Thomas Clarke
Quantum Computing Technical Foundations
September 7, 2022

# Assignment 1: Maths for Quantum Computing

Assignment Due: Monday 12th September

Solutions can be handwritten on a separate sheet of paper, typed or done on a tablet. You may print this, write the solutions on it, and then scan and upload it.
Send the completed assignment to tclarke@asesoftware.com If you have any questions or difficulties, please do reach out to the same email.
Challenge Questions are Optional

## 1. Complex Numbers

**Question 1.** Complex number algebra

Simplify the following into the form $a + bi$

1) $(6 + 4i) + (3 + 5i)$
2) $(-6 + 4i) + (-3 + 5i)$
3) $i(2 + 3i)$
4) $(6 + 4i)(6 - 4i)$

**Question 2.** Complex conjugate

Find the complex conjugate for your answers to the previous question
Hint: the complex conjugate of $z = a + ib$ is $z^* = a - ib$

**Question 3.** Euler's identity

Complex numbers can be represented using Euler's Formula as

$$a + bi = e^{i\varphi}$$

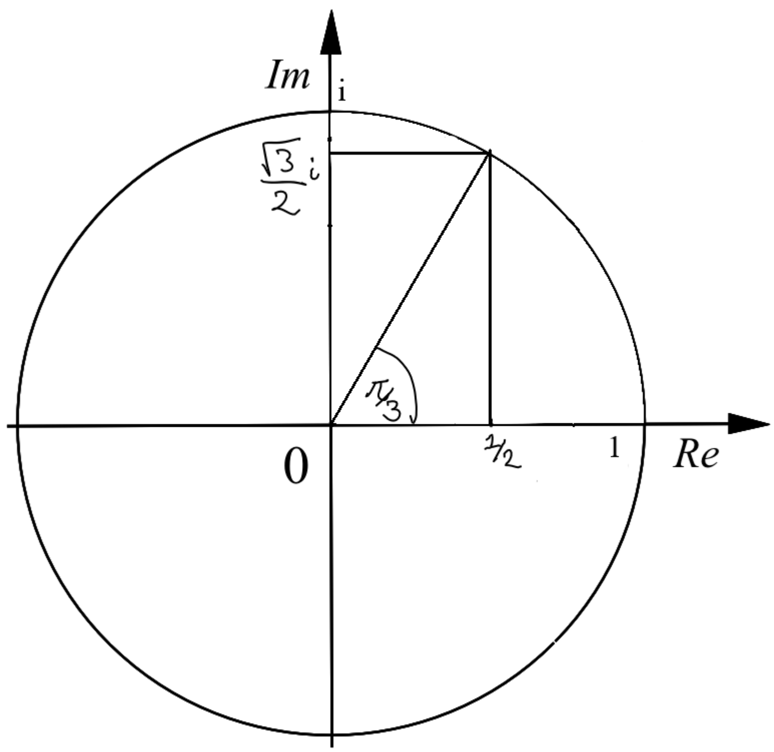
Where $a$, $b$ are real numbers that satisfy $a^2 + b^2 = 1$, and $\varphi$ is an angle in radians $0 < \varphi \leq 2\pi$.
The angle $\varphi$ is related to $a$ & $b$ through the following relations:

$$a = \sin(\varphi), b = \cos(\varphi)$$

For example, if $\varphi = \pi/3$

$e^{i\pi/3} = \cos(\pi/3) + i\sin(\pi/3)$
$= \frac{1}{2}(1 + i\sqrt{3})$

This can be represented on the Argand diagram as




Represent the following in the form $a + ib$
1) $e^{i0}$
2) $e^{i\pi}$
3) $e^{i\frac{\pi}{2}}$
4) $e^{i\frac{\pi}{4}}$

Sketch all of these on the Argand plane to check your answers

**Question 4.** Challenge Question 1: Third root of unity

Multiply the following complex numbers

$$e^{i2\pi\frac{1}{3}} \times e^{i2\pi\frac{2}{3}} \times e^{i2\pi\frac{3}{3}}$$

This is extremely easy if you use indices rules.

**Question 5.** Vector addition

Compute the following:

1) $\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}3\\4i\end{bmatrix}$

2) $\frac{1}{5}\begin{bmatrix}3\\7\end{bmatrix} - \begin{bmatrix}1\\7\end{bmatrix}$

3) $\begin{bmatrix}9 & 8\end{bmatrix} - \begin{bmatrix}7 & 6\end{bmatrix}$

**Question 6.** Conjugate Transpose

Write down the conjugate transpose for your answers to the previous question

Reminder: for any column vector $|a\rangle = \begin{bmatrix}a_0\\a_1\end{bmatrix}$, the conjugate transpose is $\langle a| = \begin{bmatrix}a_0* & a_1*\end{bmatrix}$ Where $*$ indicates the complex conjugate.The same process goes the other way from row vector back to column.

**Question 7.** Inner products

Compute the following inner product $\langle a|b\rangle$

Where $|a\rangle = \begin{bmatrix}1\\-i\end{bmatrix}$, $|b\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

And verify $\langle b|b\rangle = 1$ [1]

**Question 8.** Matrix-vector products

Compute the following matrix-vector products

1) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$

2) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$

3) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix}$

Where $\alpha, \beta$ can be any complex numbers

**Question 9.** Challenge Question 2: Entanglement

Compute the matrix-vector product of $CNOT_{b,a}\left|0+\right\rangle_{a,b}$ [2] $= |\Phi^+\rangle_{b,a}$ (i.e. compute the vector $|\Phi^+\rangle$ Where

$$CNOT_{a,b} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix}; |0+\rangle \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

Further challenge:

Express your answer in terms of the vectors

[1] Originally this read $\langle a|a\rangle$, but was changed because $|a\rangle$ is not normalised

[2] Here b is the control qubit, a is the target qubit. CNOT is usually given with a as control b as target, but is consistent with the scheme IBM use. It is better explained in chapter 6.

$$|10\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

This matrix-vector product refers to the process of generating entanglement between two qubits. It will be covered in Week 4: Multiple Qubits

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