

L-System User Notes

(Lindenmayer Systems)

Written by [Paul Bourke](#)

Version 2.5, July 1991

Introduction

This program implements some of the L-Systems discussed in "Lecture Notes in Biomathematics" by Przemyslaw Prusinkiewicz and James Hanan. A brief description of an 0L system will be presented here but for a more complete description the user should consult the literature.

The application was initially written to investigate methods of incorporating objects with a large number of drawing elements (lines, polygons) into a CAD package. L Systems is one way for example of "generating" trees at the rendering stage but not during the editing stage where the image complexity will slow down the response time.

Simple example of a 0L system

A string of characters (symbols) is rewritten on each iteration according to some replacement rules. Consider an initial string (axiom)

F+F+F+F

and a rewriting rule

F --> F+F-F-FF+F+F-F

After one iteration the following string would result

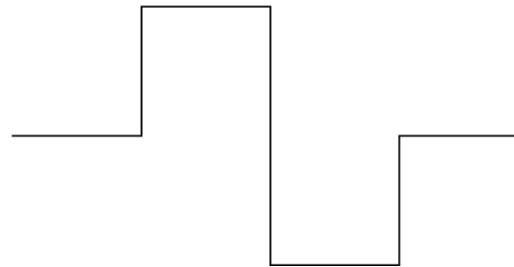
F+F-F-FF+F+F-F + F+F-F-FF+F+F-F + F+F-F-FF+F+F-F + F+F-F-FF+F+F-F

For the next iteration the same rule is applied but now to the string resulting from the last iteration

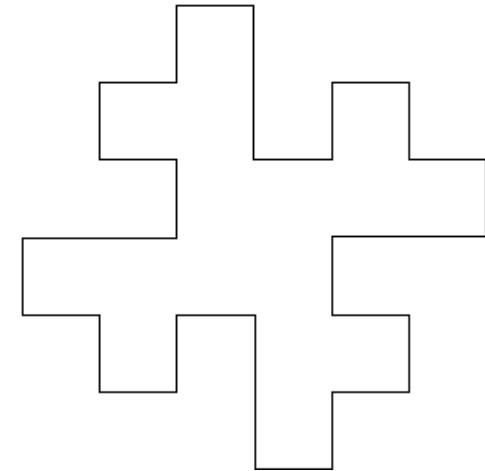
F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-
 F+ F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+ F-F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+
 F-FF+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F-F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+
 F+ F-F-F+ F-F-FF+ F+ F-F-F+ F+ F-F-F+ F+ F-F-FF+ F+ F-F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+
 F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F-F+ F+ F-F-FF+ F+ F-F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-
 FF+ F+ F-F+ F+ F-F-FF+ F+ F-F-F+ F+ F-F-FF+ F+ F-F-F+ F+ F-F-FF+ F+ F-F-FF+ F+ F-F+ F+ F-F-FF+ F+ F-F+ F+ F-F-

Some symbols are now given a graphical meaning, for example, F means move forward drawing a line, + means turn right by some predefined angle (90 degrees in this case), - means turn left.

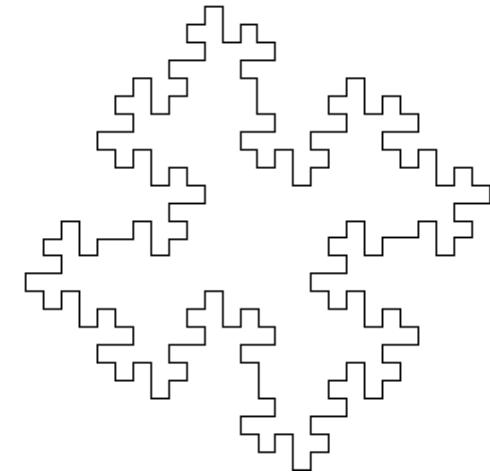
Using these symbols the initial string F+F+F+F is just a rectangle ($\theta = 90$). The replacement rule F --> F+F-F-FF+F+F-F replaces each forward movement by the following figure



The first iteration interpreted graphically is



The next iteration interpreted graphically is:



and so on.

Example:

Axiom X

F --> FF

X --> F-[X]+X]+F[+FX]-X

$\phi = 22.5$

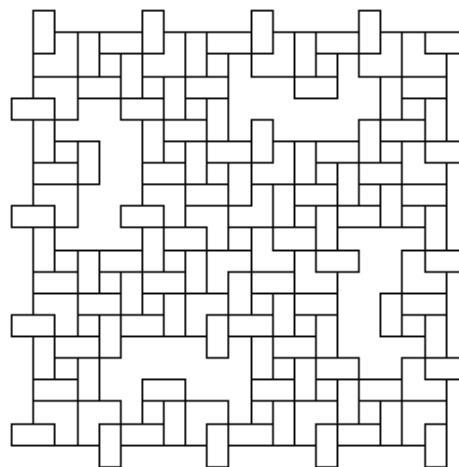


Example:

Axiom F+F+F+F

F --> FF+F-F+F+FF

$\phi = 90$

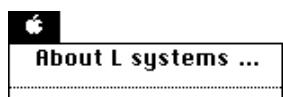


Symbols The following characters have a geometric interpretation.

Character	Meaning
F	Move forward by line length drawing a line
f	Move forward by line length without drawing a line
+	Turn left by turning angle
-	Turn right by turning angle
	Reverse direction (ie: turn by 180 degrees)
[Push current drawing state onto stack
]	Pop current drawing state from the stack
#	Increment the line width by line width increment
!	Decrement the line width by line width increment
@	Draw a dot with line width radius
{	Open a polygon
}	Close a polygon and fill it with fill colour
>	Multiply the line length by the line length scale factor
<	Divide the line length by the line length scale factor
&	Swap the meaning of + and -
(Decrement turning angle by turning angle increment
)	Increment turning angle by turning angle increment

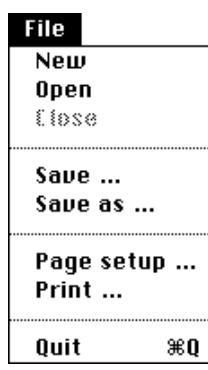
When drawing the graphical representation of the L string all other characters are ignored. The user may choose and use any other single printable characters for the replacement rules except, note: this excludes "white" characters such as spaces and tabs. For context sensitive L systems the * character is used to represent any match.

See the SYMBOL menu item for a summary of the reserved symbols. **Menus**



As with most applications the ABOUT menu item located in the Apple menu before the desk accessories displays information such as the version number and contact address of the developer.

This is the standard FILE menu although some of the items are not implemented.



NEW resets all the variables including the rules to their default values.

OPEN to load a rule description file previously saved or possibly created with a text editor.

SAVE and SAVE AS create rule description files.

PRINT and PAGESETUP control direct printing.

QUIT to exit from the program when finished.

The standard EDIT menu. The first 5 items are only included for compatibility reasons and do not apply to this application. (they are used with desk accessories in some cases)

Edit	
Undo	⌘Z
Cut	⌘H
Copy	⌘C
Paste	⌘V
Clear	⌘B
Redraw	⌘R
<hr/>	
Copy as lines (PICT)	
Copy as bitmap (PICT)	
<hr/>	
<input checked="" type="checkbox"/> Zoom to fit	
Zoom in	
Zoom out	

The two special copy items allow images to be transferred to other applications depending on the preferred data type. Line drawings should be copied for CAD packages and bitmaps for painting programs.

Normally images will be scaled to fit the window, it is possible to zoom in or out of areas with the last two items.

This is the main menu for this application.

Action	
Reset	⌘O
Next iteration	⌘I
Previous iteration	⌘B
Set iteration depth	⌘D
<hr/>	
Frame colour	▶
Fill colour	▶
Background colour	▶
<hr/>	
Productions	⌘P
Axiom	⌘A
State	⌘S
Symbol list	
<hr/>	
<input checked="" type="checkbox"/> OL	
(1,1)L	
OL Stochastic	

REDRAW updates the current display, for example, if the drawing was cancelled with clover-(period)

Menu items 2 through 4 control the iteration depth. RESET sets it to 0, the number of iterations to compute may be incremented, decremented, or set to any particular number.

FRAME, FILL and BACKGROUND are pop-up menus that allow the colour of the lines, polygon fills and background to be set.

PRODUCTIONS allows the current rules to be altered. AXIOM menu item sets the initial string. STATE menu item varies environment variables. SYMBOL LIST displays the meaning given to various predefined and reserved symbols.

The type of system is selectable from the last 3 items.

Library	
Plants	▶
Kolam patterns	▶
Bracketless	▶
Bracketed	▶
Context	▶
Stochastic	▶
Miscellaneous	▶

This contains a number of predefined image specifications grouped together in categories given by the pop-up menu names.

Users are encouraged to submit further examples that can be added to this library.

Advanced features/comments

- Always ensure that the '[' and ']' brackets match in number, ie: there should always be the same number of each type. If not a stack overflow will quickly occur, a message displayed, and the iteration depth decremented. Note: non matching brackets are not tested before the string is interpreted, a problem only occurs when a stack overflow situation is encountered.
- Polygons (using "{" and "}") cannot be nested, ie: there can only be one open at a time. If a polygon open symbol "{" is encountered while another polygon is still open it will be ignored. If a polygon is left open when the end of the string is reached it will be automatically closed.
- For a complete list of predefined symbol meanings look at the SYMBOL list under the ACTION menu.
- Best quality hardcopy line drawing images can be obtained by using MacDraw or Claris CAD. In both cases copy the image as a line drawing, paste it into MacDraw say, set the lines to black and line width 0.1mm, then print on a

LaserWriter. Note: the number of lines can quickly approach the thousands and even tens of thousands, don't expect the printing to be fast.

- Time consuming drawing may be cancelled by typing clover-. (period)
 - The maximum length of the strings generated is 300,000 symbols. Attempting to exceed this will give an error message and automatic decrementing of the iteration count.
 - The maximum stack depth is 100. This means that a bracketed string cannot have brackets nested more than 100 deep.

ie: [[[]] [[[[]] []]]]

1 2 3 2 1 2 3 4 5 4 3 4 3 2 1 0 is nested 5 deep

If the maximum stack depth is exceeded an appropriate error message will be displayed and the iteration depth decremented until the stack no longer overflows.

- When copying to the clipboard as lines make the drawing window as large as possible for maximum resolution.

References

Heinz-Otto and Deitmar Saupe. *The Science of Fractal Images*. Springer-Verlag

Przemyslaw Prusinkiewicz. *Application of L-Systems to Computer Imagery*. Lecture Notes in Computer Science #291, Pages 534-548

Przemyslaw Prusinkiewicz, James Hanan. *Lecture Notes in Biomathematics* #79

Prusinkiewicz, P. *Graphical Applications of L-Systems*. Proc. of Graphics Interface 1986 - Vision Interface, 1986, Pages 247-253.

Smith, A.R. *Plants, Fractals, and Formal Languages*. Computer Graphics, 18, 3, 1984, Pages 1-10.

Prusinkiewicz, P and Lindenmayer A. *The Algorithmic Beauty of Plants*. Springer Verleg, 1990, Pages 40-50.

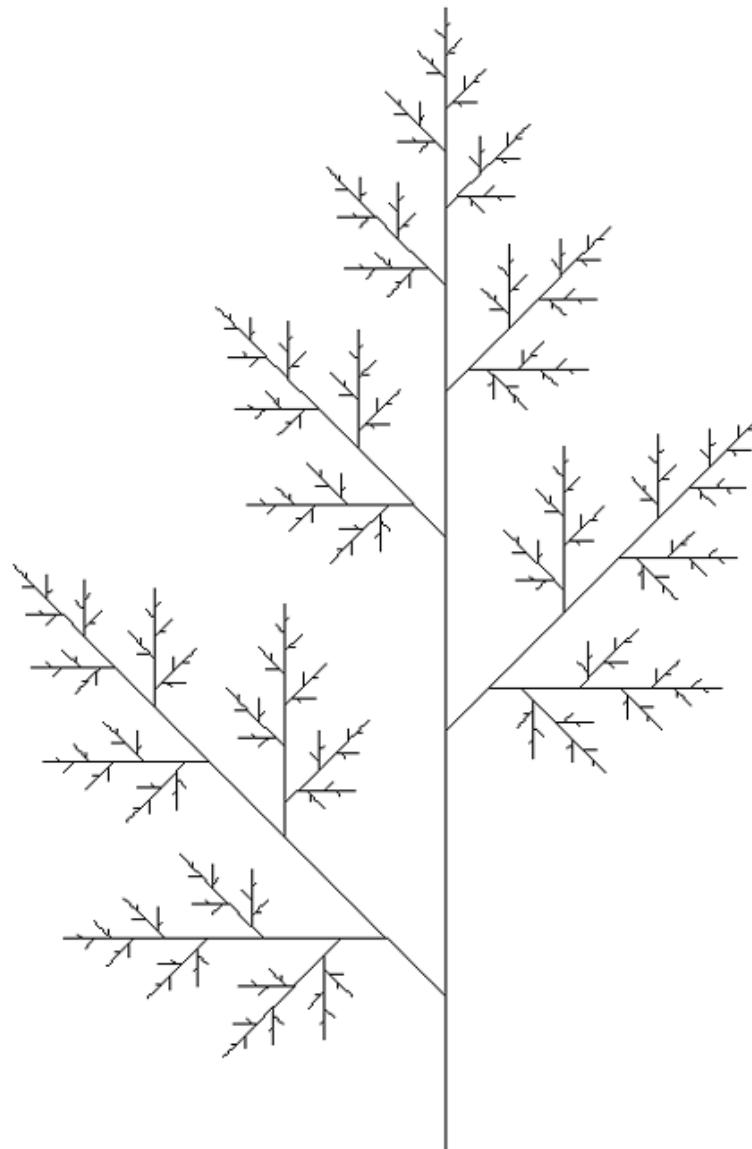
L-System Leaf

Written by Paul Bourke

axiom = a
 $F \rightarrow >F$
 $a \rightarrow F[+x]Fb$
 $b \rightarrow F[-y]Fa$
 $x \rightarrow a$
 $y \rightarrow b$

angle = 45

length factor = 1.36



L - S y s t e m B u s h e s

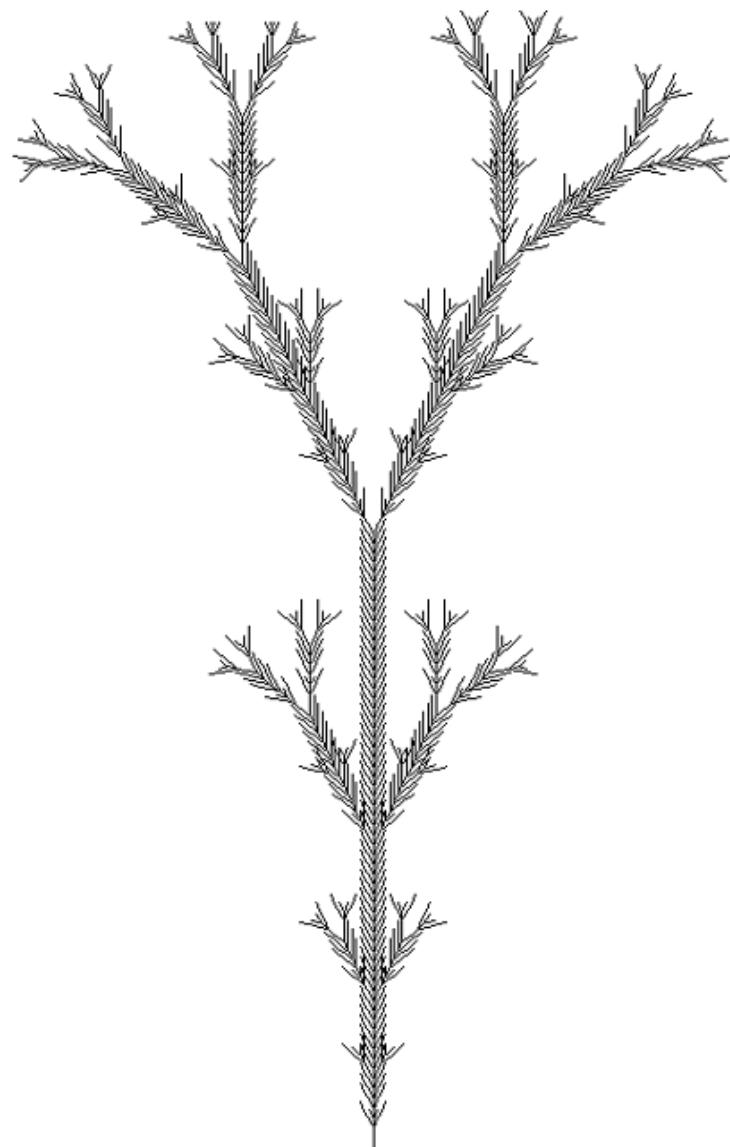
Written by [Paul Bourke](#)

axiom = Y

X -> X[-FFF][+FFF]FX

Y -> YFX[+Y][-Y]

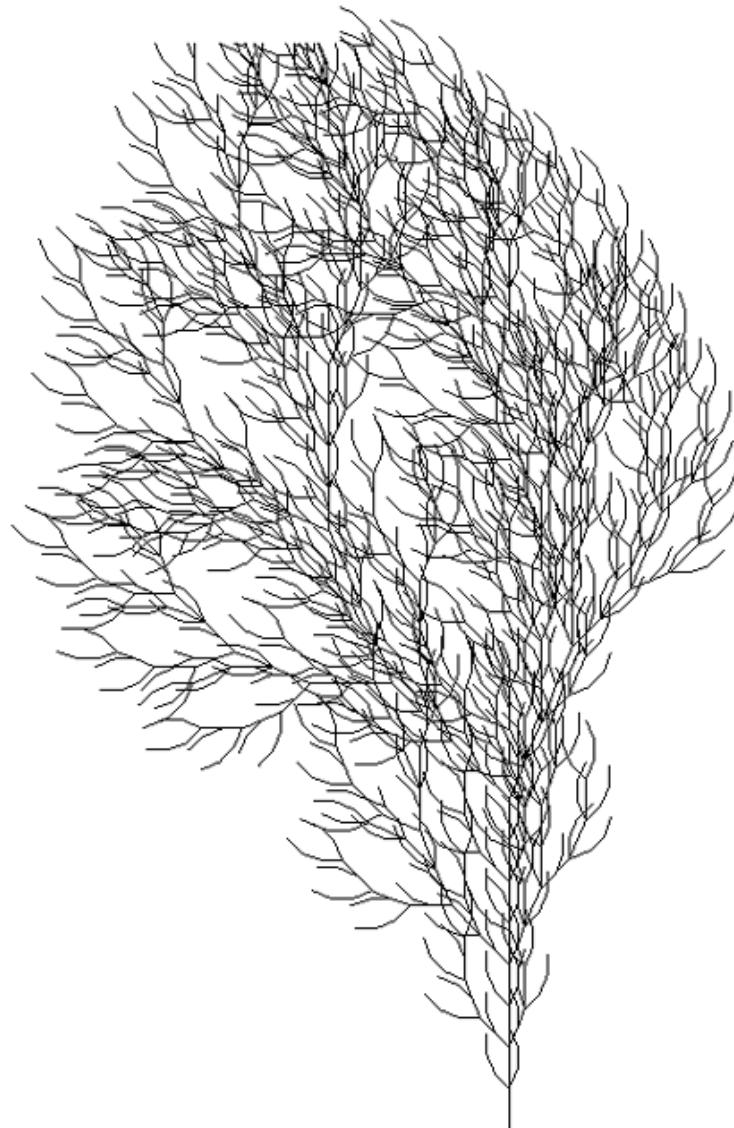
angle = 25.7



axiom = F

F -> FF+[+F-F-F]-[-F+F+F]

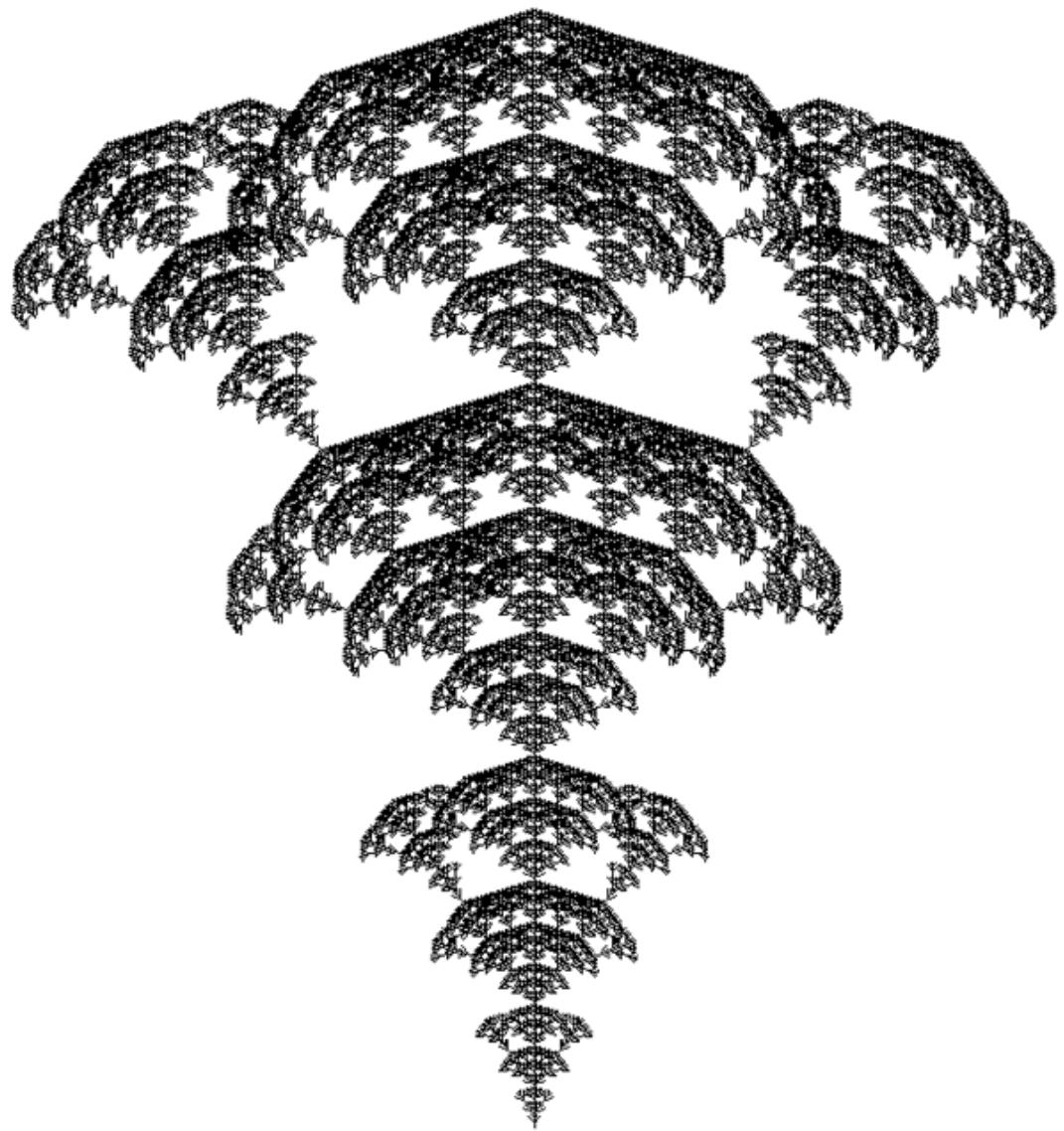
angle = 22.5



axiom = F

F -> F[+FF][-FF]F[-F][+F]F

angle = 35



Attributed to Saupe

axiom = VZFFF

V -> [+++W][---
W]YV

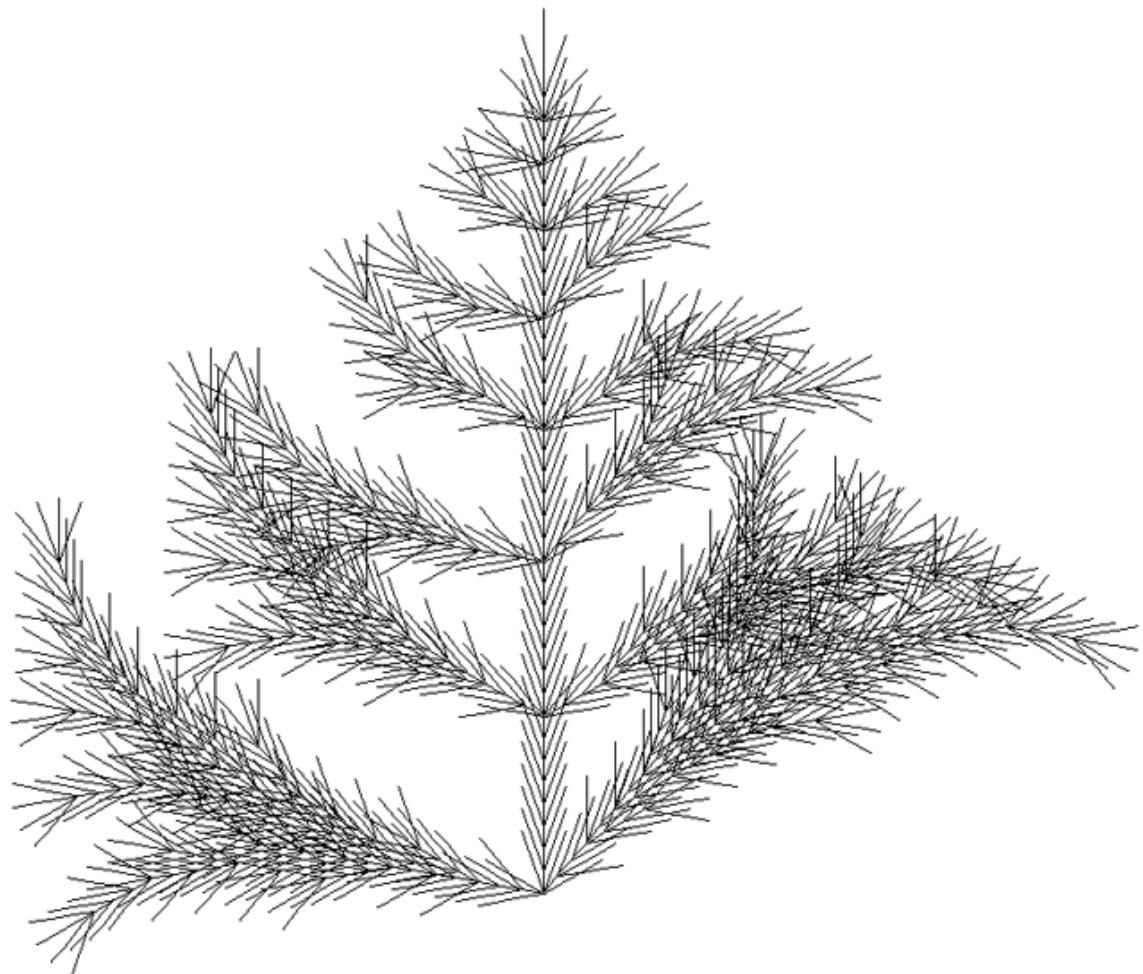
W -> +X[-W]Z

X -> -W[+X]Z

Y -> YZ

Z -> [-FFF][+FFF]F

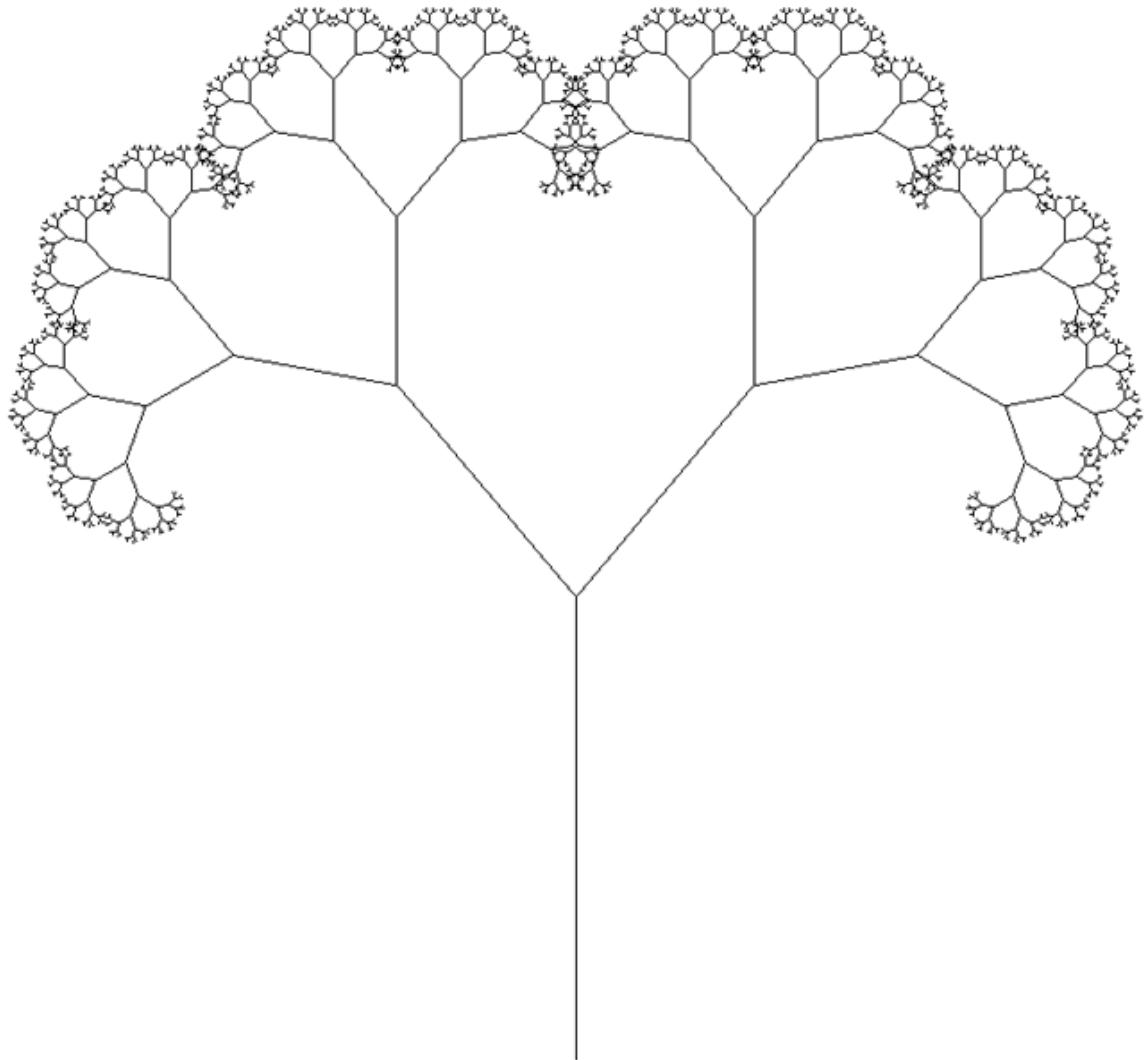
angle = 20



axiom = FX

X -> >[-FX]+FX

angle = 40

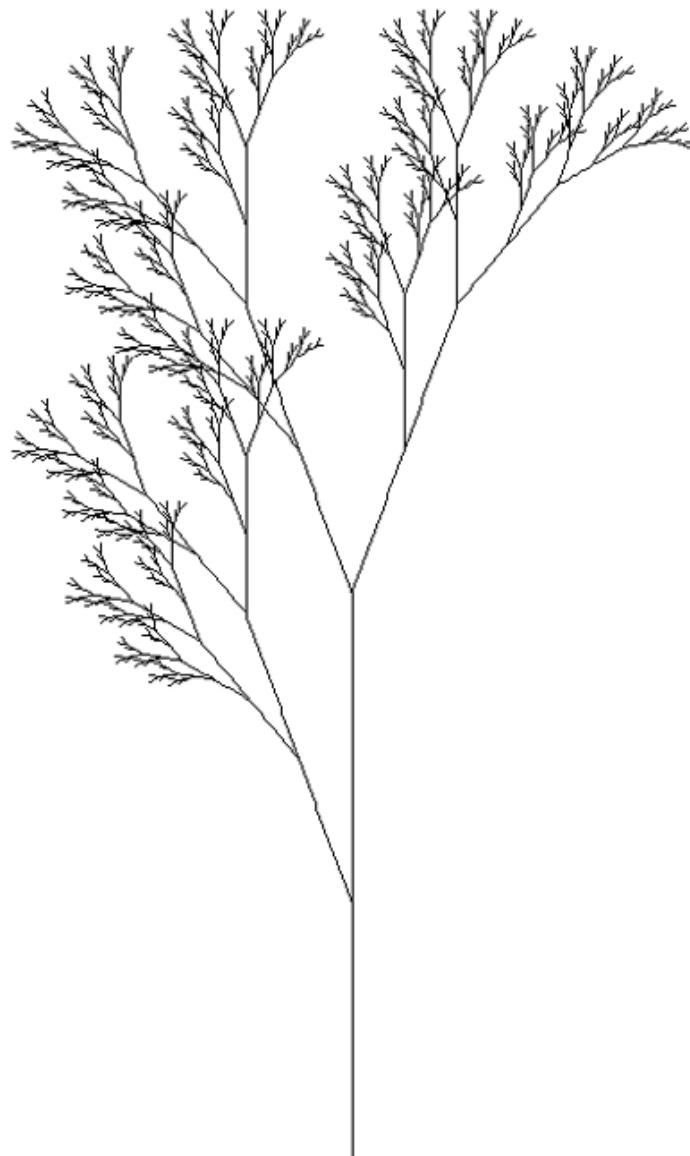




L - S y s t e m s t i c k s

Written by [Paul Bourke](#)

axiom = X
F → FF
X → F[+X]F[-X]+X
angle = 20



L - S y s t e m a l g a e

Written by [Paul Bourke](#)

axiom = aF
a → FFFFFv[++h][-

--q]fb

b -> FFFFFv[+++h][-

--q]fc

c ->

FFFFFv[+++fa]fd

d -> FFFFFv[+++h][-

--q]fe

e -> FFFFFv[+++h][-

--q]fg

g -> FFFFFv[---fa]fa

h -> ifFF

i -> fFFF[--m]j

j -> fFFF[--n]k

k -> fFFF[--o]l

l -> fFFF[--p]

m -> fFn

n -> ffO

o -> ffP

p -> ff

q -> rff

r -> fFFF[++m]s

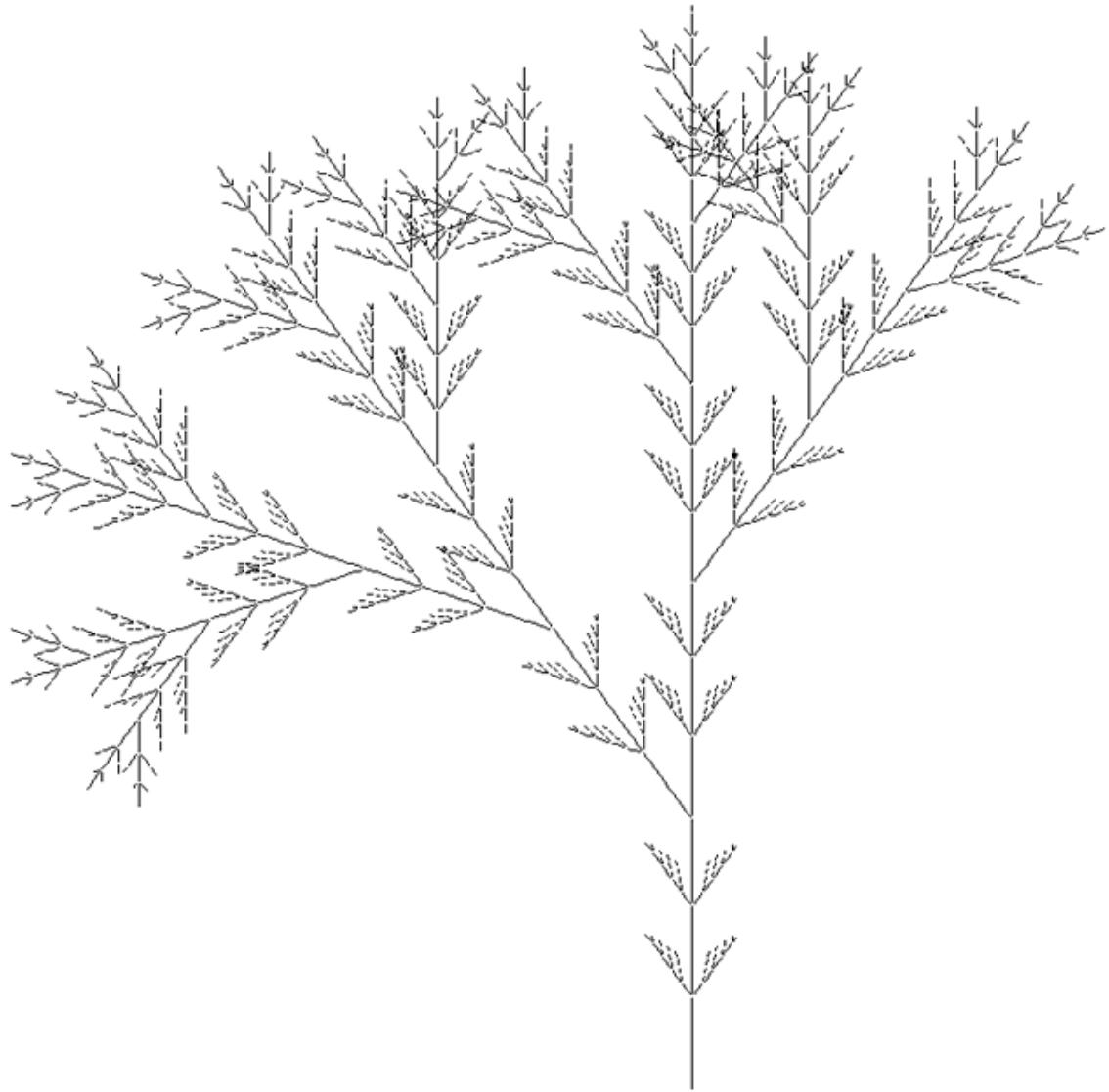
s -> fFFF[++n]t

t -> fFFF[++o]u

u -> fFFF[++p]

v -> Fv

angle = 12



axiom = aF

a -> FFFFFy[++++n]

[----t]fb

b ->

+FFFFFy[++++n][---

-t]fc

c -> FFFFFy[++++n]

[----t]fd

d -> -

FFFFFy[++++n][----

t]fe

e -> FFFFFy[++++n]

