Thermoshit of Rocket Fuckups during disasterous flight

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Abstract

1 Introduction

During the launch of a sounding rocket, the exterior of the vehicle experiences heat loading, etc etc idk

2 Modeling Nosecone Skin Heating

Objects traveling at high speeds are subject to heating and cooling as they travel through the atmosphere. As a rocket vechile travels at supersonic velocities, there are two signigigant contributions to the temperature change of the nosecone - the aerodyanmic heating from the compression of air as the nosecone pushes through it, and the heat lost to the atmosphere through radiation. For the purposes of this analysis, these will be the only sources of thermal energy change considered.

2.1 Nosecone Aerodynamic Heating

As the nosecone travels at suspersonic speeds, it forms a shock wave as it moves faster than the air can escape. This can also be considered as air being blown towards a stationary nosecone, at the speed the rocket would be travling. At the very tip of the nosecone, the air has zero relative velocity to the vehicle. This is the stagnation point [REF!].

At the stagnation point, all of the kinetic energy of the air is converted into thermal energy. Becuase this process happens so quickly, it can be modeled as an adiabatic compression of gas.

$$PV^{\gamma} = \text{constant}$$

The ideal gas approximation ($V \propto TP^{-1}$) can be applied, so

$$P^{1-\gamma}T^{\gamma} = \text{constant}$$

In our compression, this means

$$\frac{T_s}{T_0} = \left(\frac{P_s}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

With T_s, P_s as the stagnation temperature and pressure and T_0, P_0 as the static temperature and pressure. The relation between the static and stagnation pressure of a gas is sourced from a National Adivosry Committee for Aeronautics report on compressable flow:

$$\frac{P_s}{P_0} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

Where M is the mach number. This provides an equation for stagnation temperature as a function of speed:

$$\frac{T_s}{T_0} = 1 + \frac{\gamma-1}{2}M^2$$

Of course, the entire nose cone will not encounter air at the stagnation temperature. Most air will be slowed, but stopped by skin friction with the vehicle. Toft [citatation!] models the temperature of the boundary layer air (T_B) based on K, a temperature recovery factor:

$$K = \frac{T_B - T_0}{T_s - T_0}$$

Eber [citation] provides a value of $K = 0.89$ for cones with vertex angles between 20 and 50 degrees.

We then have an equation for the temperature of the boundary layer:

$$T_B = KT_s + T_0\,(1-K)$$

Eber also provides an experimentally modeled value for h, the heat transfer function:

$$h = \left(0.0071 + 0.0154\sqrt[2]{\beta}\right)\frac{k}{\mu^{0.8}l^{0.2}}\,(\rho_0 u)$$

based on k, the thermal conductivity of air; β, the vertex angle of the cone; ρ, the density of the air; μ, the dynamic viscosity of air; u the velocity of the rocket; and l, the length of the cone measured along the surface.

With all this, we can determine the heat flux into the nosecone skin from aerodynamic effects:

$$\dot{Q}_{aero} = h(T_B - T_N) \tag{1}$$

Where T_N is the actual temperaure of the nosecone skin.

2.2 Radiative Effects

Some amount of heat flux leaves the nosecone as blackbody radiation, with heat flux $\dot{Q}_rad = \epsilon\sigma(T_0^4 - T_N^4)$, where ϵ is the emmisivity of stainless steel nosecone and σ is the Stefan-Boltzman constant.

2.3 Flight Profile

In order to model the thermal behavior of the nosecone, we must provide a number of inputs to our aerodynamic heating simulation functions based on altitude and velocity. These inputs are provided by UBC Rocket's proprietary Feynman vehicle design program.

Although Feynman was originally intended for optimizing the design of rocket vechicle components, such as fins, engines, or nosecones for maximum altitude without regard for thermal effects, its outputs are helpful in the analysis of the nosecones thermal properties.

Feymnman provides data for Mach number and altitude at 10 millisecond intervals for the first approximately 160 seconds of flight, corresponding to a peak altitude of almost 100 kilometers. For this reason, we can use the output data of the Feynman simulaiton as a reasonable input flight profile for our thermal analysis.

