ADVERSARIAL SEARCH

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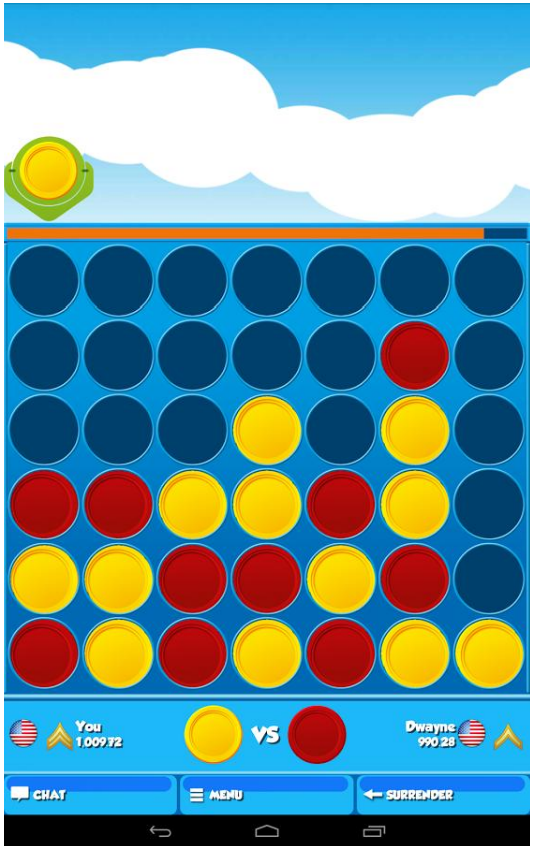
Objectives

Learning how to act when the other agents are acting against us

Games



Games vs. Search

- Search – no adversary
  - Examples: path planning, scheduling activities
- Games – adversary
  - **Unpredictable opponent(s)**
  - Solution is strategy (strategy specifies move for every possible opponent reply).
  - **Time limits force an *approximate* solution**
  - **Inefficiency is intolerable**

Zero Sum Game

- Total pay off to all players is the same for every instance of games
- Chess/Tic-tac-toe:
  - Win -> 1
  - Lose -> -1
  - Draw -> 0

Assumptions

- **Two agents acting alternately**
- **Utility values for each agent are the opposite of the other**
- Deterministic
- Fully observable
- Can generalize to stochastic games, multiple players, non zero-sum, etc
- In game theory terms:
  - "Deterministic, turn-taking, zero-sum games of perfect information"

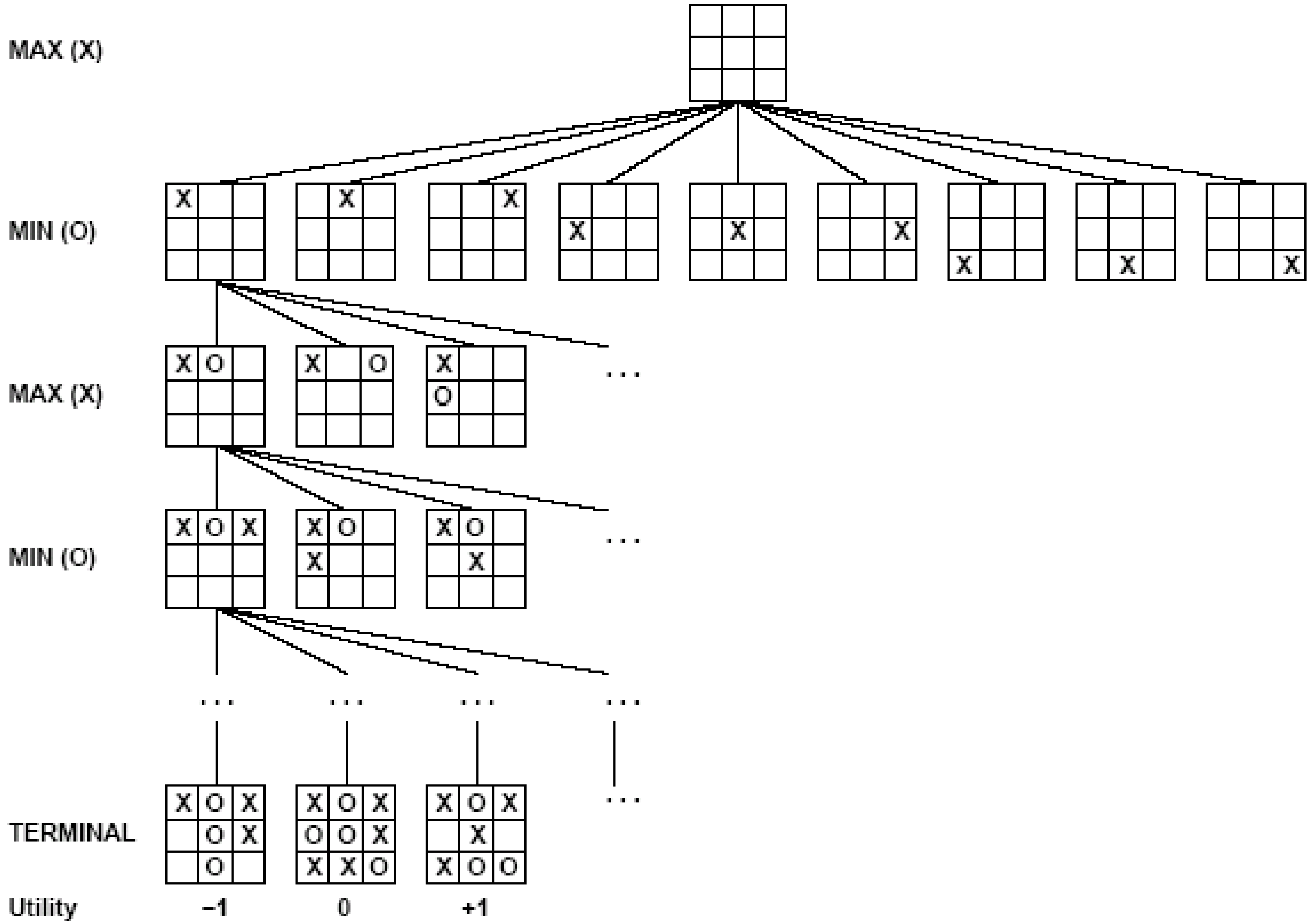
Games as search

- Initial state: e.g. board configuration of chess
- Player: which player to give the current move
- Successor function: list of (move,state) pairs specifying legal moves.
- Terminal test: Is the game finished?
- Utility function: Gives numerical value of terminal states. E.g. win (+1), lose (-1) and draw (0) in tic-tac-toe or chess

