Wordle is NP-Complete

David Cerna

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1 Set Cover

Let $\mathcal{U}$ be a set of objects and $\mathcal{S}$ a set of sets such that $\mathcal{U} = \bigcup_{S_i \in \mathcal{S}} S_i$. Does there exists a subset S' of $\mathcal{S}$, such that $|S'| \leq n < |\mathcal{S}|$ and $\mathcal{U} = \bigcup_{S_i \in S'} S_i$.

2 Wordle

Let $\mathcal{A} = \{\alpha_i\}_{i=1}^{l}$ be an alphabet, $\mathcal{A}^m = \{\alpha_{i_1} \cdots \alpha_{i_m} | 1 \leq j \leq m \ \& \ 1 \leq i_j \leq l\}$, $S \subseteq A^m$, $t \in S$, and $f_t : S \rightarrow \{0,1,2\}^m$ is a scoring function, defined as $f_t(w) = \left\langle f_t^1(w), \cdots, f_t^m(w) \right\rangle$ where

$$f_t^i(w) = \begin{cases} 0 & w_i = t_i \\ 1 & w_i \neq t_i \ \& \ \exists j(w_i = t_j \ \& \ \forall k < i(w_k = w_i \rightarrow \exists r < j(w_k = t_r))) \\ 2 & otherwise \end{cases}$$

Does there exists a subset S' of $S \setminus \{t\}$ such that $|S'| \leq n < |S| - 1$ and t is the only word not excluded by the set of constraints $\{f_t(w) | w \in S'\}$.

Definition 1 *For $w, w' \in S$, we say that $f_t(w)$ excludes a word w' if one of the following holds:*

- $f_t^i(w) = 0$ *and* $w_i \neq w_i'$.
- $f_t^i(w) = 1$ *and* $w_i \notin w'$.
- $f_t^i(w) = 2$, $\forall j(w_i = w_j \rightarrow f_t^j(w) = 2)$ *and* $w_i \in w'$.
- $f_t^i(w) = 2$, $\exists j(w_i = w_j \ \& \ f_t^j(w) < 2 \ \& \ |\{j | w_i = w_j\}| \leq |\{j | w_i = w_j'\}|)$.

3 Reduction from Set Cover to Wordle

Let $\mathcal{A} = \{\alpha_i\}_{i=1}^{|\mathcal{S}|+|\mathcal{U}|+1}$, $S = \{\alpha_1 \cdots \alpha_{|\mathcal{S}|+1}\} \cup \bigcup_{t \in \mathcal{U}} w_t$, where

- if $t \in S_i$, then $w_i^t = \alpha_i$
- if $t \notin S_i$, then $w_i^t = \beta$ for some $\beta \in \mathcal{A} \setminus \{\alpha_1 \cdots \alpha_{|\mathcal{S}|+1}\}$
- for $t, r \in \mathcal{U}$ then $w_t = w_r$ iff $t = r$.