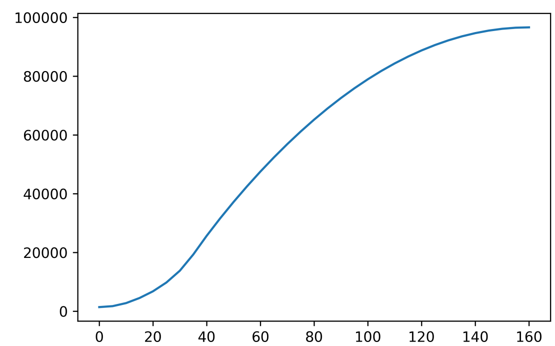

Figure 1a: Altitude vs Time of simulated vehicle trajectory

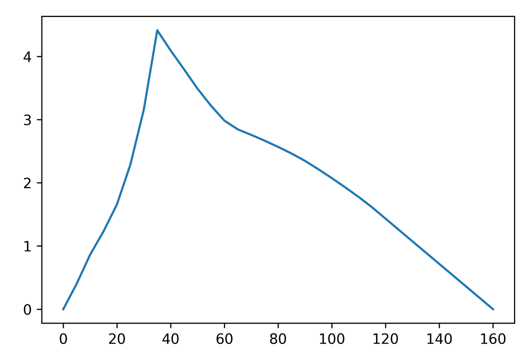

Figure 1b: Mach number vs time of simulated vechicle trajectory

Additionally, emperically-derived curves[1] for ρ_0 and T_0 in terms of altitude and of μ and k in terms of

[1] Unfortunately, some of this data was emperically-derived in Nazi Germany by scientests working on the V-2 missile.

T_0 are provided in appendix A. These provide a full picture of the conditions around the nosecone over all stages of the flight.

2.4 Heat Flux Balance

The nosecone situation through flight can be modeled by an energy balance:

$$\text{Heat Flux In} = \text{Heat Flux out}$$

This can be modeled by the heat equation:

$$GdT_N = dt(\dot{Q}_{aero} - \dot{Q}_{rad})$$

or

$$dT_N = \frac{dt(\dot{Q}_{aero} - \dot{Q}_{rad})}{G} \tag{2}$$

With a factor G, the 'skin heating capacity' [cite] determined by the specific heat of the nosecone skin c, it's thickness τ, and it's density ρ.

$$G = c\tau\rho$$

2.5 Simulation Method

Considering a small but finite change in time Δt (in our simulation case, $10ms$, from Feynman as above), we can numerically solve for the change ΔT_N in the nosecone temperature based on equation 2 above.

This solution is determined by means of a python simulation which iterates over each timestep and calcuates the change in T_N for that time. Using the atmospheric models from appendix A and the constant values from appendix B, a represntative model of the at-rest (i.e. far away from the vehicle's path of flight) is created. Then, using the Feynman-derived data for Mach number and altitude, and the derived equations from the above sections, values are determined for $u, \mu, k, h, T_s, T_B, \dot{Q}_{aero}$ and $\dot{Q}_{rad}$, which are then used to solve for ΔT_N for the timestep, which then modifies T_N for use in the simulation of the next timestep.

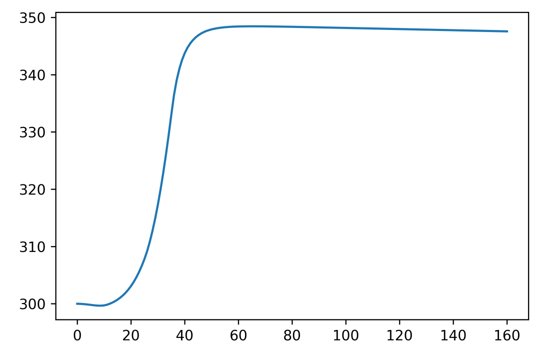

Figure 2: Temperature of nosecone skin vs time

3 Discussion of Results

Based on the above simulation, the temperature of the nosecone can be determined as a function of time as seen in figure 3 above. The peak temeperature of 348 K occurs about 65 seconds into the flight. It is interesting to note that after the rapid rise in temperature, the decline after the peak is signifigantly shallower.

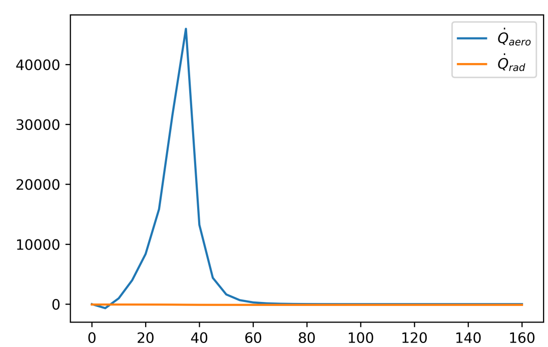

Figure 3: $\dot{Q}_{aero}$ and $\dot{Q}_{rad}$ fluxes over time

This is explained by figure 3: the vast majority of the total heat flux at the skin of the rocket is due to aerodynamic effects, with the peak of the aerodynamic flux occuring generally at the same time as the peak mach number. This would seem to be a reasonable result. Considering the fixed-vehicle frame of reference, as air particles encounter the tip of the nosecone at higher speeds, they will have more kinetic energy, all of which will be transfered to the nosecone, increasing the kinetic energy of it's constituent particles and so increasing it's temperature more than at slower velocities. Since we expect the heating of the nosecone to be positively related to the stagnation temperature, we can expect higher aerodynamic fluxes at higher speeds.

