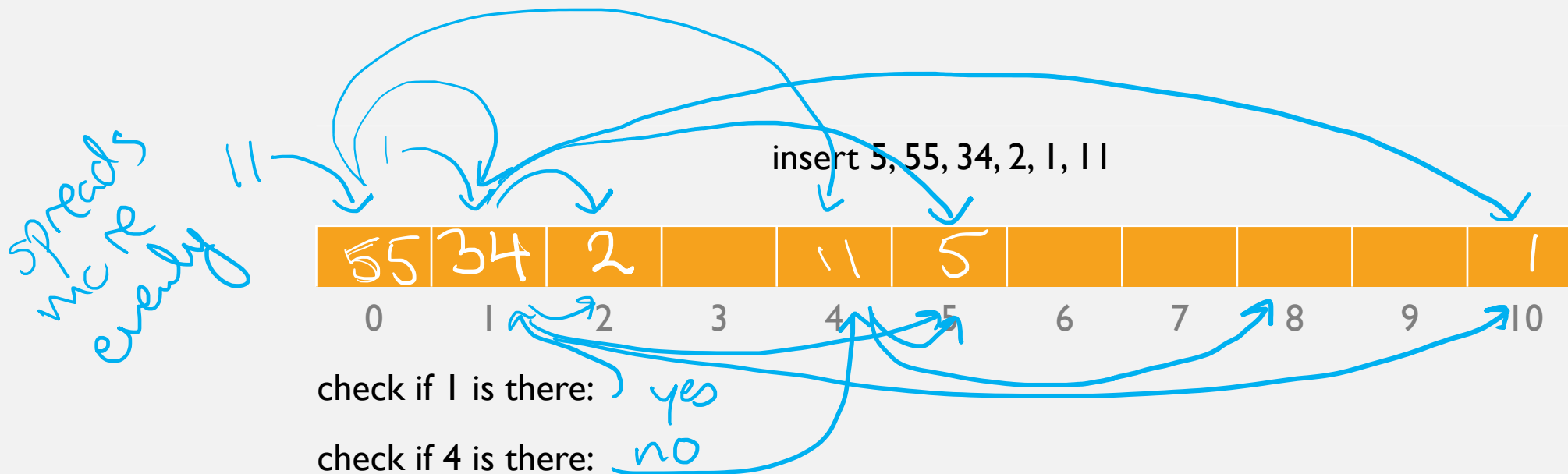
linear term  $\begin{cases} 2i, 3i, 4i, \\ 5i, \end{cases}$  could be