[----t]fg

g ->

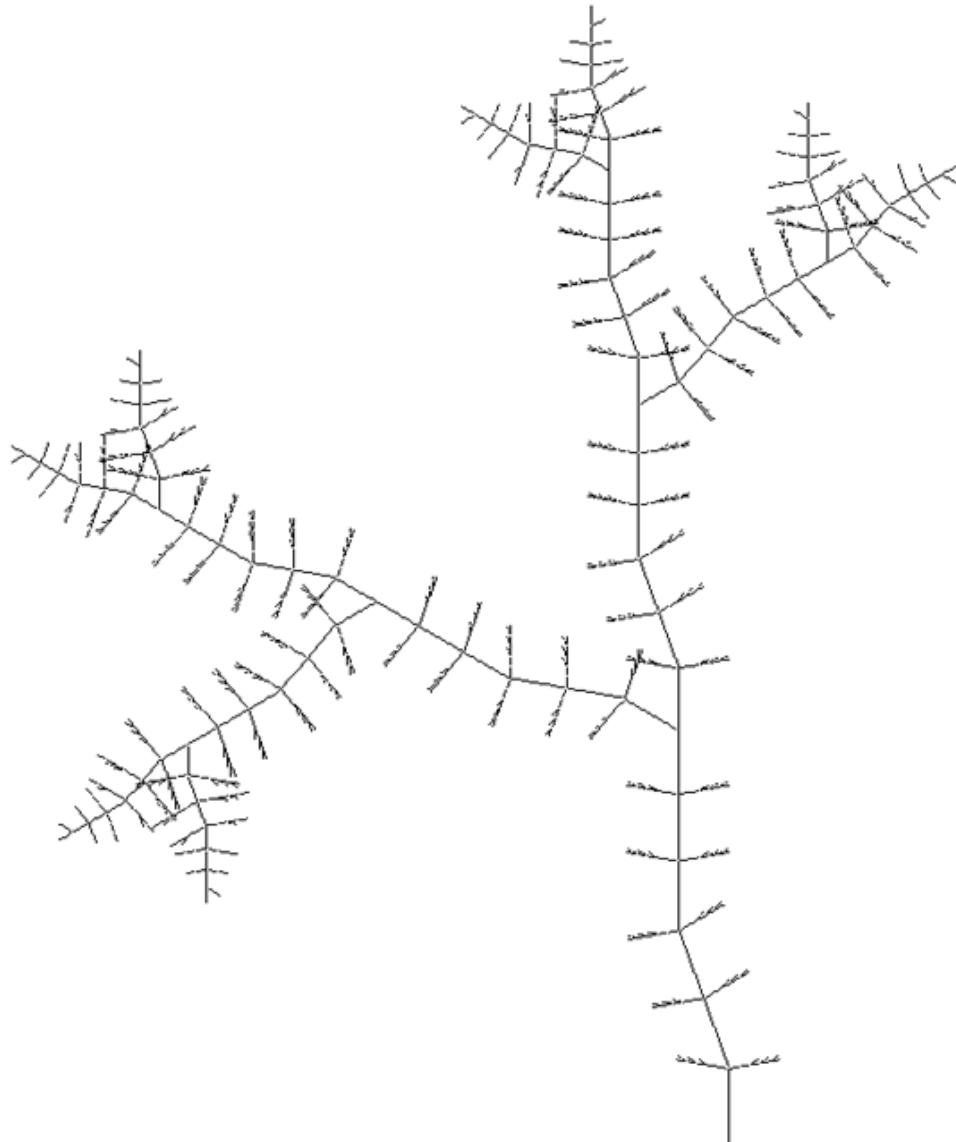
FFFFFy[+++fa]fh

h -> FFFFFy[++++n]

```

[---t]fi
i ->
+FFFFFy[++++n][---
-t]fj
j -> FFFFFy[++++n]
[---t]fk
k -> -
FFFFFy[++++n][---
t]fl
l -> FFFFFy[++++n]
[---t]fm
m -> FFFFFy[---fa]fa
n -> offF
o -> fFFFp
p -> fFFF[-s]q
q -> fFFF[-s]r
r -> fFFF[-s]
s -> fFff
t -> uFFF
u -> fFFFv
v -> fFFF[+s]w
w -> fFFF[+s]x
x -> fFFF[+s]
y -> Fy
angle = 12

```



L - S y s t e m W e e d

Written by [Paul Bourke](#)

```

axiom = F
F -> FF-[XY]+[XY]
X -> +FY
Y -> -FX
angle = 22.5

```



T r i a n g l e

Written by [Paul Bourke](#)

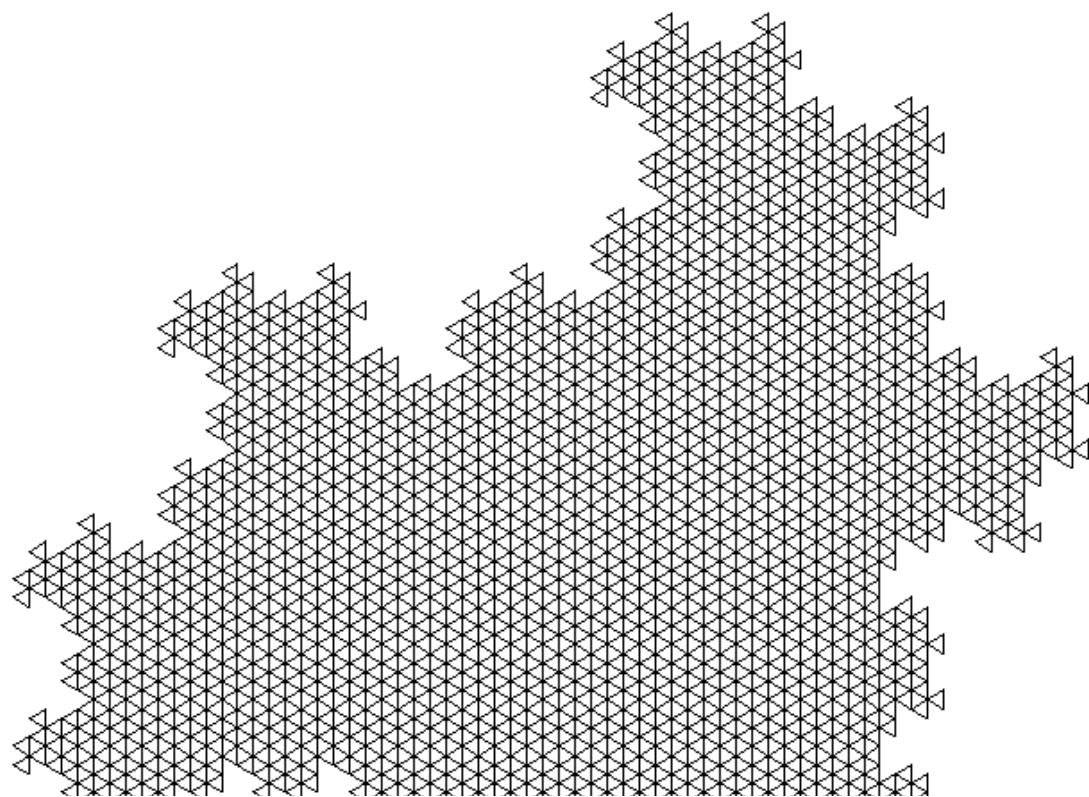
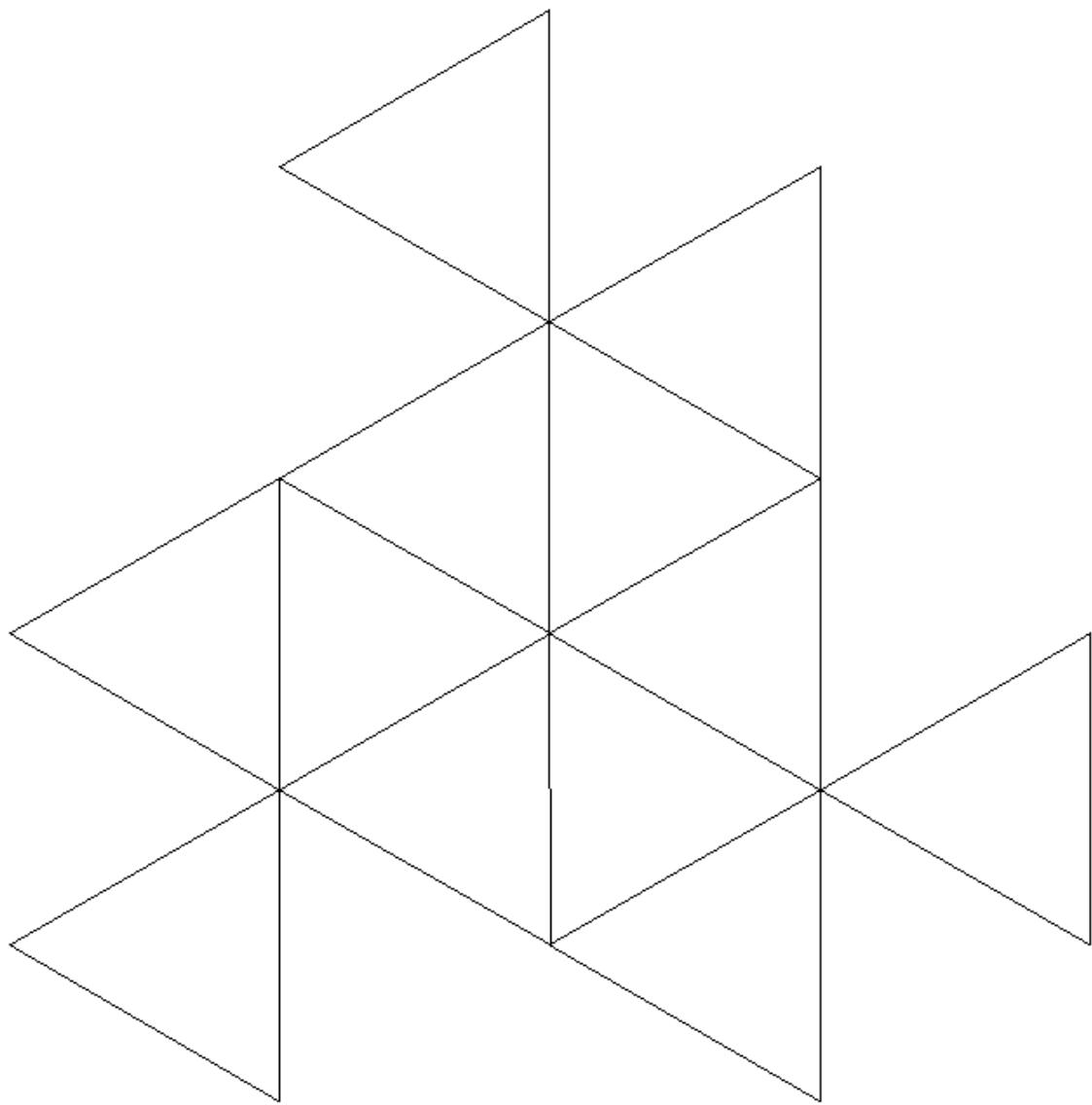
July 1990

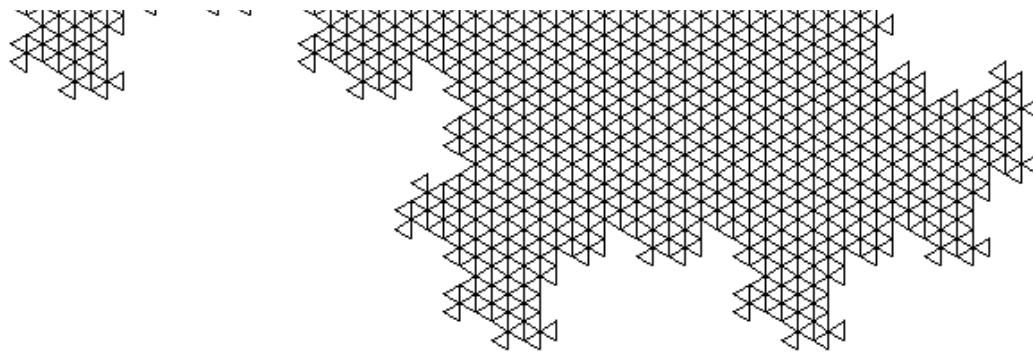
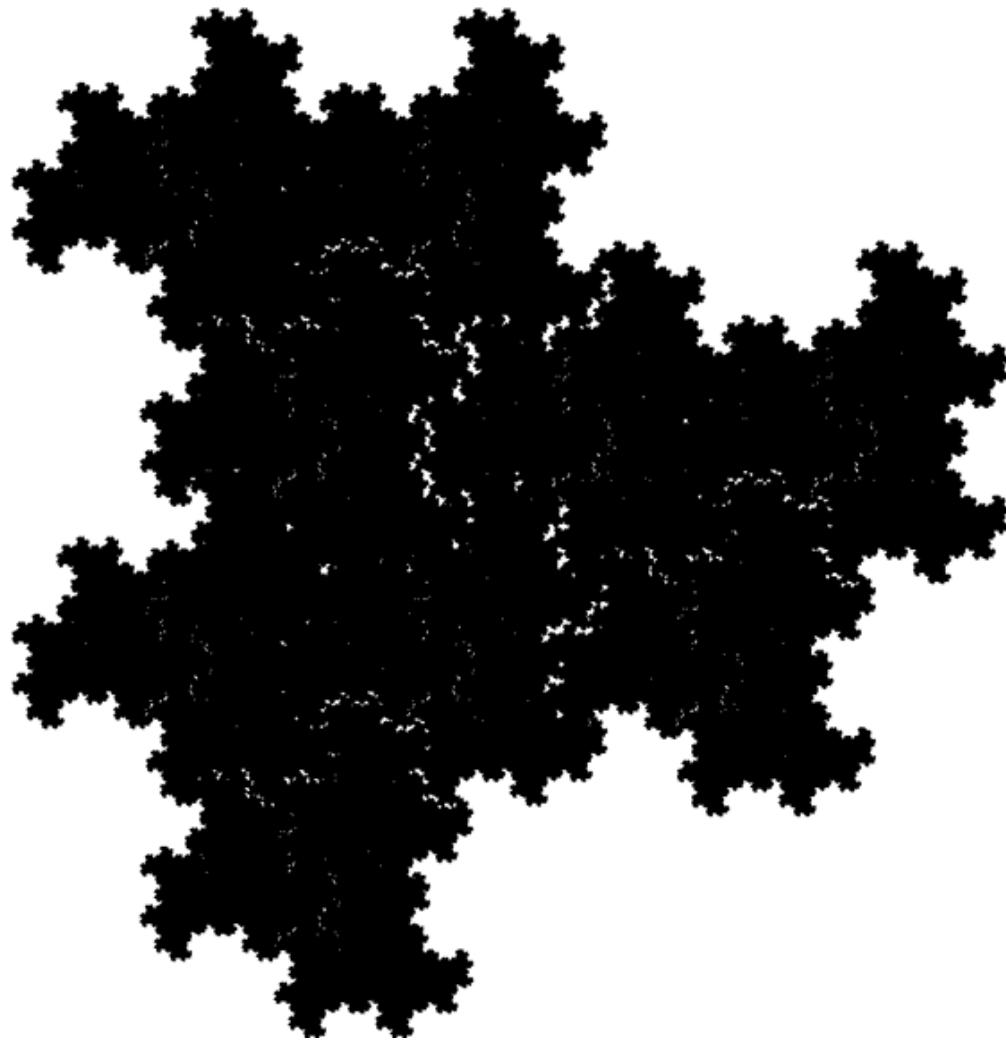
L-Systems

axiom = F+F+F

F -> F-F+F

angle = 120



**As an IFS**

The IFS equations are as follows

$$x_{n+1} = a x_n + b y_n + e$$

$$y_{n+1} = c x_n + d y_n + f$$

The parameter table:

	set 1	set 2	set 3
a	0.0	0.0	0.0
b	0.577	0.577	0.577
c	-0.577	-0.577	-0.577

d	0.0	0.0	0.0
e	0.0951	0.4413	0.0952
f	0.5983	0.7893	0.9893
probability	1/3	1/3	1/3

Quadratic Gosper

Written by [Paul Bourke](#)

June 1990

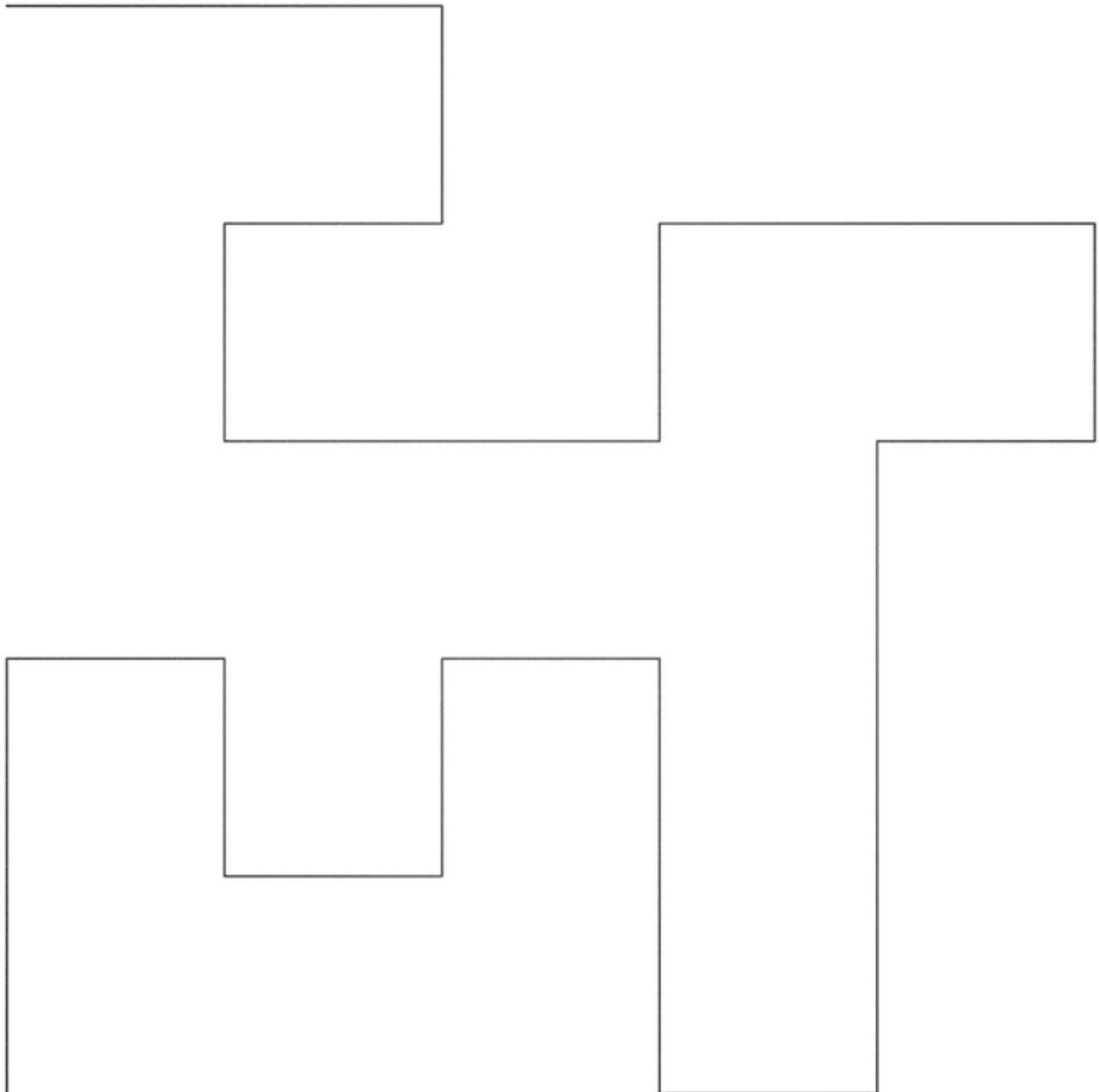
Attributed to Dekking, 1982

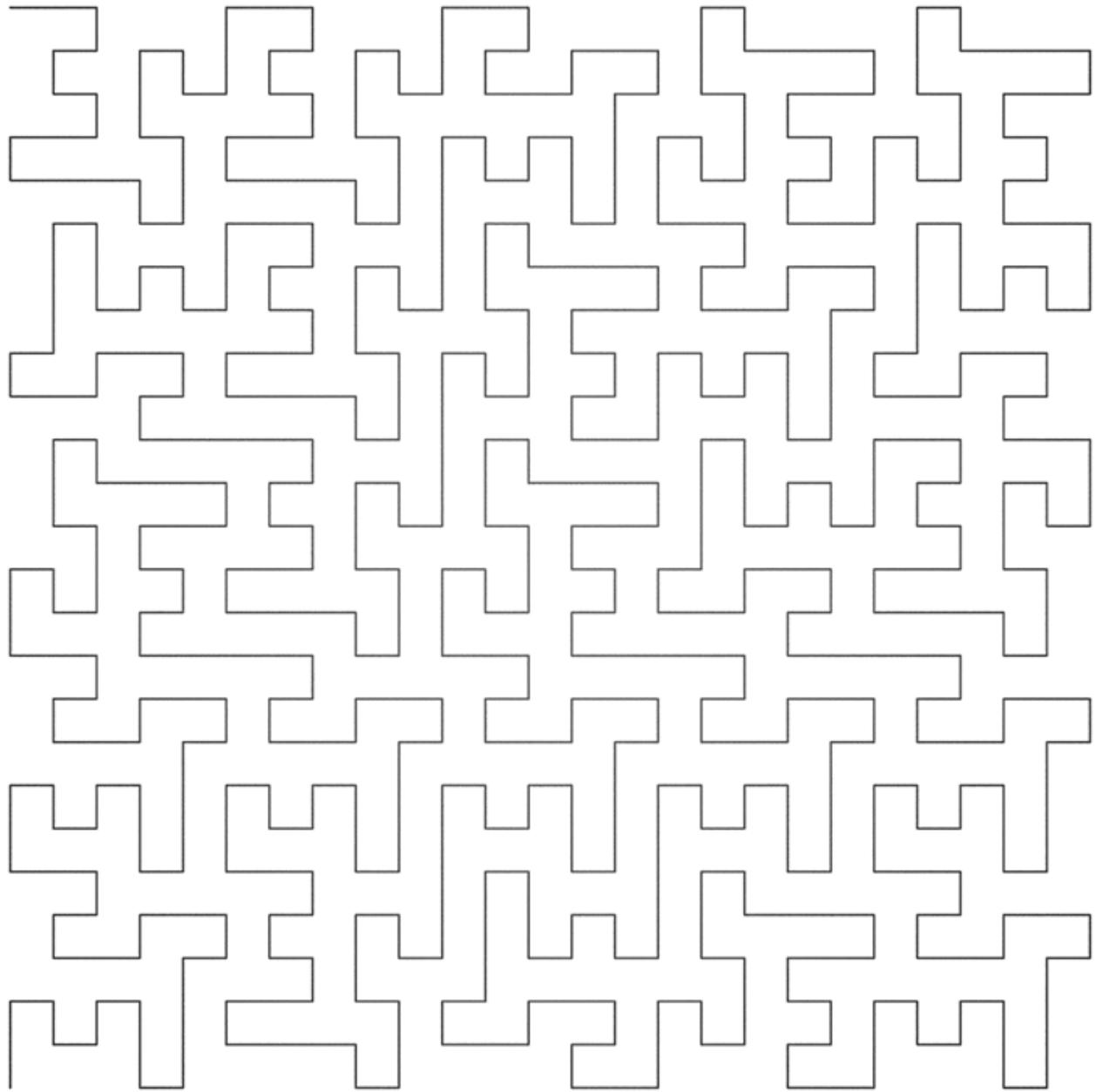
axiom = -YF

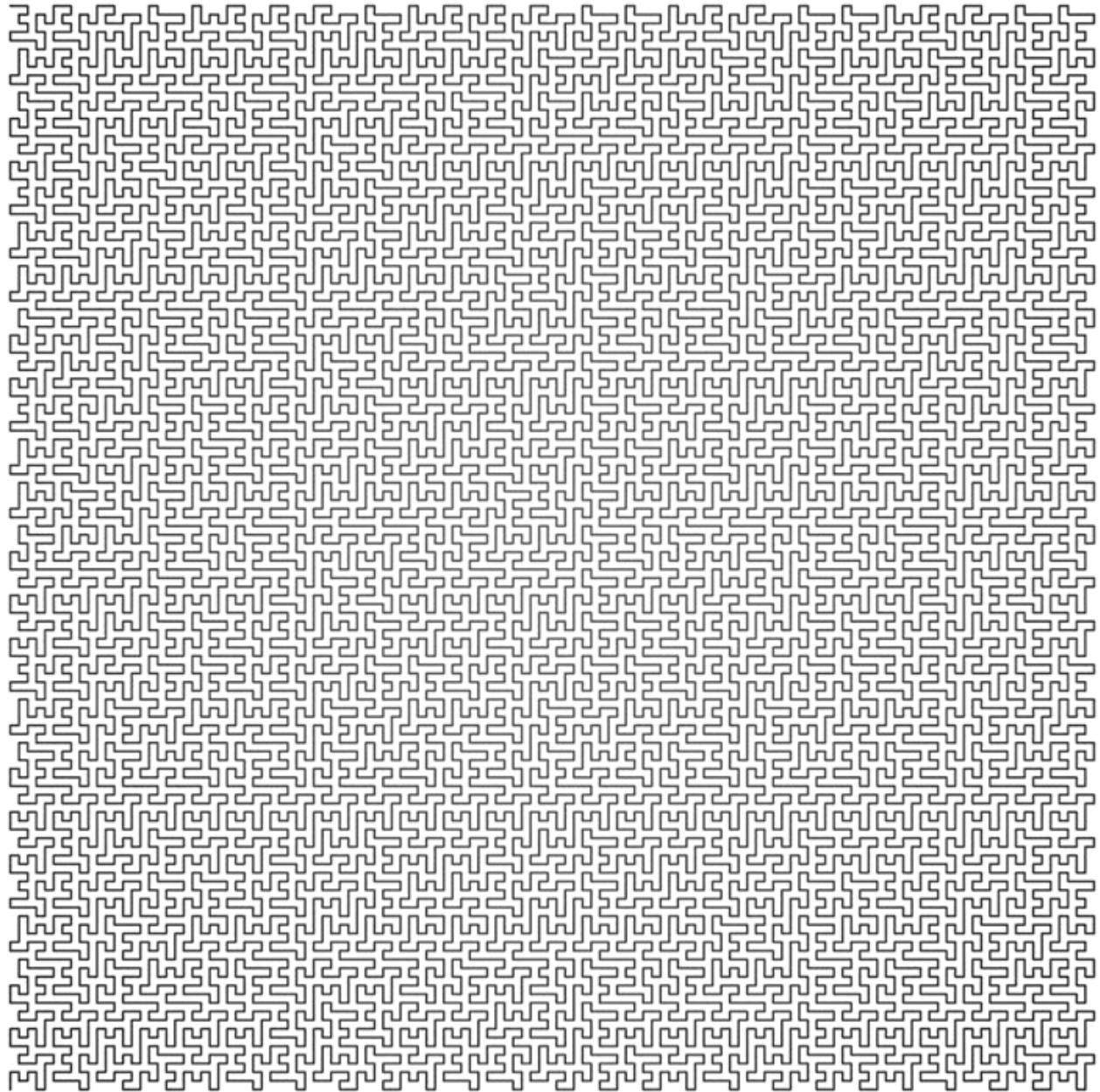
X -> XFX-YF-YF+FX+FX-YF-YFFX+YF+FXFXYF-FX+YF+FXFX+YF-FXYF-YF-FX+FX+YFYF-

Y -> +FXFX-YF-YF+FX+FXYF+FX-YFYF-FX-YF+FXYFYF-FX-YFFX+FX+YF-YF-FX+FX+YFY

angle = 90







S q u a r e S i e r p i n s k i

Written by [Paul Bourke](#)