Game Setup

- **Both player wants to maximize the utility value earned at the end of the game**
- **We "see" the utility score as seen by MAX**
- **So, we can designate two players as MAX and MIN**
- **MAX tries to maximize its utility**
- **MIN tries to minimize MAX's utility**
- MAX uses search tree to determine next move.
- We play as MAX

Partial Game Tree for Tic-Tac-Toe



Size of search trees

- b = branching factor
- d = number of moves by both players
- Search tree is O(b^d)

- Chess
  - b ~ 35
  - D ~100
    - search tree is ~ 10^{154} (!!)
    - completely impractical to search this

- Game-playing emphasizes being able to make optimal decisions in a finite amount of time

Optimal Strategy

An optimal strategy leads to outcomes at least as good as any other strategy when the opponent plays optimally

Strategy 1

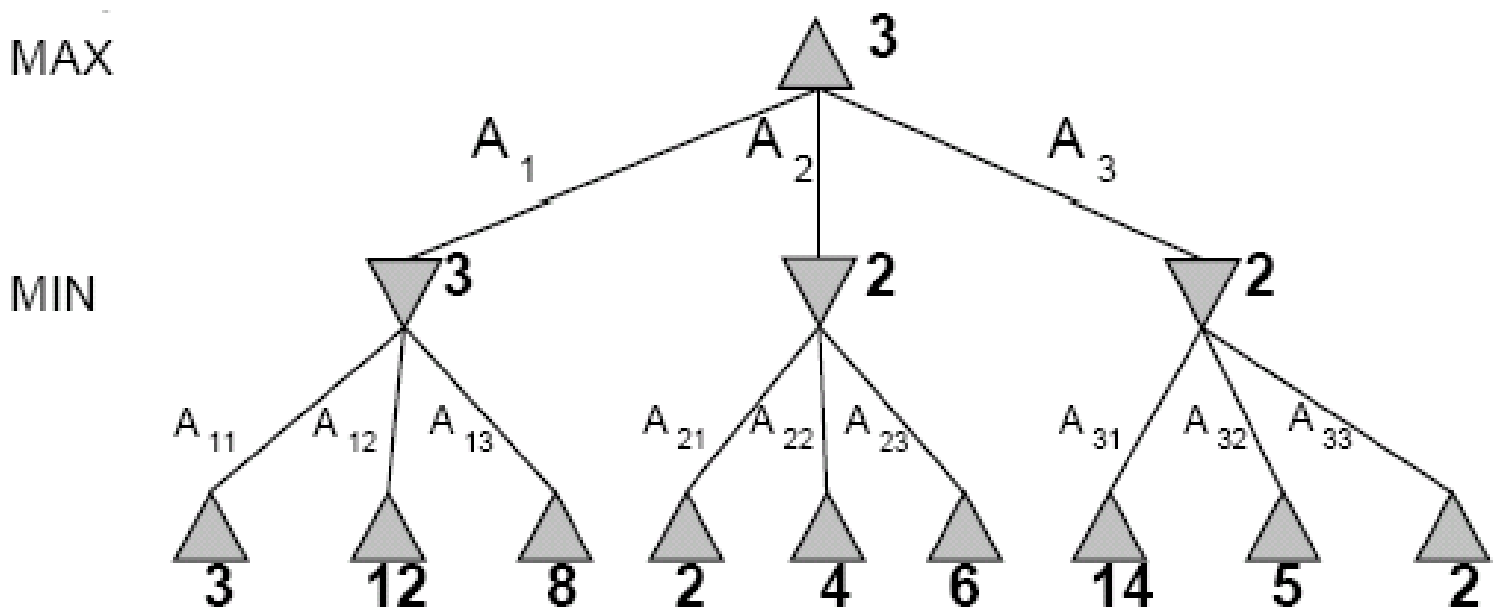
The minimax algorithm

The minimax algorithm

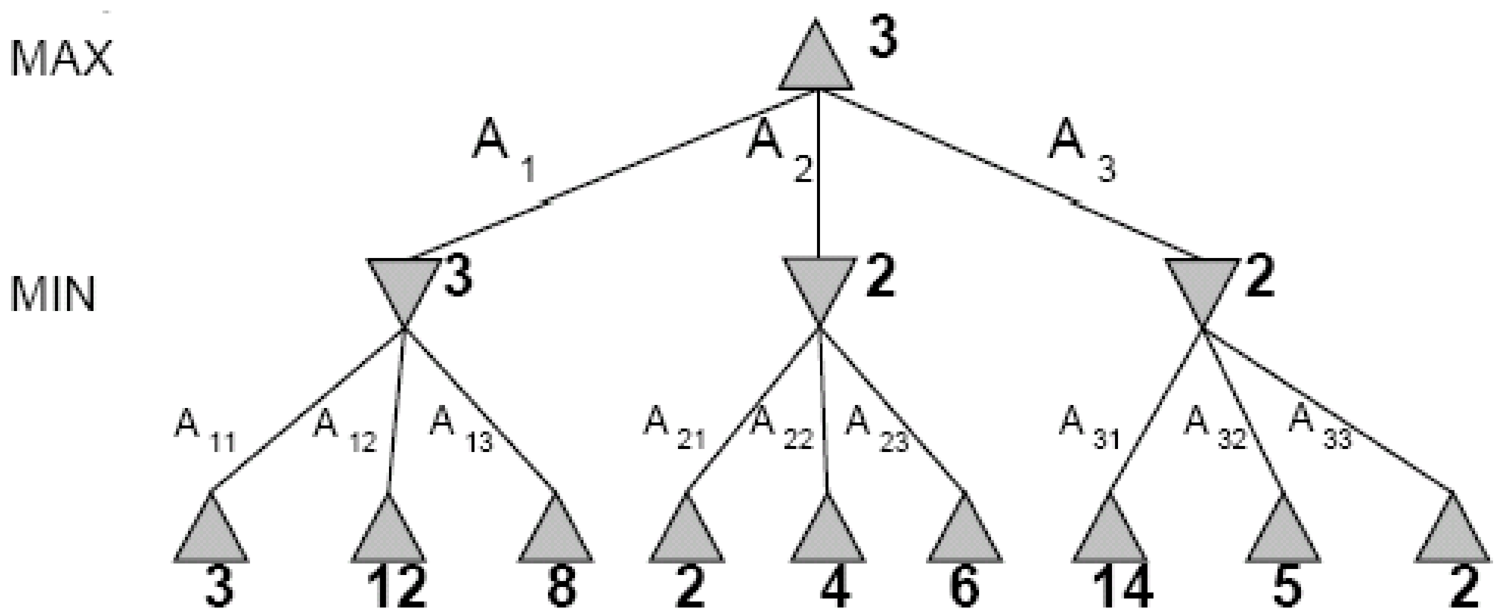
- Find the optimal *strategy* for MAX assuming an optimal MIN opponent

- Assumption: Both players play optimally!

