Thomas Clarke
Quantum Computing Technical Foundations
September 7, 2022

Assignment 1: Maths for Quantum Computing

Assignment Due: Monday 12th September

Solutions can be handwritten on a separate sheet of paper, typed or done on a tablet. You may print this, write the solutions on it, and then scan and upload it.
Send the completed assignment to tclarke@asesoftware.com If you have any questions or difficulties, please do reach out to the same email.
Challenge Questions are Optional

1. Complex Numbers

Question 1. Complex number algebra

Simplify the following into the form $a + bi$

1) $(6 + 4i) + (3 + 5i)$
2) $(-6 + 4i) + (-3 + 5i)$
3) $i(2 + 3i)$
4) $(6 + 4i)(6 - 4i)$

Question 2. Complex conjugate

Find the complex conjugate for your answers to the previous question
Hint: the complex conjugate of $z = a + ib$ is $z^* = a - ib$

Question 3. Euler's identity

Complex numbers can be represented using Euler's Formula as

$$a + bi = e^{i\varphi}$$

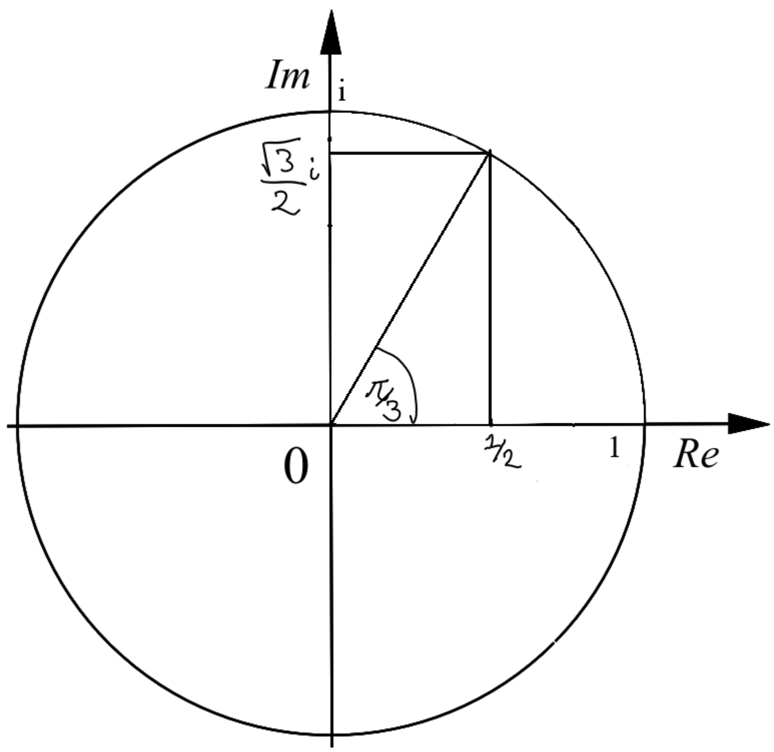
Where a, b are real numbers that satisfy $a^2 + b^2 = 1$, and φ is an angle in radians $0 < \varphi \leq 2\pi$.
The angle φ is related to a & b through the following relations:

$$a = \sin(\varphi), b = \cos(\varphi)$$

For example, if $\varphi = \pi/3$

$e^{i\pi/3} = \cos(\pi/3) + i\sin(\pi/3)$
$= \frac{1}{2}(1 + i\sqrt{3})$

This can be represented on the Argand diagram as



Represent the following in the form $a + ib$

1) e^{i0}
2) $e^{i\pi}$
3) $e^{i\frac{\pi}{2}}$
4) $e^{i\frac{\pi}{4}}$

Sketch all of these on the Argand plane to check your answers

Question 4. Challenge Question 1: Third root of unity

Multiply the following complex numbers

$$e^{i2\pi\frac{1}{3}} \times e^{i2\pi\frac{2}{3}} \times e^{i2\pi\frac{3}{3}}$$

This is extremely easy if you use indices rules.

Question 5. Vector addition

Compute the following:

1) $\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}3\\4i\end{bmatrix}$

2) $\frac{1}{5}\begin{bmatrix}3\\7\end{bmatrix}-\begin{bmatrix}1\\7\end{bmatrix}$

3) $\begin{bmatrix}9 & 8\end{bmatrix}-\begin{bmatrix}7 & 6\end{bmatrix}$

Question 6. Conjugate Transpose

Write down the conjugate transpose for your answers to the previous question

Reminder: for any column vector $|a\rangle = \begin{bmatrix}a_0\\a_1\end{bmatrix}$, the conjugate transpose is $\langle a| = \begin{bmatrix}a_0* & a_1*\end{bmatrix}$ Where $*$ indicates the complex conjugate.The same process goes the other way from row vector back to column.

Question 7. Inner products

Compute the following inner product $\langle a|b\rangle$

Where $|a\rangle = \begin{bmatrix}1\\-i\end{bmatrix}$, $|b\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

And verify $\langle b|b\rangle = 1$ [1]

Question 8. Matrix-vector products

Compute the following matrix-vector products

1) $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$

2) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$

3) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix}$

Where α, β can be any complex numbers

Question 9. Challenge Question 2: Entanglement

Compute the matrix-vector product of $CNOT_{b,a}\,|0+\rangle_{a,b}$ [2] $= |\Phi^+\rangle_{b,a}$ (i.e. compute the vector $|\Phi^+\rangle$ Where

$$CNOT_{a,b} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\end{bmatrix};\ |0+\rangle\ \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\\0\end{bmatrix}$$

Further challenge:

Express your answer in terms of the vectors

[1] Originally this read $\langle a|a\rangle$, but was changed because $|a\rangle$ is not normalised

[2] Here b is the control qubit, a is the target qubit. CNOT is usually given with a as control b as target, but is consistent with the scheme IBM use. It is better explained in chapter 6.

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix} \; |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix} \; |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

This matrix-vector product refers to the process of generating entanglement between two qubits. It will be covered in Week 4: Multiple Qubits

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