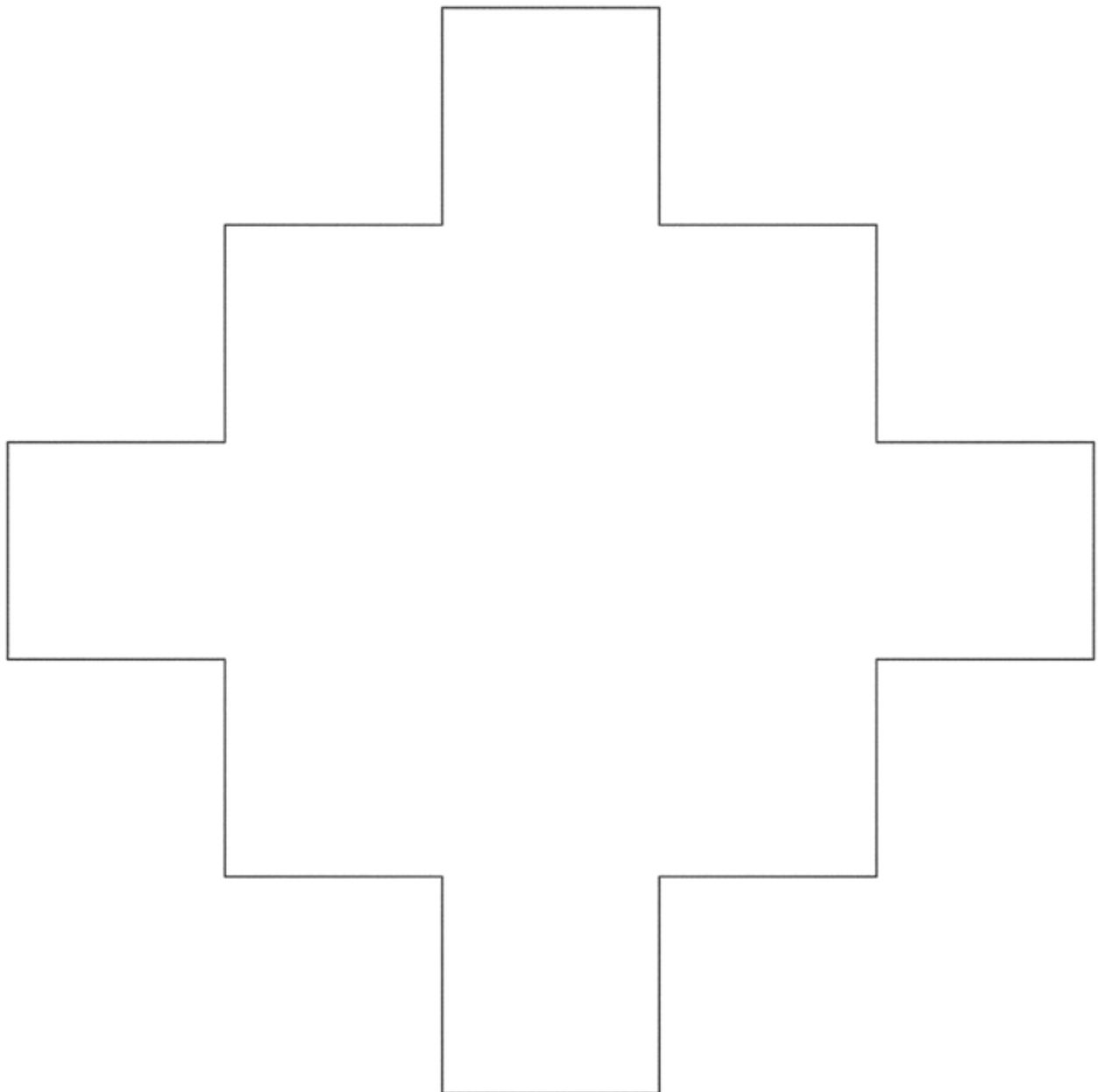
August 1990

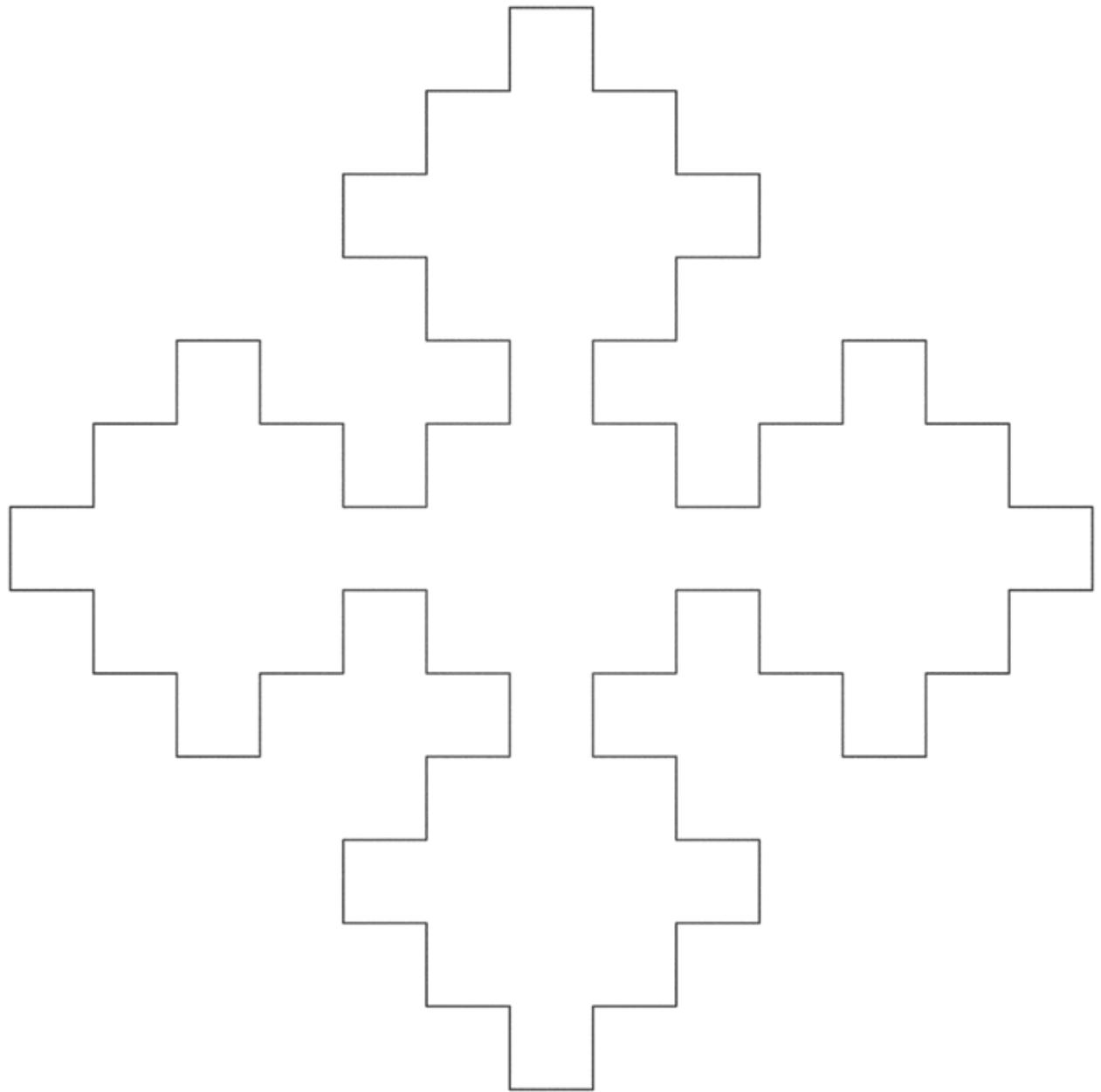
First reported circa 1912

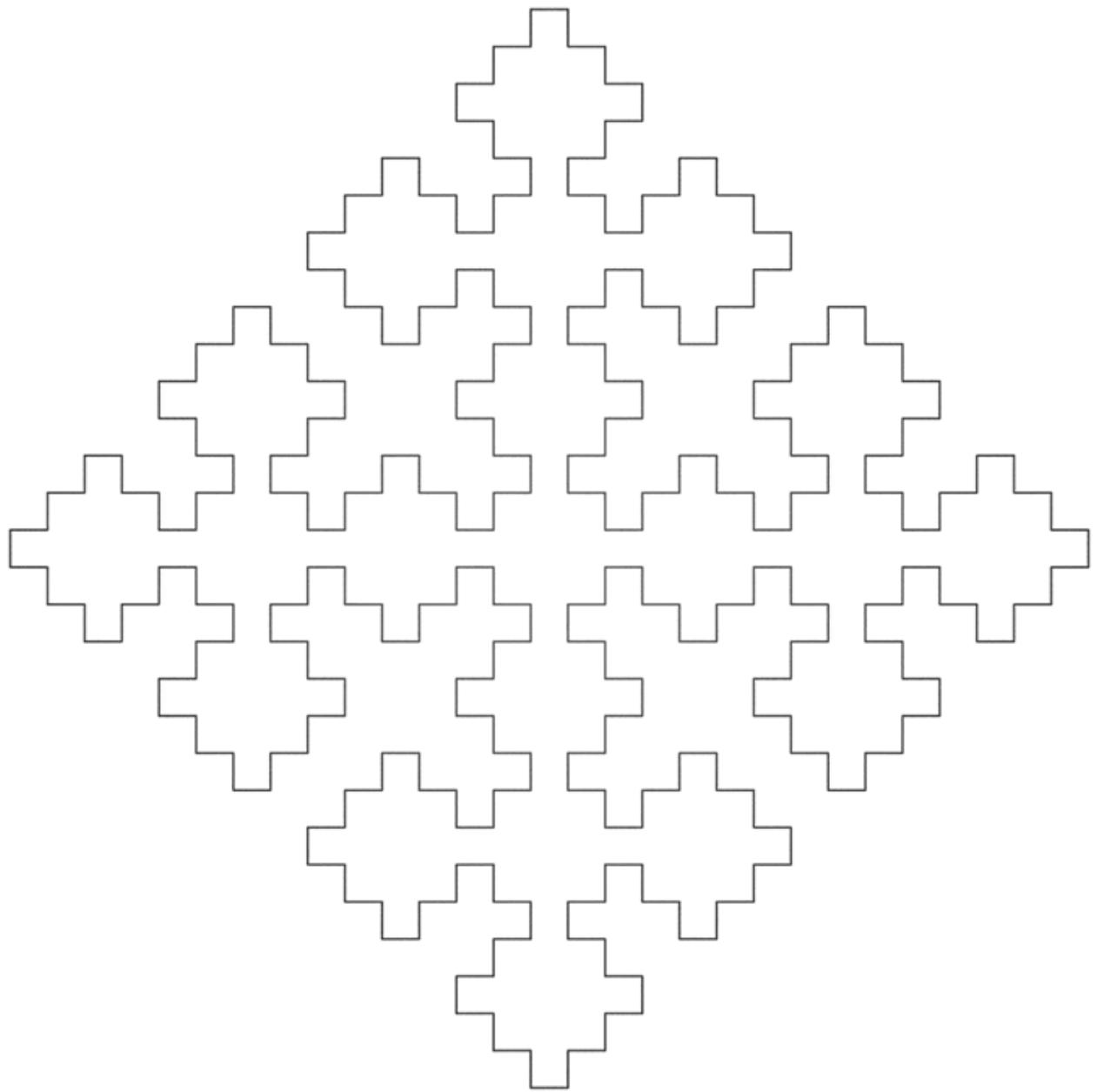
axiom = F+XF+F+XF

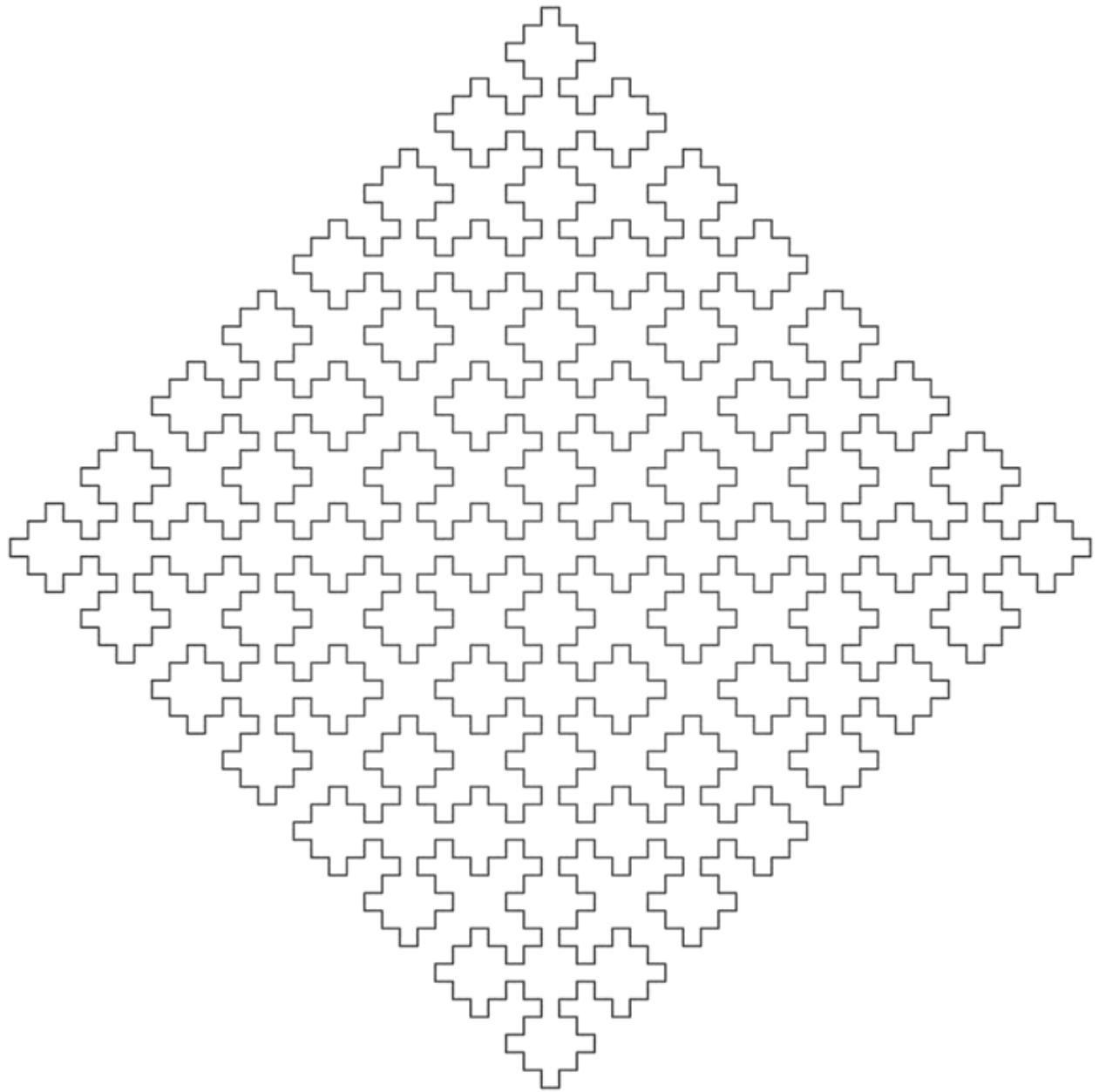
X -> XF-F+F-XF+F+XF-F+F-X

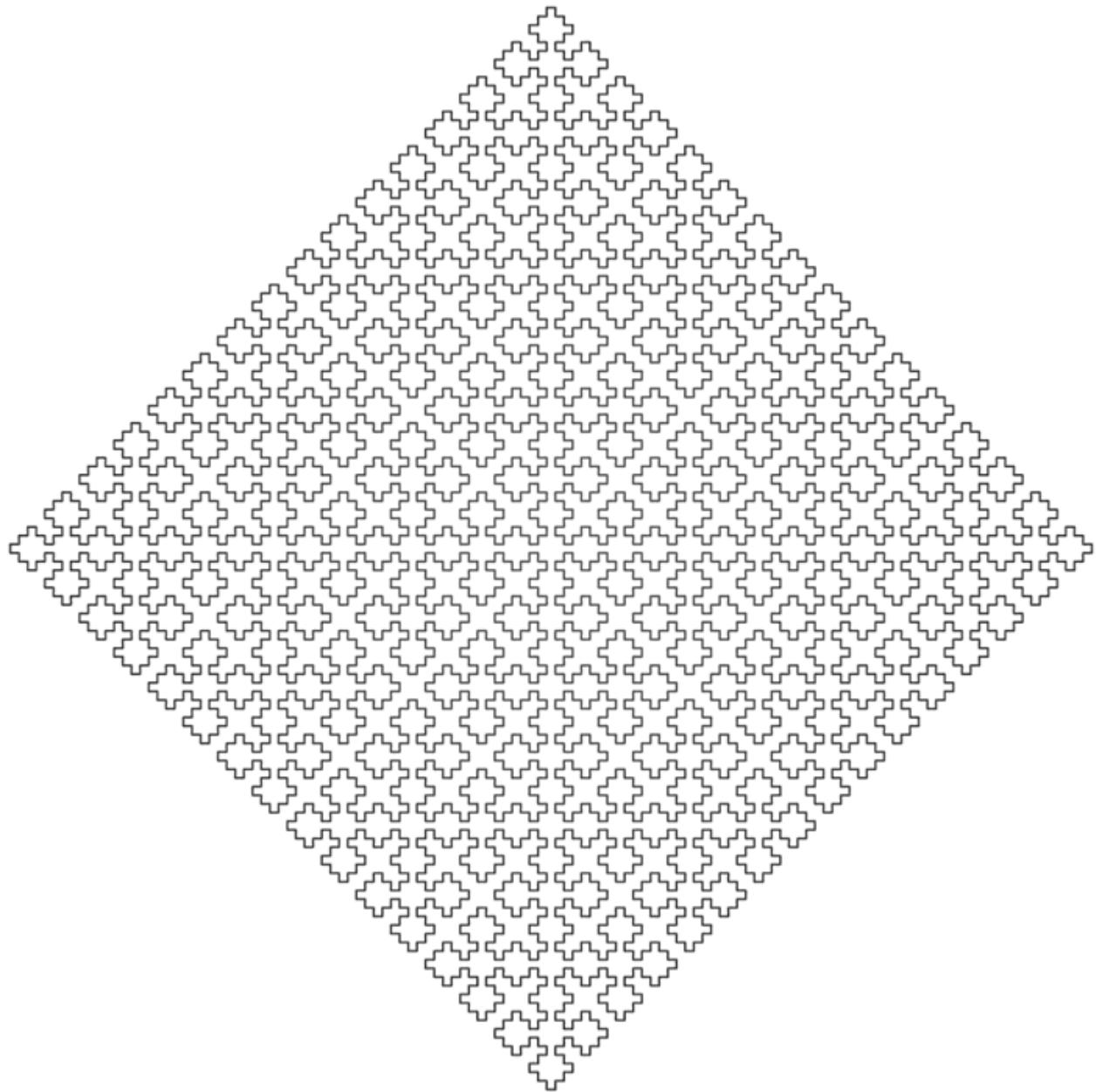
angle = 90











C r y s t a l

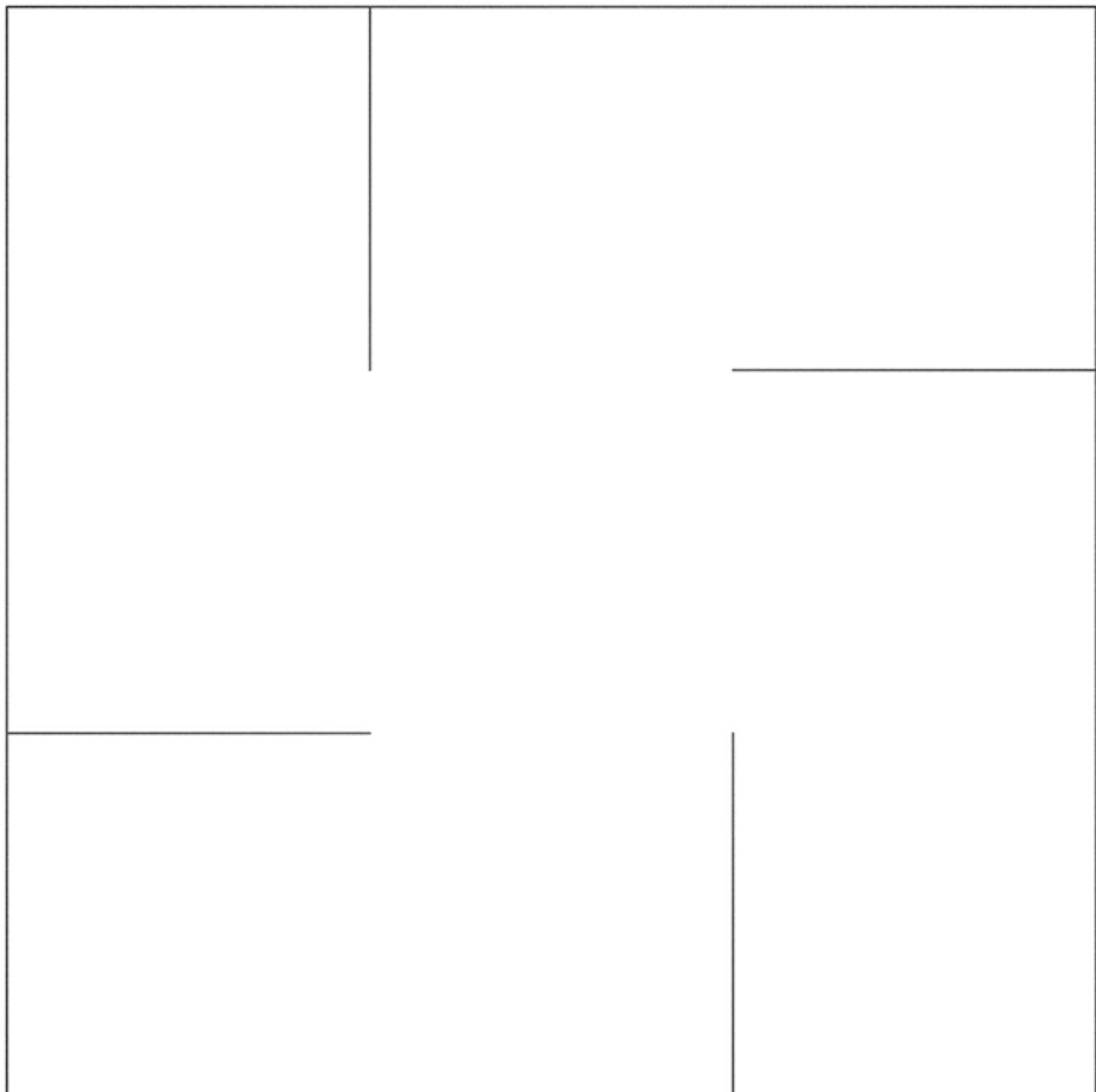
Written by [Paul Bourke](#)

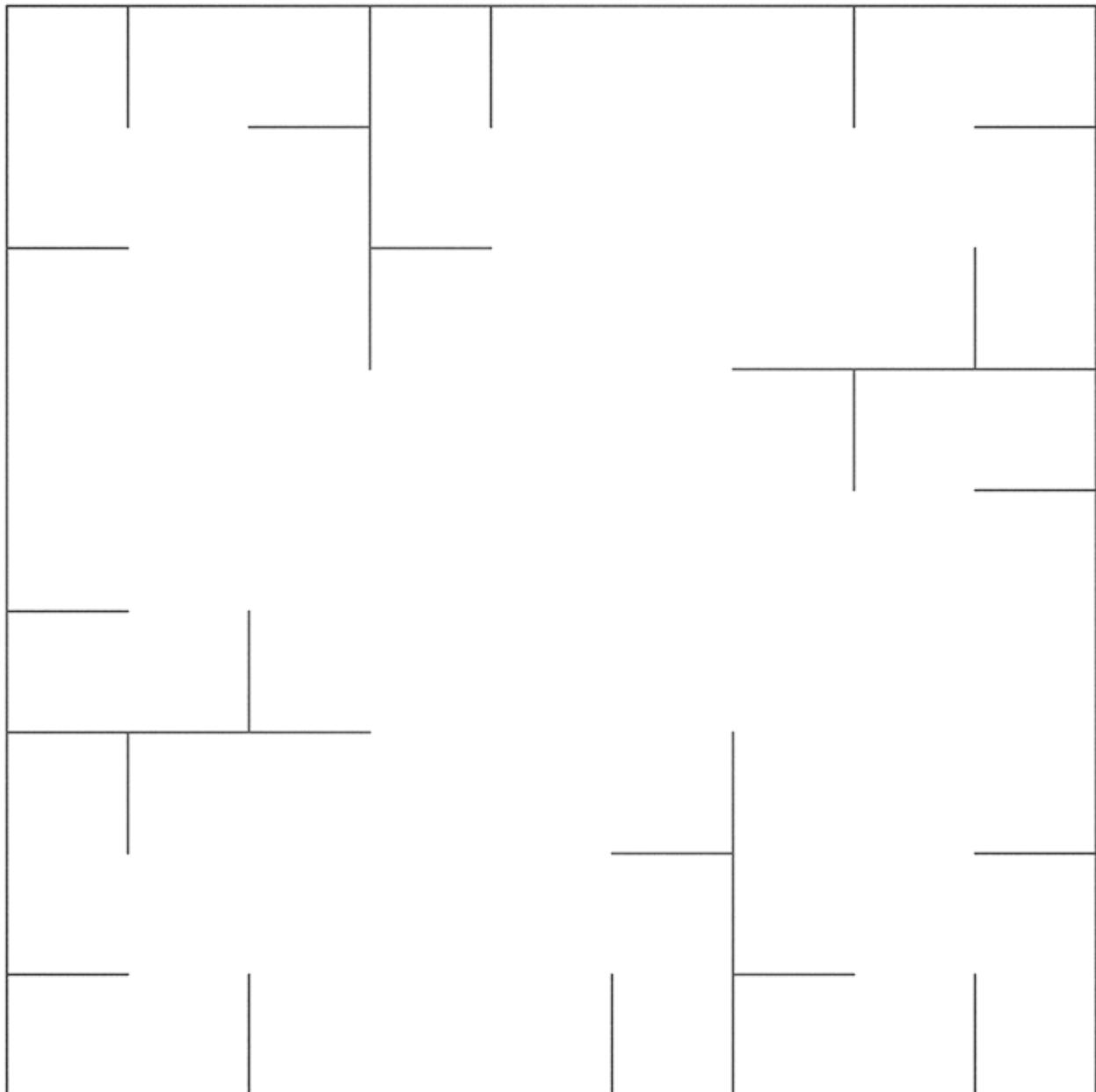
June 1990

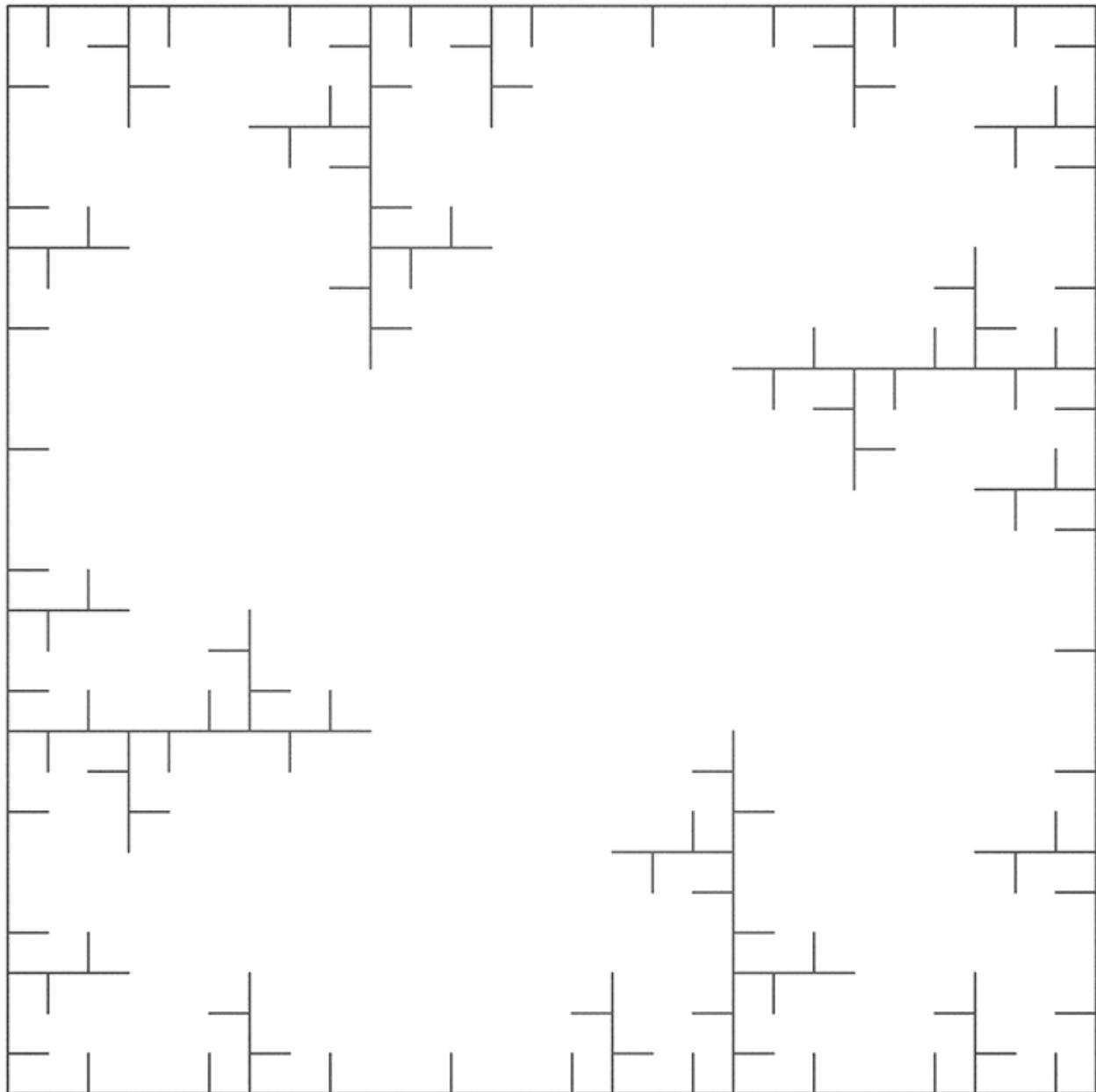
axiom = F+F+F+F

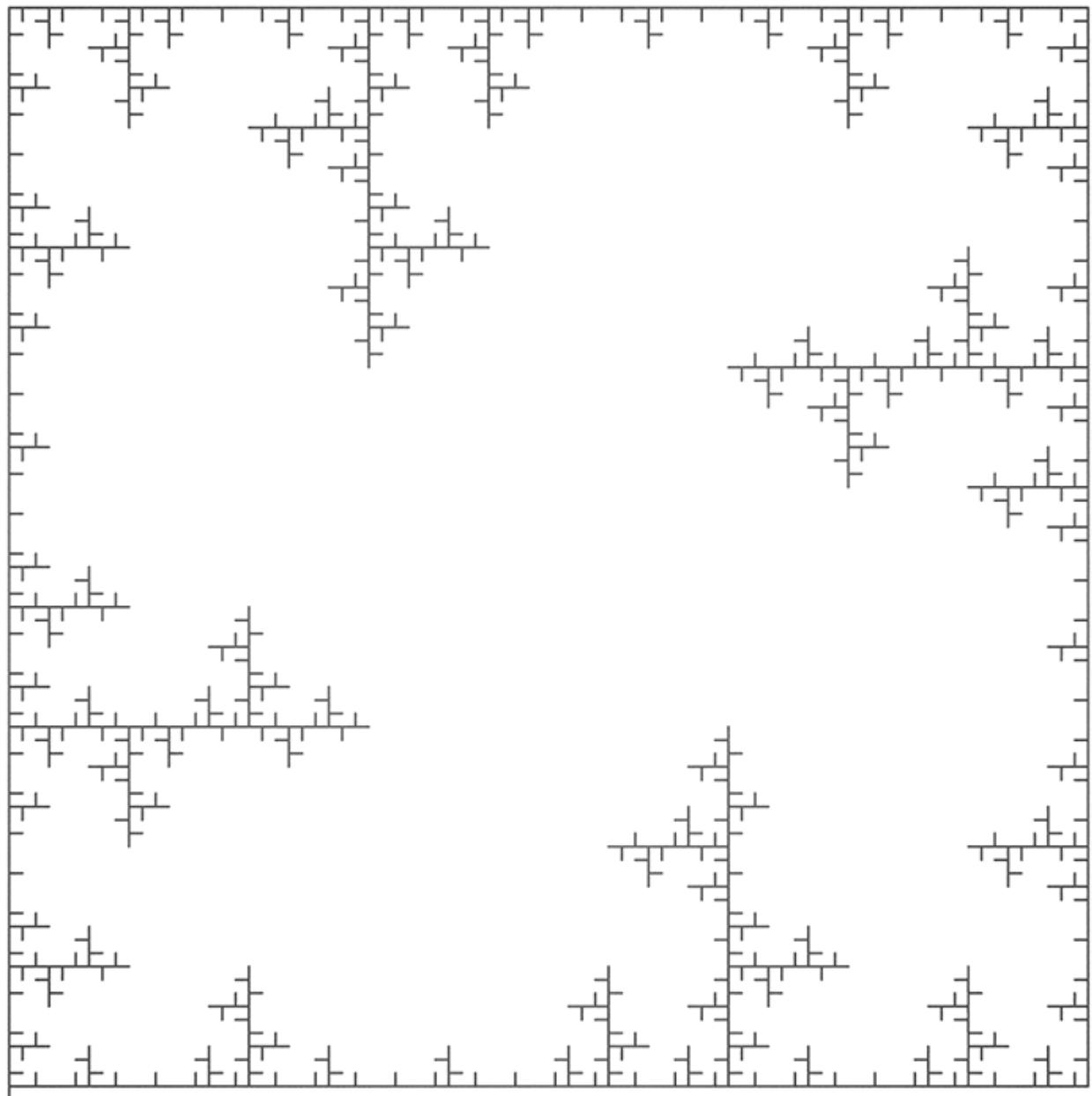
F -> FF+F++F+F

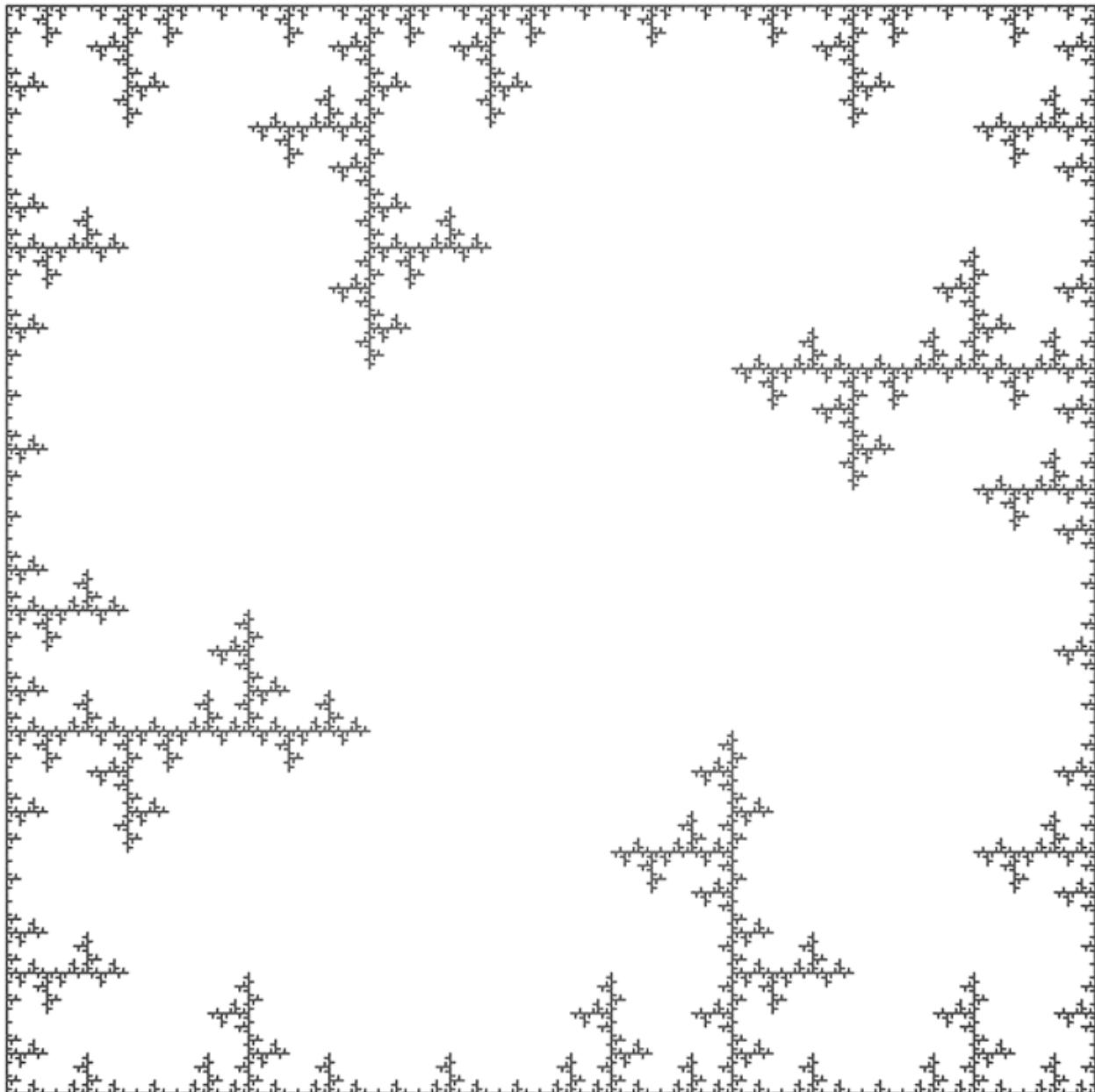
angle = 90













Derby, Western Australia

Peano Curve

Written by [Paul Bourke](#)

June 1990

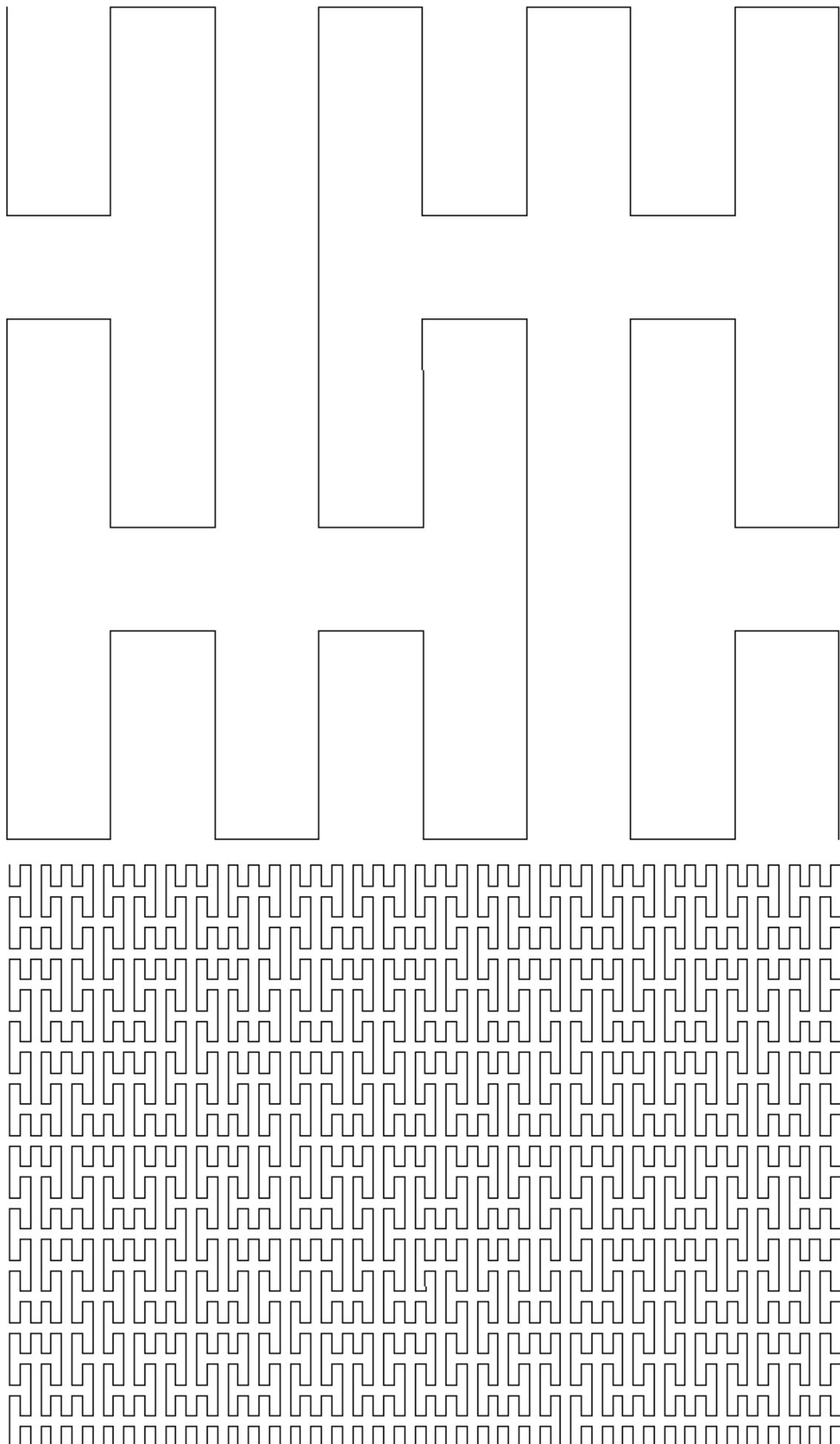
First documented circa 1890

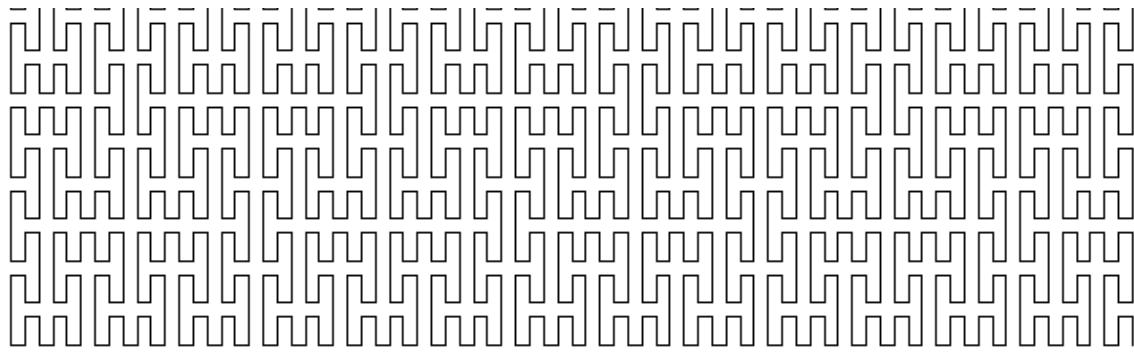
axiom = X

X -> XYFYX+F+YFXFY-F-XFYFX

Y -> YFXFY-F-XFYFX+F+YFXFY

angle = 90





Quadratic Snowflake

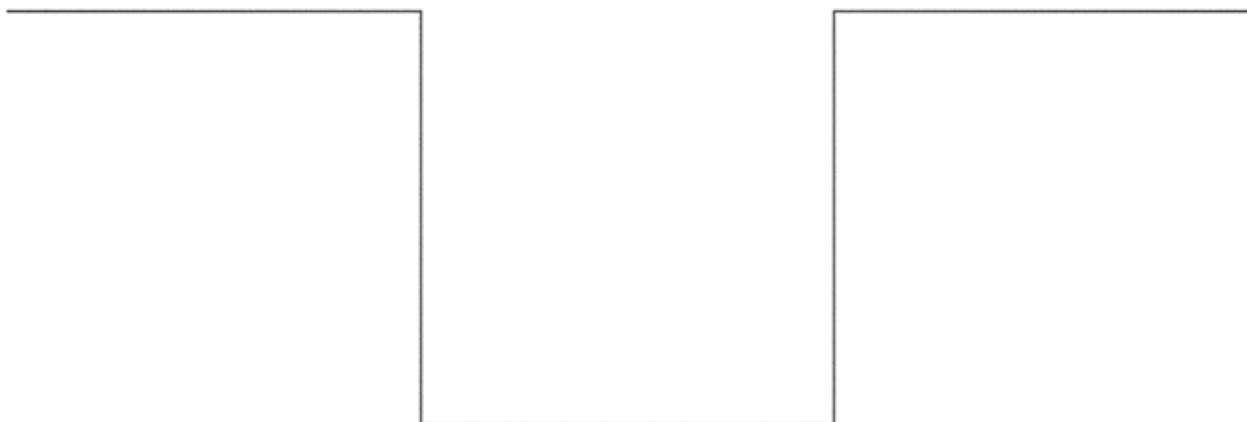
Written by [Paul Bourke](#)

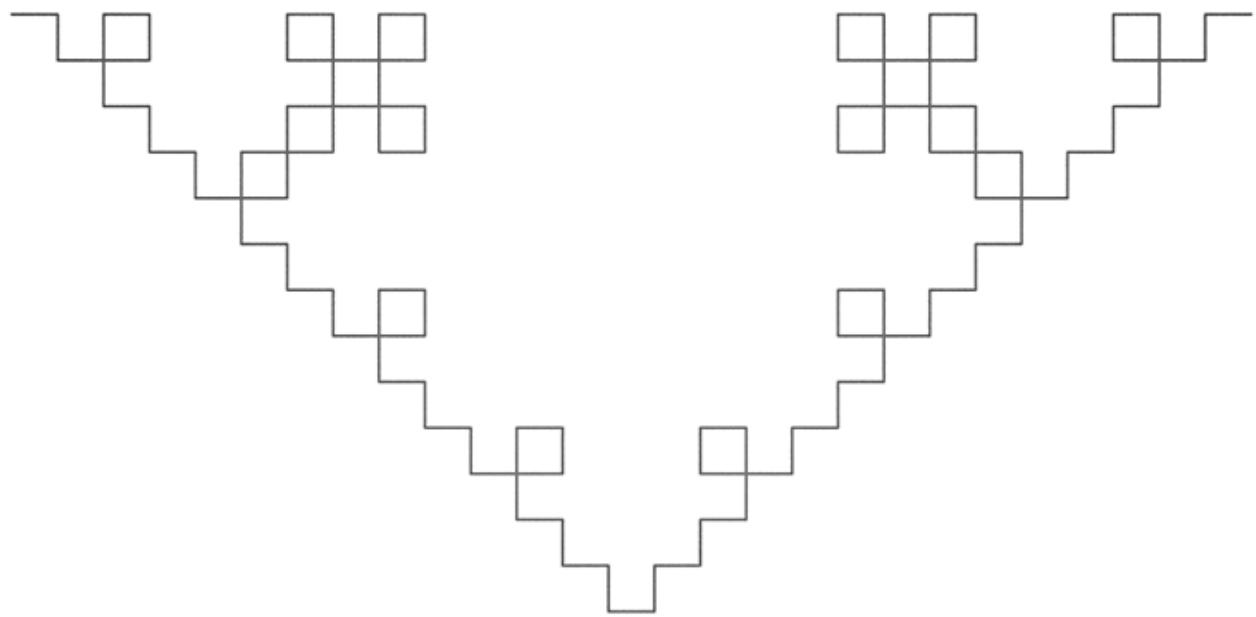
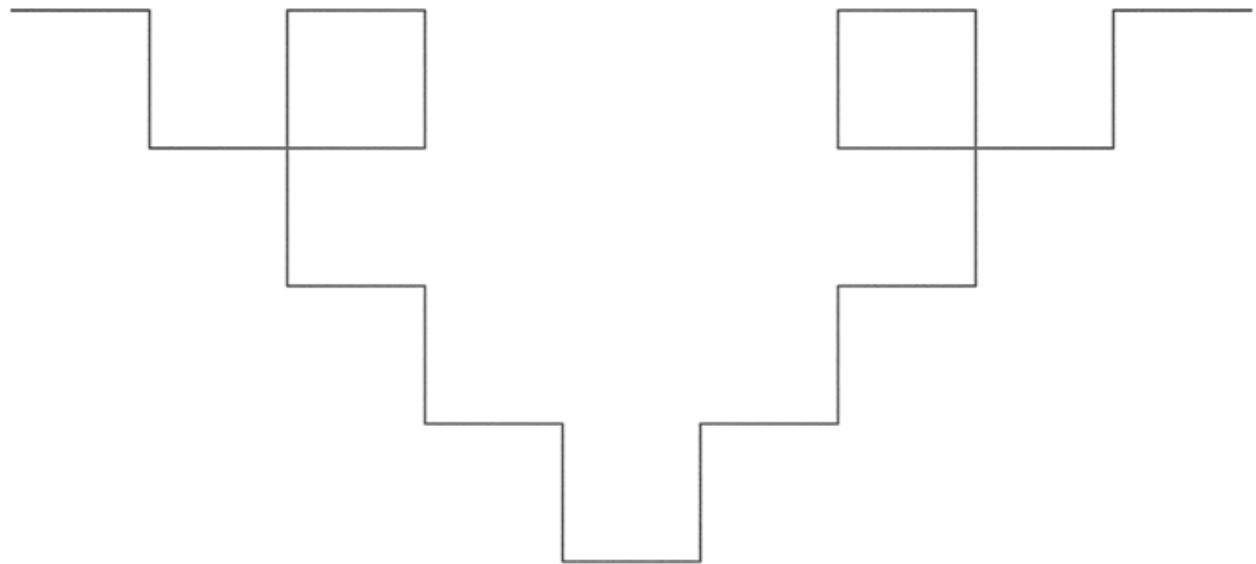
July 1990