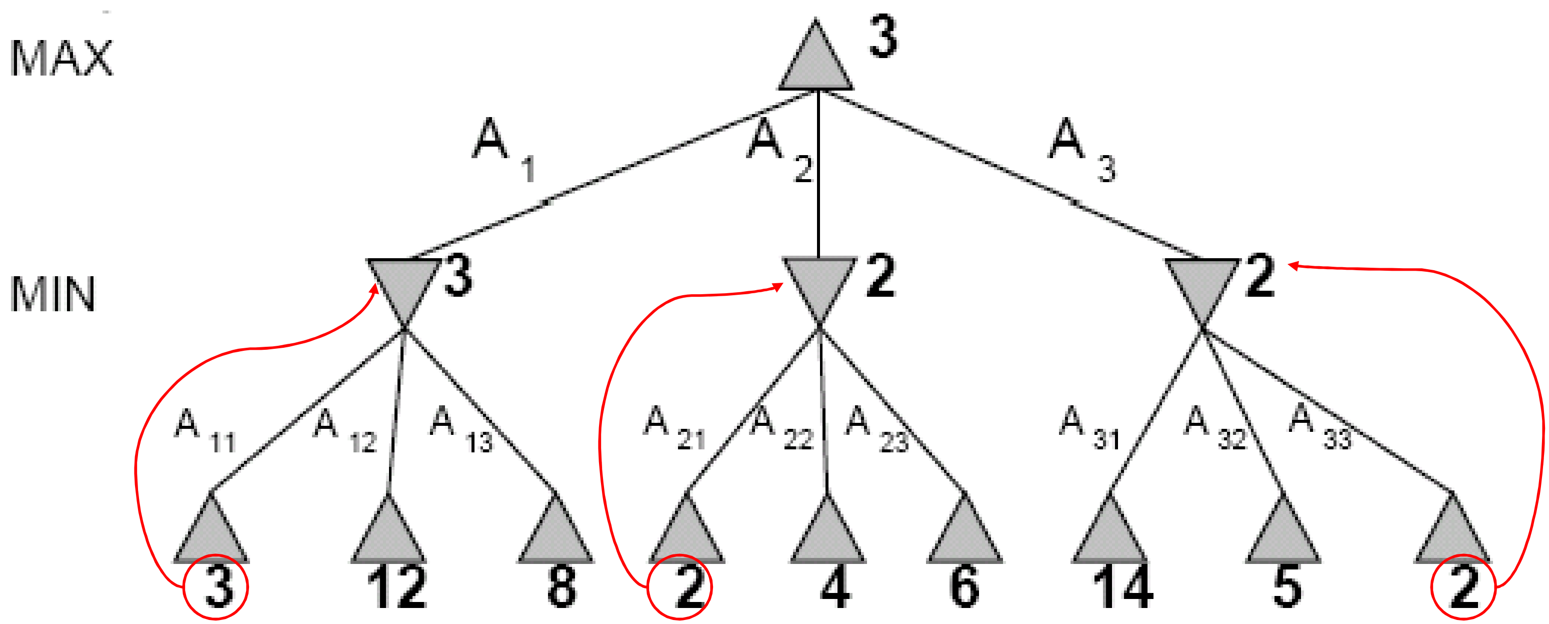
Two-Ply Game Tree



Two-Ply Game Tree

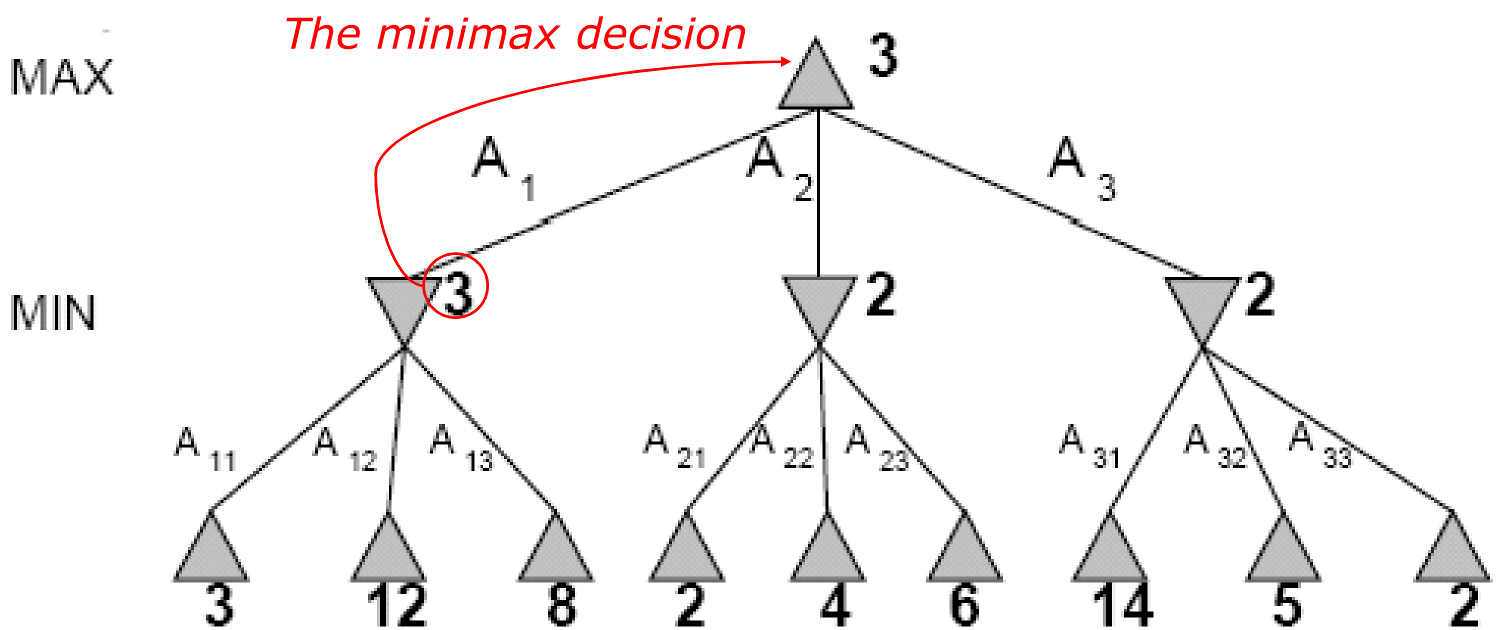


Two-Ply Game Tree



Two-Ply Game Tree

Minimax maximizes the utility for the worst-case outcome for max



The minimax algorithm

- Minimax value is the utility of MAX for being in the corresponding state

MINIMAX-VALUE(*n*) =

UTILITY(*n*) **If *n* is a terminal**

$\mathbf{max}_{s \in successors(n)}$ **MINIMAX-VALUE(*s*)** **If *n* is a max node**