## OPEN ADDRESSING II: QUADRATIC PROBING

$$h_1(x) = k \bmod 11$$
$$h(x) = h_1(x) + i^2$$

or some other quadratic function of  $i$

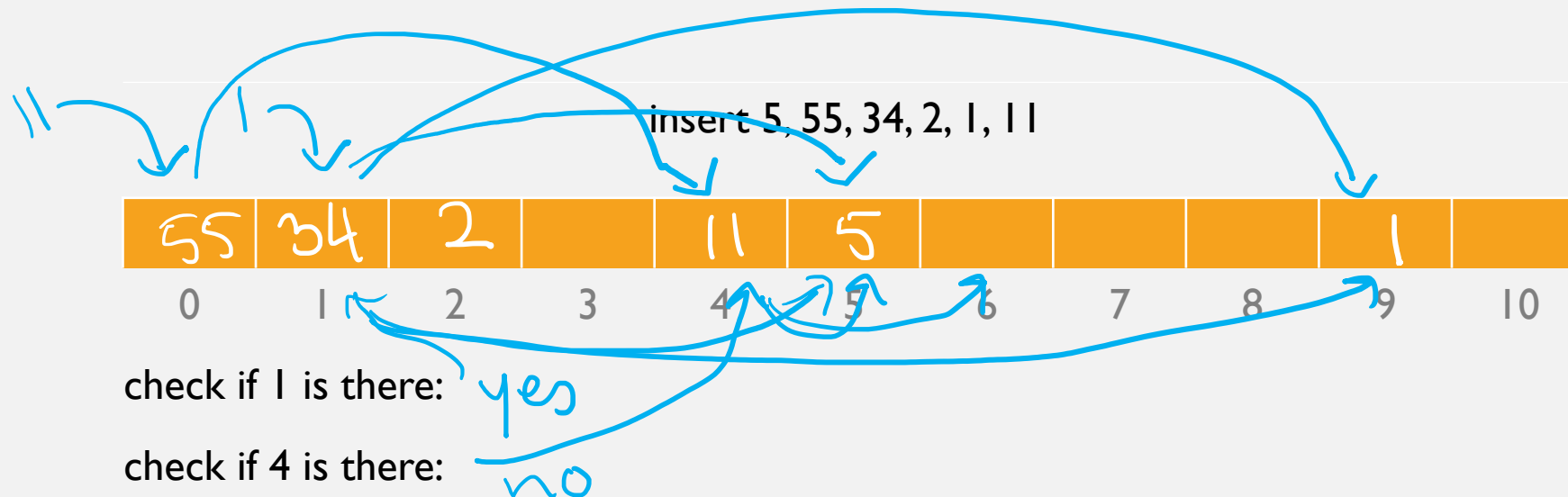


## OPEN ADDRESSING III: DOUBLE HASHING

$$h_1(k) = k \bmod 11$$

$$h(k) = h_1(k) + \text{some other function}$$

$$\text{e.g.} = h_1(k) + i(5 - k \bmod 5)$$



## WHY ARE HASH TABLES IDEAL FOR IMPLEMENTING SETS AND MAPS?

Hash table  $\rightarrow$  no particular order

Insertion  
Searching

Hash table  
 $O(1)$   
 $O(1)$

Sorted array  
 $O(n)$   
 $O(\log n)$

# PERFORMANCE OF HASH TABLES

Load factor

$$L = \frac{\text{\#filled cells}}{\text{\#cells}}$$

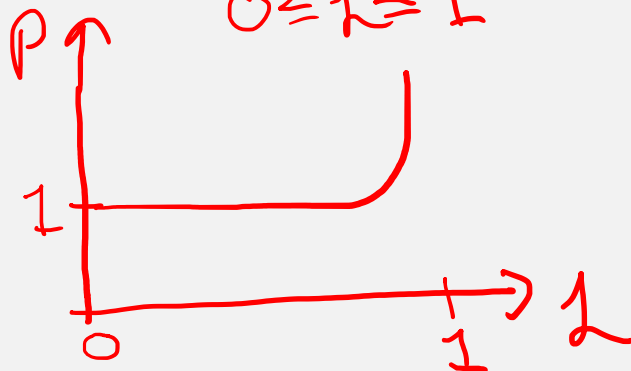
Expected #probes

$p$  for

Linear probing

$$p = \frac{1}{2} \left( 1 + \frac{1}{1-L} \right)$$

$$0 \leq L \leq 1$$

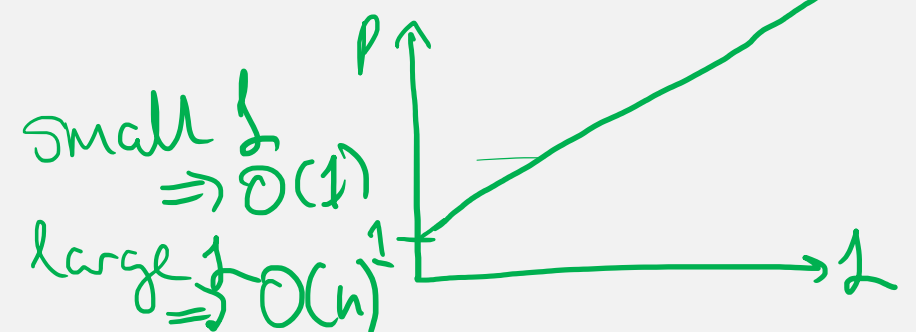


Small load factor  
 $\Rightarrow O(1)$

Separate chaining

$$p = 1 + \frac{L}{2}$$

$L$  can be  $> 1$



Small  $L$   
 $\Rightarrow O(1)$   
 large  $L$   
 $\Rightarrow O(L)$