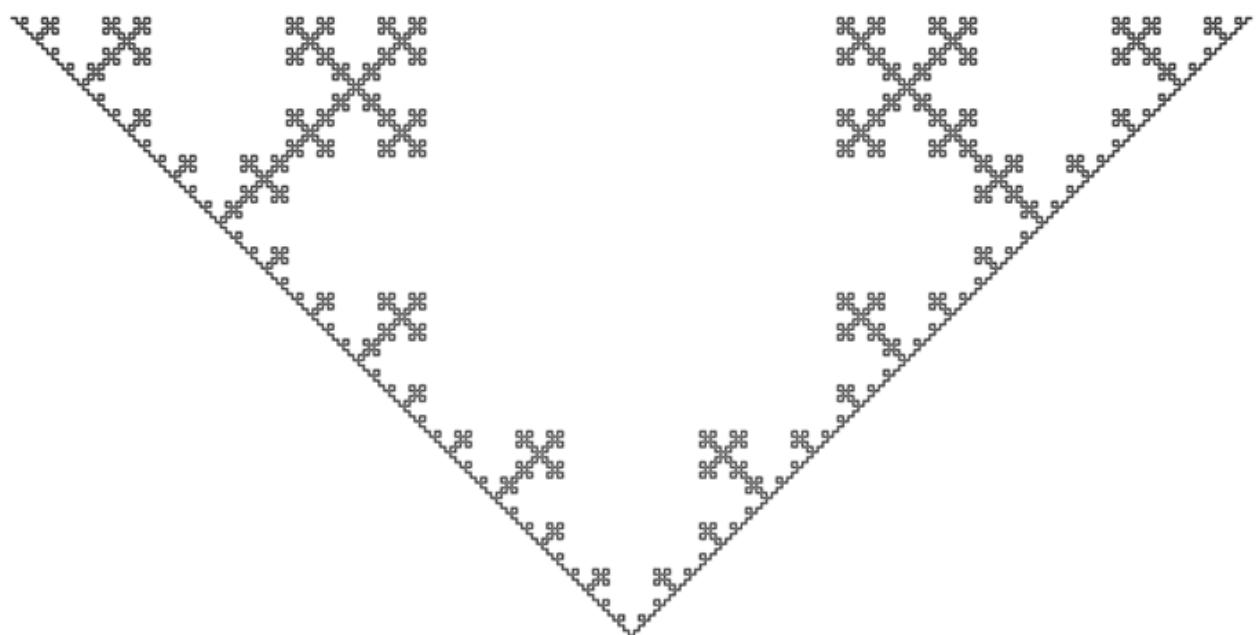
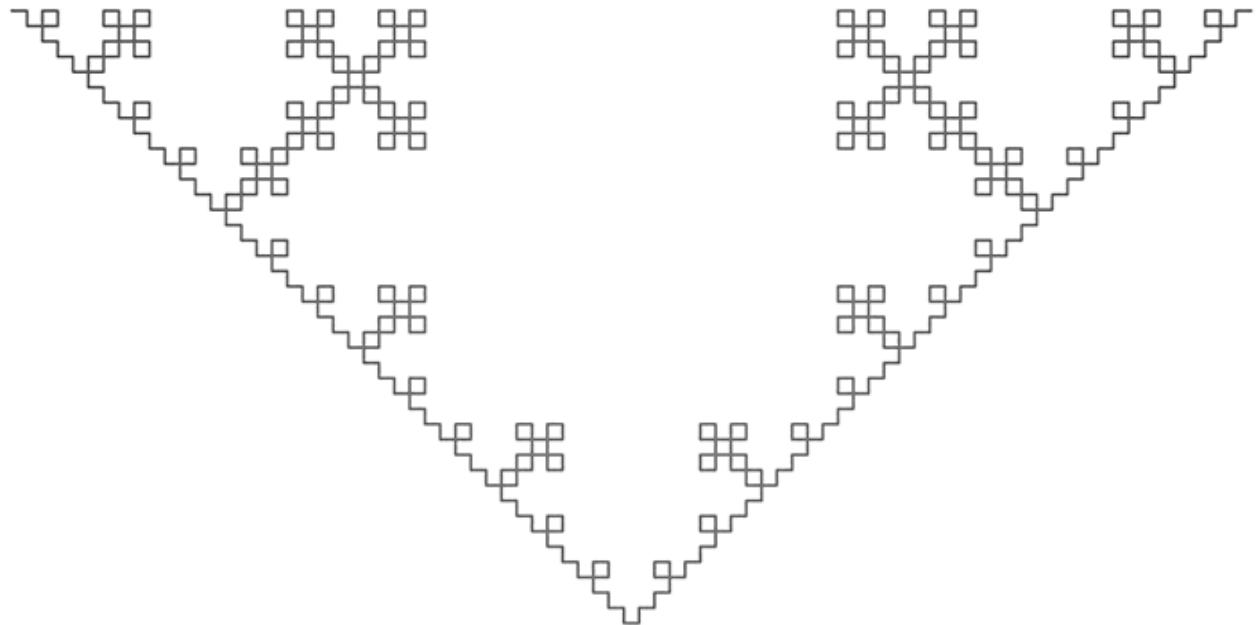
axiom = F

F -> F-F+F-F-F

angle = 90







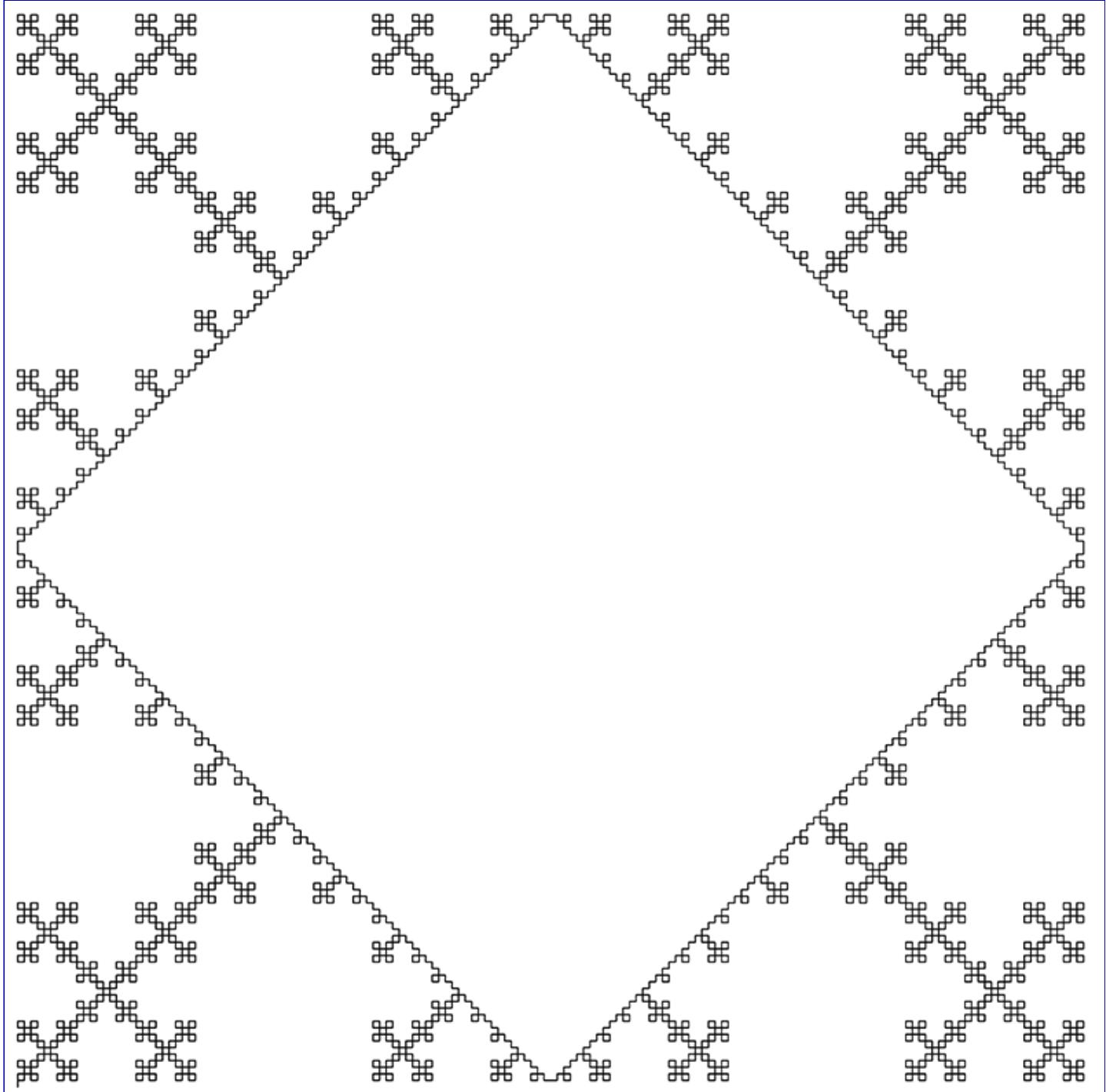
Variation by Hasan Hosam.

December 2018

axiom = FF+FF+FF+FF

F -> F+F-F-F+F

angle = 90



Quadratic Koch Island

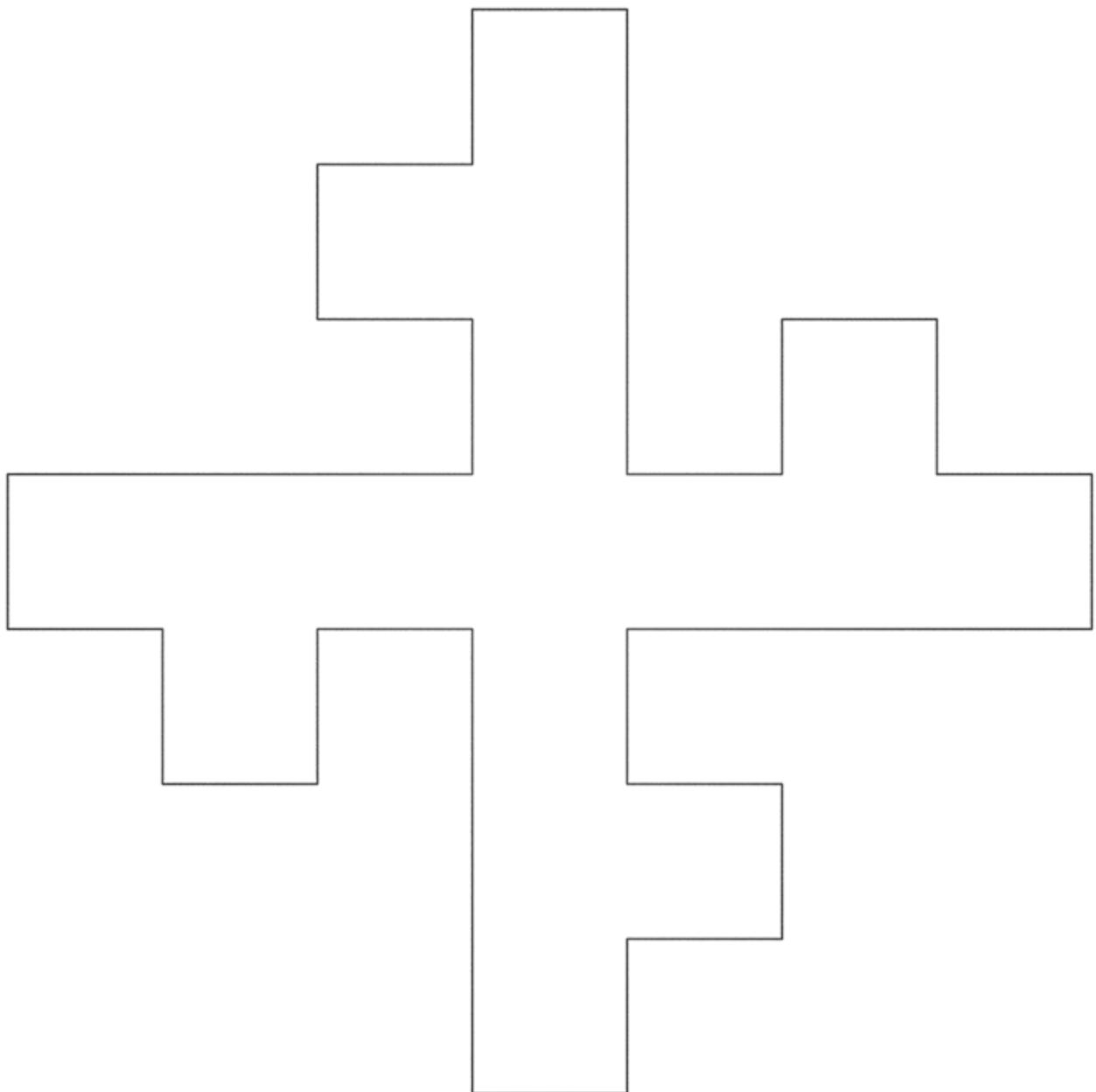
Written by [Paul Bourke](#)

July 1990

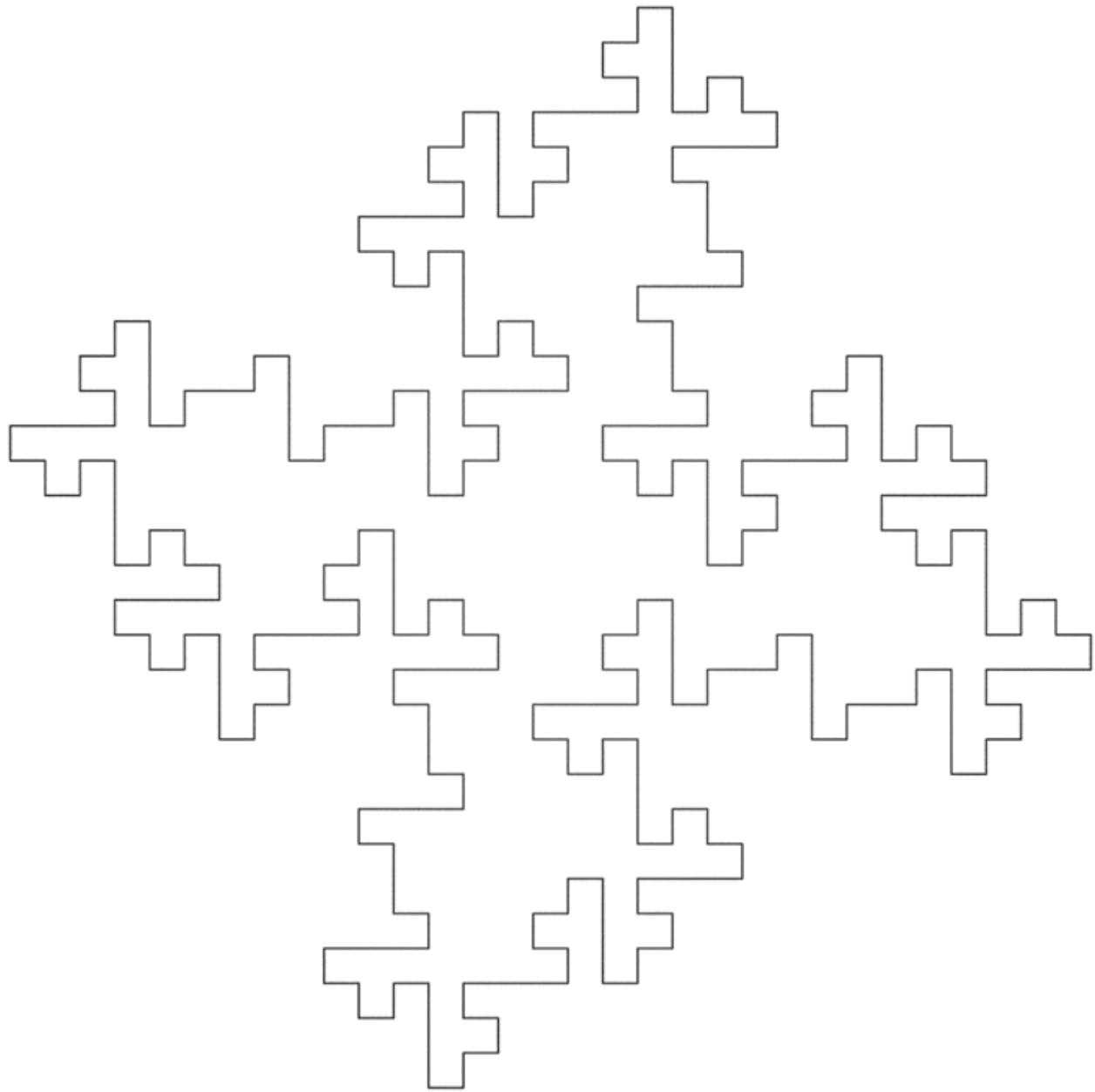
axiom = F+F+F+F
F -> F+F-F-FFF+F+F-F

angle = 90

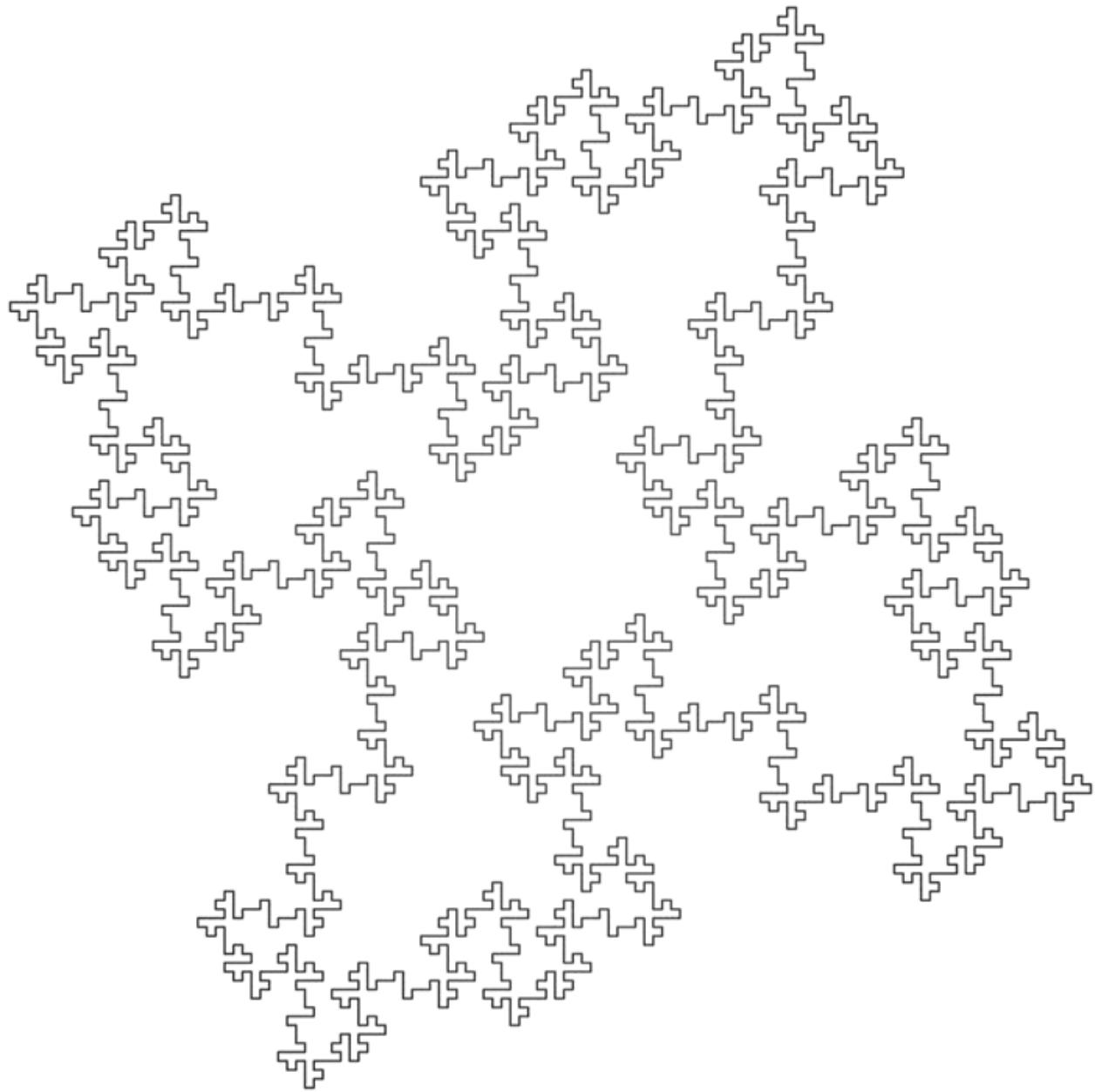
String length: 63



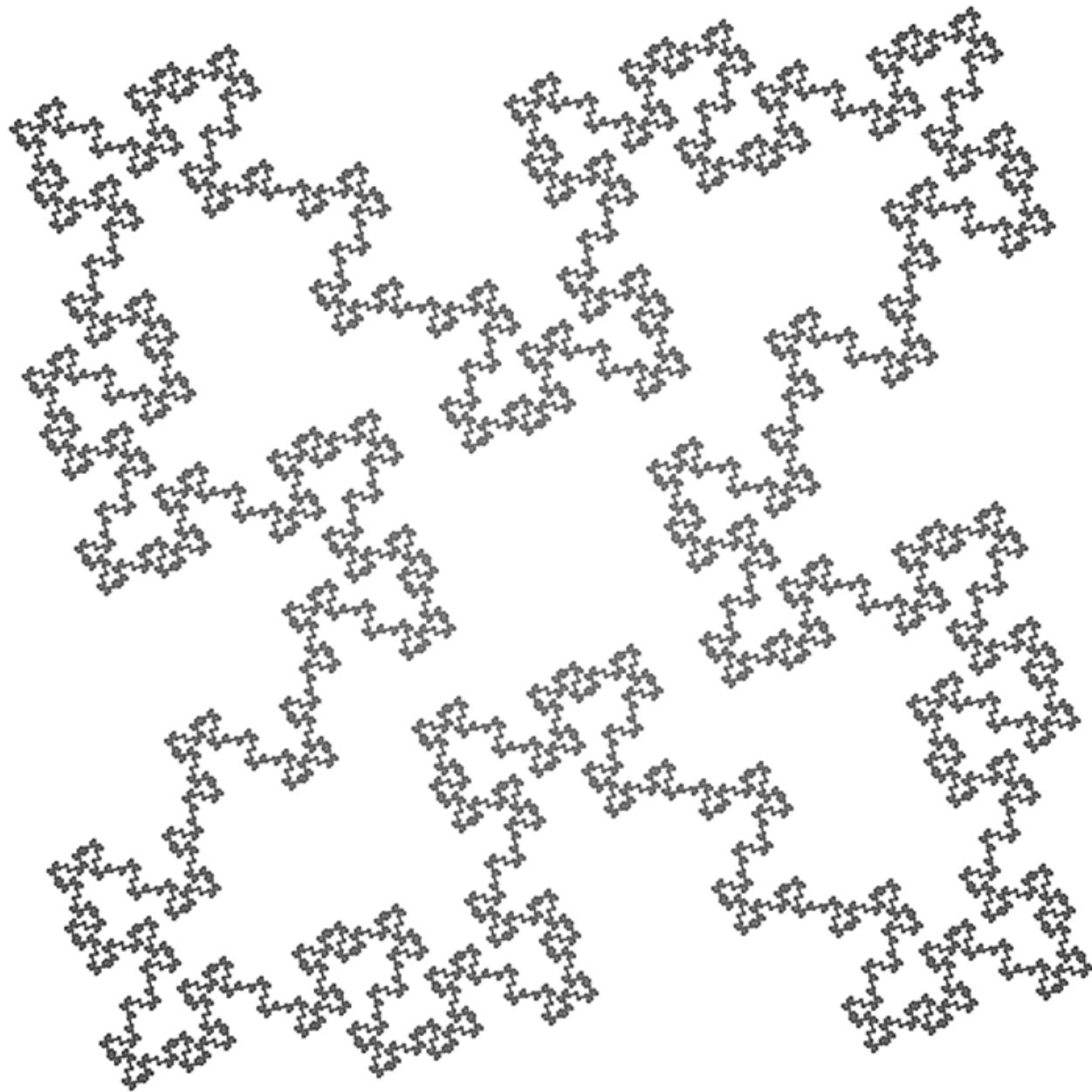
String length: 567



String length: 5103



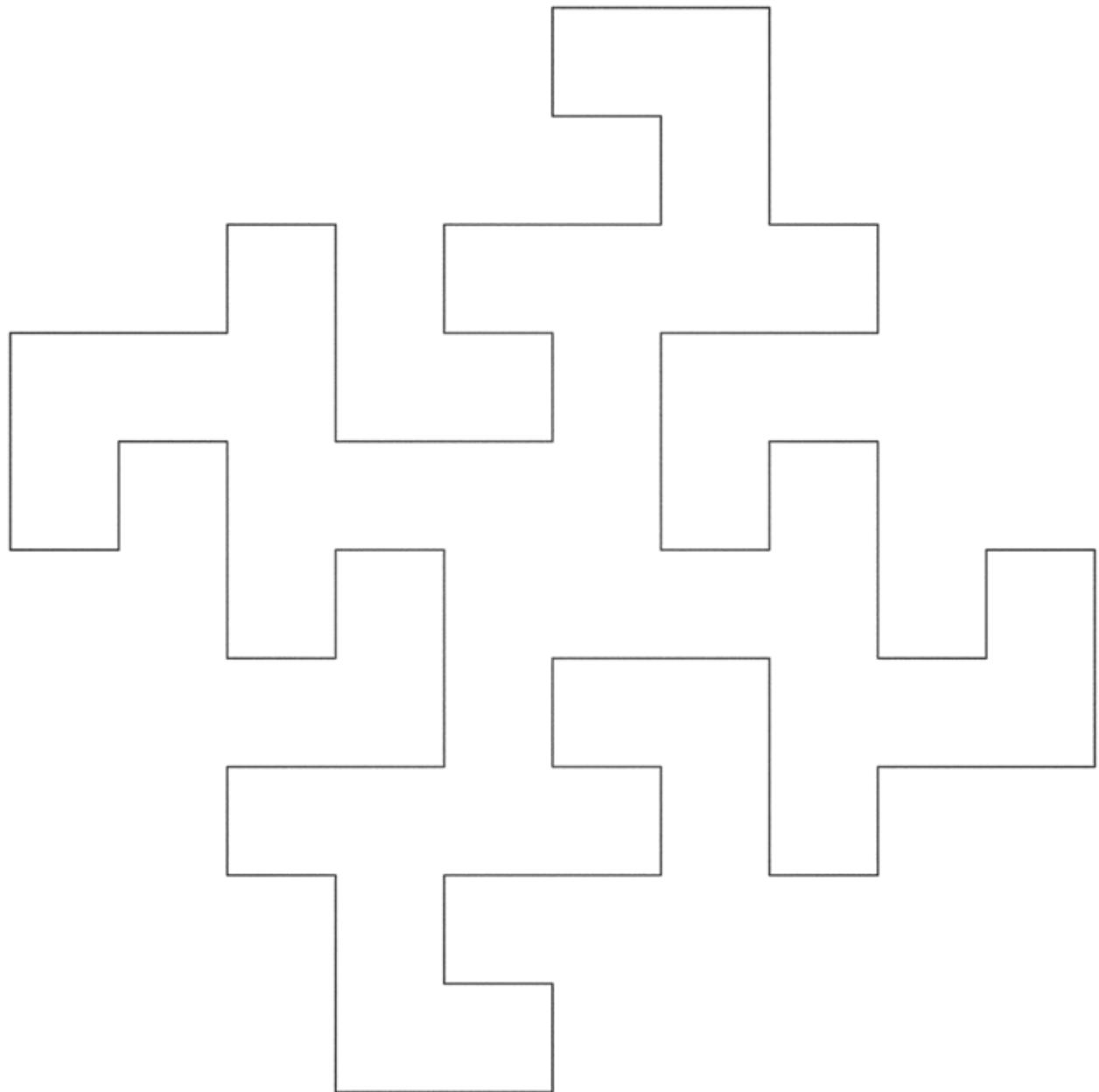
String length: 45927

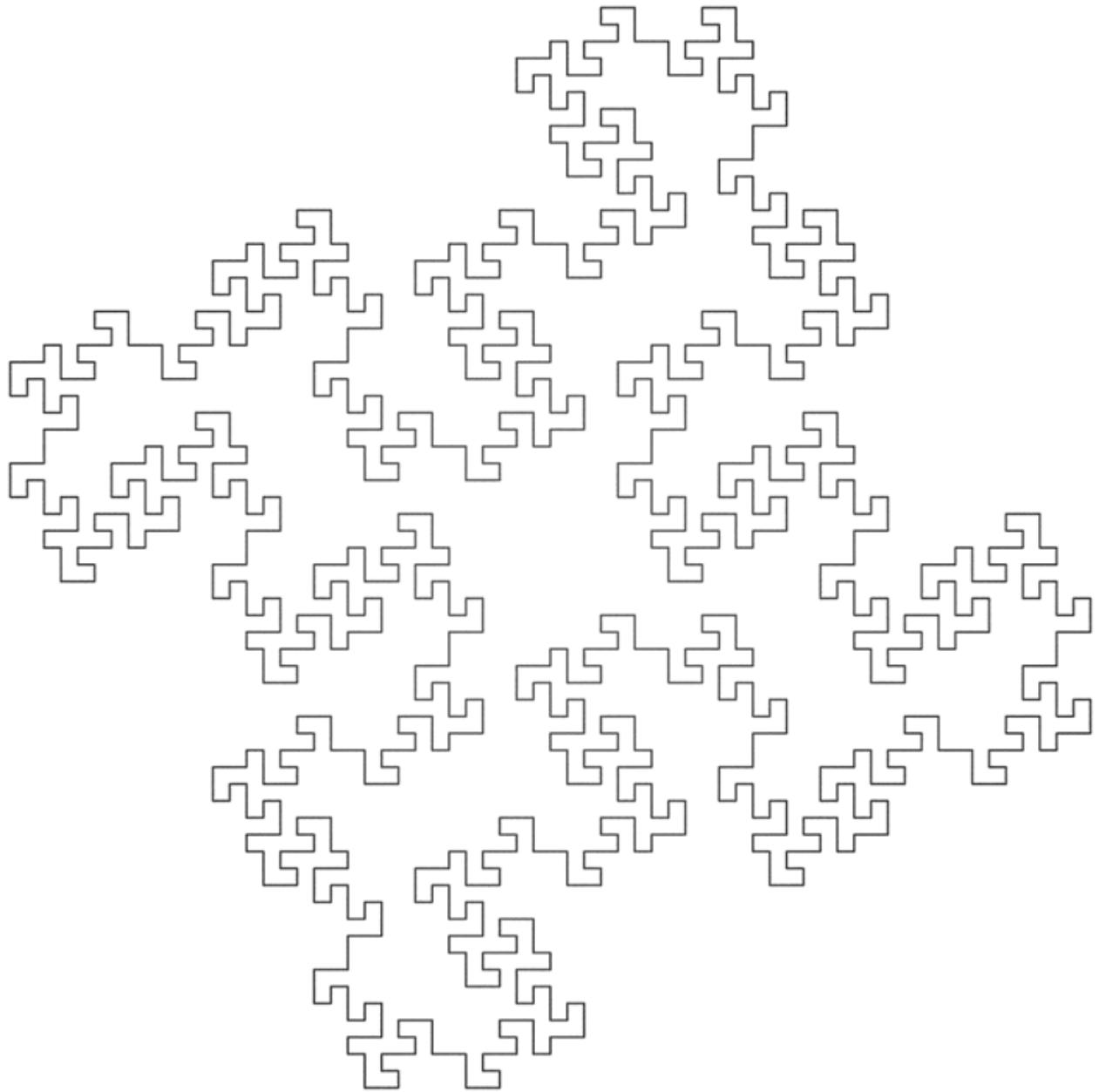


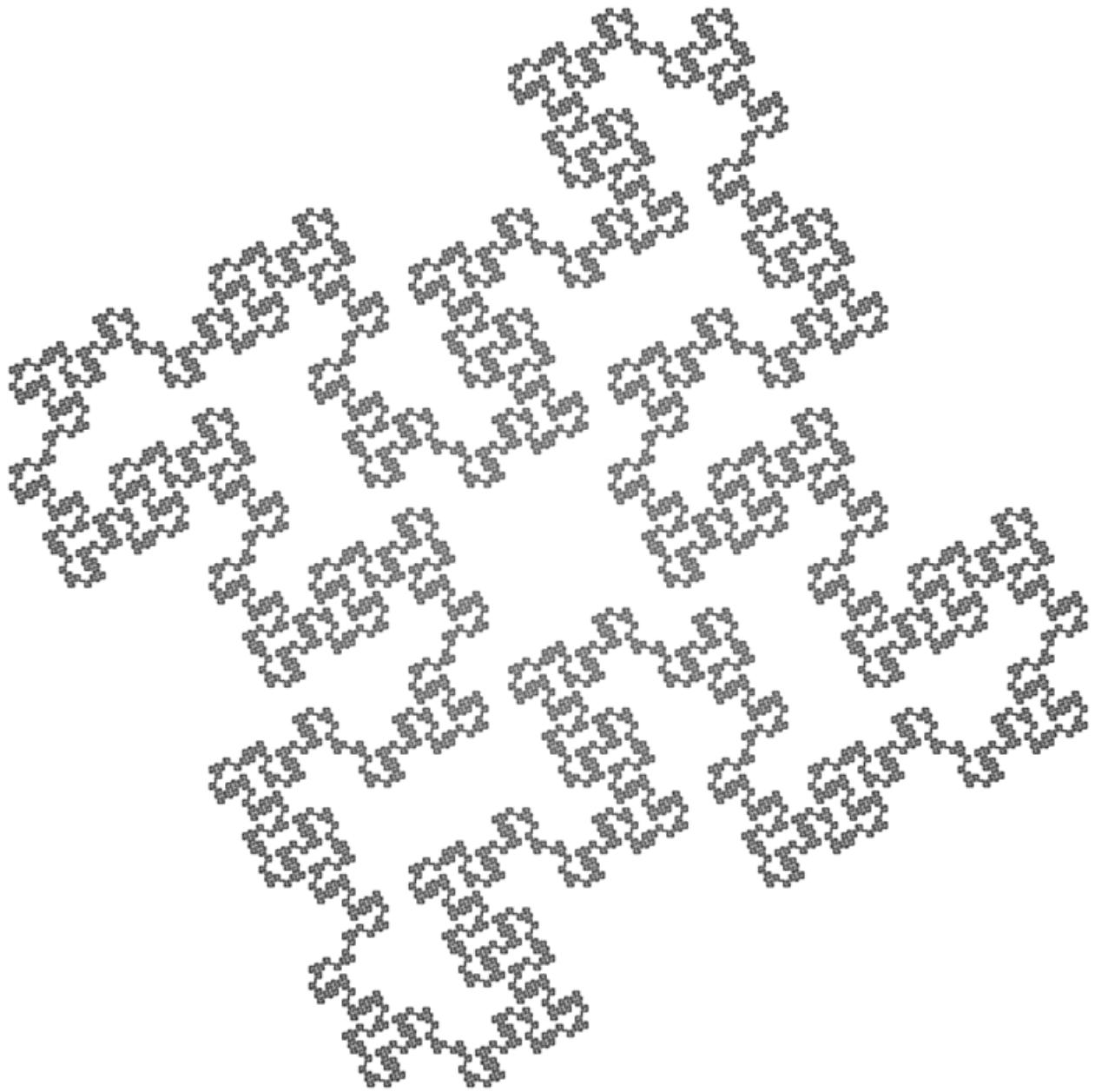
axiom = F+F+F+F

F -> F-FF+FF+F+F-F-FF+F+F-F-F-FF+FF+F

angle = 90







Variation by Hasan Hosam.