$\mathbf{min}_{s \in successors(n)}$ **MINIMAX-VALUE(*s*)** **If *n* is a min node**

Minimax algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*

  $v \leftarrow$ MAX-VALUE(*state*)
  return the *action* in SUCCESSORS(*state*) with value v

---

function MAX-VALUE(*state*) **returns** *a utility value*

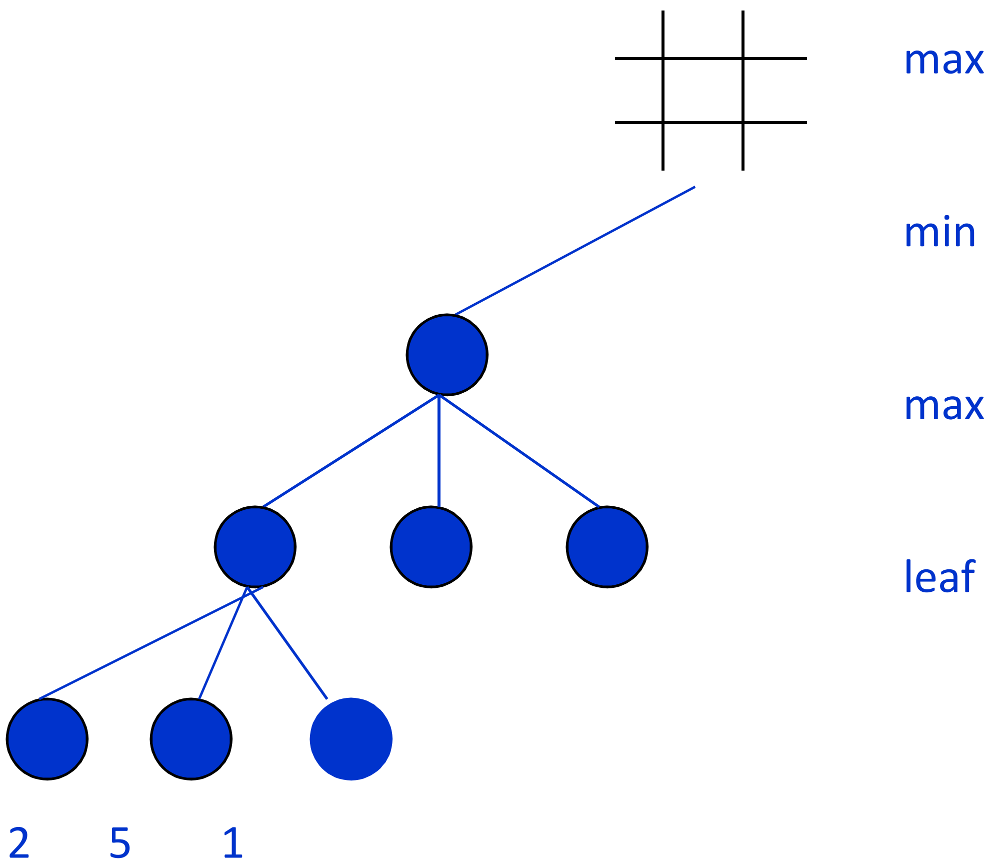
  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow -\infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MAX(v, MIN-VALUE(s))
  return v

---

function MIN-VALUE(*state*) **returns** *a utility value*

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
  $v \leftarrow \infty$
  for a, s in SUCCESSORS(*state*) **do**
    $v \leftarrow$ MIN(v, MAX-VALUE(s))
  return v

Minimax is done depth-first



Properties of Minimax

- Complete? Yes (if tree is finite)
- Optimal? Yes (against an optimal opponent)
- Time complexity? $O(b^m)$
- Space complexity? O(bm) (depth-first exploration)

- For chess, $b \approx 35$, $m \approx 100$
  → *exact solution completely infeasible*

Need to speed it up.

Strategy 2

Alpha Beta Pruning

Strategy 2

Alpha Beta Pruning

We need not visit every node

Alpha-Beta Procedure

- The alpha-beta procedure can speed up a depth-first minimax search.
- **Alpha:** a lower bound on the value that a max node may ultimately be assigned

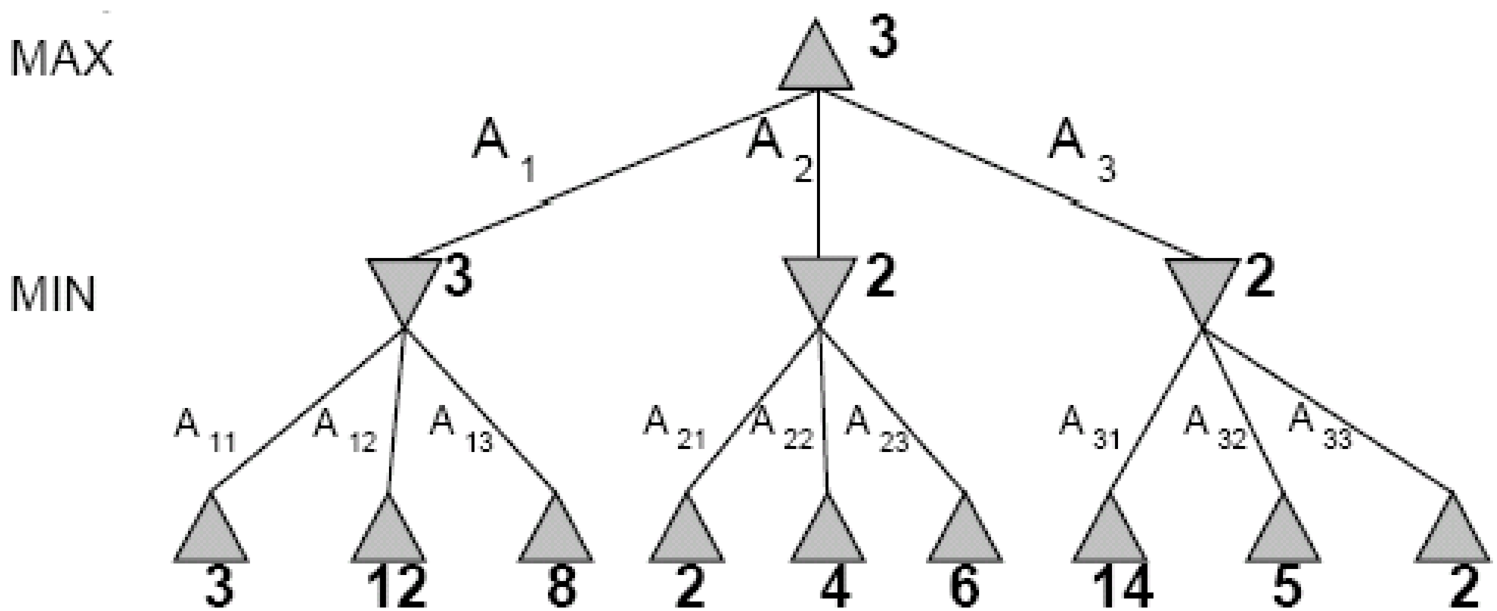
  $v \geq \alpha$

- **Beta:** an upper bound on the value that a minimizing node may ultimately be assigned $v \leq \beta$

