

# TBarrier notebooks

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## 1 Overview *TBarrier* notebooks

The notebooks are stored in the *TBarrier* repository on github. By clicking on the underlined headers you are redirected to the corresponding folder on the github repository.

### 1. 2D

#### (a) data

##### i. AVISO

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/AVISO](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/AVISO)

##### ii. Bickley

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/Bickley](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/Bickley)

##### iii. Isotropic two-dimensional turbulence

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/Turbulence](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/data/Turbulence)

#### (b) demos

##### i. AdvectiveBarriers

- FTLE2D: (see *Hyperbolic LCS from the finite-time Lyapunov exponent*)

FTLE-field applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FTLE2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FTLE2D)

- PRA2D: (see *Elliptic LCSs from the polar rotation angle*)

PRA-field applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PRA2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PRA2D)

- LAVD2D: (see *Elliptic LCSs from the Lagrangian-averaged vorticity deviation and LAVD for 2D flows*)

LAVD-field applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/LAVD2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/LAVD2D)

- TRA2D: (see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

TRA-field applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TRA2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TRA2D)

- TSE2D: (see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

TSE-field applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TSE2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/TSE2D)

- HyperbolicLCS: (see *Local variational theory of hyperbolic LCS Hyperbolic LCSs in 2D flows*)

Hyperbolic LCS from tensorlines (shrinklines/stretchlines) of the Cauchy-Green strain tensor applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicLCS](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicLCS)

- HyperbolicOECS: (see *Shearless OECSs and objective saddle points in 2D flows*)

Hyperbolic OECS from (local) tensorlines launched from objective saddle points applied to AVISO.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicOECS](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/HyperbolicOECS)

- EllipticLCS: (see *Computing elliptic LCSs as closed null-geodesics*)

Elliptic LCS as closed null geodesics from the Cauchy-Green strain tensor applied to AVISO, Bickley and 2D turbulence data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticLCS](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticLCS)

- EllipticOECS: (*Computing elliptic LCSs as closed null-geodesics*)

Elliptic OECS as closed null geodesics from the rate of strain tensor applied to AVISO.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticOECS](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/EllipticOECS)

- FastTensorlineComputation:

Computation of tensorlines using the newly proposed algorithm with the re-parametrization of the eigenvectors from AVISO data. The Fast Tensorline Computation (FTC) is applied to the rate of strain tensor to (locally) extract hyperbolic OECS away from tensorline singularities.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FastTensorlineComputation](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/FastTensorlineComputation)

- PoincareMap2D: (see *Poincaré maps*)

Poincare map applied to periodic Bickley-jet.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PoincareMap2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/AdvectiveBarriers/PoincareMap2D)

## ii. DiffusionBarriers:

Here, we discuss diffusive Lagrangian and Eulerian transport barriers. The analysis and algorithm are similar to the ones discussed for the advective Barriers.

- DBS: (see *Unconstrained diffusion barriers in 2D flows*)

Diffusion Barrier Sensitivity (DBS) applied to AVISO and Bickley data

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/DBS](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/DBS)

- EllipticLagrangianDiffusionBarriers: (see *Unconstrained diffusion barriers in 2D flows*)

Elliptic Lagrangian Diffusion Barriers (using null-geodesics algorithm) extracted from AVISO and Bickley data.

- [github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticLagrangianDiffusionBarriers](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticLagrangianDiffusionBarriers)

- EllipticEulerianDiffusionBarriers: (see *Unconstrained diffusion barriers in 2D flows*)

Elliptic Eulerian Diffusion Barriers extracted from AVISO and Bickley data.

NOTE: There is no mention of this part in the book and no figures are provided. Stergios, computed, it however in the BarrierTool and it has been mentioned in the original paper (see remark 1 in <http://georgehaller.com/reprints/mbarriers.pdf>). That is why I thought I should also include it.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticEulerianDiffusionB](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/DiffusionBarriers/EllipticEulerianDiffusionB)

iii. StochasticBarriers: (see *Transport barriers in stochastic velocity fields*)

Barriers in stochastic velocity field extracted from AVISO data. We specifically focus on elliptic Lagrangian stochastic Barriers.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/StochasticBarriers](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/StochasticBarriers)

iv. ActiveBarriers: (see *2D homogeneous, isotropic turbulence*)

Active barriers extracted from the two-dimensional turbulence simulation. Lagrangian and Eulerian active barriers to vorticity and linear momentum extracted using the active FTLE, active PRA, active TSE, active TRA and the Hamiltonian based formulation.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers)