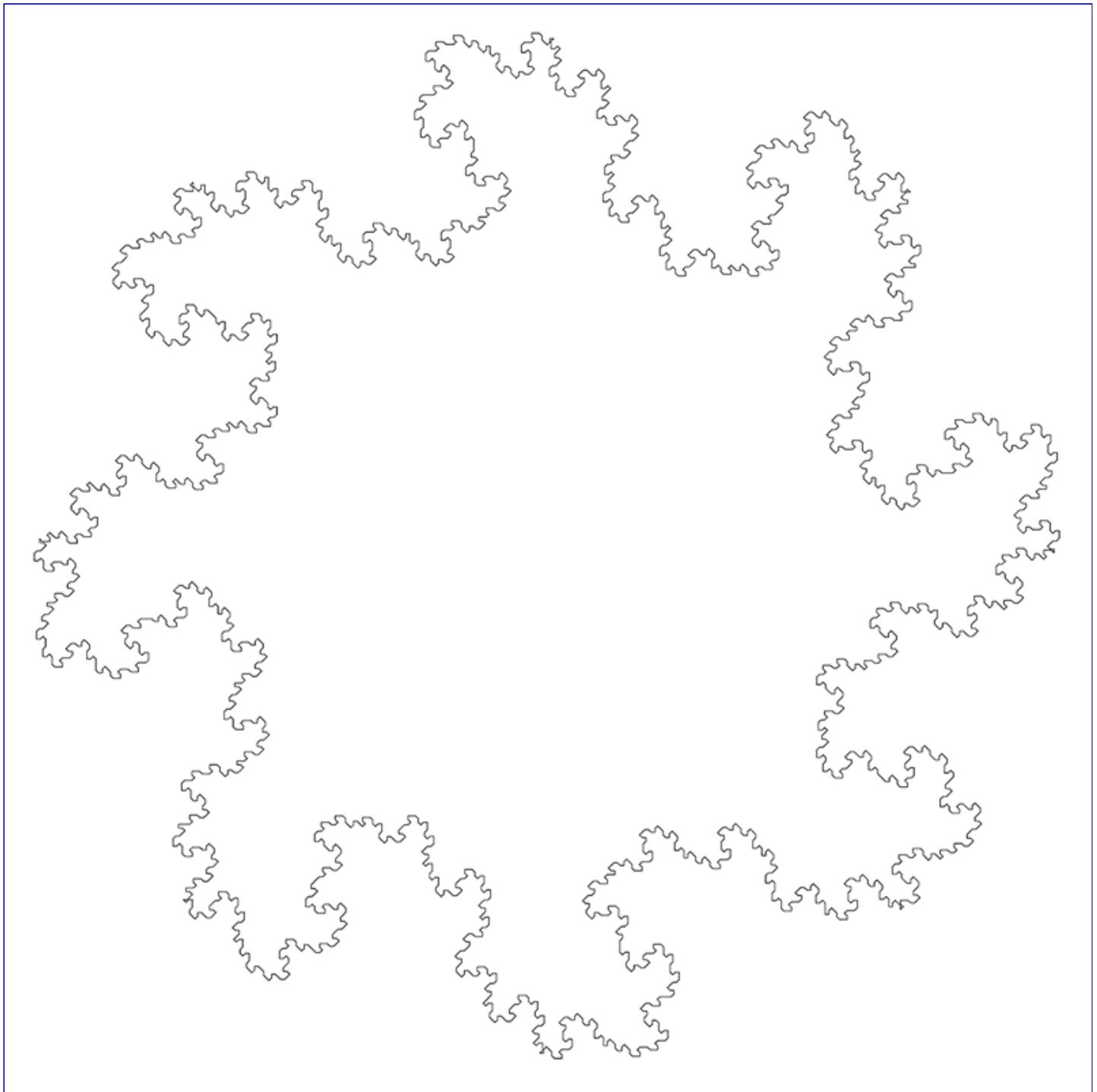
December 2018

axiom = X+X+X+X+X+X+X+X

X -> X+YF++YF-FX--FXFX-YF+X

Y -> -FX+YFYF++YF+FX--FX-YF

angle = 45



K o c h C u r v e

Written by [Paul Bourke](#)

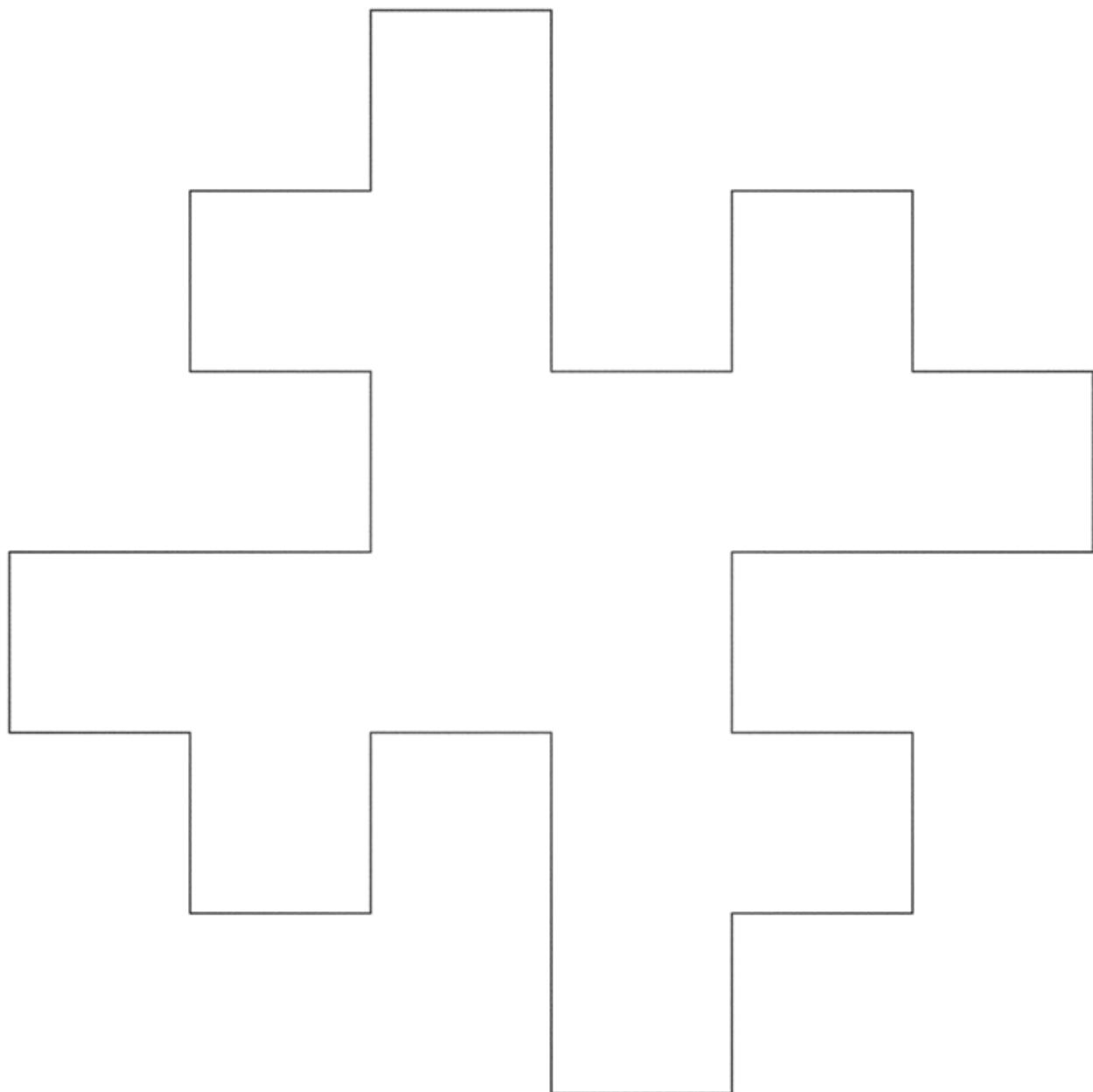
June 1990

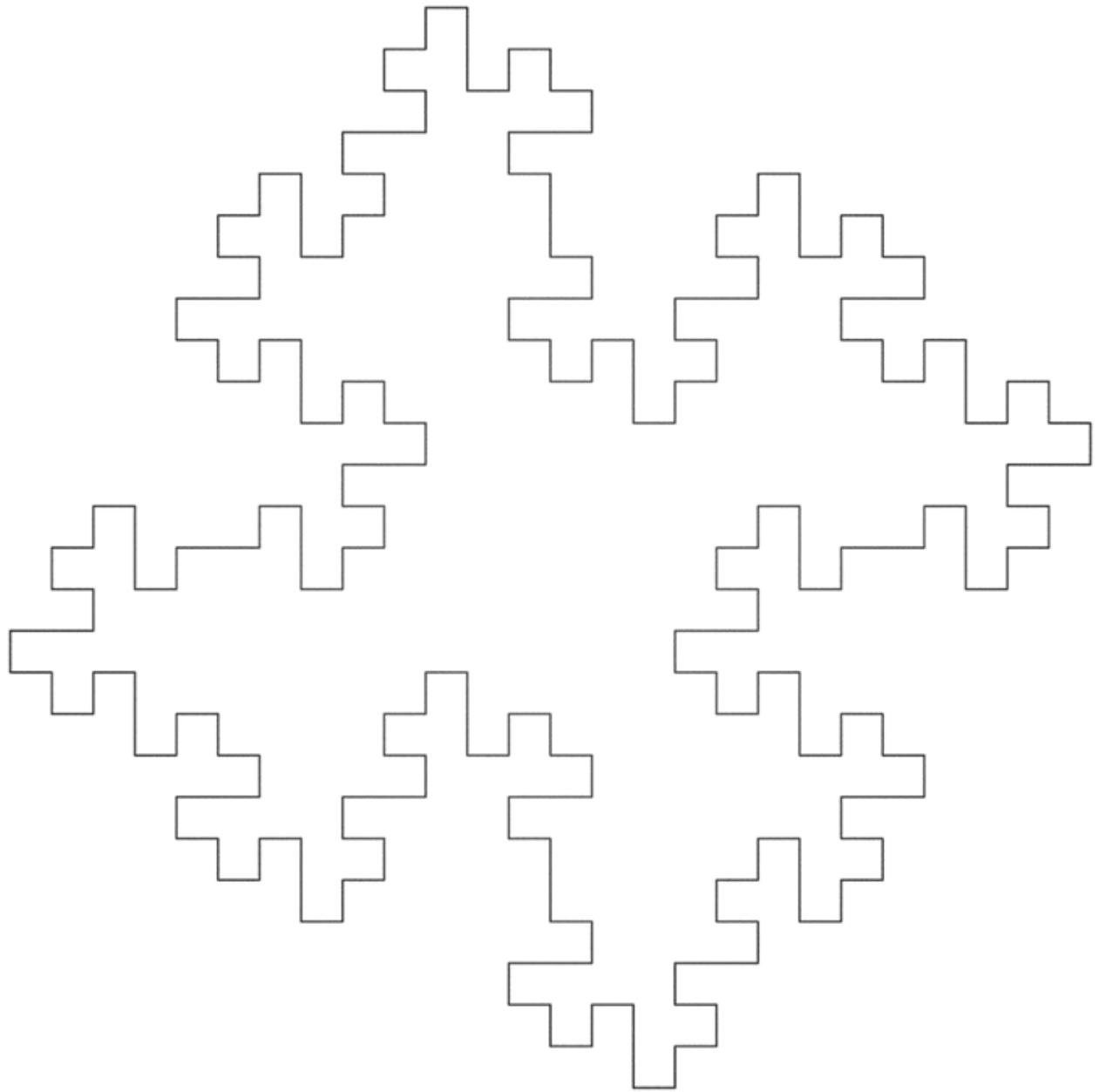
Fractal Dimension: 1.5

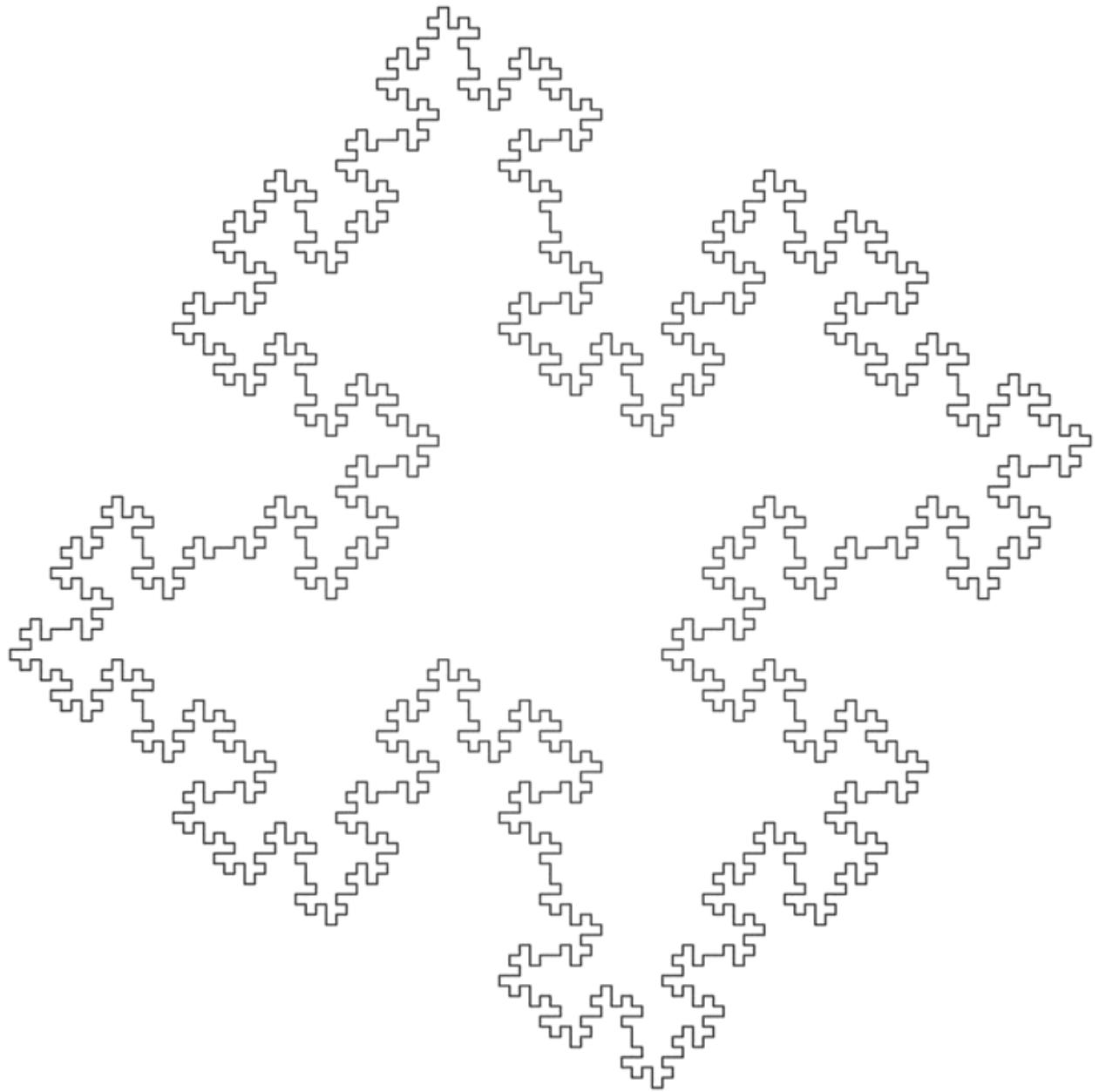
axiom = F+F+F+F

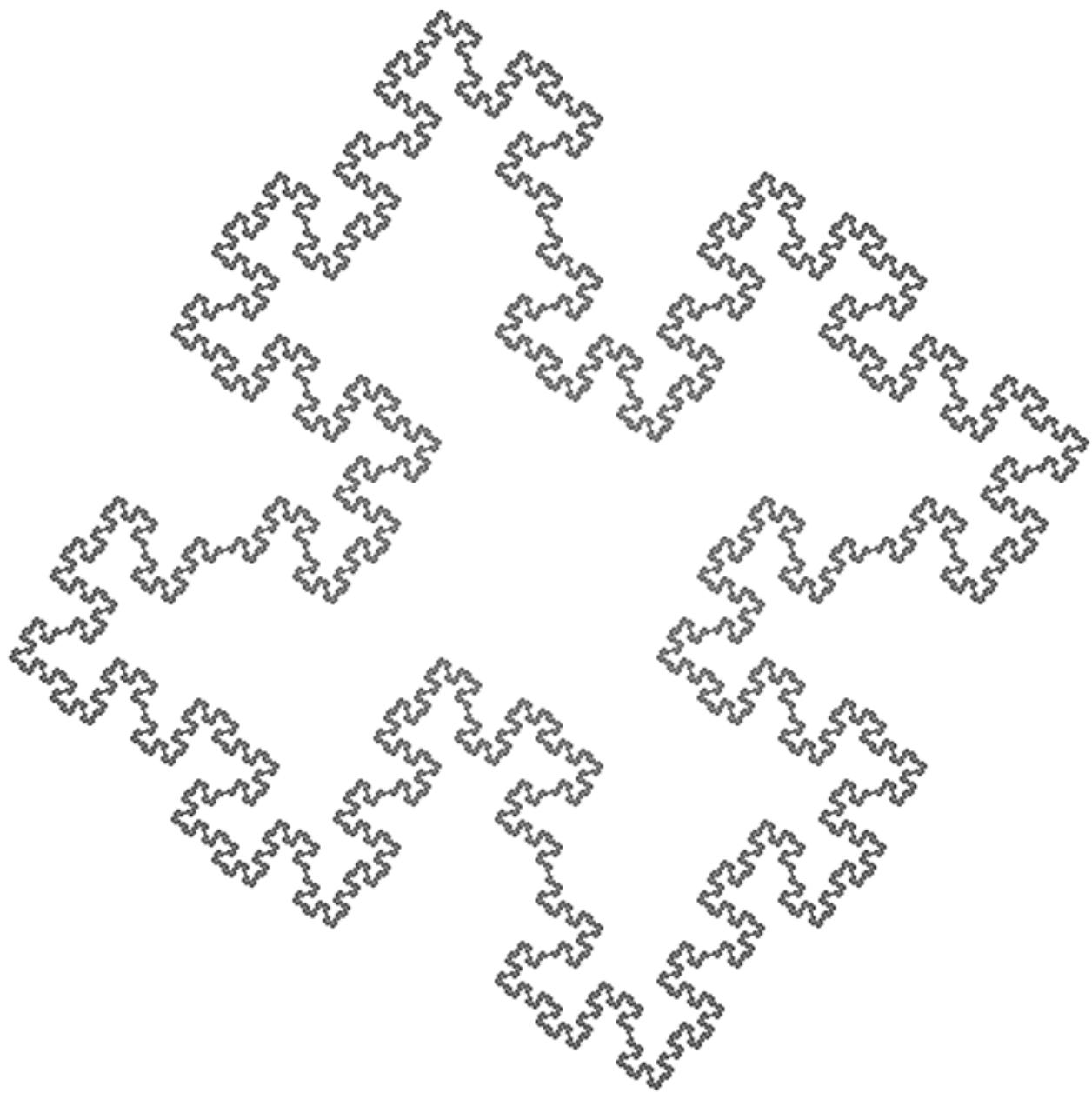
F -> F+F-F-FF+F+F-F

angle = 90









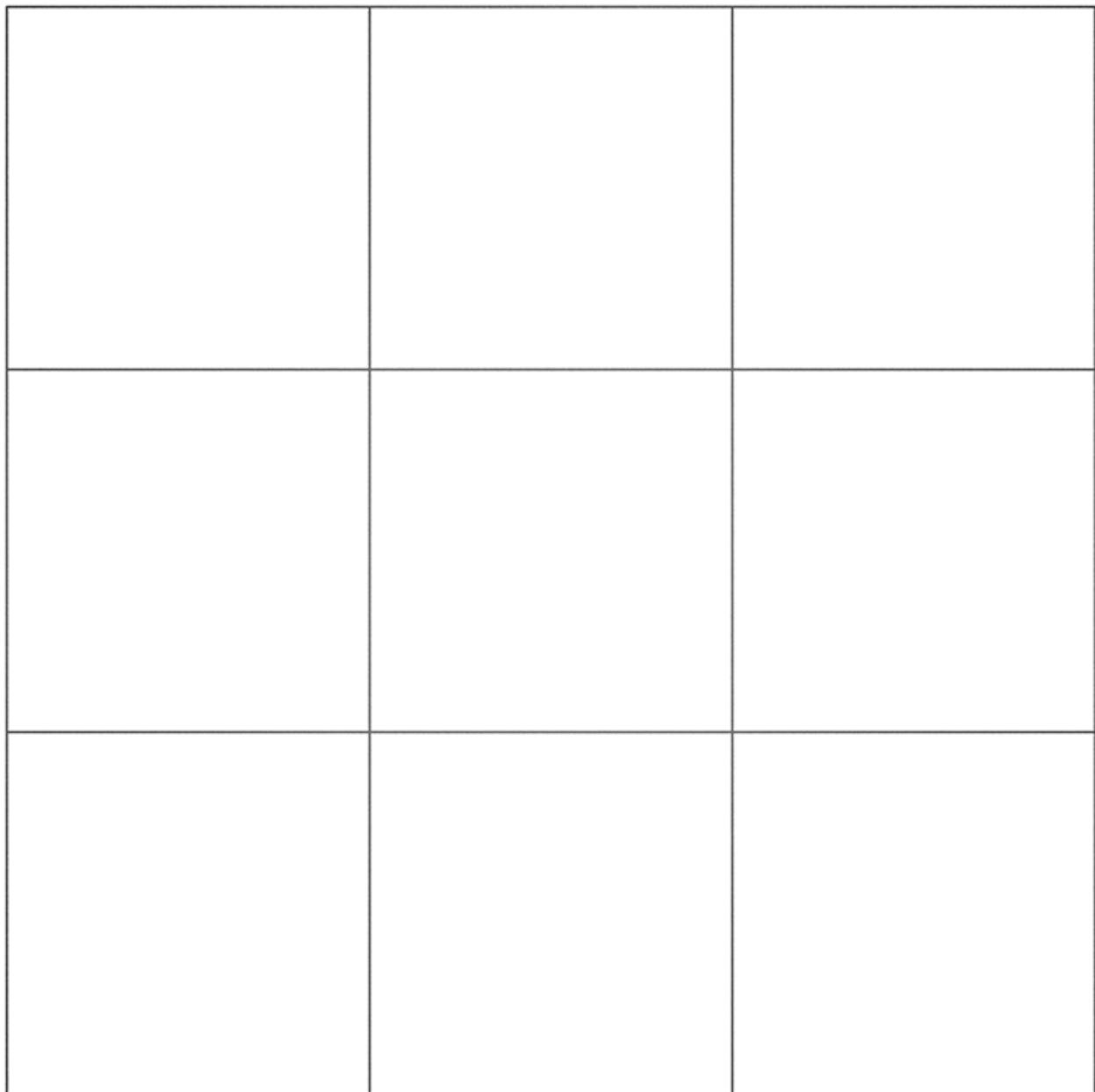
B o a r d

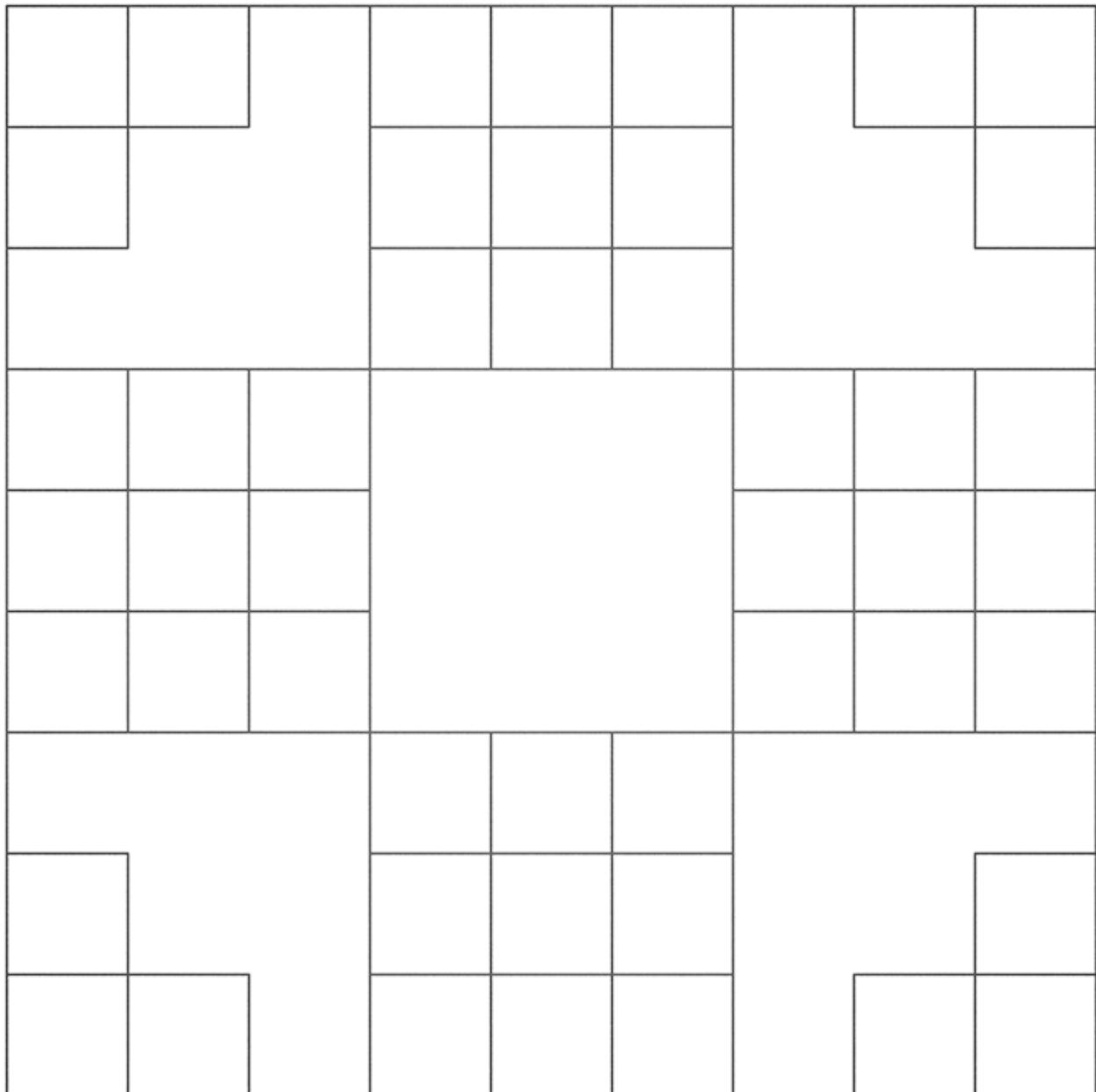
Written by [Paul Bourke](#)

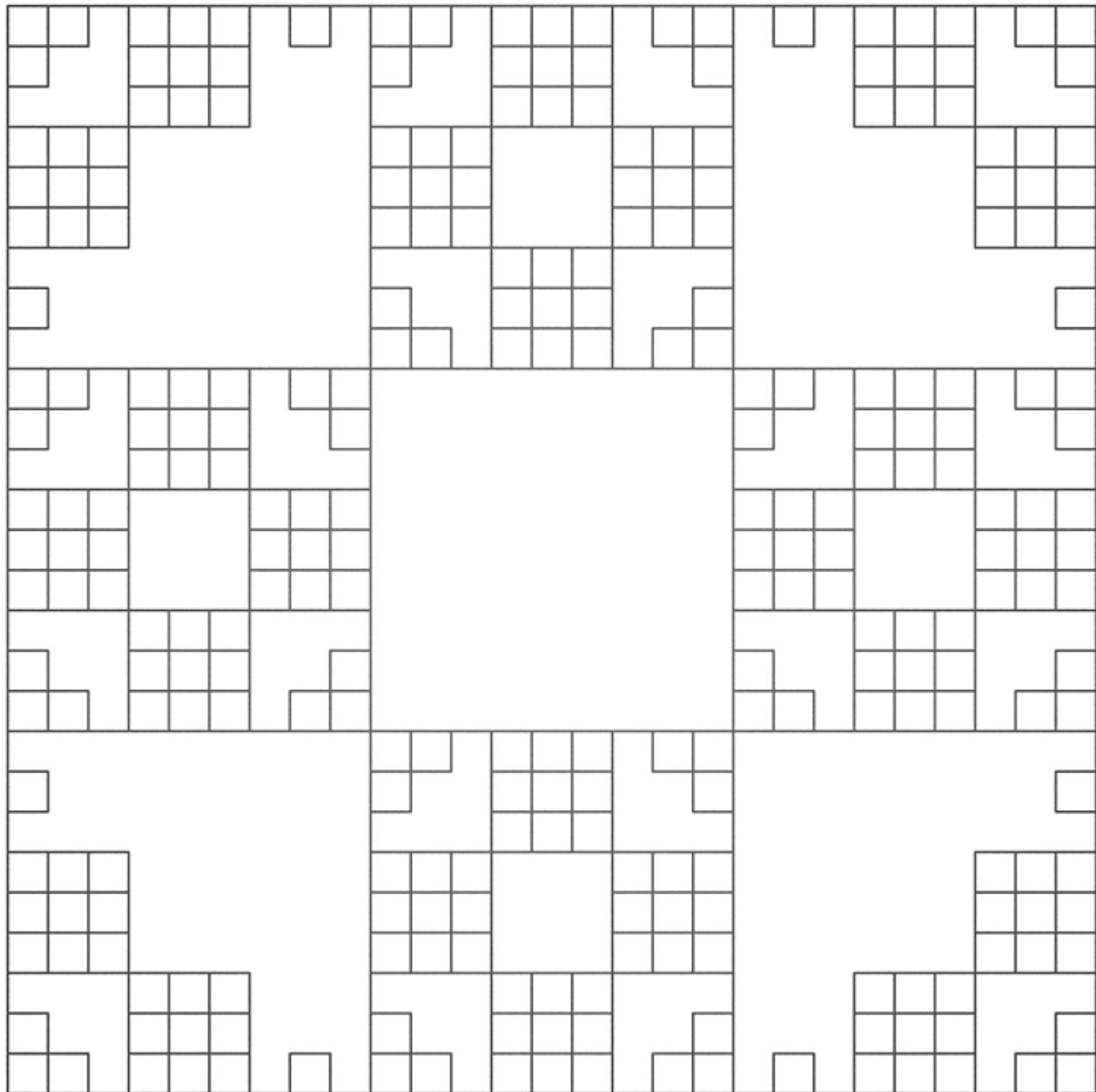
June 1990

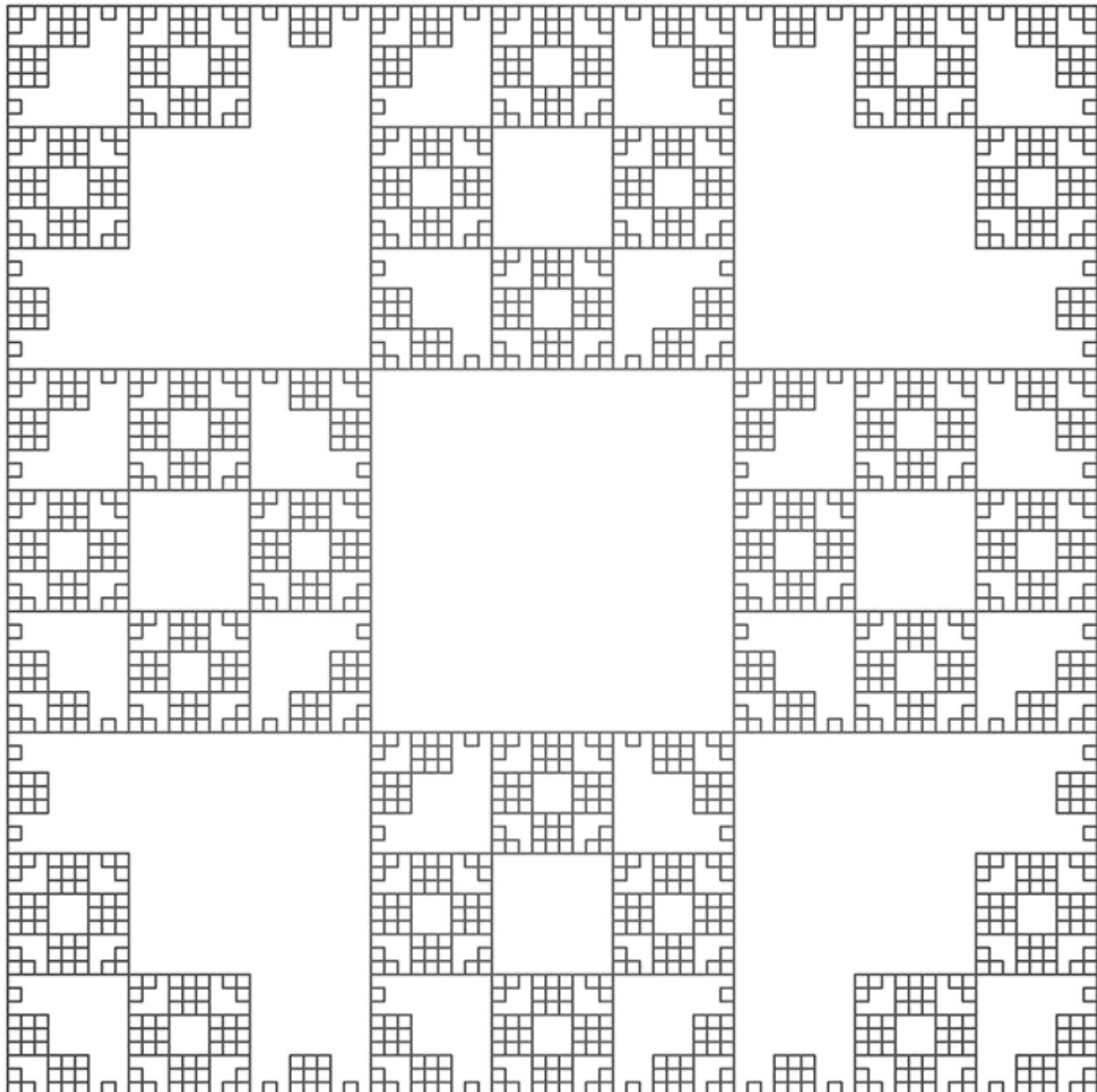
axiom = F+F+F+F
F -> FF+F+F+F+FF

angle = 90









Hilbert

Written by [Paul Bourke](#)

June 1990

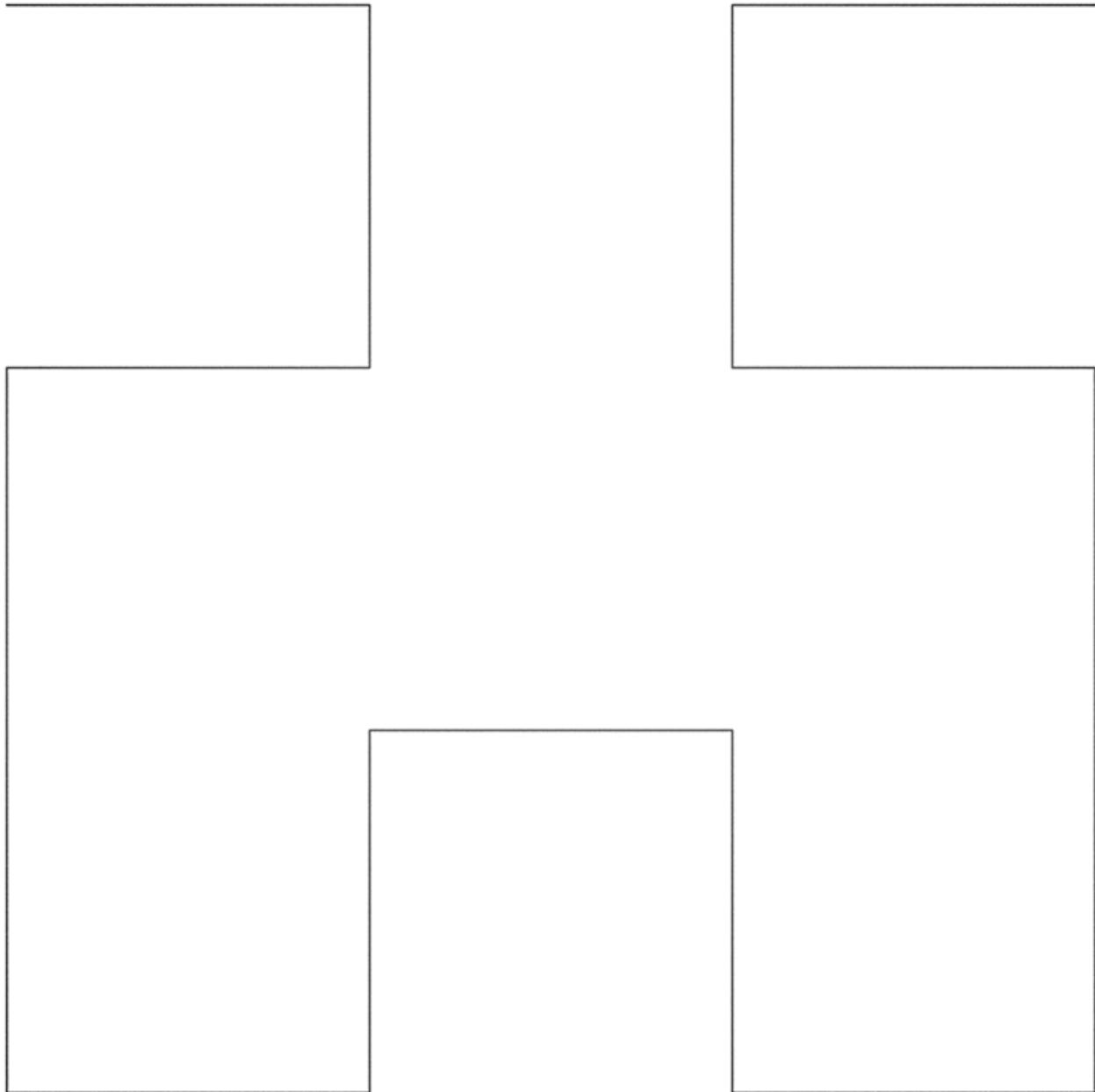
Attributed to the German mathematician David Hilbert, circa 1891