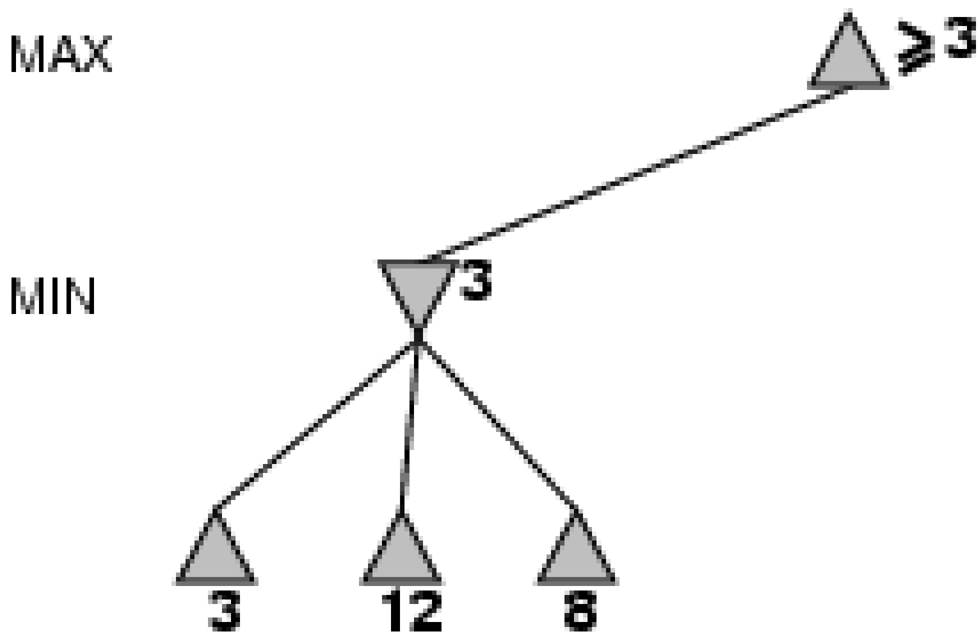
Alpha-Beta Procedure

- At MAX node, compare the utility estimate of current node (**v**), with the β value, and reason weather MIN will let the game to follow this path, if not prune (i.e. **v >=** β)
- At MIN node, compare the utility estimate of current node (**v**), with the α value, and reason weather MAX will let the game to follow this path , if not prune (i.e. **v <=** α)