Appendices

A Atmospheric Models

A.1 Air Pressure as a function of Altitude

Air pressure P_0 as a funciton of altitude s [reference]

$$P_0(s) = \begin{cases} \exp\left(4.43165 \cdot 10^{-14}s^3 - 2.28553 \cdot 10^{-9}s^2 - 1.14097 \cdot 10^{-4}s + 6.95109\right) & 0\text{km} < s \leq 25\text{km} \\ \exp\left(-2.28179 \cdot 10^{-14}s^3 + 3.34063 \cdot 10^{-}9s^2 - 2.84655 \cdot 10^{-4}s + 8.73033\right) & 25\text{km} < s \leq 75\text{km} \\ \exp\left(4.44813 \cdot 10^{-14}s^3 - 1.13434 \cdot 10^{-}9s^2 + 7.62651 \cdot 10^{-4}s - 15.5981\right) & 75\text{km} < s \leq 120\text{km} \end{cases}$$

A.2 Air Density as a function of Altitude

Air pressure ρ_0 as a function of altidue s [reference]

$$\rho_0(s) = \begin{cases} \exp\left(4.88158 \cdot 10^{-18}s^4 - 1.808 \cdot 10^{-13}s^3 + 2.432 \cdot 10^{-11}s^2 - 9.693 \cdot 10^{-5}s + 0.1922\right) & 0\text{km} < s \leq 25\text{km} \\ \exp\left(-6.034 \cdot 10^{-19}s^4 - 1.035 \cdot 10^{-13}s^3 - 5.746 \cdot 10^{-9}s^2 - 2.21 \cdot 10^{-5} - 0.396\right) & 25\text{km} < s \leq 75\text{km} \\ \exp\left(-1.004 \cdot 10^{-18}s^4 + 4.440 \cdot 10^{-13}s^3 - 7.137 \cdot 10^{-8}s^2 + 4.773 \cdot 10^{-5} - 121.84\right) & 75\text{km} < s \leq 120\text{km} \end{cases}$$

A.3 Air Temperature as a function of Altitude

Air Temperature, T_0, as a function of altitude s [ref]

$$T_0(s) = \begin{cases} 287.954 - 5.03015 \cdot 10^{-3}s - 1.2859 \cdot 10^{-7}s^2 & 0\text{km} < s \leq 10\text{km} \\ 225.15 & 10\text{km} < s \leq 23\text{km} \\ 242.057 - 2.33854 \cdot 10^{-3}s + 7.08133 \cdot 10^{-8}s^2 & 23\text{km} < s \leq 42\text{km} \\ -534.104 + 3.95468 \cdot 10^{-2}s - 6.0177 \cdot 10^{-7}s^2 + 2.71838 \cdot 10^{-12}s^3 & 42\text{km} < s \leq 81.5\text{km} \\ 867.12 - 9.78603 \cdot 10^{-3}s - 5.75164 \cdot 10^{-8}s^2 + 8.81316 \cdot 10^{-13}s^3 & 81.5\text{km} < s \leq 120\text{km} \end{cases}$$

A.4 Dynamic Viscocity of Air as a function of Temperature

Dynamic Viscocity of Air, μ, as a function of Temperature T_0 [reference]

$$\mu(T_0) = -1.00 \cdot 10^{-5} - 1.47 \cdot 10^{-9}T + 1.68 \cdot 10^{-6}T^{\frac{1}{2}}$$

A.5 Thermal Conductivity of Air as a funciton of Temperature

Thermal Conductivity of Air, k, as a funciton of Temperature T_0 [ref]

$$k(T_0) = -1.29 \cdot 10^{-2} + 2.43 \cdot 10^{-5}T - 3.39 \cdot 10^{-9}T^2 + 1.88 \cdot 10^{-3}T^{\frac{1}{2}}$$

B Simulation Constants