ENGG1300

Notes for HKU · Spring 2024

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MORE notes on my website!

1 Vectors

Basics

Vector can be represented by $\vec{F}$ or $\boldsymbol{F}$. The magnitude can be represented by $|F|$ or F.

Unit vectors

They are vectors with magnitude 1. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

1.1 Coplanar vectors

Cartesian vector notation

In two dimensions, the Cartesian unit vectors $\boldsymbol{i}, \boldsymbol{j}$ are used to designate the directions of the x and y axes respectively.

$$F = F_x\boldsymbol{i} + F_y\boldsymbol{j}$$

Where $F_{x/y}$ is the x/y component of F. And to find the x/y components we can use trigonometry:

$$F_x = F\sin\theta, \quad F_y = F\cos\theta$$

Resultant force

The resultant force F_R can be found by the sum of the components of F:

$$F_R = \sum F$$

In Cartesian form, it's the same as adding all the terms together: $F_R = (F_{x1} + F_{x2})\boldsymbol{i} + (F_{y1} + F_{y2})\boldsymbol{j}$

Orientation of vector

We always consider the angle between F & F_x. It can be found by $\theta = \tan^{-1}\frac{F_y}{F_x}$.

Magnitude of forces

The magnitude will simply be the square root of the sum of squared components of the force:

$$|F| = \sqrt{F_x^2 + F_y^2 + \ldots}$$

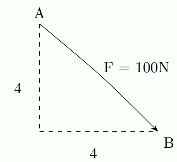
Moving vectors by their position vectors

We can move a force F to a point using its position vector r (pointing to their tail), using the following definition:

$$\vec{F} = |F| \times \hat{F} = |F| \times \frac{r}{|r|}$$

The position vectors are in Cartesian form, so the moved vectors will also be in Cartesian form.

Consider the following graph:



First, we consider r_{AB}. From the graph:

$$r_{\vec{AB}} = 4\boldsymbol{i} + 4\boldsymbol{j}$$

Then, we find the magnitude of r_{AB}:

$$|r_{AB}| = \sqrt{4^2 + 4^2} = 5.65\ldots$$

Finally, we apply our formula for F:

$$\begin{aligned}\vec{F} = |F| \times \frac{r_{AB}}{|r_{AB}|} &= 100 \times (\frac{r_x}{5.65}\boldsymbol{i} + \frac{r_y}{5.65}\boldsymbol{j}) \\ &= 100 \times (\frac{4}{5.65}\boldsymbol{i} + \frac{4}{5.65}\boldsymbol{j}) \\ &= 70.7\boldsymbol{i} + 70.7\boldsymbol{j}N\end{aligned}$$

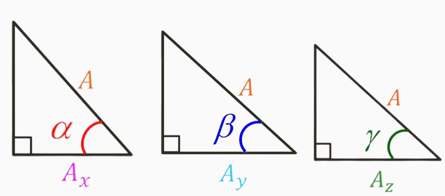
1.2 Vectors in 3D

The concepts above can be extended to 3D simply by adding another variable to the system.

Coordinate direction angles in 3D

The direction of A is defined by the *coordinate direction angles*: α, β, γ, which are measured between the tail of A and the positive x, y, z axes.

$$\alpha = \cos^{-1}\frac{A_x}{A}, \quad \beta = \cos^{-1}\frac{A_y}{A}, \quad \gamma = \cos^{-1}\frac{A_z}{A}$$



2 Moment of forces

Definition of moment

The moment of a force is a measure of its *tendency* to cause a body to rotate about a specific point. The moment about a point O, when F is applied a distance d from the point is:

$$M_O = F \times d$$

Keep in mind that *positive* moment is *anti-clockwise*.

