

Wordle is NP-Complete

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1 Set Cover

Let $k \geq 1$, $\mathcal{U} = \{t^i\}_{i=1}^k$ and \mathcal{S} a set of sets such that $\mathcal{U} = \bigcup_{S_i \in \mathcal{S}} S_i$. Does there exists a subset S' of \mathcal{S} , such that $|S'| \leq n < |\mathcal{S}|$ and $\mathcal{U} = \bigcup_{S_i \in S'} S_i$?

2 Wordle

Let $\mathcal{A} = \{\alpha_i\}_{i=1}^l$ be an alphabet, $\mathcal{A}^m = \{\alpha_{i_1} \cdots \alpha_{i_m} | 1 \leq j \leq m \text{ \& } 1 \leq i_j \leq l\}$, $S \subseteq \mathcal{A}^m$, $t \in S$, and $f_t : S \rightarrow \{0, 1, 2\}^m$ is a scoring function, defined as $f_t(w) = \langle f_t^1(w), \dots, f_t^m(w) \rangle$ where

$$f_t^i(w) = \begin{cases} 0 & w_i = t_i \\ 1 & w_i \neq t_i \text{ \& } \exists j(w_i = t_j \text{ \& } \forall k < i(w_k = w_i \rightarrow \exists r < j(w_k = t_r))) \\ 2 & \text{otherwise} \end{cases}$$

Does there exists a subset S' of $S \setminus \{t\}$ such that $|S'| \leq n < |S| - 1$ and t is the only word not excluded by the set of constraints $\{f_t(w) | w \in S'\}$?

Definition 1 For $w, w' \in S$, we say that $f_t(w)$ excludes a word w' if one of the following holds:

- $f_t^i(w) = 0$ and $w_i \neq w'_i$.
- $f_t^i(w) = 1$ and $w_i \notin w'$.
- $f_t^i(w) = 2$, $\forall j(w_i = w_j \rightarrow f_t^j(w) = 2)$ and $w_i \in w'$.
- $f_t^i(w) = 2$, $\exists j(w_i = w_j \text{ \& } f_t^j(w) < 2 \text{ \& } |\{j | w_i = w_j\}| \leq |\{j | w_i = w'_j\}|)$.

3 Reduction from Set Cover to Wordle

Let $\mathcal{A} = \{\alpha_i\}_{i=1}^{|\mathcal{S}|+|\mathcal{U}|+1}$, $S = \{\alpha_1 \cdots \alpha_{|\mathcal{S}|+1}\} \cup \bigcup_{1 \leq j \leq k} w^{t^j}$, where

- if $t^j \in S_i$, then $w_i^{t^j} = \alpha_i$
- if $t^j \notin S_i$ or $i = |\mathcal{S}| + 1$, then $w_i^{t^j} = \alpha_{j+|\mathcal{S}|+1}$