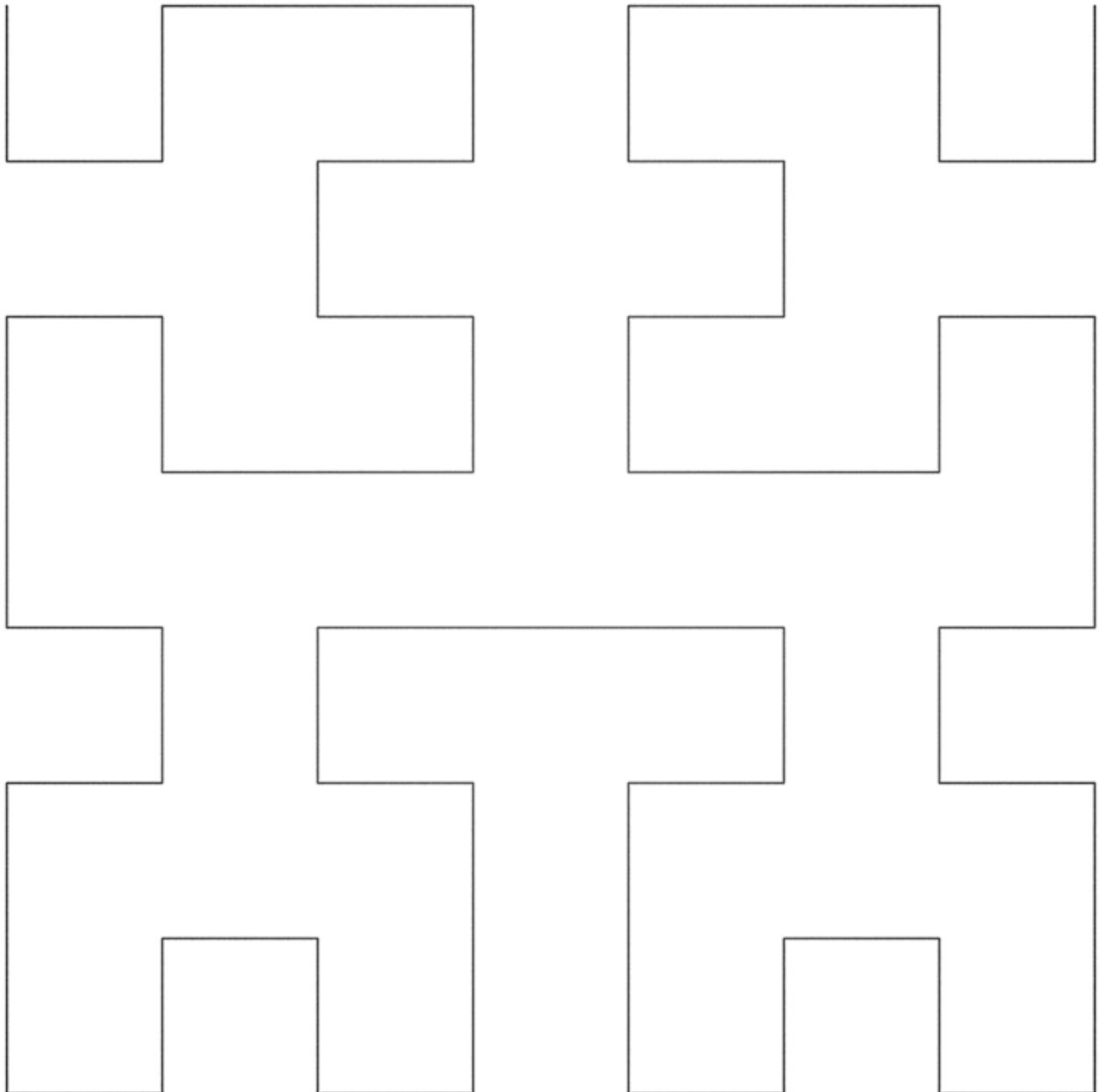
axiom = X

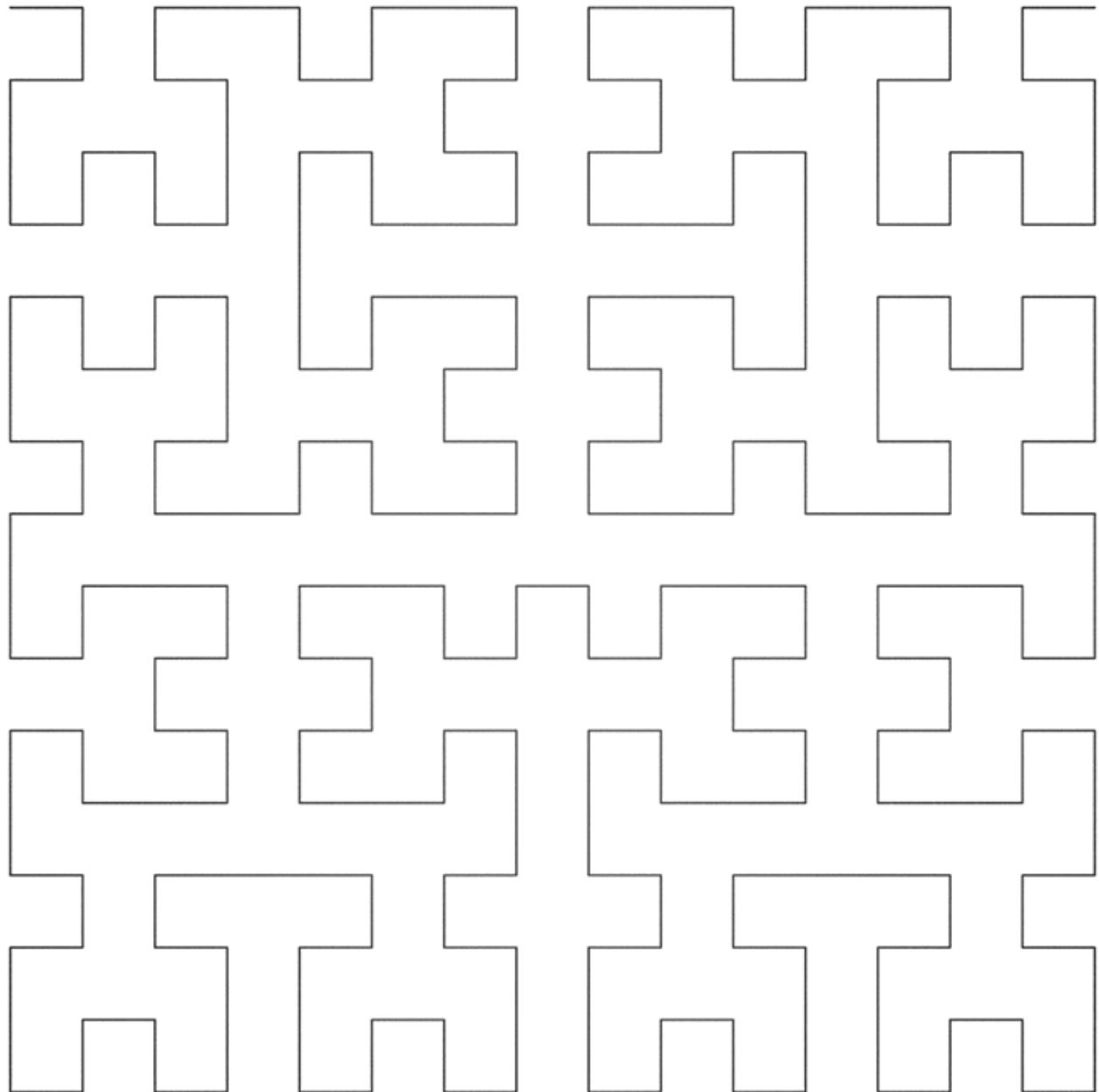
X -> -YF+XFX+FY-

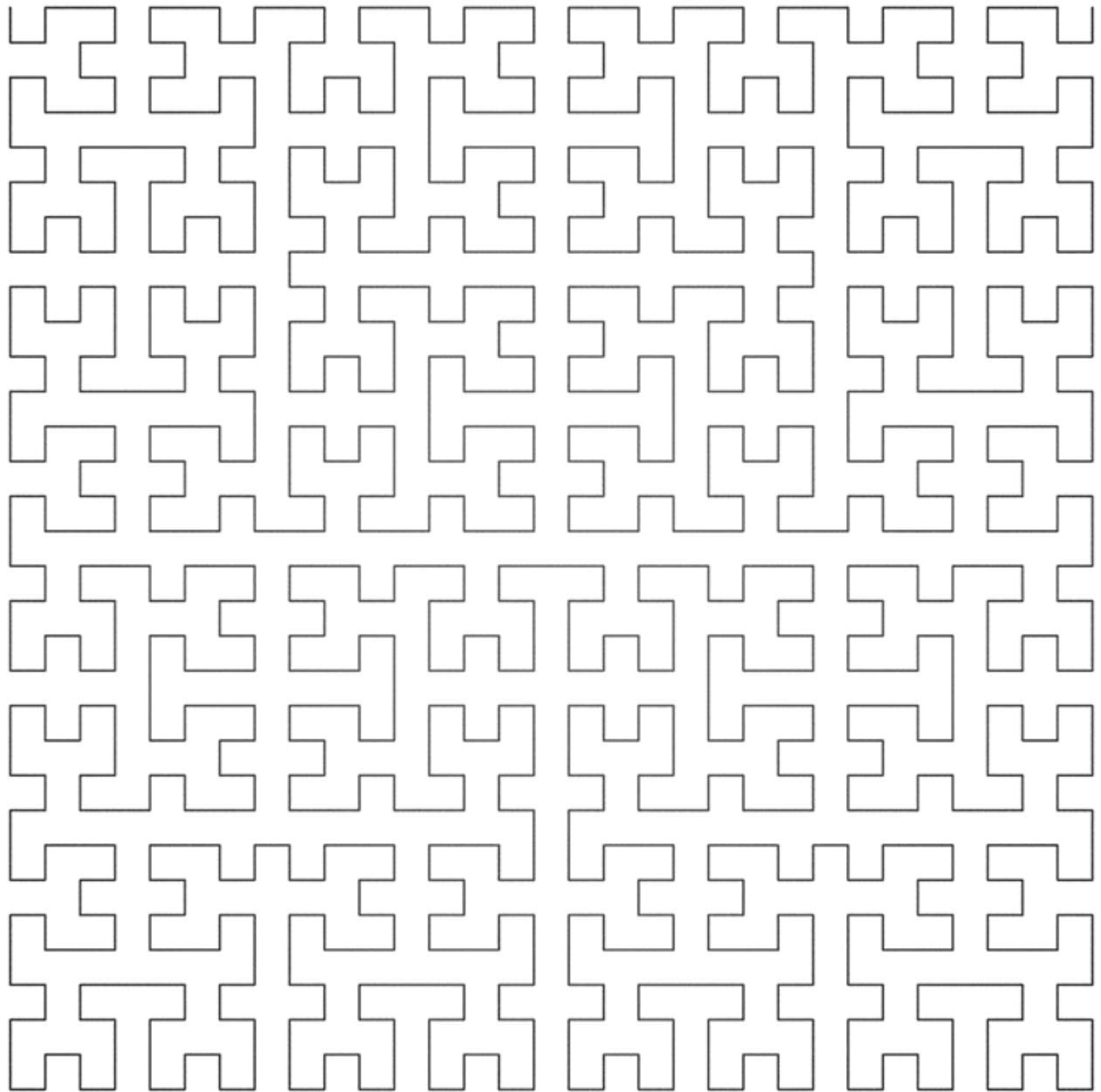
Y -> +XF-YFY-FX+

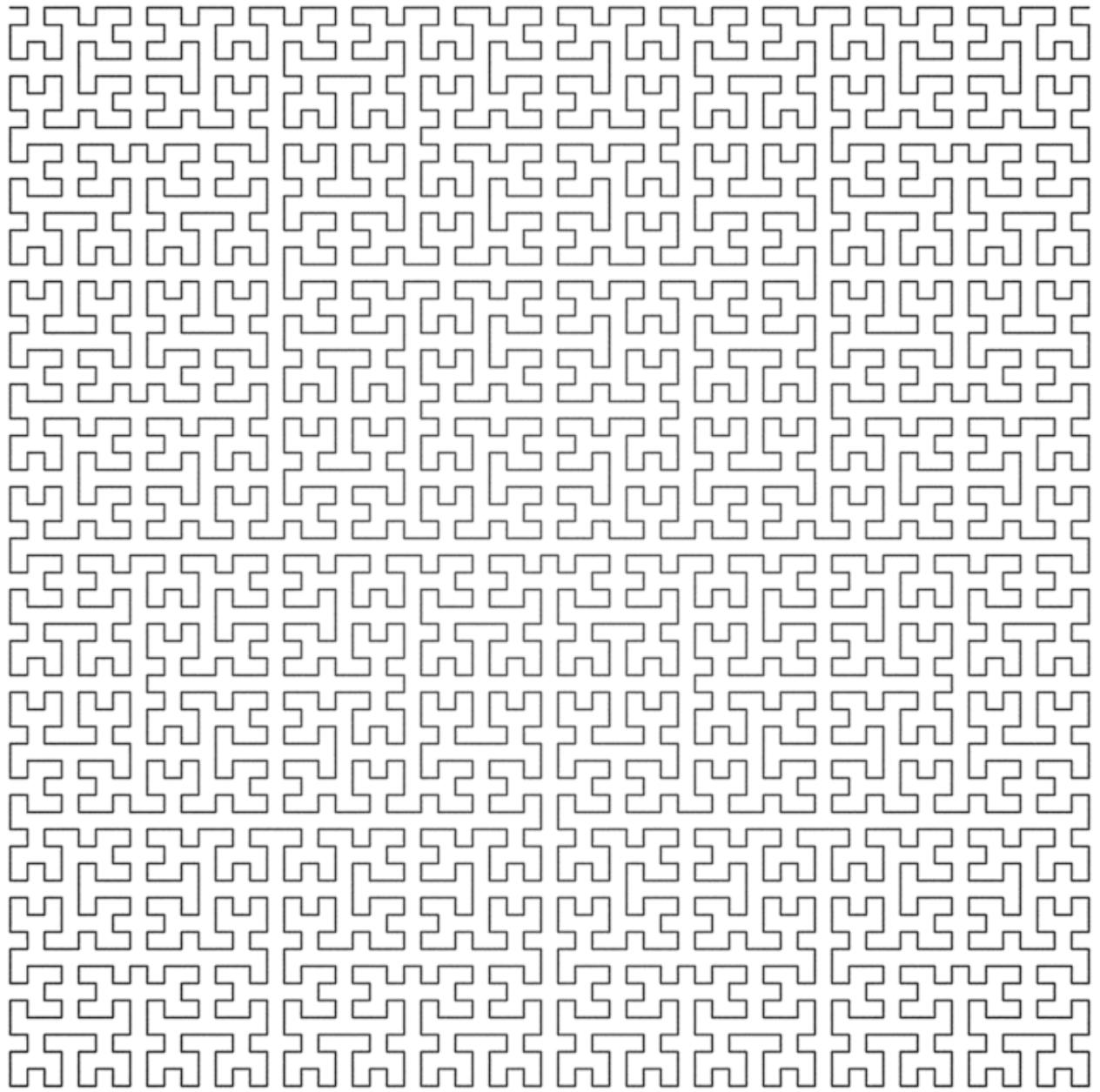
angle = 90











Sierpinski Arrowhead

Written by [Paul Bourke](#)

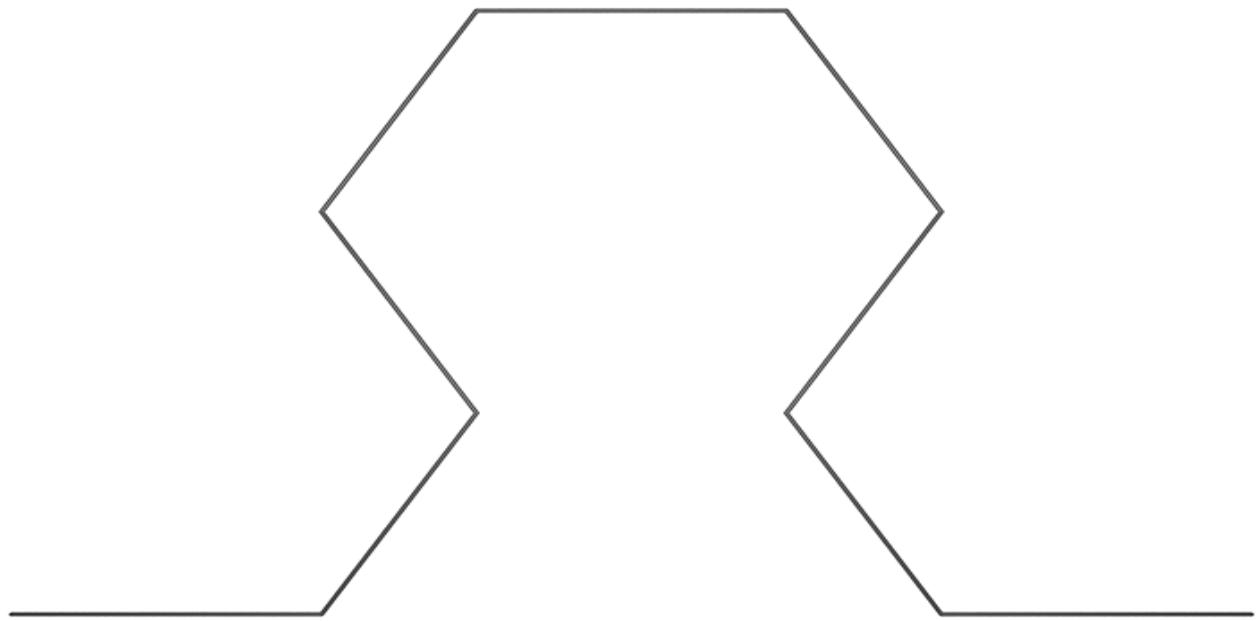
June 1990

axiom = YF
X -> YF+XF+Y

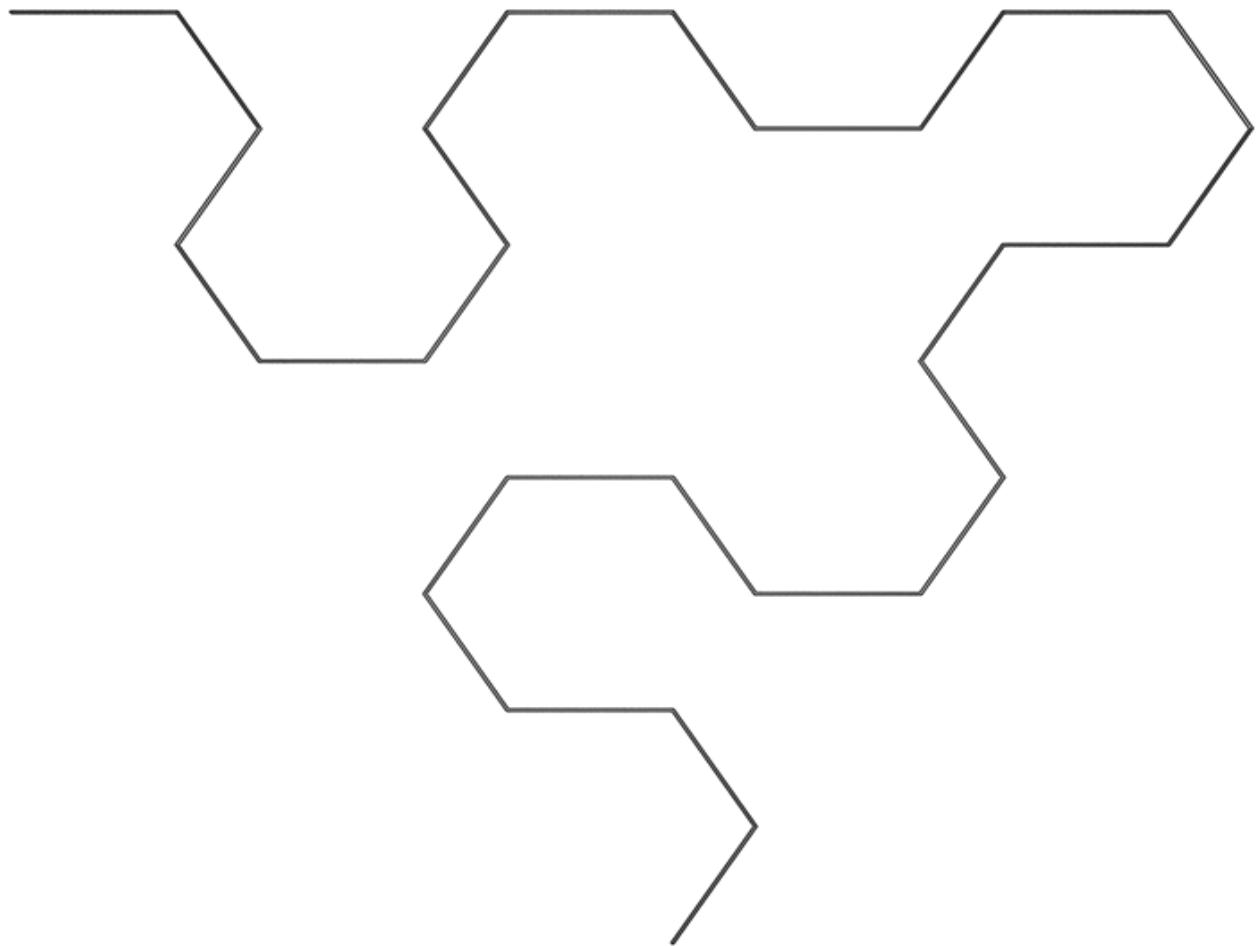
$Y \rightarrow XF-YF-X$

angle = 60

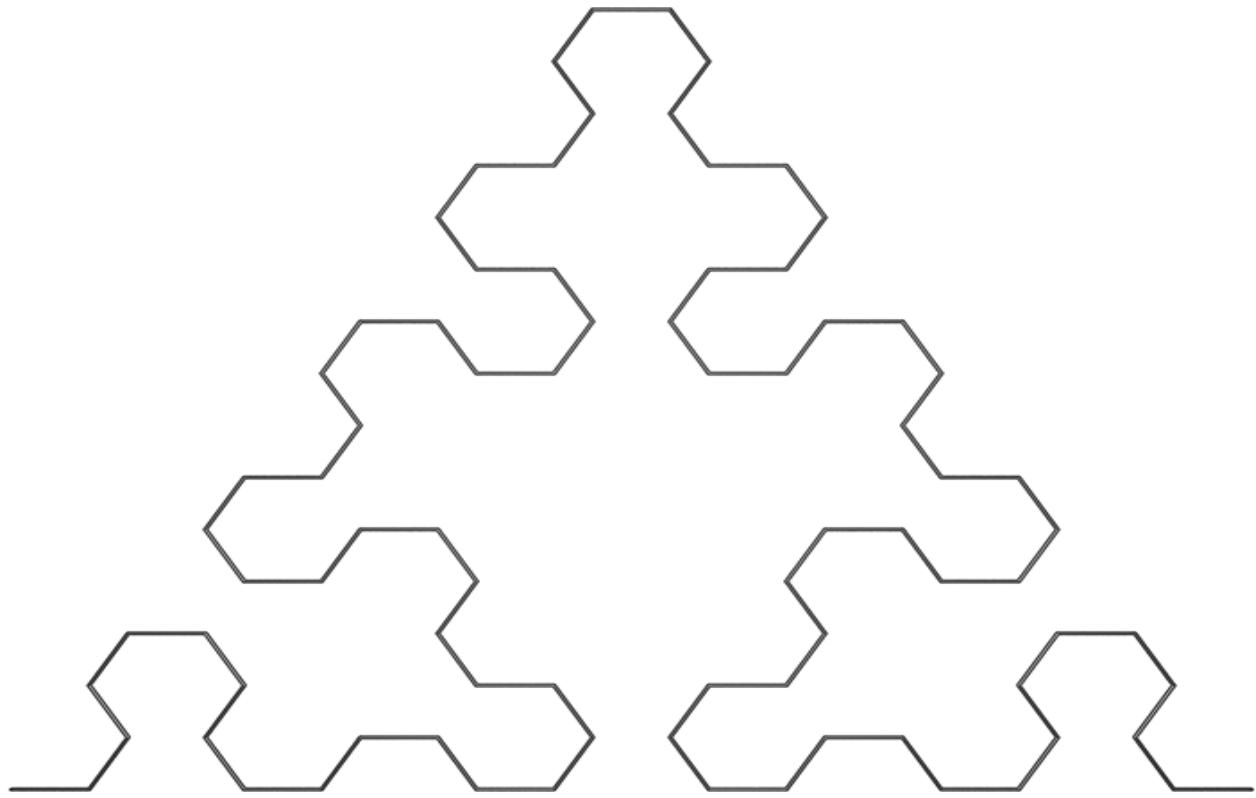
$YF+XF+YF-XF-YF-XF-YF+XF+YF$

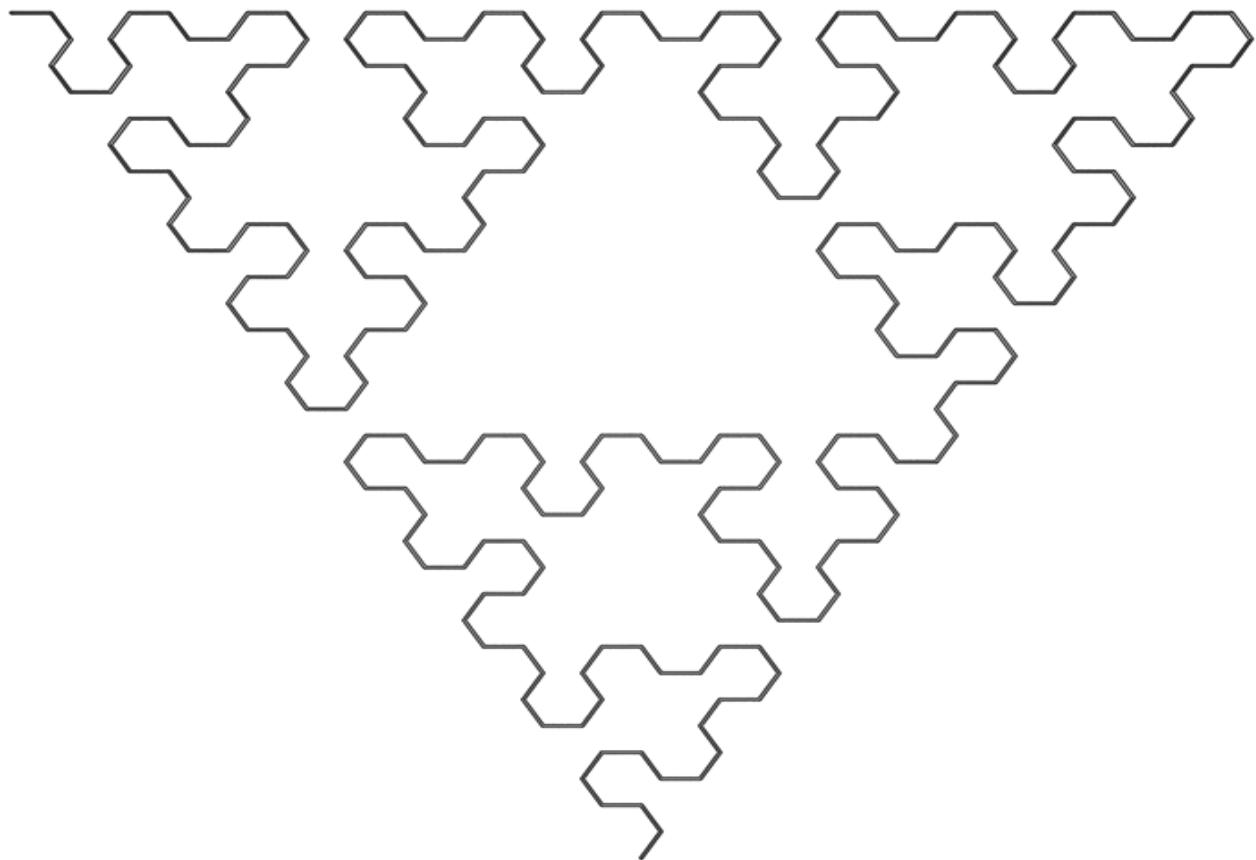


$XF-YF-XF+YF+XF+YF+XF-YF-XF-YF+XF+YF-XF-YF-XF-YF+XF+YF+XF+YF+XF-YF-XF$

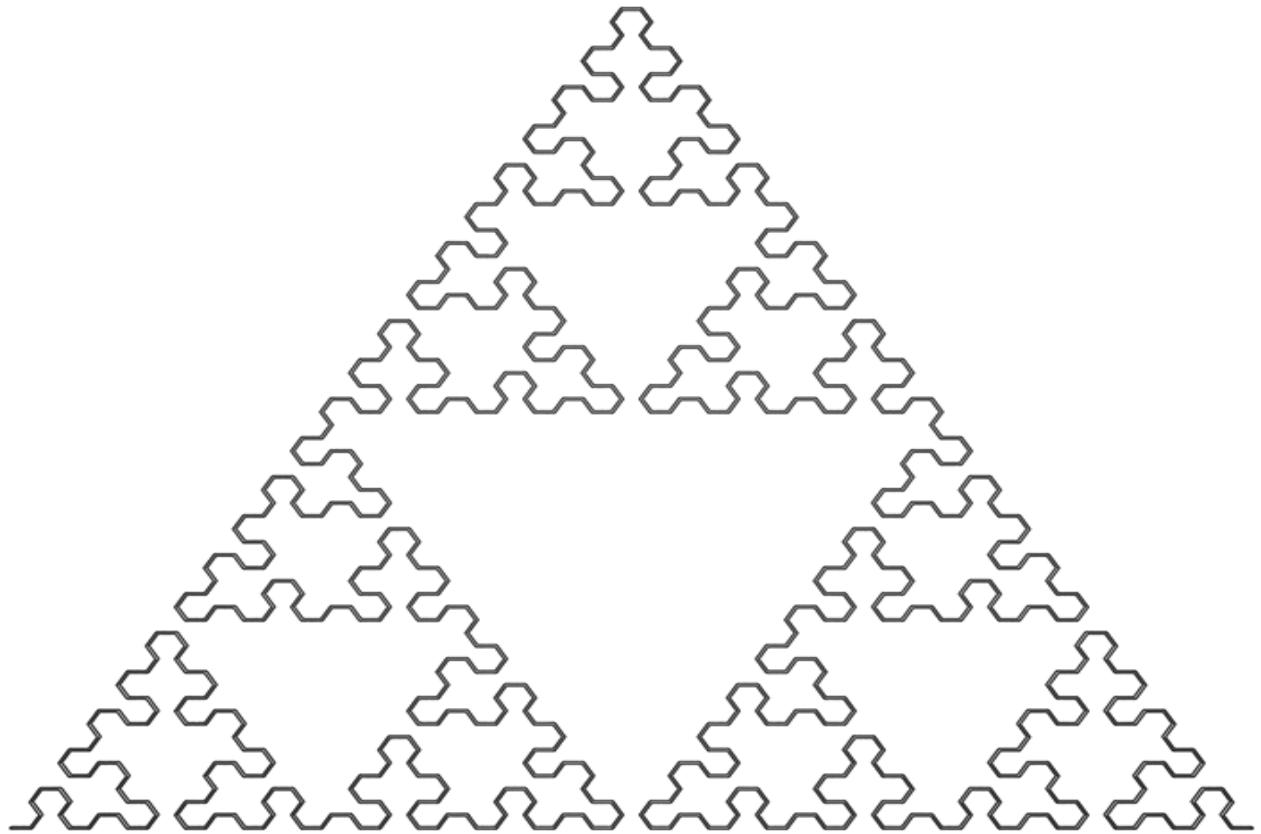


YF+XF+YF-XF-YF-XF-YF+XF+YF+XF-YF-XF+YF+XF+YF+XF-YF-XF+YF+XF+YF-XF-YF-XF-YF+XF+YF -XF-YF-XF+YF+XF+YF+XF-YF-XF-YF+XF+YF-XF-YF-XF-YF+XF+YF-XF-YF-XF+YF+XF+YF+XF-YF-XF

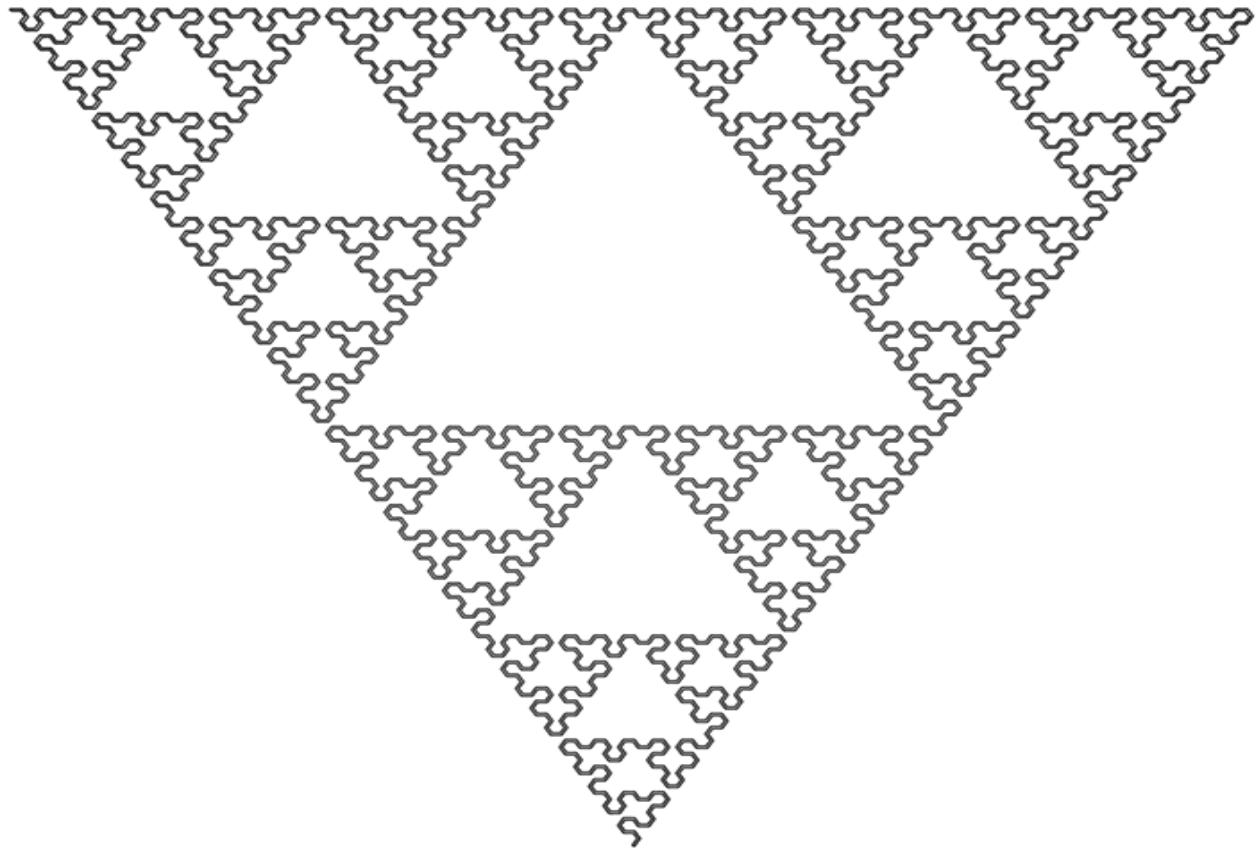




String length: 2186



String length: 6560



Von Koch Snowflake

Written by [Paul Bourke](#)

June 1990

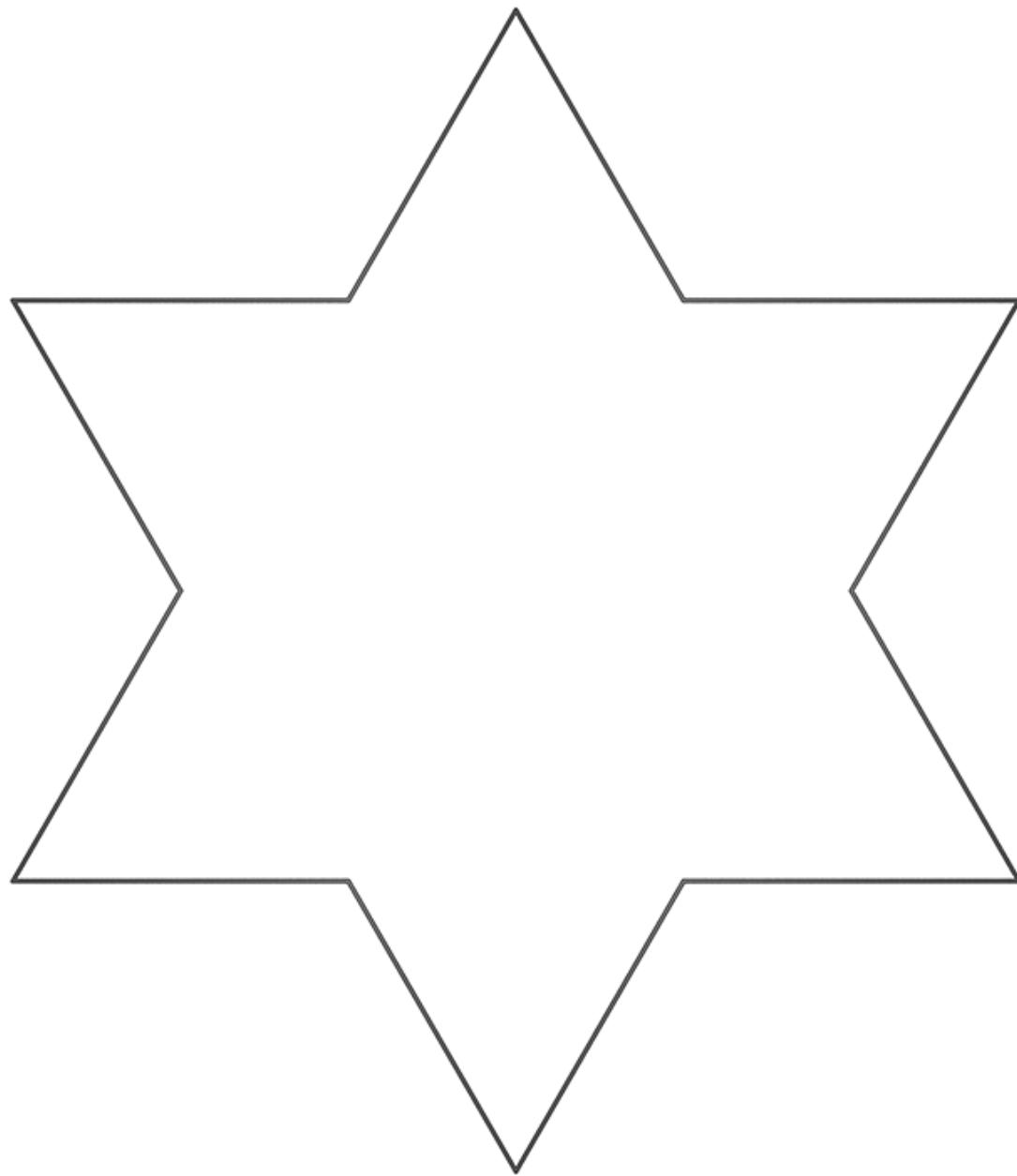
Fractal Dimension: $\log(4)/\log(3) = 1.262$