2.1 Coplanar / 2D moment

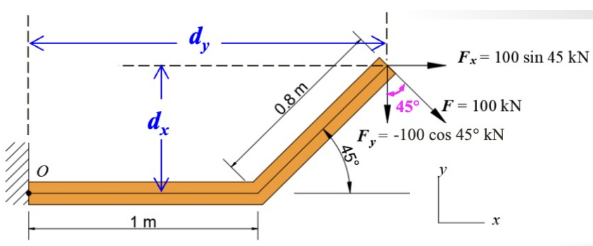
Resultant moments

The resultant moment is the **sum** of all moments present on the point, given by:

$$M_R = \sum M_O$$

2.1.1 The moment of a non-linearly attached force

One simple way is to find the *components* of the force, and sum their individual moments together. The following is a simple example:



After finding the component forces of F, we can deduce the resultant moment to be:

$$\begin{aligned} M_O &= F_x \times 0.8 \sin 45 \deg + F_y \times (1 + 0.8 \cos 45 \deg) \\ &= -150.7kNm \end{aligned}$$

2.2 Non-coplanar / 3D moment

Moments in a 3D system

Consider position vector $\vec{r}$ drawn from O to any point on the *line of action* of F. The moment can hence be given by:

$$M_O = r \times F$$

Finding the moment via cross products Cartesian vectors

The cross product C given by A and B is:

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = C$$

The cross-product for vectors going in the *same direction* is 0. (i.e. $n\boldsymbol{k} \times m\boldsymbol{k} = 0$)

Resultant moments

The resultant moment is simply the **sum** of *couple moments* and moments of forces:

$$(M_R)_O = \sum M_O + \sum M$$

You can interpret $(M_R)_O$ as the resultant moment about point O.

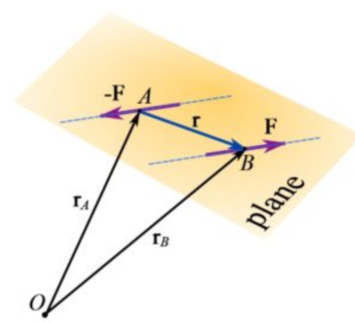
2.3 Couple moments

Couples are *two parallel forces* that have the same magnitude but have *opposite directions*, separated by a *perpendicular distance* d. The magnitude of the moment is given by:

$$M = Fd$$

Notice that there's no point mentioned so far. For couple moment, it is **always the same about any point**. Let's assume for any point O (refer to graph), the moment is:

$$\begin{aligned} M_O &= r_B \times F + r_A \times -F \\ &= (r_B - r_A) \times F \\ &= r \times F \quad \text{which is independent of } O \end{aligned}$$



Hence, we can say that couple moments are **free vectors**.

3 Equilibrium of rigid bodies

Conditions for rigid body equilibriums

A rigid body is in equilibrium if:

$$\sum F = 0, \quad \sum M_O = 0$$

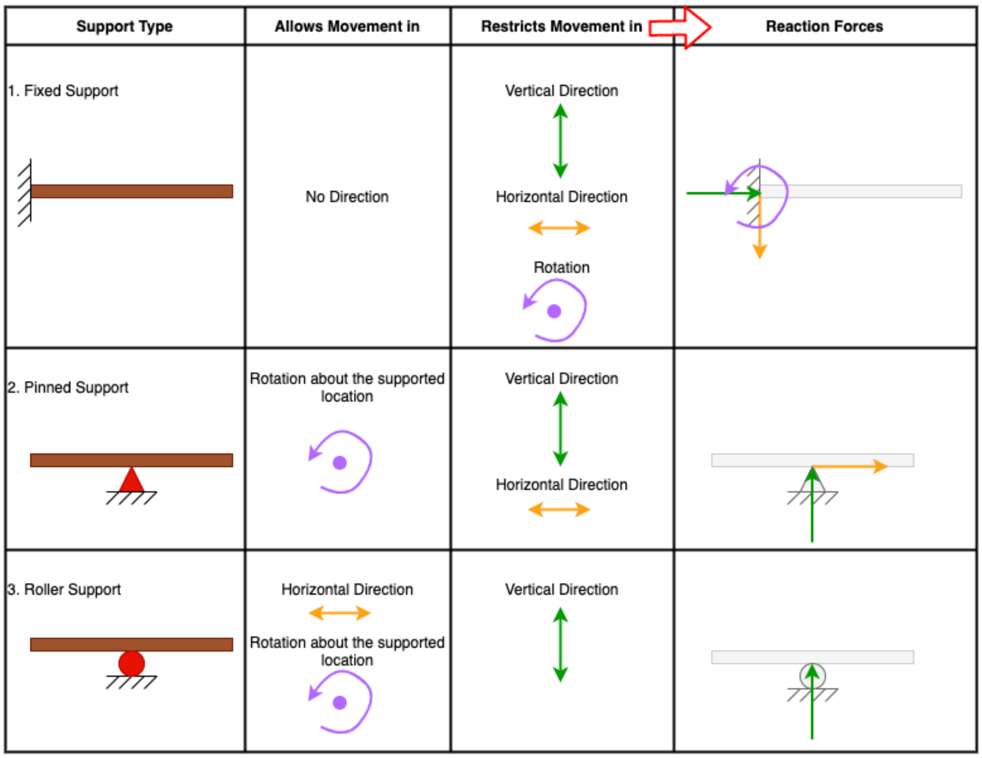
Where O is any point.