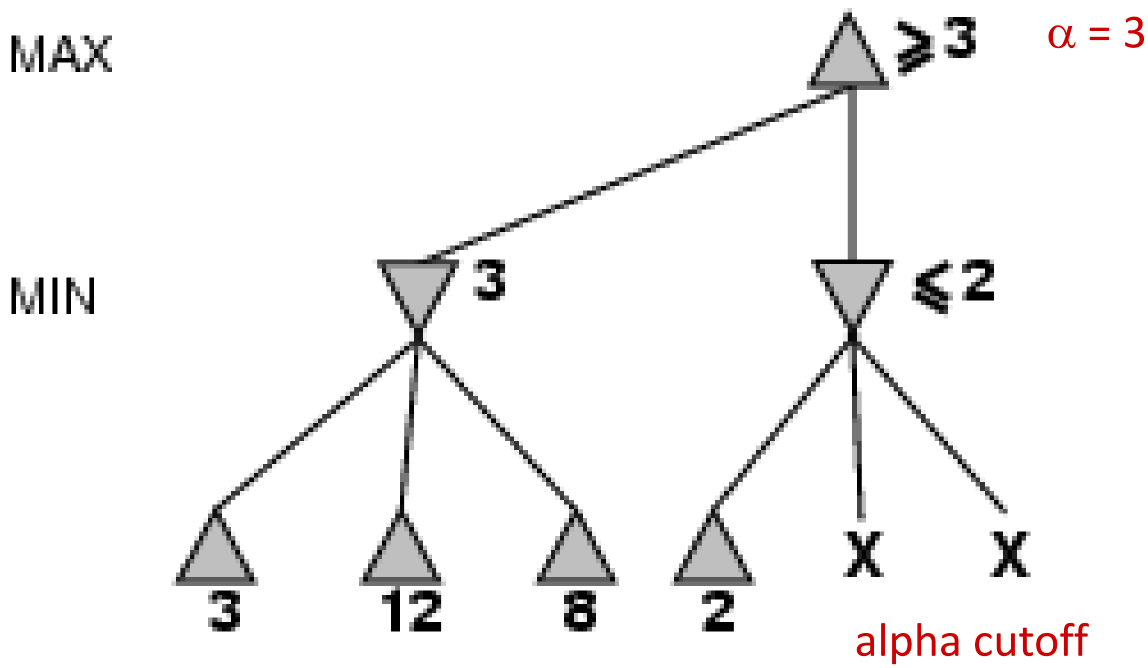
α-β pruning example



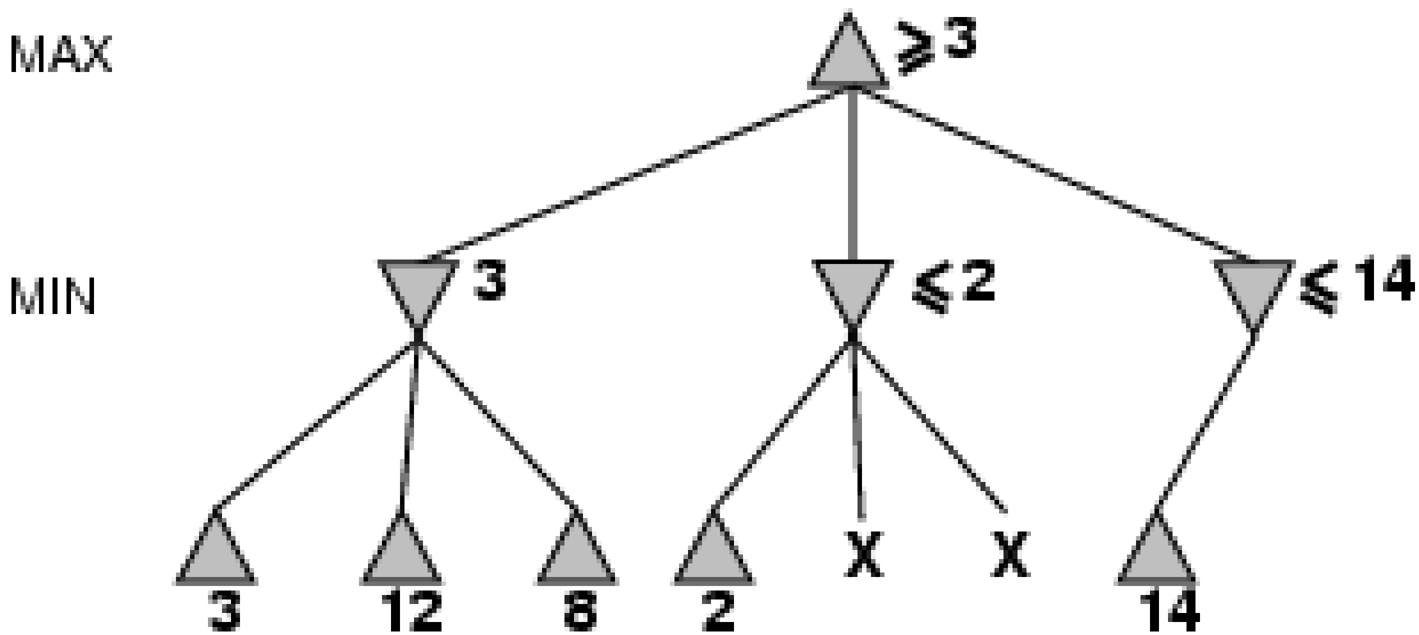
α-β pruning example



α-β pruning example



α-β pruning example



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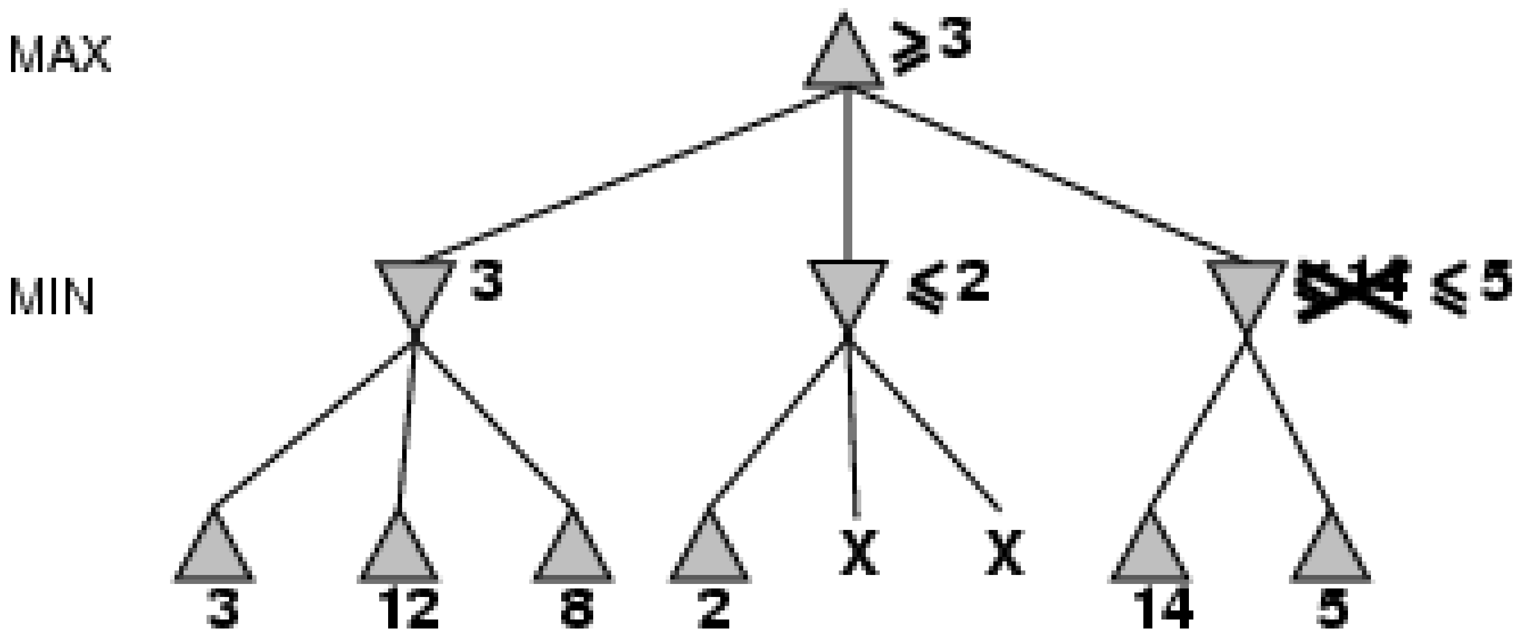
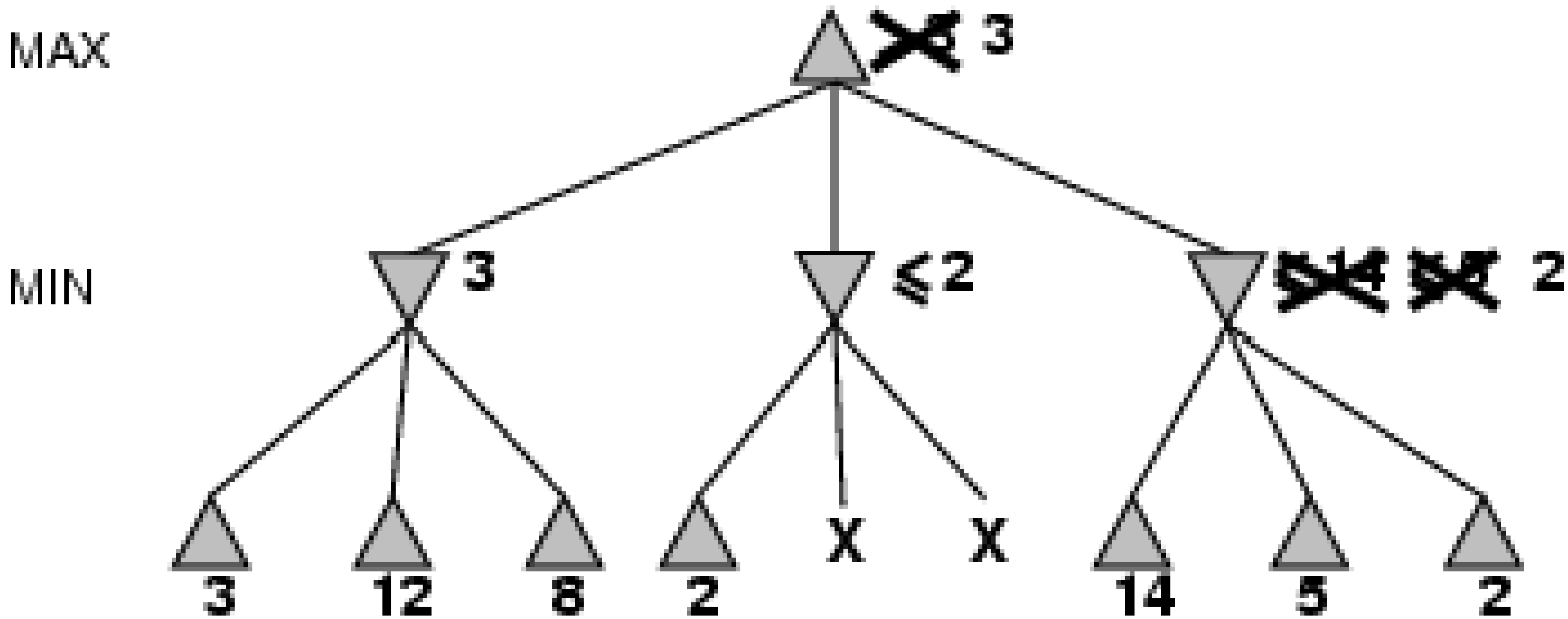


