

# Wordle is NP-Complete

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## 1 Set Cover

Let  $k \geq 1$ ,  $\mathcal{U} = \{t^i\}_{i=1}^k$  and  $\mathcal{S}$  a set of sets such that  $\mathcal{U} = \bigcup_{S_i \in \mathcal{S}} S_i$ . Does there exists a subset  $S'$  of  $\mathcal{S}$ , such that  $|S'| \leq n < |\mathcal{S}|$  and  $\mathcal{U} = \bigcup_{S_i \in S'} S_i$ ?

## 2 Wordle

Let  $\mathcal{A} = \{\alpha_i\}_{i=1}^l$  be an alphabet,  $\mathcal{A}^m = \{\alpha_{i_1} \cdots \alpha_{i_m} | 1 \leq j \leq m \text{ \& } 1 \leq i_j \leq l\}$ ,  $S \subseteq \mathcal{A}^m$ ,  $t \in S$ , and  $f_t : S \rightarrow \{0, 1, 2\}^m$  is a scoring function, defined as  $f_t(w) = \langle f_t^1(w), \dots, f_t^m(w) \rangle$  where

$$f_t^i(w) = \begin{cases} 0 & w_i = t_i \\ 1 & w_i \neq t_i \text{ \& } \exists j(w_i = t_j \text{ \& } \forall k < i(w_k = w_i \rightarrow \exists r < j(w_k = t_r))) \\ 2 & \text{otherwise} \end{cases}$$

Does there exists a subset  $S'$  of  $S \setminus \{t\}$  such that  $|S'| \leq n < |S| - 1$  and  $t$  is the only word not excluded by the set of constraints  $\{f_t(w) | w \in S'\}$ ?

**Definition 1** For  $w, w' \in S$ , we say that  $f_t(w)$  excludes a word  $w'$  if one of the following holds:

- $f_t^i(w) = 0$  and  $w_i \neq w'_i$ .
- $f_t^i(w) = 1$  and  $w_i \notin w'$ .
- $f_t^i(w) = 2$ ,  $\forall j(w_i = w_j \rightarrow f_t^j(w) = 2)$  and  $w_i \in w'$ .
- $f_t^i(w) = 2$ ,  $\exists j(w_i = w_j \text{ \& } f_t^j(w) < 2 \text{ \& } |\{j | w_i = w_j\}| \leq |\{j | w_i = w'_j\}|)$ .

## 3 Reduction from Set Cover to Wordle

Let  $\mathcal{A} = \{\alpha_i\}_{i=1}^{|\mathcal{S}|+|\mathcal{U}|+1}$ ,  $S = \{\alpha_1 \cdots \alpha_{|\mathcal{S}|+1}\} \cup \bigcup_{1 \leq j \leq k} w^{t^j}$ , where

- if  $t^j \in S_i$ , then  $w_i^{t^j} = \alpha_i$
- if  $t^j \notin S_i$ , then  $w_i^{t^j} = \alpha_{j+|\mathcal{S}|+1}$