3.1 Free body diagrams

The procedure to draw a free body diagram (FBD) is as follows:

1. Draw the outlined shape of the rigid body.
2. Show all forces acting on the rigid body. (Weight, reaction, friction, etc.)
3. Identification and labelling.

3.2 Support reactions

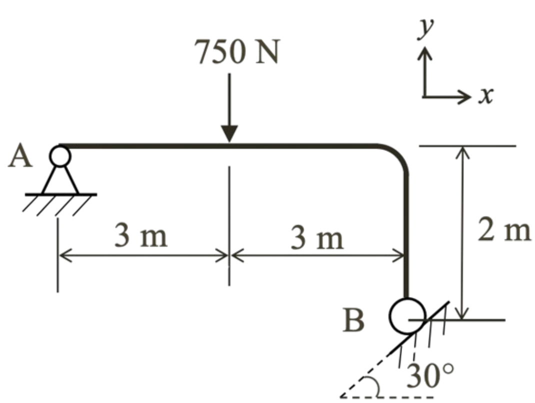


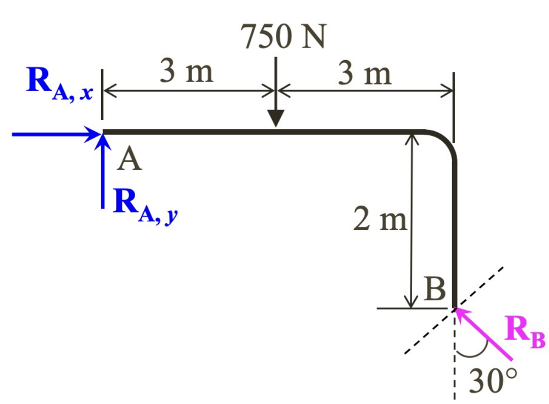
Support Type	Allows Movement in	Restricts Movement in	Reaction Forces
1. Fixed Support	No Direction	Vertical Direction Horizontal Direction Rotation	
2. Pinned Support	Rotation about the supported location	Vertical Direction Horizontal Direction	
3. Roller Support	Horizontal Direction Rotation about the supported location	Vertical Direction	

Source: https://clearcalcs.com/blog/support-connection-types

3.3 Finding forces in equilibrium systems - an example

Consider this equilibrium system. Let's find the reaction at supports A and B. First, we draw it's FBD as according to support reactions:





Then, we start by applying equilibrium equations. We consider the moment about A to find R_B, then find the component support forces at A:

$$\begin{aligned}\sum M_A &= 0\\ 6R_{Bx} &= 2R_{By} + 3 \times 750\\ 6R_B \sin 60\deg &= 2R_B \cos 60\deg + 3 \times 750\\ R_B &= 536.2N\end{aligned}$$

We know at any point the component resultant forces must be 0, so:

$$\begin{aligned}&\sum F_x = 0\\ &R_{Ax} = 536.2 \cos 60\deg\\ &R_{Ax} = 268.1N\\ \\ &R_{Ay} = 750 - 536.2 \sin 60\deg\\ &R_{Ay} = 285.6N\end{aligned}$$