α-β pruning example



Properties of α-β

- Pruning does not affect final result. This means that it gets the exact same result as does full minimax.
- Good move ordering improves effectiveness of pruning
- With "perfect ordering," time complexity = $O(b^{m/2})$
  → doubles depth of search

The α-β algorithm

```
function ALPHA-BETA-SEARCH(state) returns an action
   inputs: state, current state in game
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in SUCCESSORS(state) with value v
```

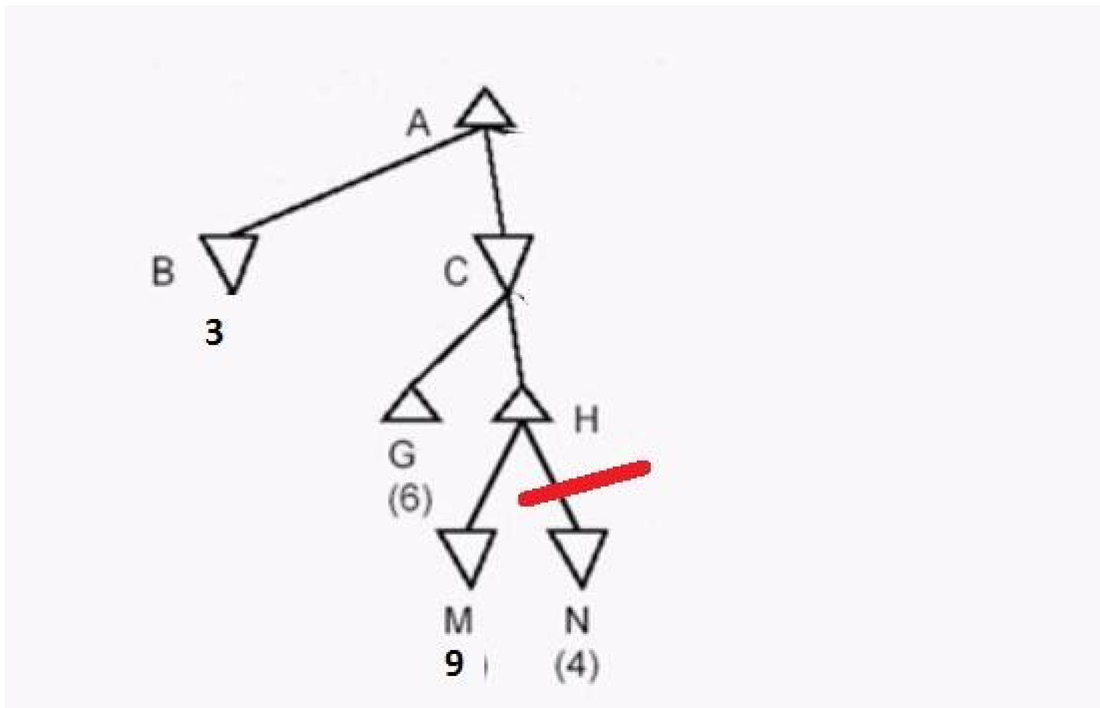
```
function MAX-VALUE(state, α, β) returns a utility value
   inputs: state, current state in game
           α, the value of the best alternative for  MAX along the path to state
           β, the value of the best alternative for  MIN along the path to state
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for a, s in SUCCESSORS(state) do
      v ← MAX(v, MIN-VALUE(s, α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

The α-β algorithm

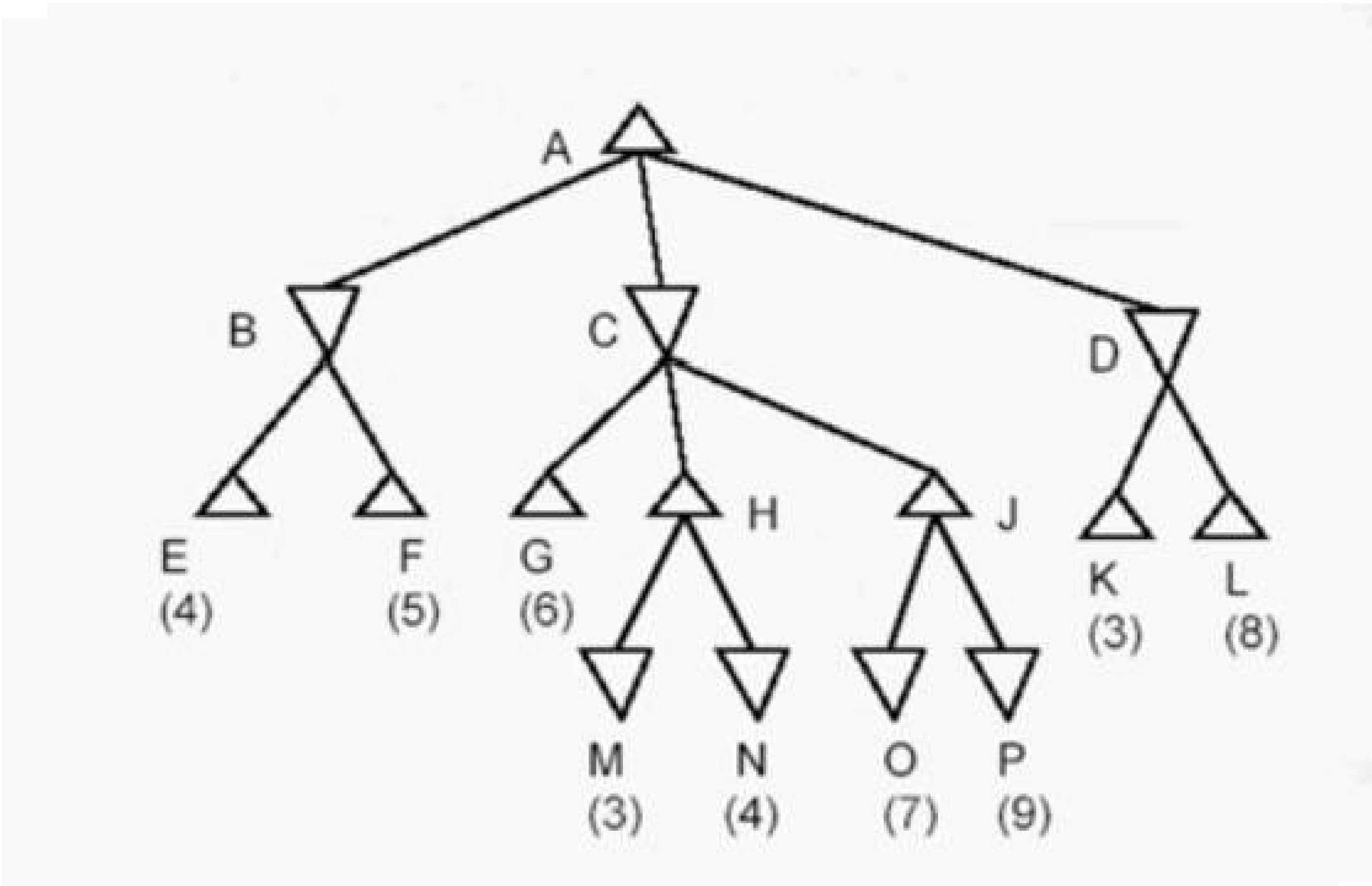
```
function MIN-VALUE(state, α, β) returns a utility value
    inputs: state, current state in game
            α, the value of the best alternative for  MAX along the path to state
            β, the value of the best alternative for  MIN along the path to state

