TBarrier notebooks

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Contents

1 Introduction

TBarrier contains a series of jupyter notebooks that guide you through methods used to extract advective, diffusive, stochastic and active transport barriers from discrete velocity data.

The collection of notebooks implement the methods explained in the book Transport Barriers and Coherent Structures in Flow Datawritten by George Haller. We address both two-dimensional and three dimensional steady/unsteady velocity fields. Hence, *TBarrier* is designed for fluid dynamicists in general, with a special focus on oceanographers and CFD analysts.

1.1 Programming Language

The jupyter notebooks are written in Python. It was, however, designed in a way so that it could easily be translated into other languages such as Juliaand Matlab. Table 1 includes a qualitative (and hence subjective) comparison of Python, Julia and Matlab.

The decision on the programming language was based on various factors. Although computational speed is definitely a criterion that has to be considered when implementing the code, other factors such as the ease of access, user friendliness and versatility of the programming language have to be included. Although Python seems to be behind Julia and Matlab in terms of computational speed (especially when solving a large number of ODEs), it outperforms them when it comes to user friendliness. Apart from academia, Python is also widely used in the industry as it is open source. Due to the limitations on the ODE solver, the computational speed of the computations in the *TBarrier* notebook is lower compared to Matlab or Julia. However, the main purpose of the notebook series is to explain the methods highlighted in the book with clarity on a very user friendly and easily accessible platform.

Qualitative Comparison of programming languages			
Feature	Python	Julia	Matlab
Access	Open Source	Open Source	Closed Source
Support from Community	Good	Bad	Moderate
ODE solver	Bad	Good	Moderate
Computational speed	Moderate	Good	Moderate
User friendly	Good	Bad	Moderate

Table 1: Qualitative Comparison of programming languages

1.2 Existing Software

We highlight three major publicly available existing software packages designed to extract transport barriers from discrete velocity data.

The toolbox *CoherentStructures.jl* has been developed in Oliver Junge's research group at TUM, Germany. It is written in Julia and computationally very efficient. It focuses, however, principally on advective transport barriers in two dimensional flows and does not explicitly address methods for computing diffusive, stochastic and active barriers.

Secondly, we point out the *LCStool* and the *BarrierTool* developed by George Haller's research groupat ETHZ. Both tools are written in Matlab. The former tool focuses on the extraction of advective Lagrangian Coherent Structures from two-dimensional discrete velocity data, whereas the latter includes a GUI addressing both advective and diffusive Lagrangian and Eulerian transport barriers in two-dimensional flows.

2 Overview *TBarrier* notebooks

The notebooks are stored in the *TBarrier*repository on github. By clicking on the underlined headers you are redirected to the corresponding folder on the github repository.

1. 2D
   (a) data
      i. AVISO

         github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/AVISO

      ii. BickleyJet

         github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/BickleyJet

iii. Isotropic two-dimensional turbulence

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/Turbulence

(b) demos

i. AdvectiveBarriers

- FTLE: (see *Hyperbolic LCS from the finite-time Lyapunov exponent*)

  FTLE-field applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FTLE

- PRA: (see *Elliptic LCSs from the polar rotation angle*)

  PRA-field applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PRA

- LAVD: (see *Elliptic LCSs from the Lagrangian-averaged vorticity deviation* and *LAVD for 2D flows*)

  LAVD-field applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/LAVD

- TRA: (see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

  TRA-field applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TRA

- TSE: (see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

  TSE-field applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TSE

- HyperbolicLCS: (see *Local variational theory of hyperbolic LCS Hyperbolic LCSs in 2D flows*)

  Hyperbolic LCS from tensorlines (shrinklines/stretchlines) of the Cauchy-Green strain tensor applied to AVISO, Bickley and 2D turbulence data.

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicLCS

- HyperbolicOECS: (see *Shearless OECSs and objective saddle points in 2D flows*)

  Hyperbolic OECS from (local) tensorlines launched from objective saddle points applied to AVISO.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicOECS

- EllipticLCS: (see *Computing elliptic LCSs as closed null-geodesics*)

  Elliptic LCS as closed null geodesics from the Cauchy-Green strain tensor applied to AVISO, Bickley and 2D turbulence data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticLCS

- EllipticOECS: (*Computing elliptic LCSs as closed null-geodesics*)

  Elliptic OECS as closed null geodesics from the rate of strain tensor applied to AVISO.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticOECS

- FastTensorlineComputation:

  Computation of tensorlines using the newly proposed algorithm with the reparametrization of the eigenvectors from AVISO data. The Fast Tensorline Computation (FTC) is applied to the rate of strain tensor to (locally) extract hyperbolic OECS away from tensorline singularities.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FastTensorlineComputati

- PoincareMap: (see *Poincaré maps*)

  Poincare map applied to periodic Bickley-jet.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PoincareMap

ii. DiffusionBarriers:

Here, we discuss diffusive Lagrangian and Eulerian transport barriers. The analysis and algorithm are similar to the ones discussed for the advective Barriers.

