MAT 458-Design of Experiments

(II) Simple comparative experiments

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Outline

1. Introduction

2.Basic statistical concepts and Sampling distributions

3. Simple comparative experiments

4. Paired comparison design

1. Introduction

We consider experiments to compare two conditions (sometimes called treatments). These are often called simple comparative experiments.

For the above discussion, we first need a review of several basic statistical concepts and sampling distributions.

2. Basic statistical concepts and Sampling distributions

In the experiment, the experimental error (or simply error) is a statistical error, meaning that it arises from variation that is uncontrolled and generally unavoidable.

The presence of error or noise implies that the response variable is a random variable. A random variable may be either **discrete** or **continuous**.

Let's review some basic statistical concepts and sampling distributions.

3. Simple comparative experiments

We will use a procedure known as the two-sample t-test.

Statistical hypothesis testing is a useful framework for many experimental situations.

The Z-test would work perfectly if we knew the two population variances.

Since we usually don't know the true population variances, what would happen if we just plugged in the sample variances?

The answer is that if the sample sizes were large enough (say both $n > 40$) the Z-test would work just fine. It is a good large-sample test for the difference in means.

What if the sample size is small?

It turns out that if the sample size is small we can no longer use the N(0,1) distribution as the reference distribution.

Let's review

(i) Hypothesis testing when the variances are known.

(ii) Hypothesis testing when the variances are unknown and the sample sizes are large.

(iii) Hypothesis testing when the variances are unknown and at least one of the sample sizes is small.

(iv) Hypothesis testing when the variances are equal and unknown.

(v) Hypothesis tests on variances (F tests)

P-value

Importance of the t-test

Provides an objective framework for simple comparative experiments.

Could be used to test all relevant hypotheses in a two-level factorial design.

Completely randomized experiment design (CRD):

For comparing mean response to two treatments, a pool of experiment unit is assigned randomly to one or other treatments.

The CRD model:

$$Y_{ij} = \mu_i + \varepsilon_{ij}\,, \quad \begin{cases} i = 1, 2 \\ j = 1, 2, \cdots, n_i \end{cases}$$

Confidence Intervals (CI's)

Hypothesis testing gives an objective statement concerning the difference in means, but it doesn't specify how "different" they are. We can use confidence intervals.

Power and Sample size issues

Checking assumptions

4. Paired comparison design

Paired experiments — this is an example of blocking.

Let's review Paired comparison experiments.

The Model is

$$Y_{ij} = \mu_i + \beta_j + \varepsilon_{ij}\,, \quad \{ \begin{array}{l} i = 1, 2 \\ j = 1, 2, \cdots, n \end{array}$$