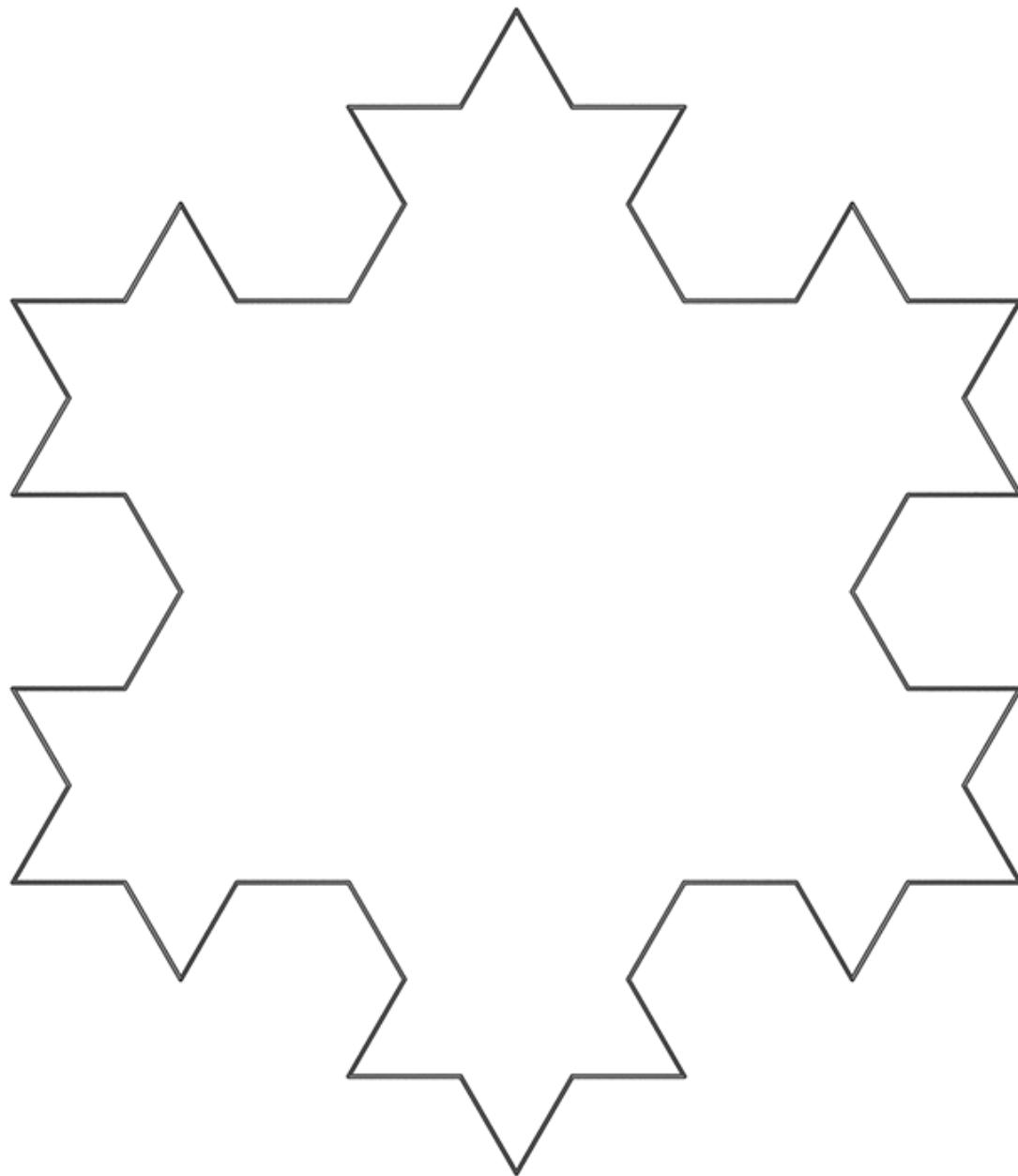
L-Systems

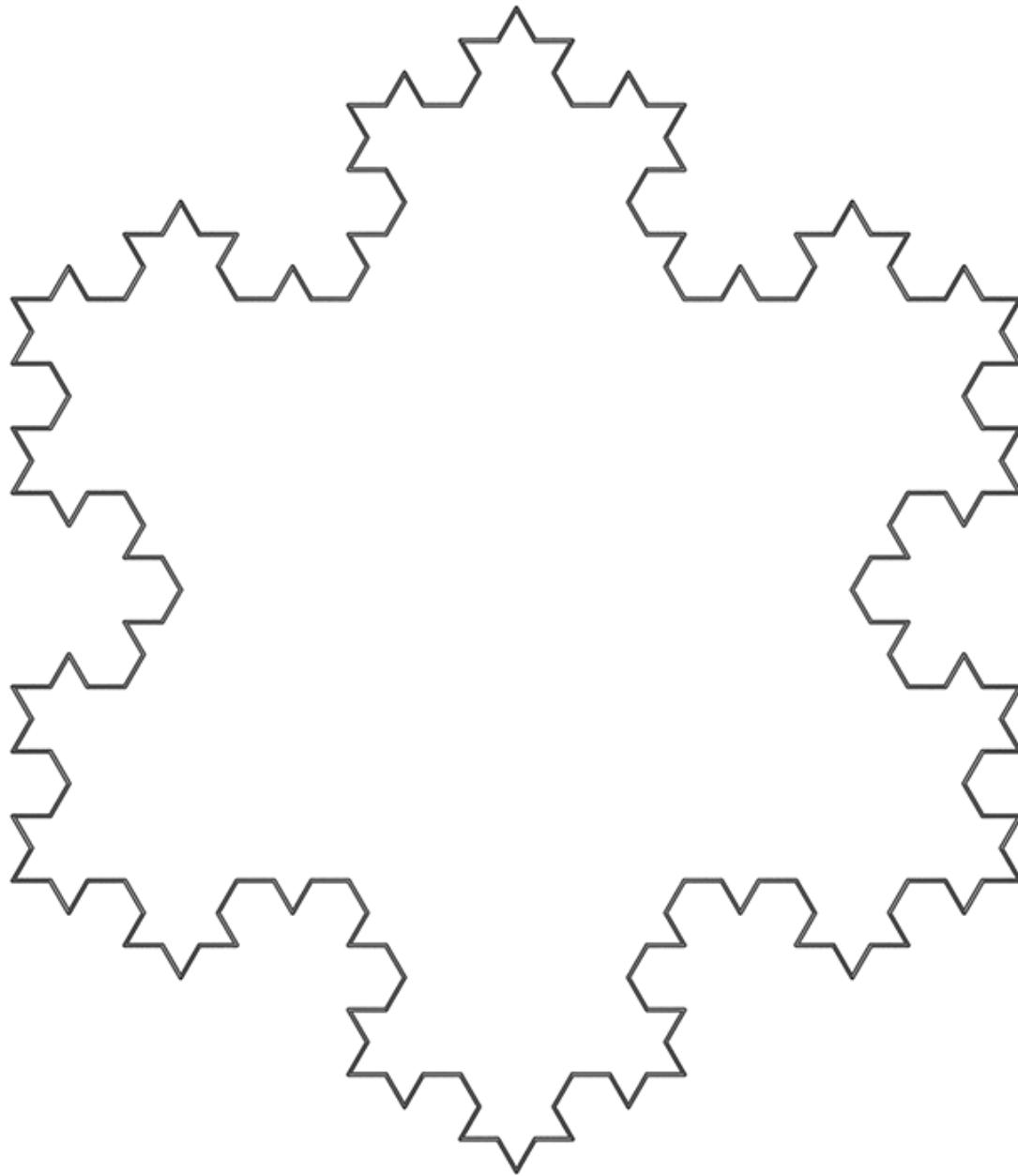
axiom = F++F++F

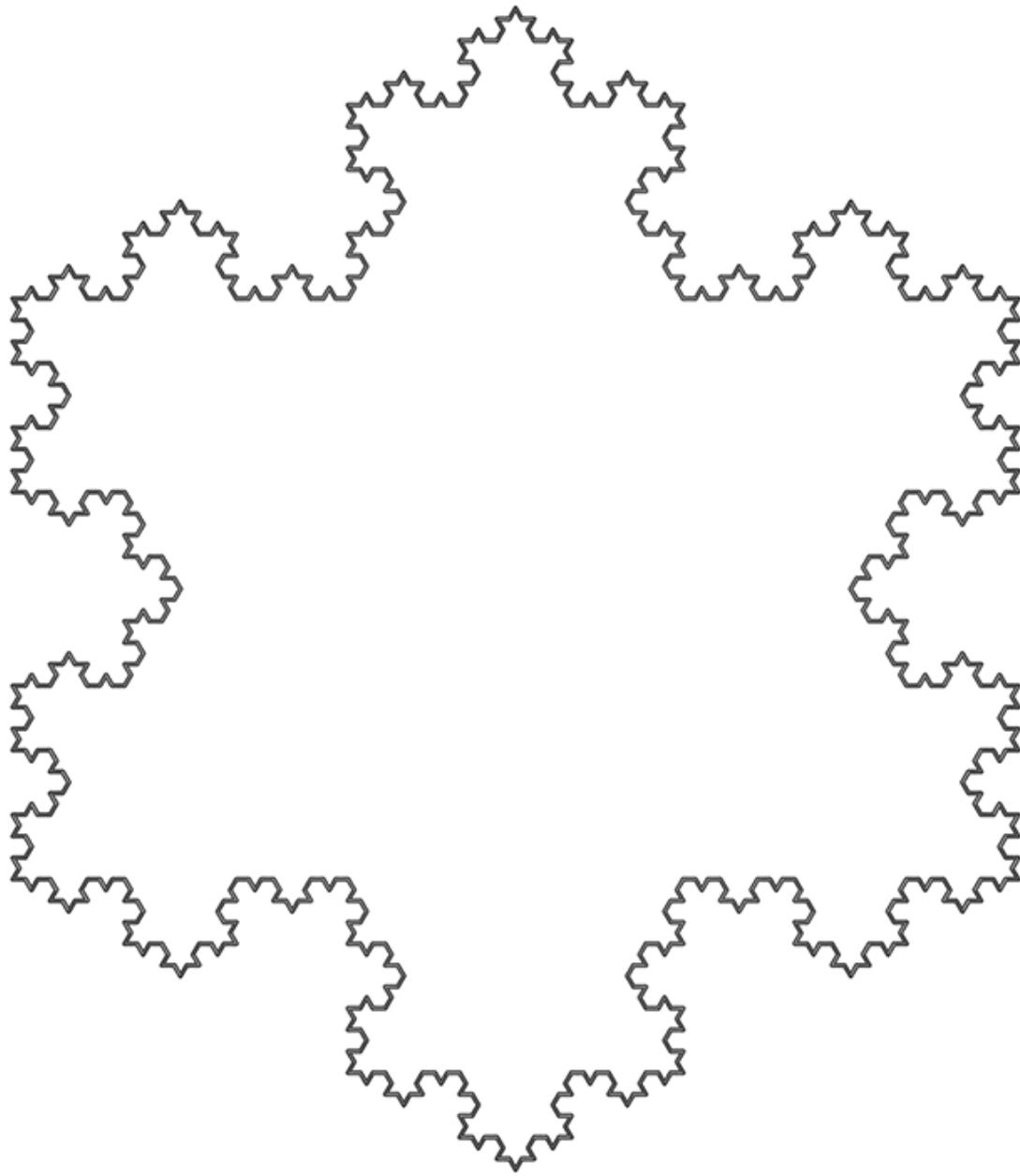
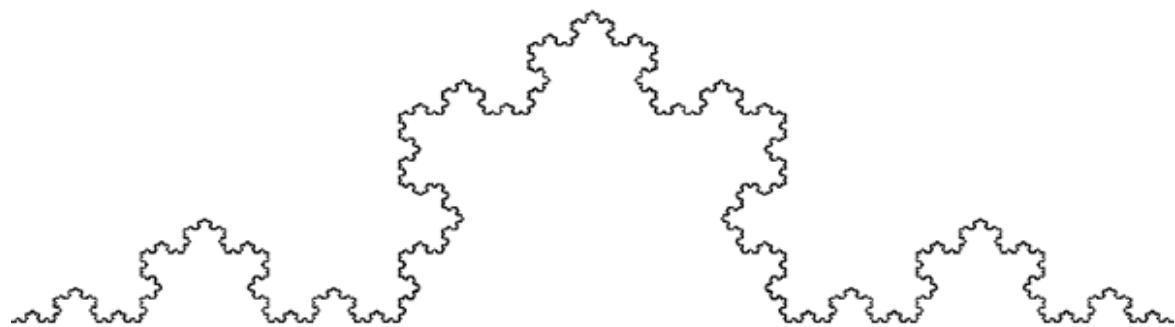
F -> F-F++F-F

angle = 60







**IFS**

The IFS equations are as follows

$$x_{n+1} = a x_n + b y_n + e$$

$$y_{n+1} = c x_n + d y_n + f$$

The parameter table:

	set 1	set 2	set 3	set 4
a	0.3330	0.3330	0.1670	0.1670
b	0.0000	0.0000	-0.2890	0.2890
c	0.0000	0.0000	0.2890	-0.2890
d	0.3330	0.3330	0.1670	0.1670
e	-0.3330	0.3330	-0.0830	0.0830
f	0.0000	0.0000	0.1440	0.1440

C r o s s

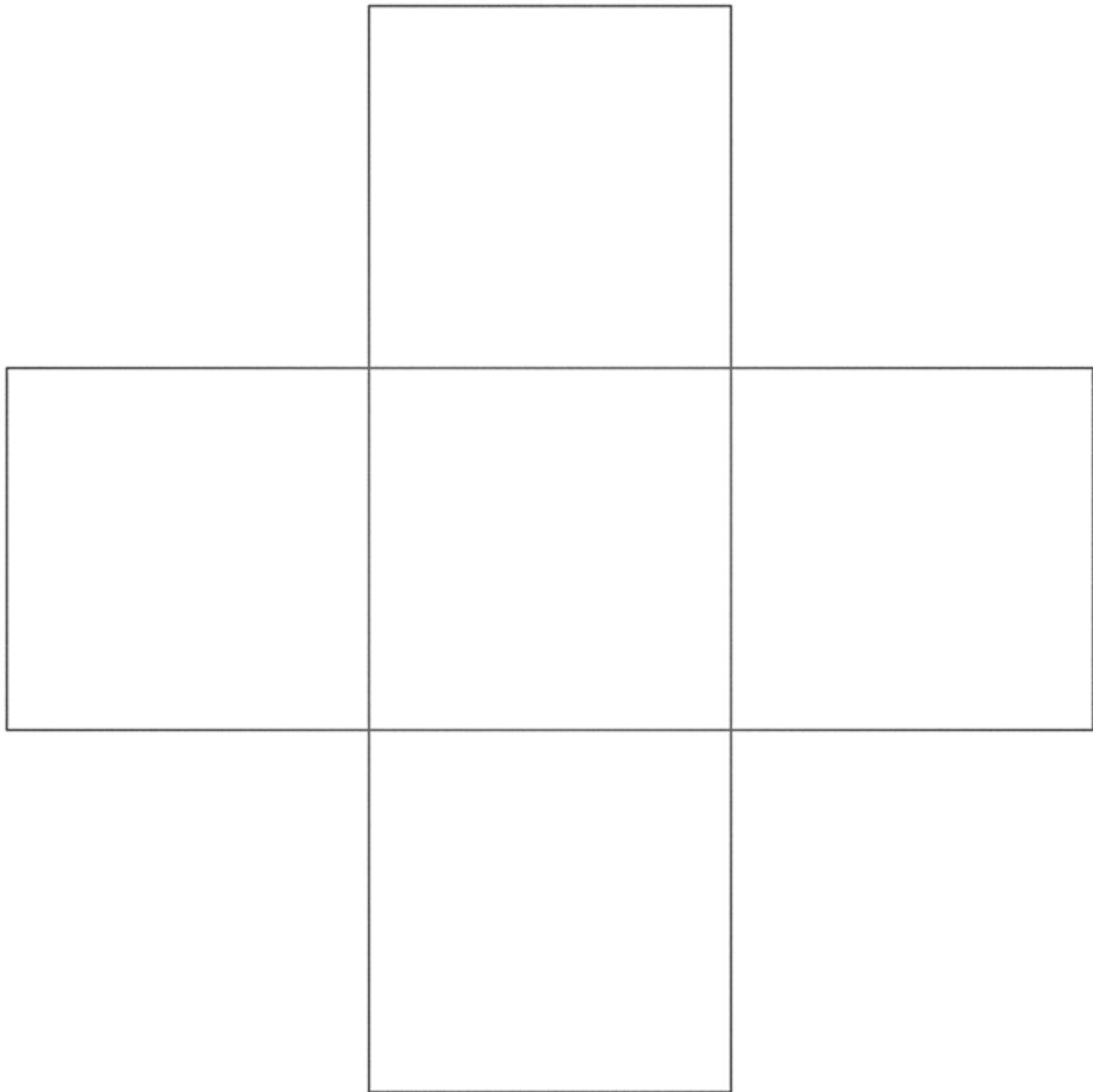
Written by [Paul Bourke](#)