- DBS: (see *Unconstrained diffusion barriers in 2D flows*)

  Diffusion Barrier Sensitivity (DBS) applied to AVISO and Bickley data

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/DBS

- EllipticLagrangianDiffusionBarriers: (see *Unconstrained diffusion barriers in 2D flows*)

  Elliptic Lagrangian Diffusion Barriers (using null-geodesics algorithm) extracted from AVISO and Bickley data.

- github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticLagrangianDiffusio

- EllipticEulerianDiffusionBarriers: (see *Unconstrained diffusion barriers in 2D flows*)

  Elliptic Eulerian Diffusion Barriers extracted from AVISO and Bickley data.

  NOTE: There is no mention of this part in the book and no figures are provided. Stergios, computed, it however in the BarrierTool and it has been mentioned in the original paper (see remark 1 in http://georgehaller.com/reprints/mbarriers.pdf). That is why I thought I should also include it.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticEulerianDiffusionB

iii. StochasticBarriers: (see *Transport barriers in stochastic velocity fields*)

Barriers in stochastic velocity field extracted from AVISO data. We specifically focus on elliptic Lagrangian stochastic Barriers.

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/StochasticBarriers

iv. ActiveBarriers: (see *2D homogeneous, isotropic turbulence*)

Active barriers extracted from the two-dimensional turbulence simulation. Lagrangian and Eulerian active barriers to vorticity and linear momentum extracted using the active FTLE, active PRA, active TSE, active TRA and the Hamiltonian based formulation.

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers

- aFTLE: (see *Active FTLE (aFTLE) and active TSE (aTSE)*) github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aFTLE

- aTSE: (see *Active FTLE (aFTLE) and active TSE (aTSE)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTSE

- aTRA: (see *Active PRA (aPRA) and active TRA (aTRA)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTRA

- aPRA: (see *Active PRA (aPRA) and active TRA (aTRA)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aPRA

- Hamiltonian: (see *Active transport barriers in general 2D Navier–Stokes flow*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/Hamiltonian

(c) subfunctions:

Folder containing frequently used functions to compute trajectories from two-dimensional velocity data, evaluate the gradient of the flowmap/velocity, the classic Cauchy-Green strain tensor, Singular Value Decomposition, Dynamic Polar Decomposition, Polar, Decomposition, ...

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/subfunctions

2. 3D

(a) data

i. Arnold-Beltrami-Childress (ABC) flow:

Classic spatially periodic ABC flow. We consider both the steady and unsteady version.

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/ABC

ii. Turbulence:

Three dimensional turbulent channel flow data from John Hopkins Research Center

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/Turbulence

(b) demos

i. AdvectiveBarriers

- FTLE(see *Hyperbolic LCS from the finite-time Lyapunov exponent*)

  FTLE-field applied to ABC data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/FTLE

- TSE(see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

  TSE field applied to ABC and turbulent channel flow data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TSE

- TRA(see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

  TRA field applied to ABC and turbulent channel flow data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TRA

- LAVD: (see *Elliptic LCSs from the Lagrangian-averaged vorticity deviation* and *LAVD for 3D flows*)

  LAVD field applied to ABC data and turbulent channel flow data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/LAVD

- UnifiedLCSTheory: (see *Unified variational theory of elliptic and hyperbolic LCS in 3D*)

  Extract LCS from the ξ_2 eigenvector field of the Cauchy-Green strain tensor from the ABC data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/UnifiedLCSTheory

- PoincareMap:

  Classic Poincare map applied to ABC data.

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/PoincareMap

ii. ActiveBarriers

Active barriers extracted from the three-dimensional channel flow data (see *3D turbulent channel flow*) and/or from the ABC data
(see *Quasi-objective, single-trajectory diagnostics for transport barriers*).

Lagrangian and Eulerian active barriers to vorticity and linear momentum extracted using the active FTLE, active PRA, active TSE and active TRA.

- aFTLE(see *Active FTLE (aFTLE) and active TSE (aTSE)*) github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aFTLE

- aTSE(see *Active FTLE (aFTLE) and active TSE (aTSE)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTSE

- aPRA(see *Active PRA (aPRA) and active TRA (aTRA)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aPRA

- aTRA(see *Active PRA (aPRA) and active TRA (aTRA)*)

  github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTRA

(c) subfunctions:

Folder containing frequently used functions to compute trajectories from two-dimensional velocity data, evaluate the gradient of the flowmap/velocity, the classic Cauchy-Green strain tensor, etc...

github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/subfunctions