    if TERMINAL-TEST(state) then return UTILITY(state)
    v ← +∞
    for a, s in SUCCESSORS(state) do
        v ← MIN(v, MAX-VALUE(s, α, β))
        if v ≤ α then return v
        β ← MIN(β, v)
    return v
```

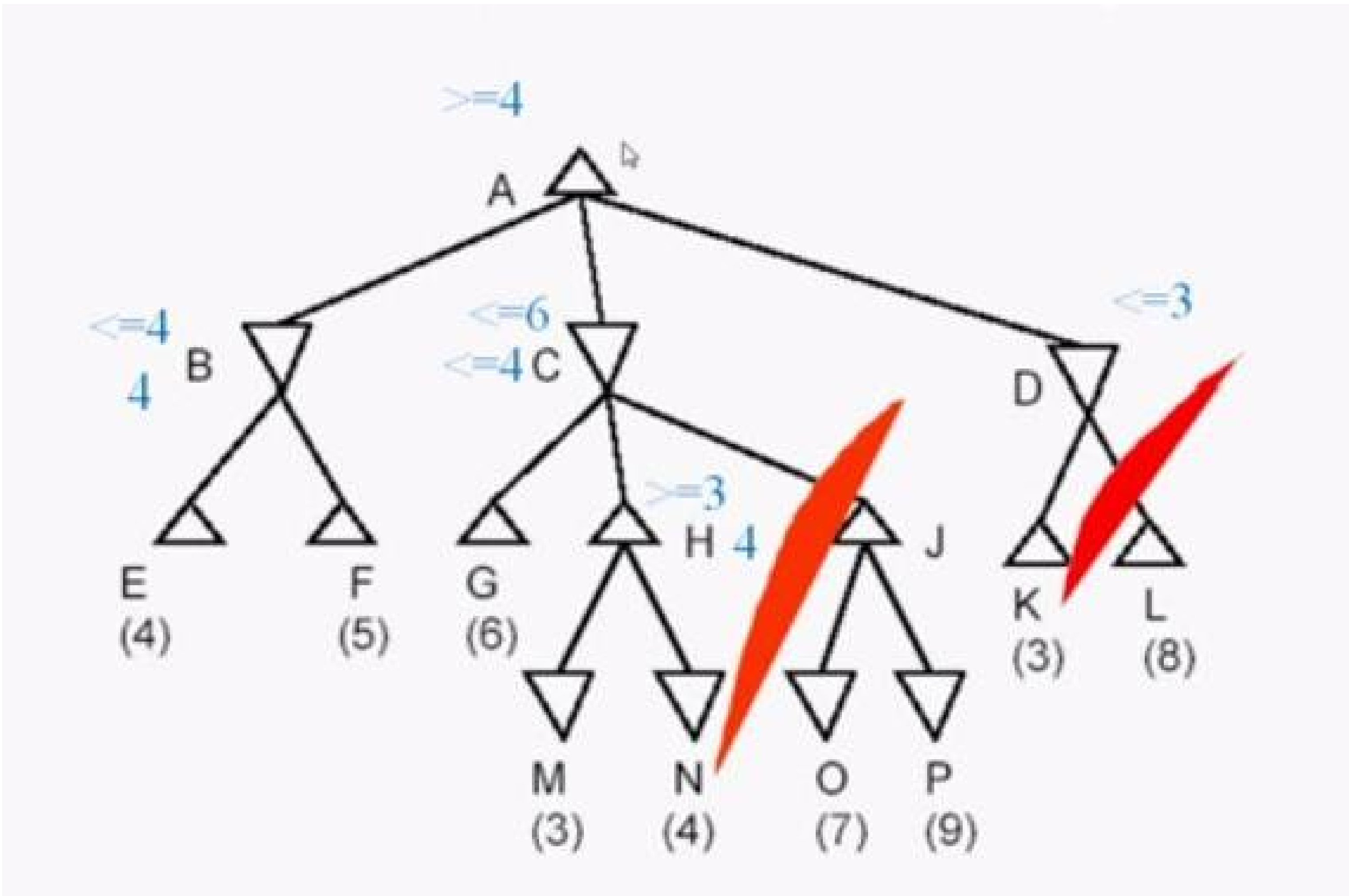
Example



Example



Example



What if the depth is still very large???

What if the de
larg

Limit the search only to a certain depth and use a **heuristic** to provide an **estimate** of the **MINIMAX value** of that node

Some heuristics for Tic-Tac-Toe

- +100 for EACH 3-in-a-line for computer.
- +10 for EACH two-in-a-line (with a empty cell) for computer.
- +1 for EACH one-in-a-line (with two empty cells) for computer.
- Negative scores for opponent, i.e., -100, -10, -1 for EACH opponent's 3-in-a-line, 2-in-a-line and 1-in-a-line.
- 0 otherwise (empty lines or lines with both computer's and opponent's seeds).

Resource

- Chapter 5
  - 5.1, 5.2, 5.3, 5.4

- Animation
  - http://alphabeta.alekskamko.com/
  - http://homepage.ufp.pt/jtorres/ensino/ia/alfabeta.html