- aFTLE2D: (see *Active FTLE (aFTLE) and active TSE (aTSE)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aFTLE2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aFTLE2D)

- aTSE2D: (see *Active FTLE (aFTLE) and active TSE (aTSE)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTSE2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTSE2D)

- aTRA2D: (see *Active PRA (aPRA) and active TRA (aTRA)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTRA2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aTRA2D)

- aPRA2D: (see *Active PRA (aPRA) and active TRA (aTRA)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aPRA2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/aPRA2D)

- Hamiltonian: (see *Active transport barriers in general 2D Navier–Stokes flow*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/Hamiltonian](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/ActiveBarriers/Hamiltonian)

- Decompositions:

- SVD2D(see *Singular Value Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/SVD2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/SVD2D)

- PD2D(see *Polar Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/PD2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/PD2D)

- DPD2D(see *Dynamic Polar Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/DPD2D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/demos/Decompositions/DPD2D)

(c) subfunctions:

Folder containing frequently used functions to compute trajectories from two-dimensional velocity data, evaluate the gradient of the flowmap/velocity, the classic Cauchy-Green strain tensor, etc...

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/subfunctions](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/subfunctions)

## 2. 3D

(a) data

i. Arnold-Beltrami-Childress (ABC) flow:

Classic spatially periodic ABC flow. We consider both the steady and unsteady version.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/ABC](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/ABC)

ii. Turbulence:

Three dimensional turbulent channel flow data from John Hopkins Research Center

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/Turbulence](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/data/Turbulence)

(b) demos

i. AdvectiveBarriers

- FTLE3D(see *Hyperbolic LCS from the finite-time Lyapunov exponent*)

FTLE-field applied to ABC data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/FTLE3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/FTLE3D)

- TSE3D(see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

TSE field applied to ABC and turbulent channel flow data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TSE3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TSE3D)

- TRA3D(see *Quasi-objective, single-trajectory diagnostics for transport barriers*)

TRA field applied to ABC and turbulent channel flow data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TRA3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/TRA3D)

- LAVD3D: (see *Elliptic LCSs from the Lagrangian-averaged vorticity deviation and LAVD for 3D flows*)

LAVD field applied to ABC data and turbulent channel flow data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/LAVD3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/LAVD3D)

- UnifiedLCSTheory: (see *Unified variational theory of elliptic and hyperbolic LCS in 3D*)

Extract LCS from the  $\xi_2$  eigenvector field of the Cauchy-Green strain tensor from the ABC data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/UnifiedLCSTheory](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/UnifiedLCSTheory)

- PoincareMap3D:

Classic Poincare map applied to ABC data.

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/PoincareMap3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/AdvectiveBarriers/PoincareMap3D)

## ii. ActiveBarriers

Active barriers extracted from the three-dimensional channel flow data (see *3D turbulent channel flow*) and/or from the ABC data

(see *Quasi-objective, single-trajectory diagnostics for transport barriers*).

Lagrangian and Eulerian active barriers to vorticity and linear momentum extracted using the active FTLE, active PRA, active TSE and active TRA.

- aFTLE3D(see *Active FTLE (aFTLE) and active TSE (aTSE)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aFTLE3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aFTLE3D)

- aTSE3D(see *Active FTLE (aFTLE) and active TSE (aTSE)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTSE3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTSE3D)

- aPRA3D(see *Active PRA (aPRA) and active TRA (aTRA)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aPRA3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aPRA3D)

- aTRA3D(see *Active PRA (aPRA) and active TRA (aTRA)*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTRA3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/ActiveBarriers/aTRA3D)

### iii. Decompositions:

- SVD3D(see *Singular Value Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/SVD3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/SVD3D)

- PD3D(see *Polar Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/PD3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/PD3D)

- DPD3D(see *Dynamic Polar Decomposition*)

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/DPD3D](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/demos/Decompositions/DPD3D)

### (c) subfunctions:

Folder containing frequently used functions to compute trajectories from two-dimensional velocity data, evaluate the gradient of the flowmap/velocity, the classic Cauchy-Green strain tensor, etc...

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/subfunctions](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/3D/subfunctions)

The SVD, PD and DPD for 2D data is under:

[github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/subfunctions/Decompositions](https://github.com/EncinasBartos/TBarrier/tree/main/TBarrier/2D/subfunctions/Decompositions)