ENGG1300

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1 Vectors

Basics

Vector can be represented by $\vec{F}$ or $\boldsymbol{F}$. The magnitude can be represented by $|F|$ or F.

Unit vectors

They are vectors with magnitude 1. $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

1.1 Coplanar vectors

Cartesian vector notation

In two dimensions, the Cartesian unit vectors $\boldsymbol{i}, \boldsymbol{j}$ are used to designate the directions of the x and y axes respectively.

$$F = F_x\boldsymbol{i} + F_y\boldsymbol{j}$$

Where $F_{x/y}$ is the x/y component of F. And to find the x/y components we can use trigonometry:

$$F_x = F\sin\theta, \quad F_y = F\cos\theta$$

Resultant force

The resultant force F_R can be found by the sum of the components of F:

$$F_R = \sum F$$

In Cartesian form, it's the same as adding all the terms together: $F_R = (F_{x1} + F_{x2})\boldsymbol{i} + (F_{y1} + F_{y2})\boldsymbol{j}$

Orientation of vector

We always consider the angle between F & F_x. It can be found by $\theta = \tan^{-1}\frac{F_y}{F_x}$.

Magnitude of forces

The magnitude will simply be the square root of the sum of squared components of the force:

$$|F| = \sqrt{F_x^2 + F_y^2 + \dots}$$

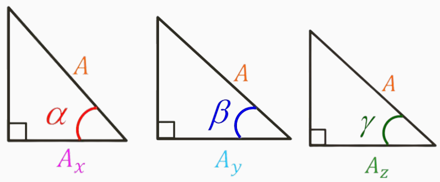
1.2 Vectors in 3D

The concepts above can be extended to 3D simply by adding another variable to the system.

Coordinate direction angles in 3D

The direction of A is defined by the *coordinate direction angles*: α, β, γ, which are measured between the tail of A and the positive x, y, z axes.

$$\alpha = \cos^{-1} \frac{A_x}{A}, \quad \beta = \cos^{-1} \frac{A_y}{A}, \quad \gamma = \cos^{-1} \frac{A_z}{A}$$



1.3 Moving vectors

Moving vectors by their position vectors

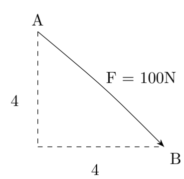
We can move a force F to a point using its position vector r (pointing to their tail), using the following definition:

$$\vec{F} = |F| \times \hat{F} = |F| \times \frac{r}{|r|}$$

The position vectors are in Cartesian form, so the moved vectors will also be in Cartesian form.

1.3.1 Simple example

Consider the following graph:



First, we consider r_{AB}. From the graph:

$$r_{\vec{AB}} = 4\boldsymbol{i} + 4\boldsymbol{j}$$

Then, we find the magnitude of r_{AB}:

$$|r_{AB}| = \sqrt{4^2 + 4^2} = 5.65\ldots$$

Finally, we apply our formula for F:

$$\begin{aligned}\vec{F} = |F| \times \frac{r_{AB}}{|r_{AB}|} &= 100 \times (\frac{r_x}{5.65}\boldsymbol{i} + \frac{r_y}{5.65}\boldsymbol{j}) \\ &= 100 \times (\frac{4}{5.65}\boldsymbol{i} + \frac{4}{5.65}\boldsymbol{j}) \\ &= 70.7\boldsymbol{i} + 70.7\boldsymbol{j}N\end{aligned}$$

2 Moment of forces

Definition of moment

The moment of a force is a measure of its *tendency* to cause a body to rotate about a specific point. The moment about a point O, when F is applied a distance d from the point is:

$$M_O = F \times d$$

Keep in mind that *positive* moment is *anti-clockwise*.

2.1 Coplanar / 2D moment

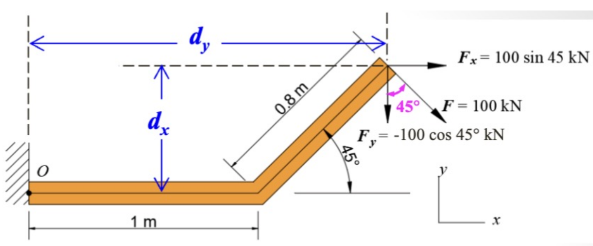
Resultant moments

The resultant moment is the **sum** of all moments present on the point, given by:

$$M_R = \sum M_O$$

2.1.1 The moment of a non-linearly attached force

One simple way is to find the *components* of the force, and sum their individual moments together. The following is a simple example:



After finding the component forces of F, we can deduce the resultant moment to be:

$$\begin{aligned} M_O &= F_x \times 0.8 \sin 45 \deg + F_y \times (1 + 0.8 \cos 45 \deg) \\ &= -150.7 kNm \end{aligned}$$

2.2 Non-coplanar / 3D moment

Moments in a 3D system

Consider position vector $\vec{r}$ drawn from O to any point on the *line of action* of F. The moment can hence be given by:

$$M_O = r \times F$$

Finding the moment via cross products Cartesian vectors

The cross product C given by A and B is:

$$A \times B = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = C$$

The cross-product for vectors going in the *same direction* is 0. (i.e. $n\boldsymbol{k} \times m\boldsymbol{k} = 0$)

Resultant moments

The resultant moment is simply the **sum** of *couple moments* and moments of forces:

$$(M_R)_O = \sum M_O + \sum M$$

You can interpret $(M_R)_O$ as the resultant moment about point O.

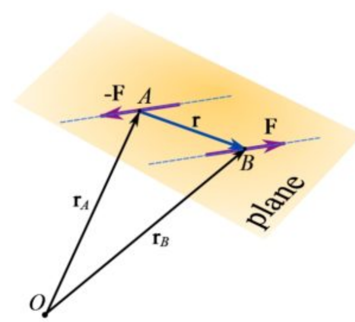
2.3 Couple moments

Couples are *two parallel forces* that have the same magnitude but have *opposite directions*, separated by a *perpendicular distance* d. The magnitude of the moment is given by:

$$M = Fd$$

Notice that there's no point mentioned so far. For couple moment, it is **always the same about any point**. Let's assume for any point O (refer to graph), the moment is:

$$\begin{aligned} M_O &= r_B \times F + r_A \times -F \\ &= (r_B - r_A) \times F \\ &= r \times F \quad \text{which is independent of } O \end{aligned}$$



Hence, we can say that couple moments are **free vectors**.

3 Equilibrium of rigid bodies