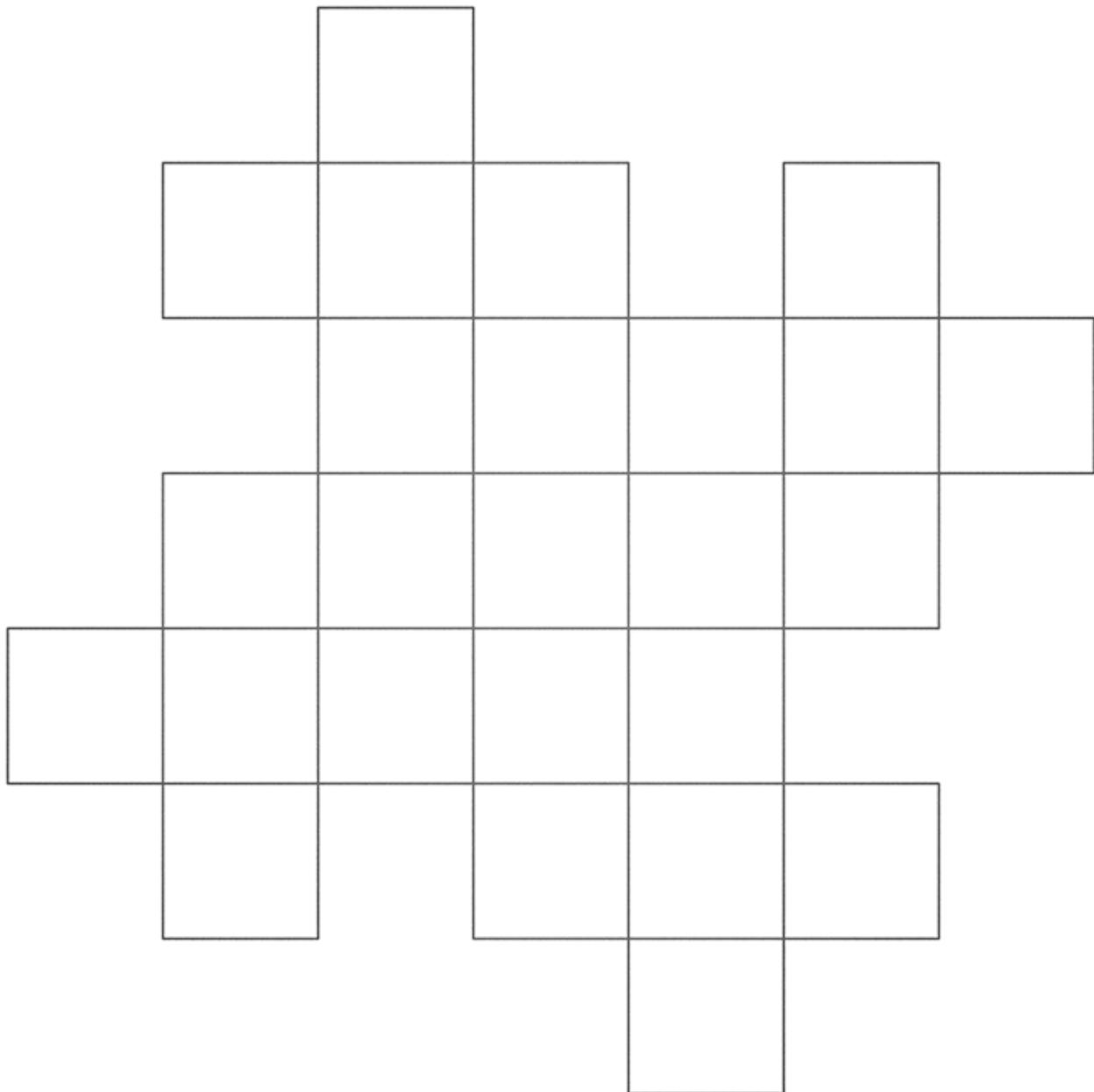
June 1990

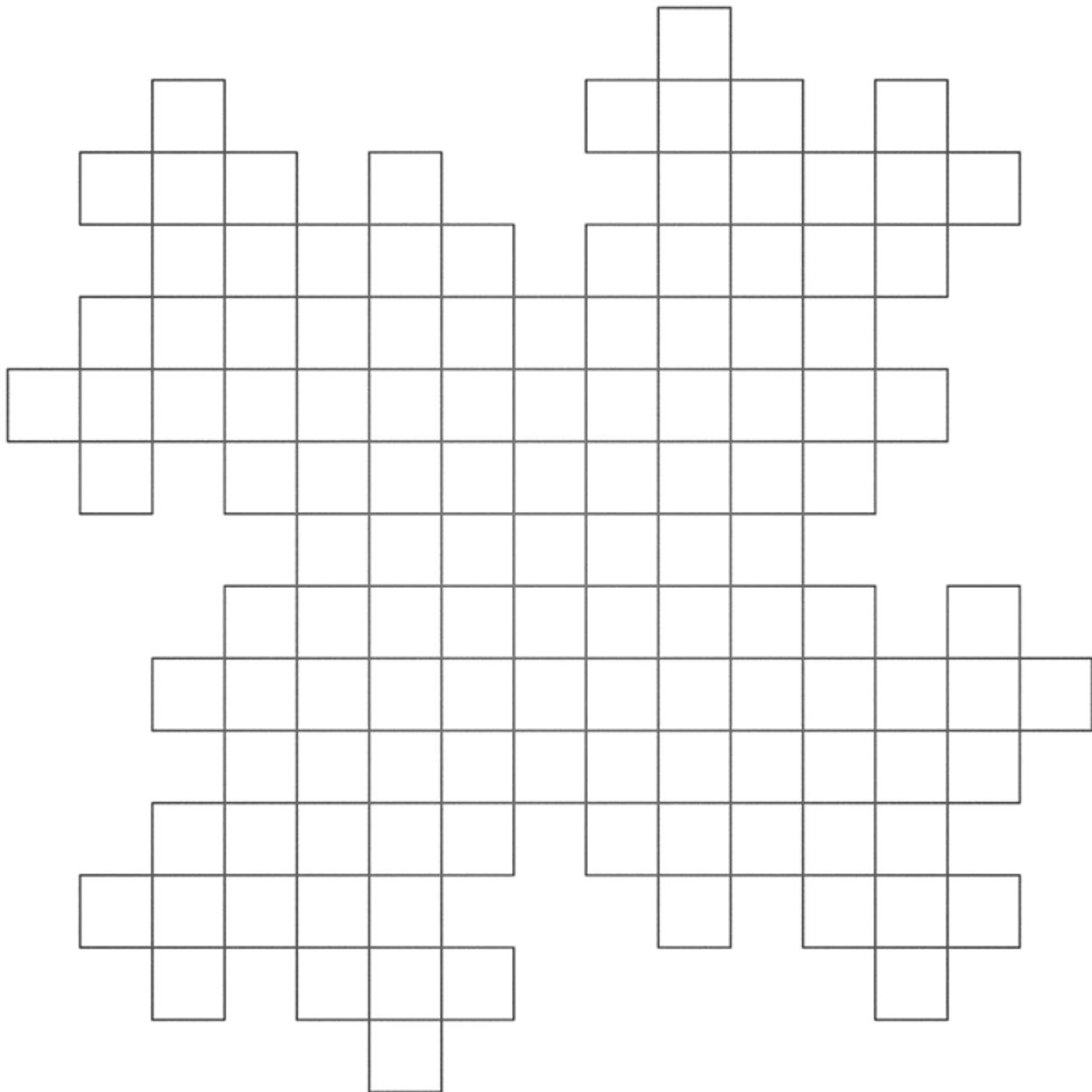
axiom = F+F+F+F

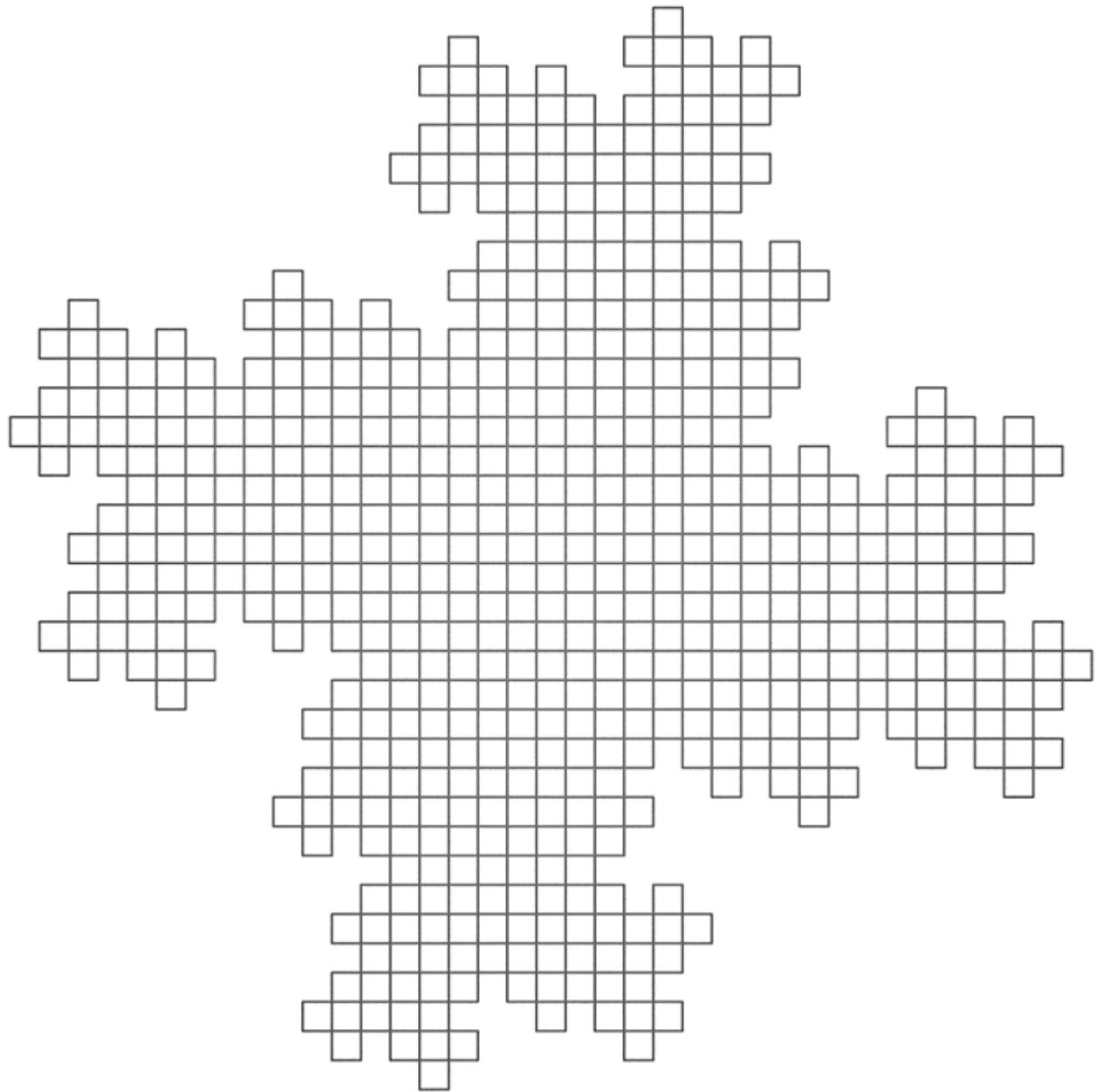
F -> F+FF++F+F

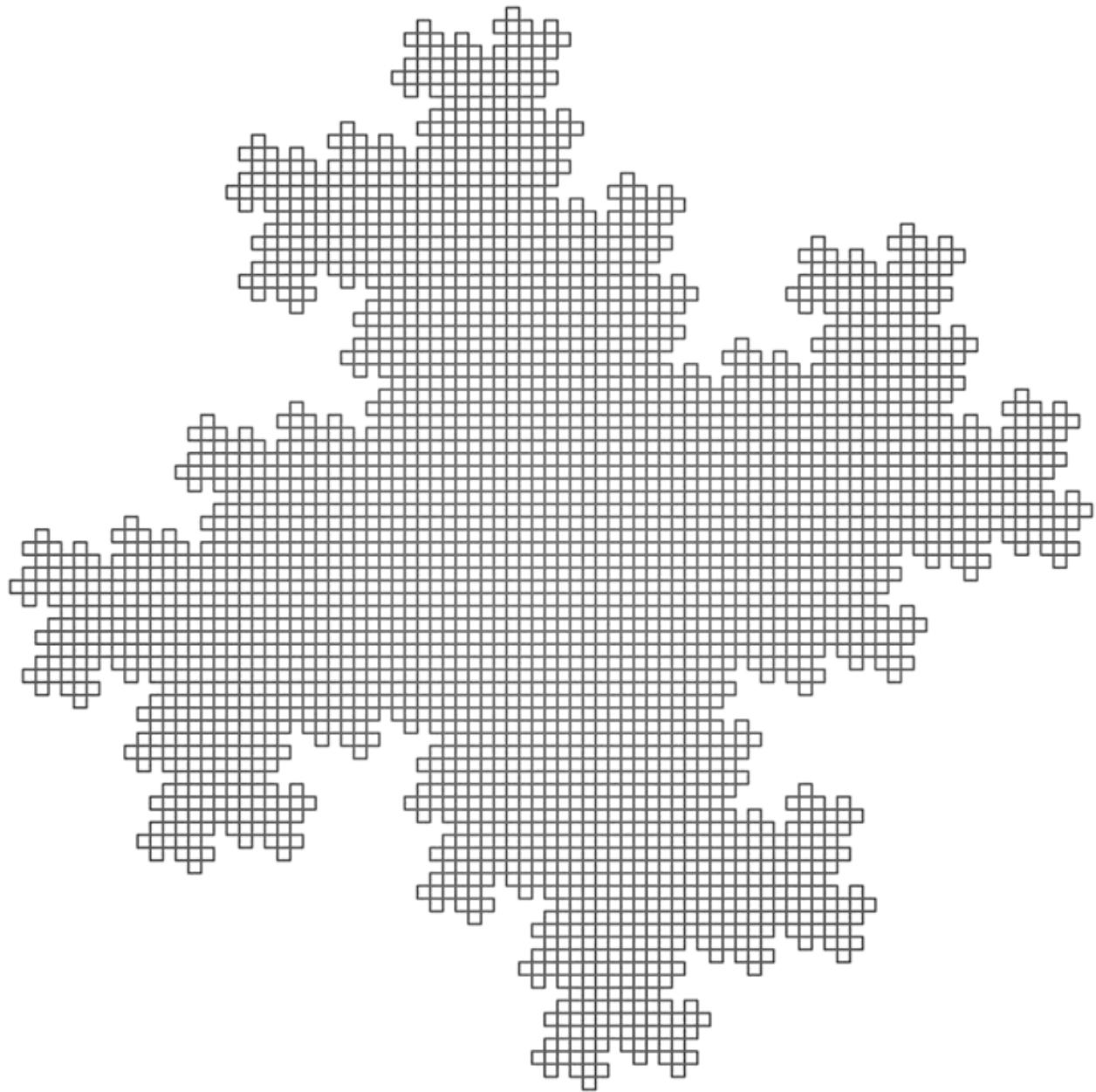
angle = 90









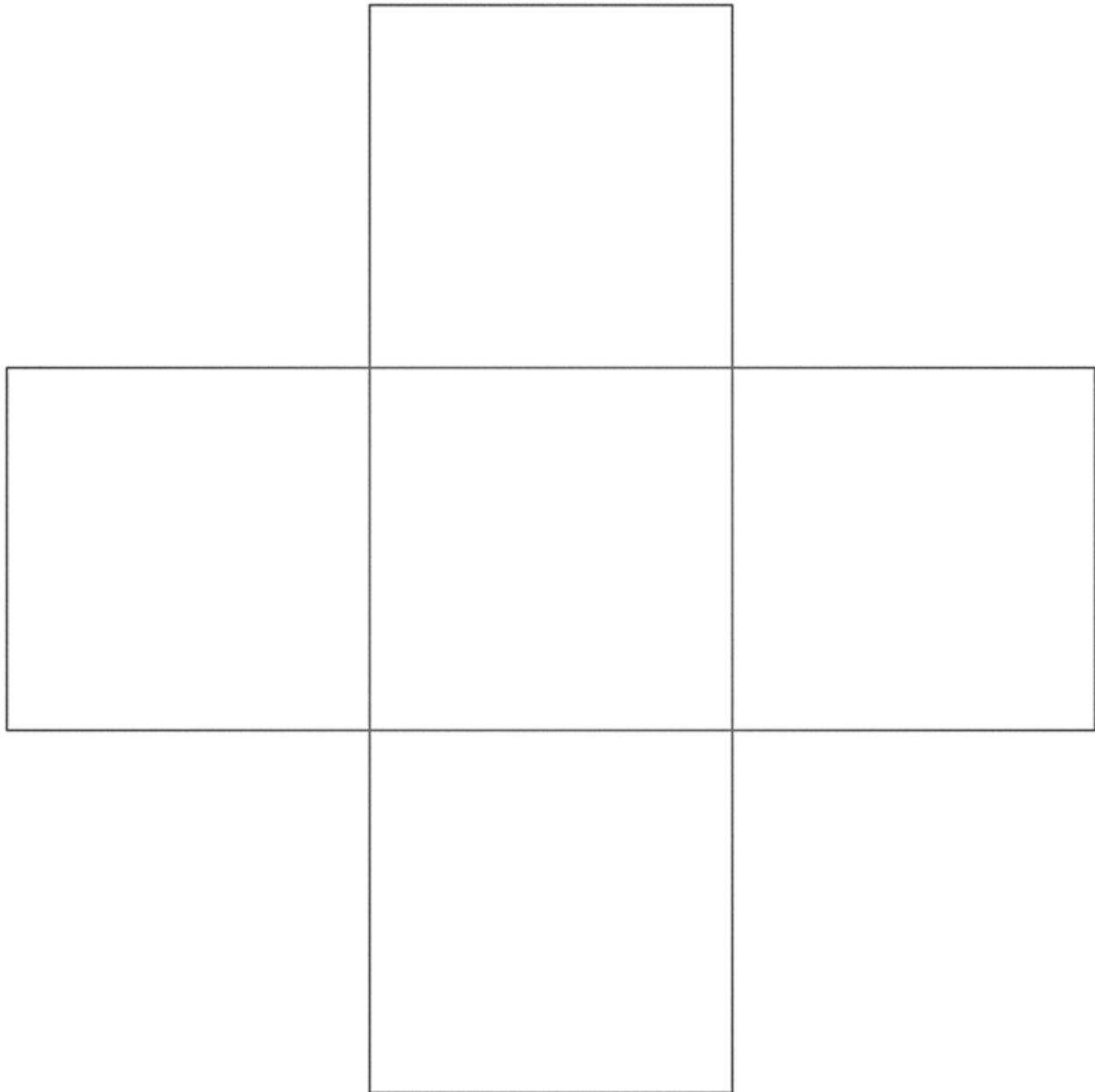


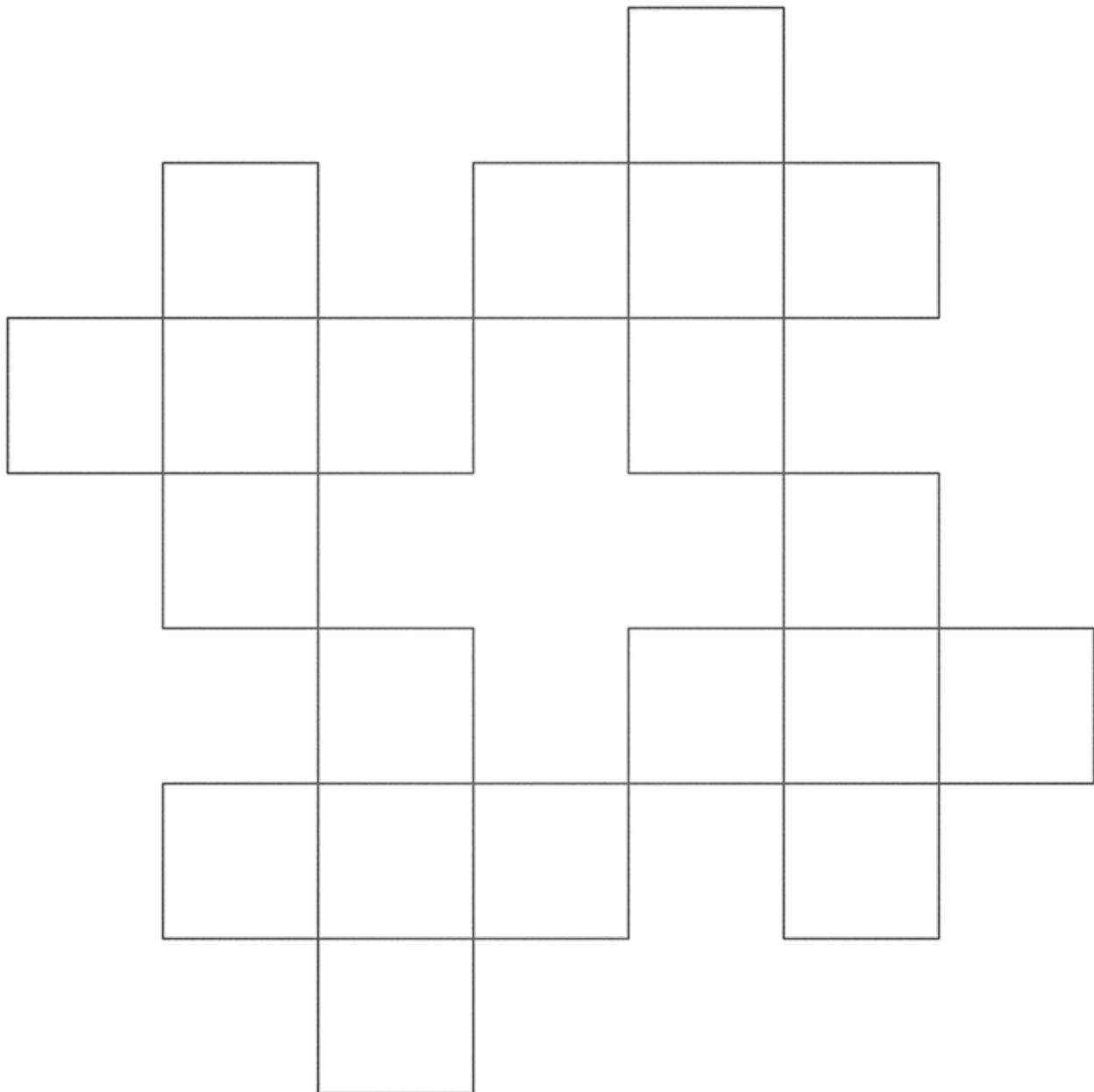
C r o s s

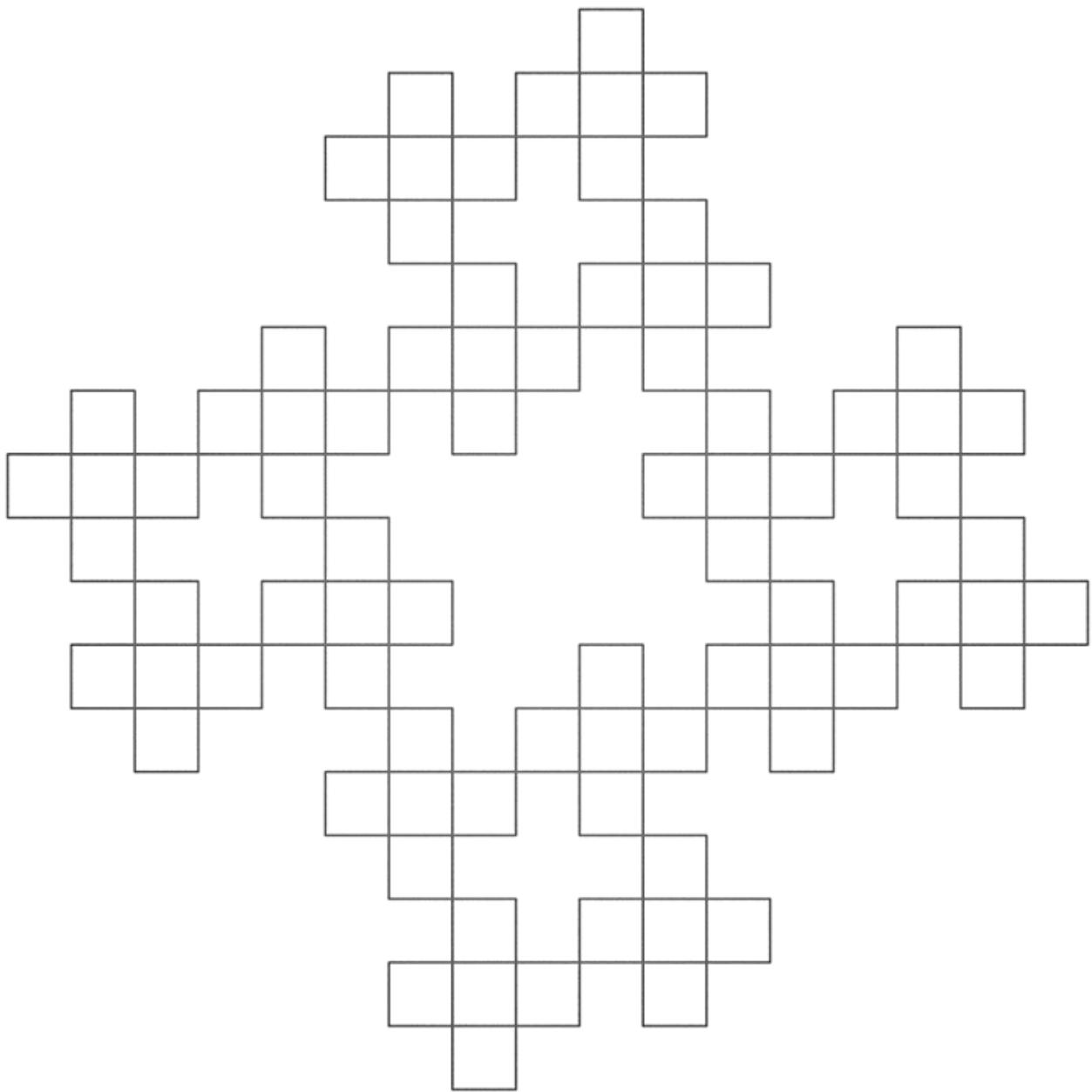
Written by [Paul Bourke](#)
August 1990

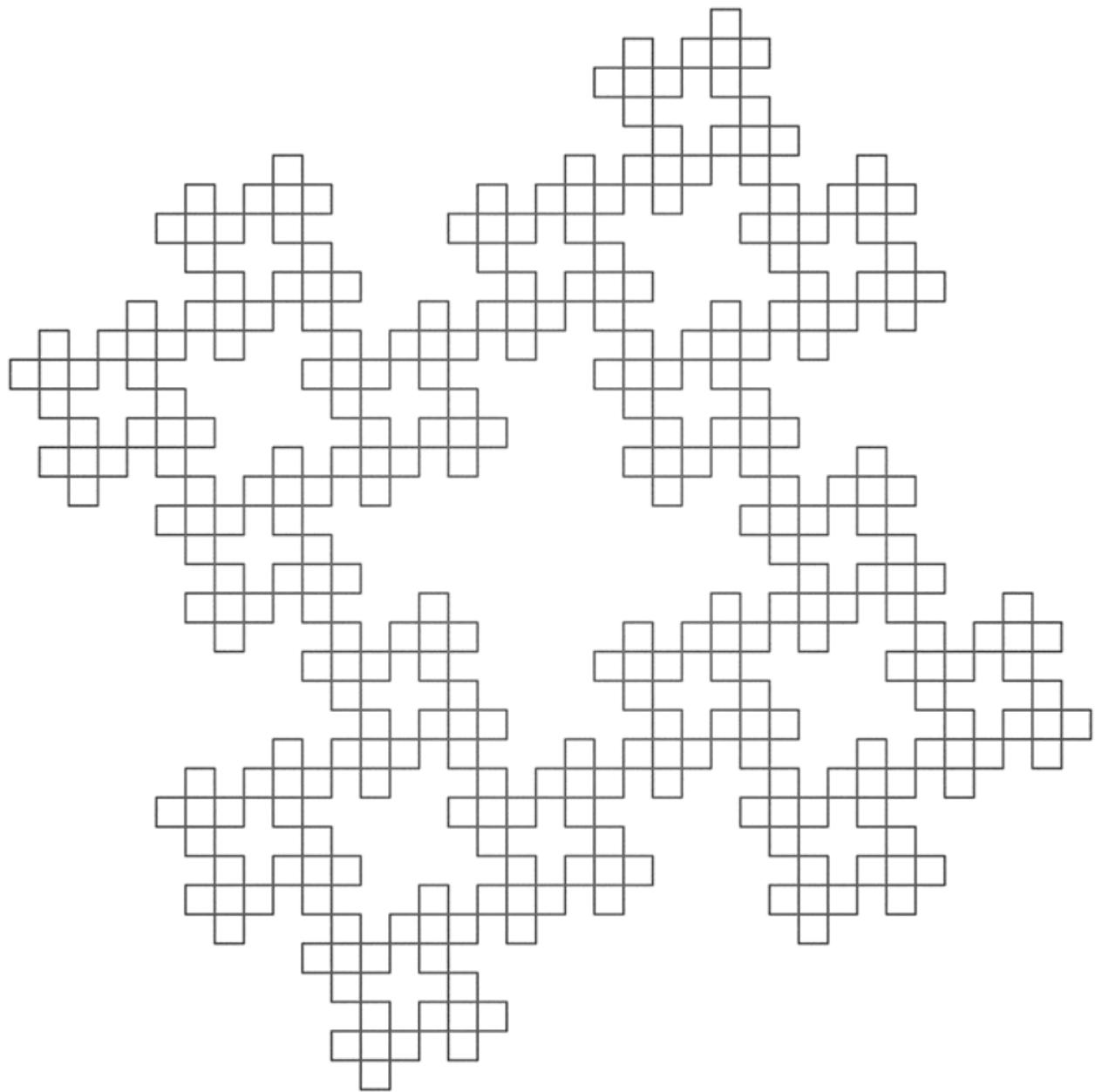
axiom = F+F+F+F
F -> F+F-F+F+F

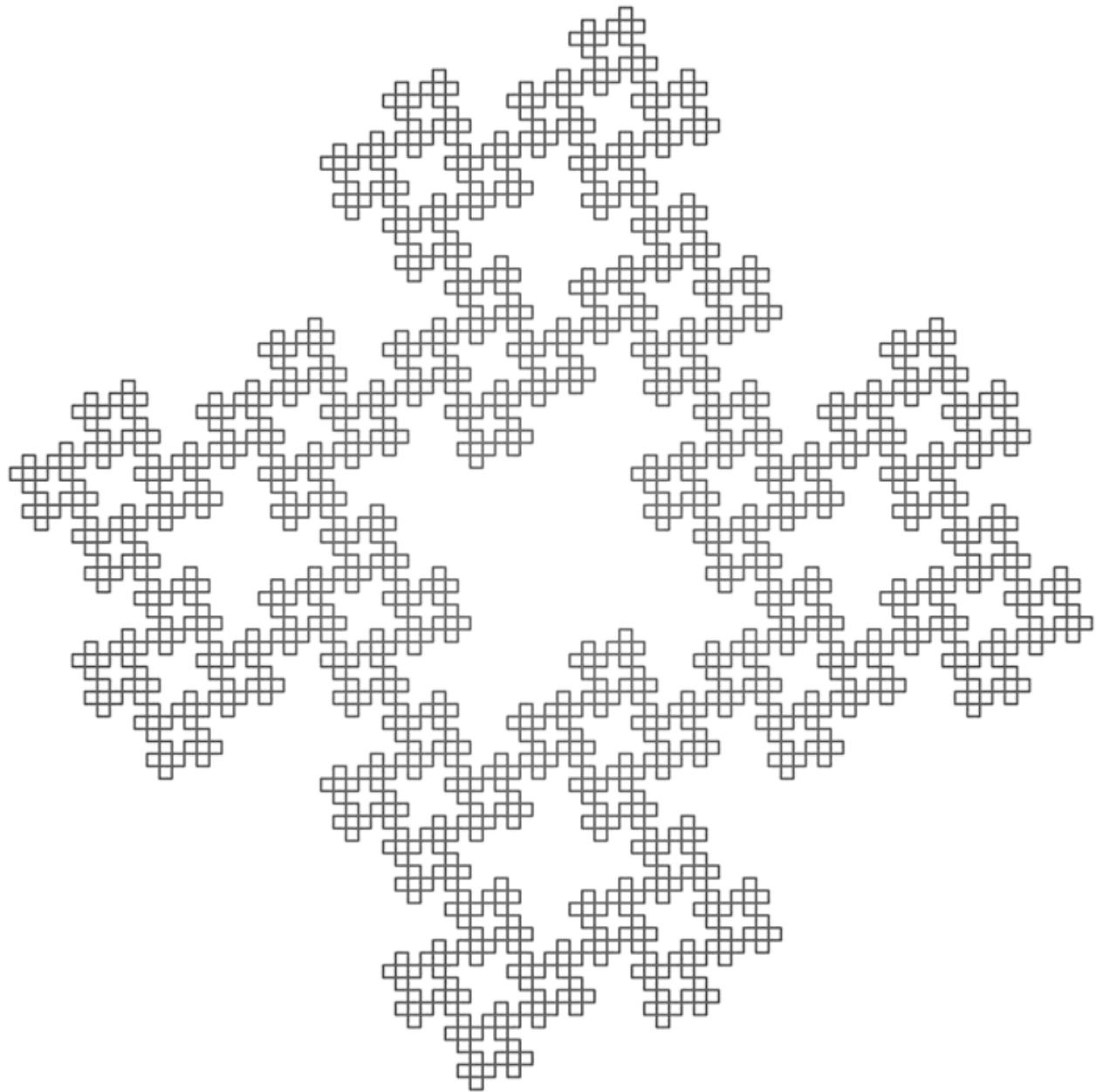
angle = 90











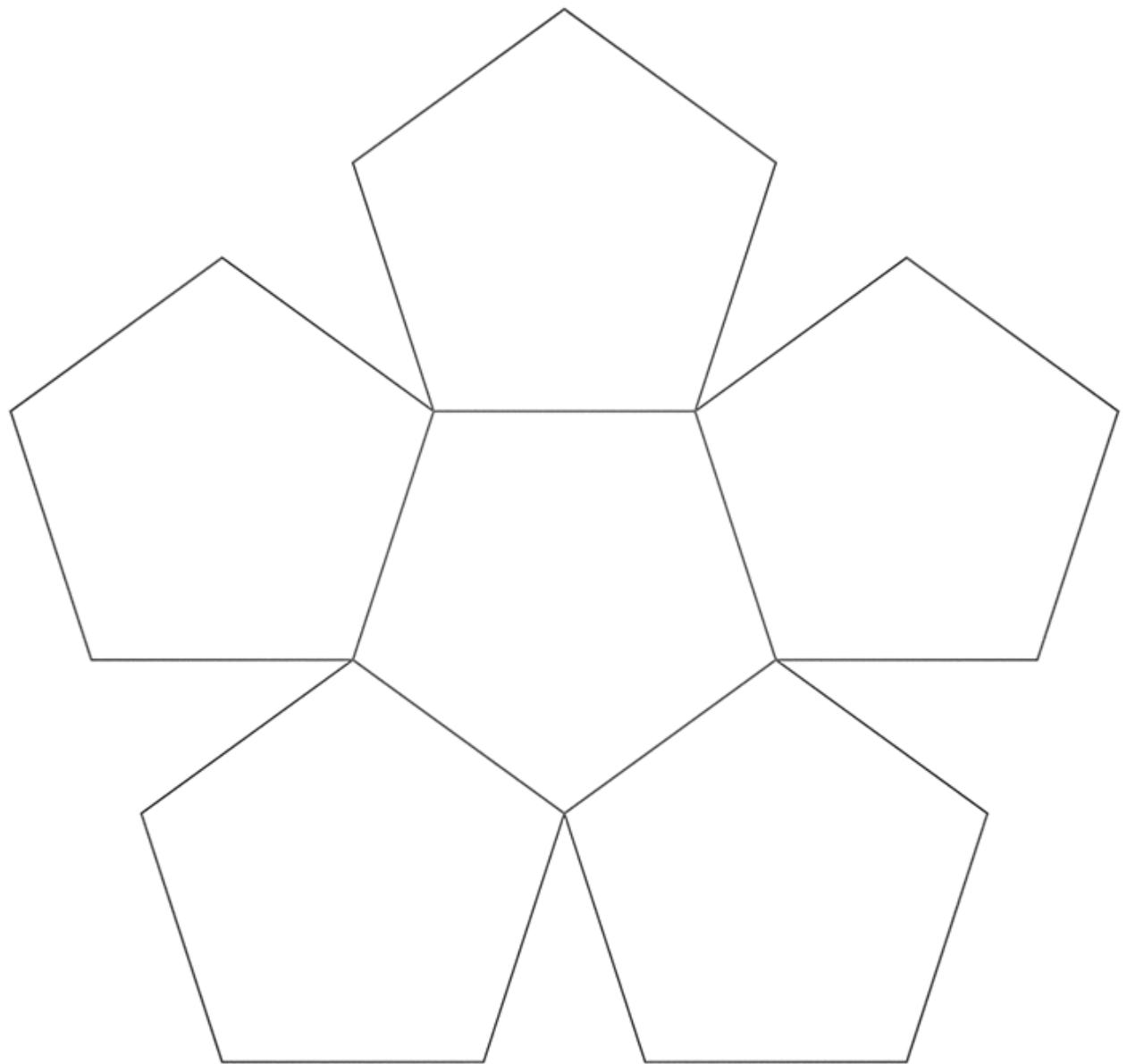
Pentaplexity

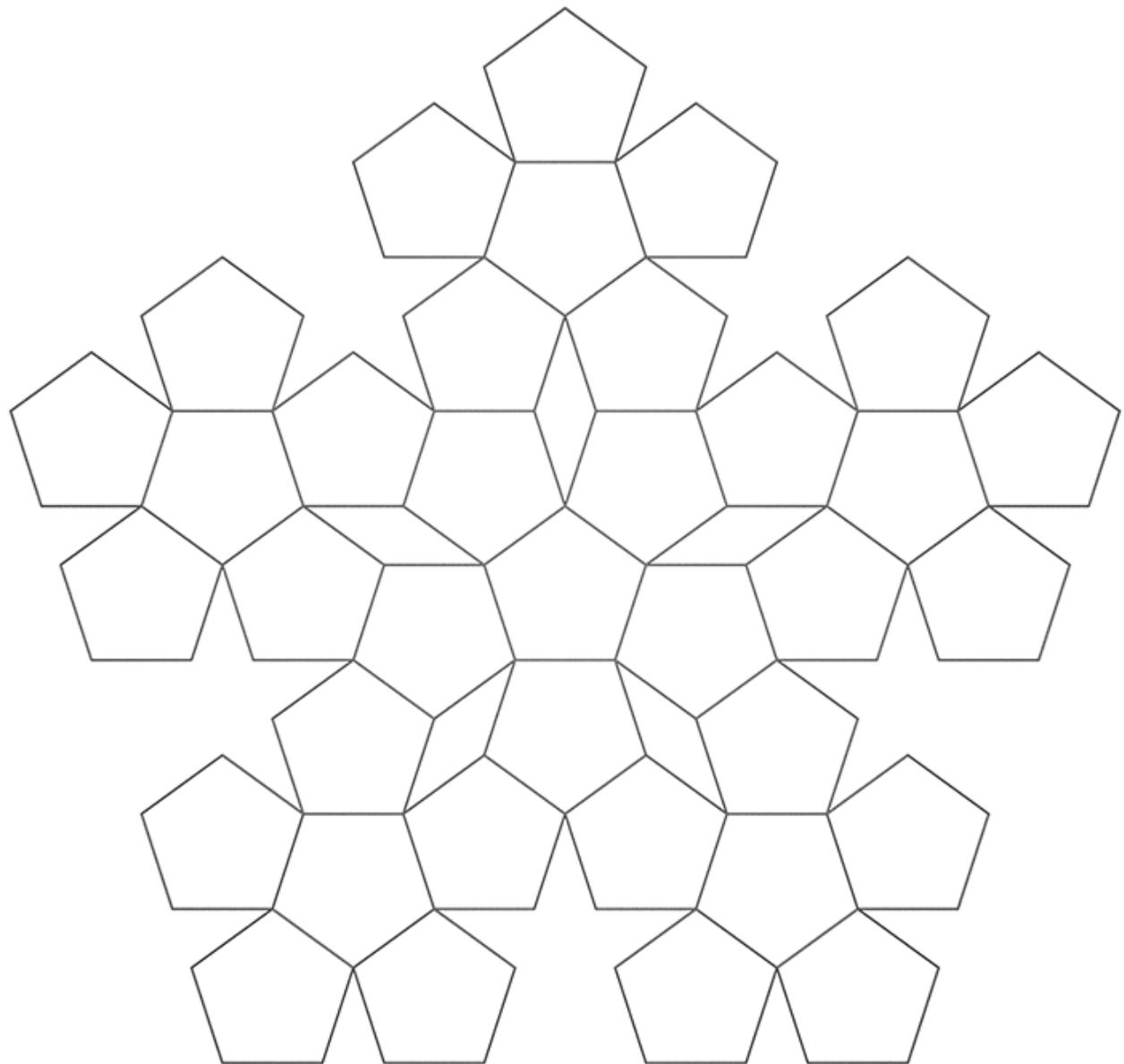
Written by [Paul Bourke](#)

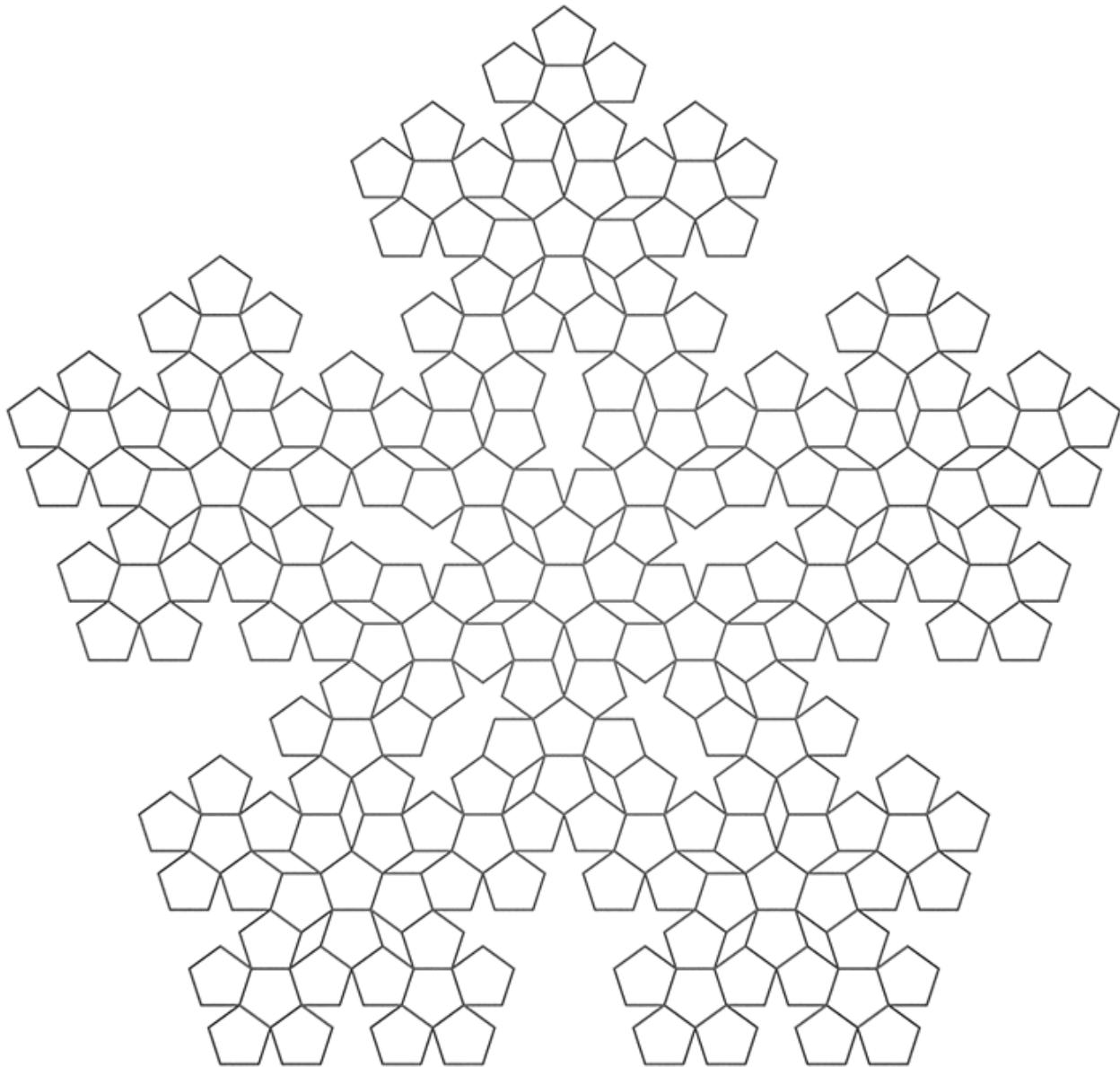
September 1990

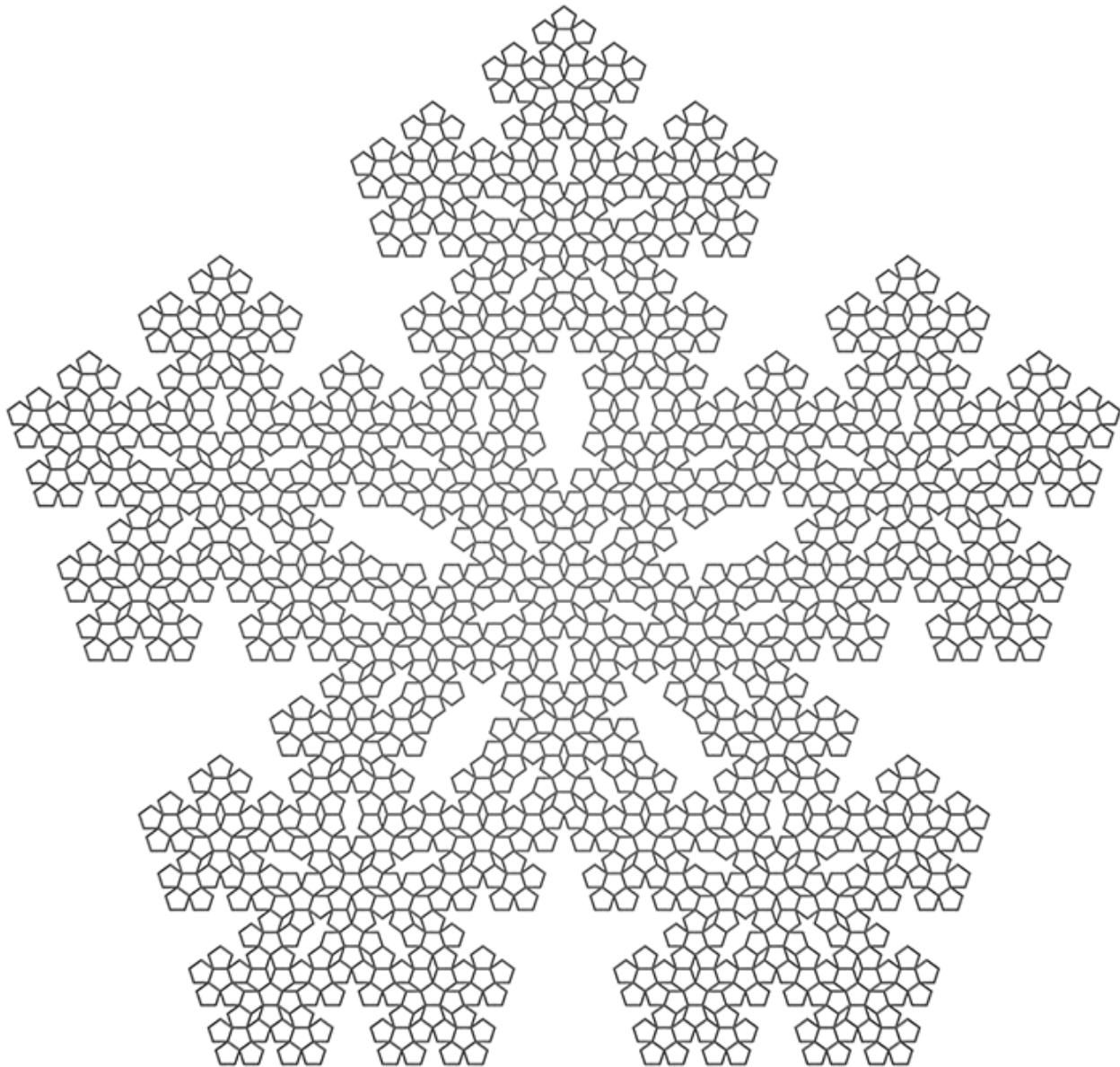
axiom = F++F++F++F++F
F -> F++F++F|F-F++F

angle = 36









Tiles

Written by [Paul Bourke](#)

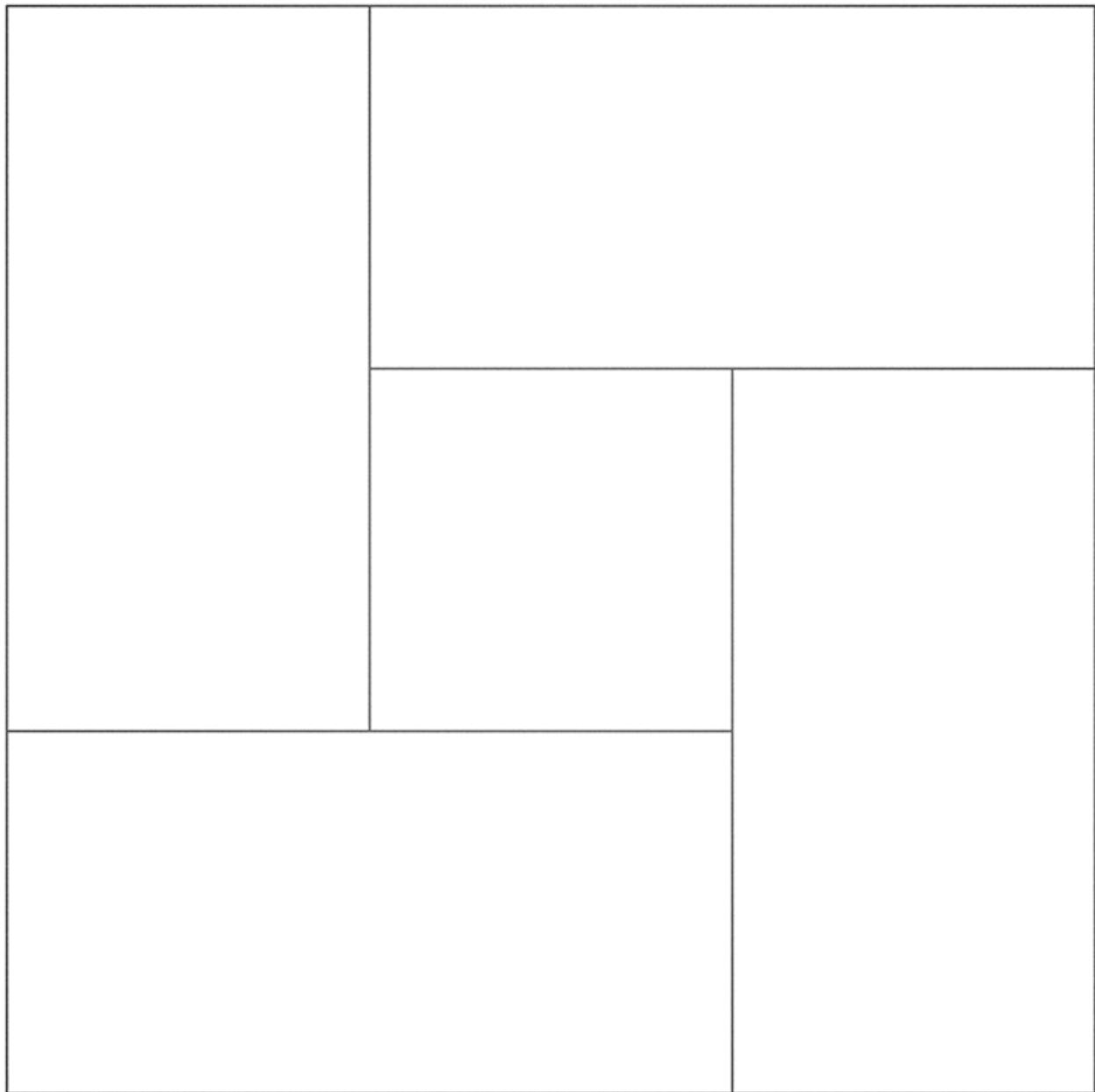
June 1990

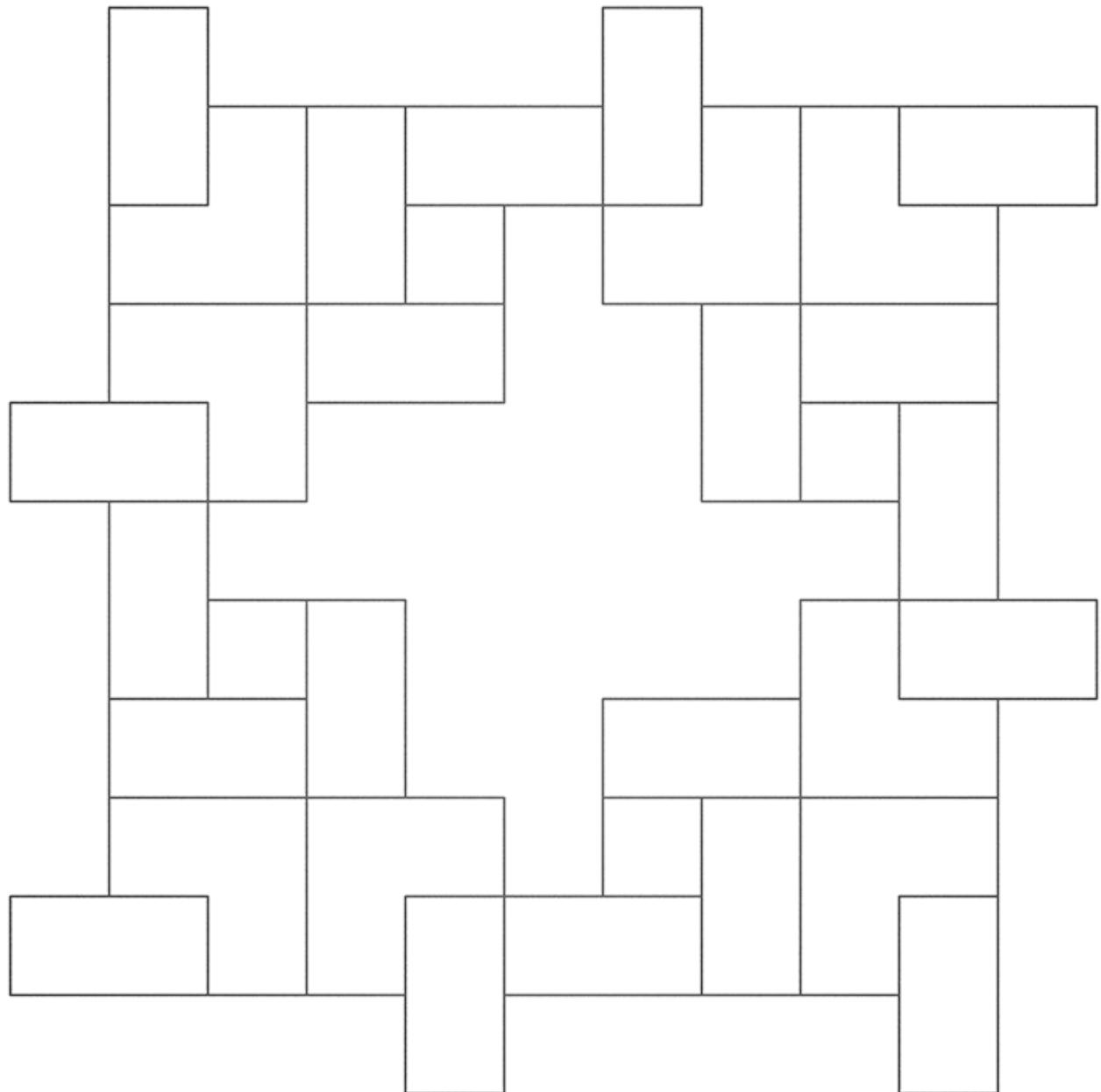
Fractal Dimension: $\log(7)/\log(3) = 1.771$

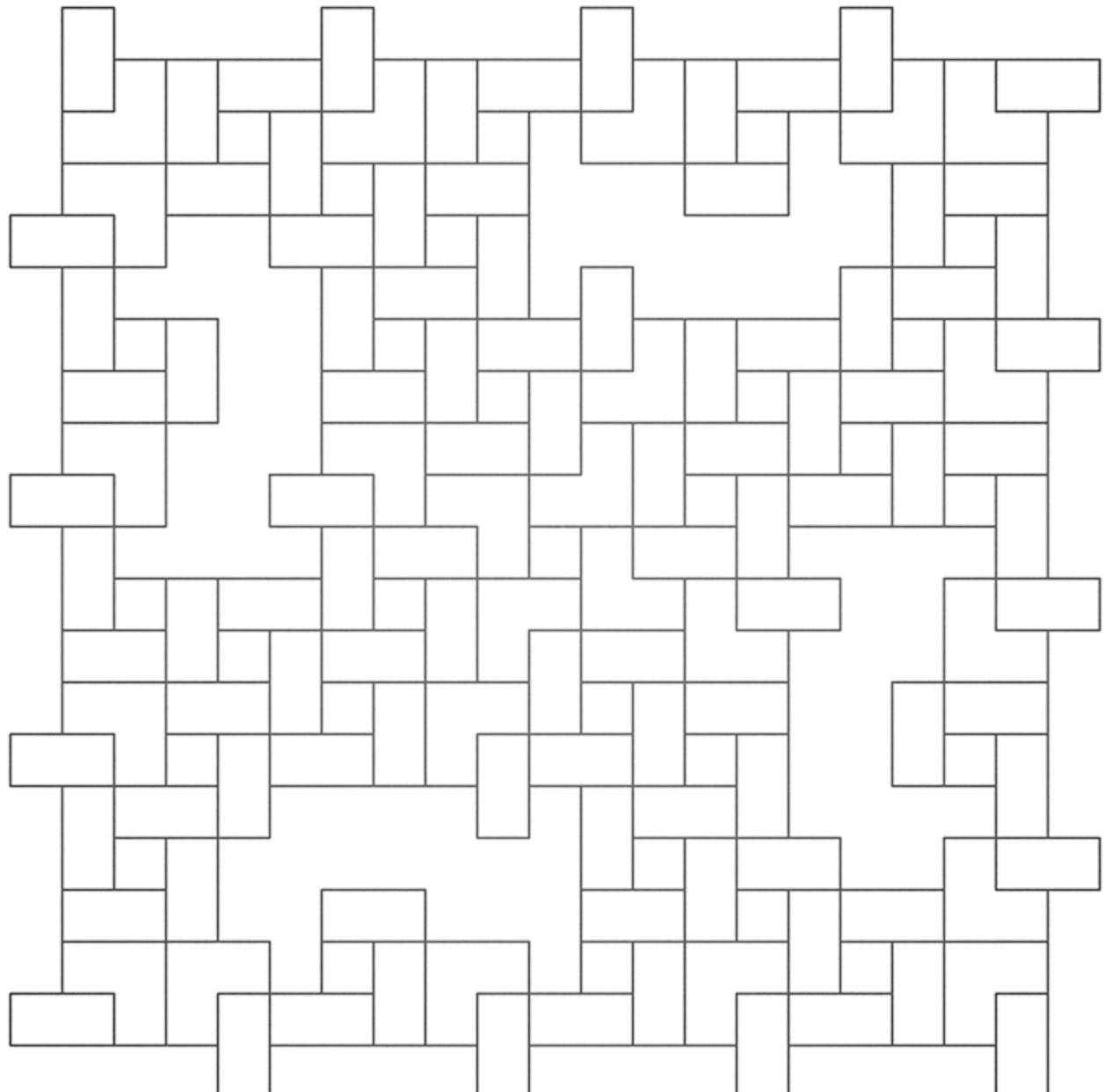
axiom = F+F+F+F

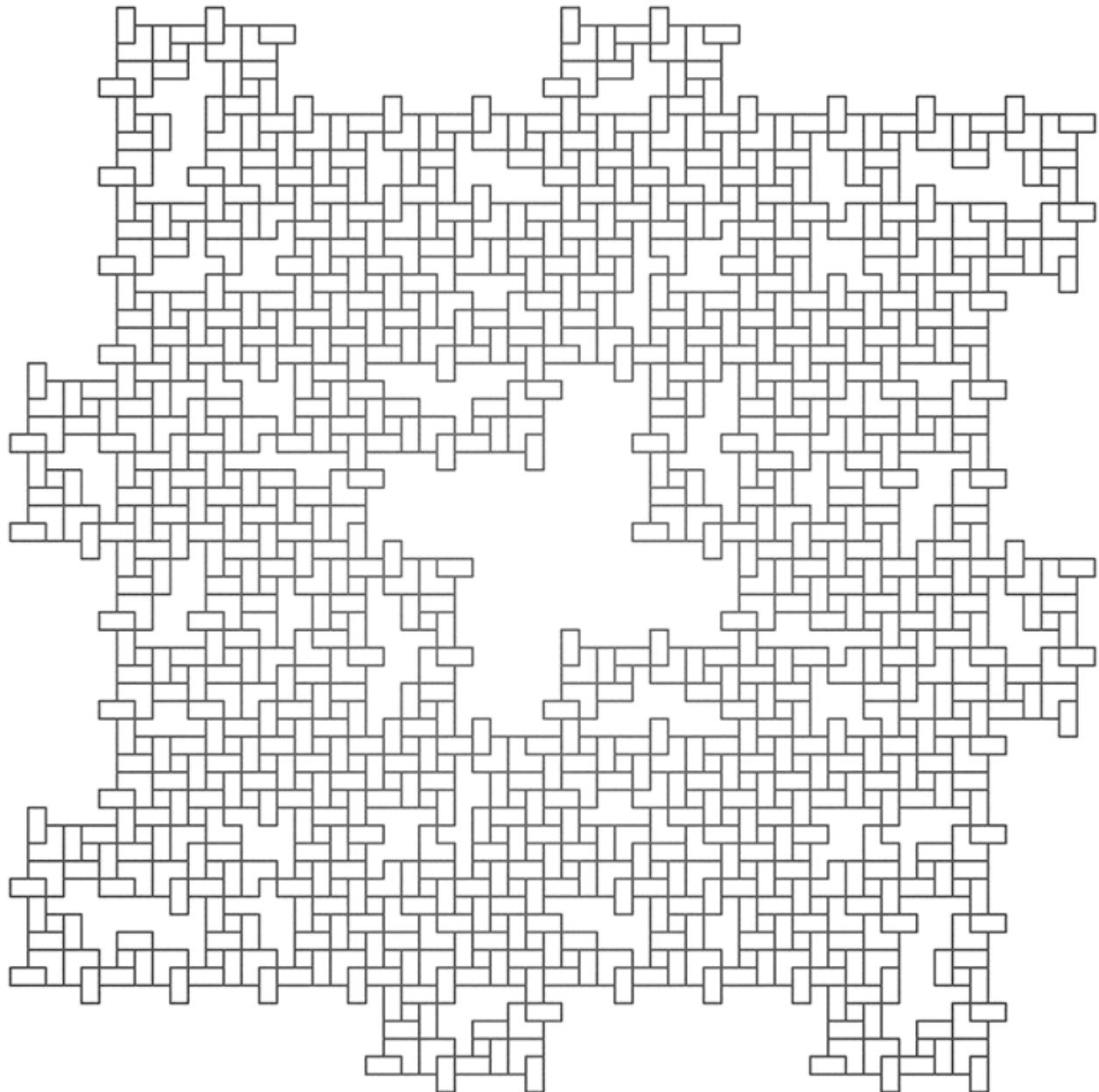
F -> FF+F-F+F+FF

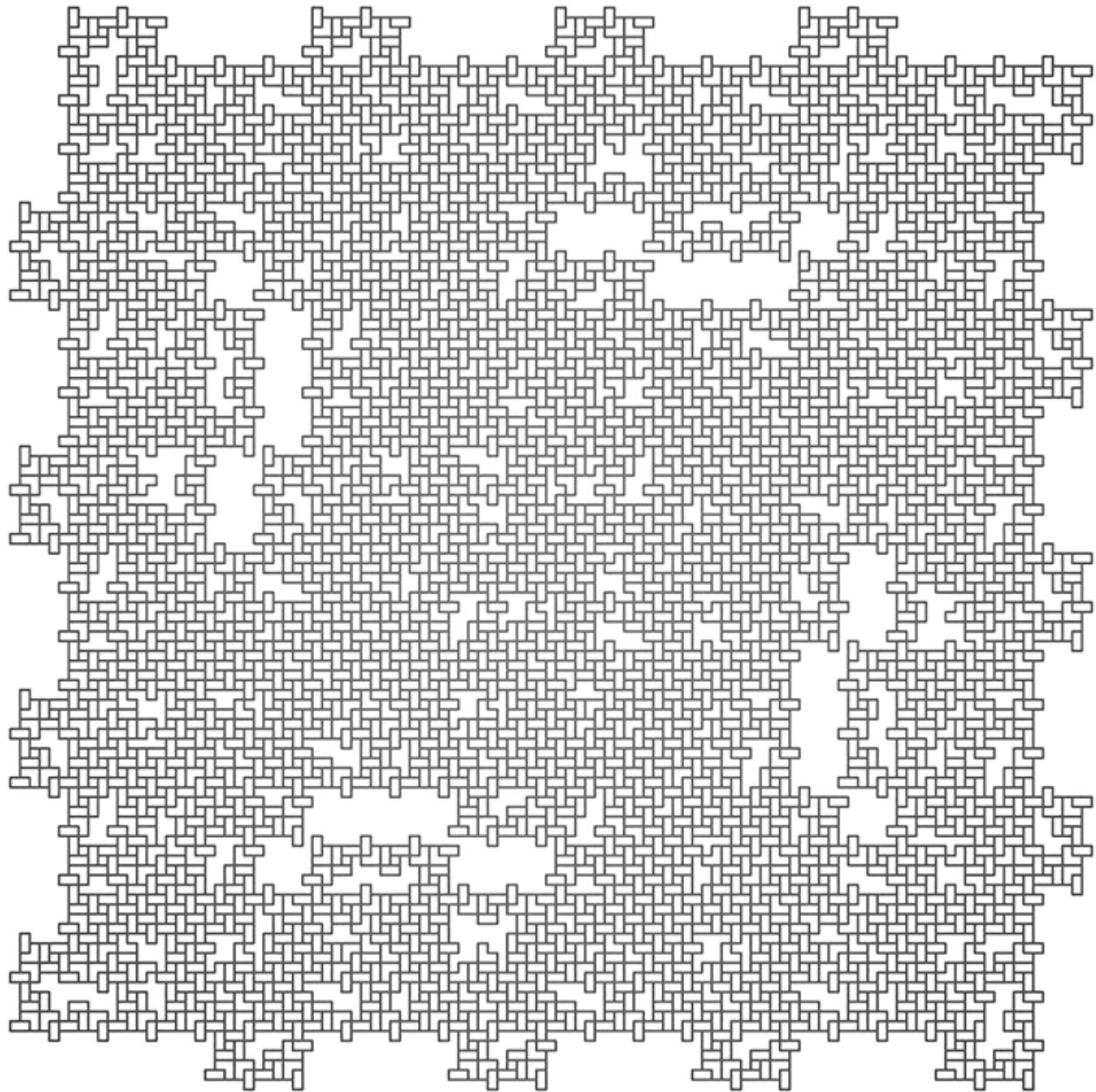
angle = 90











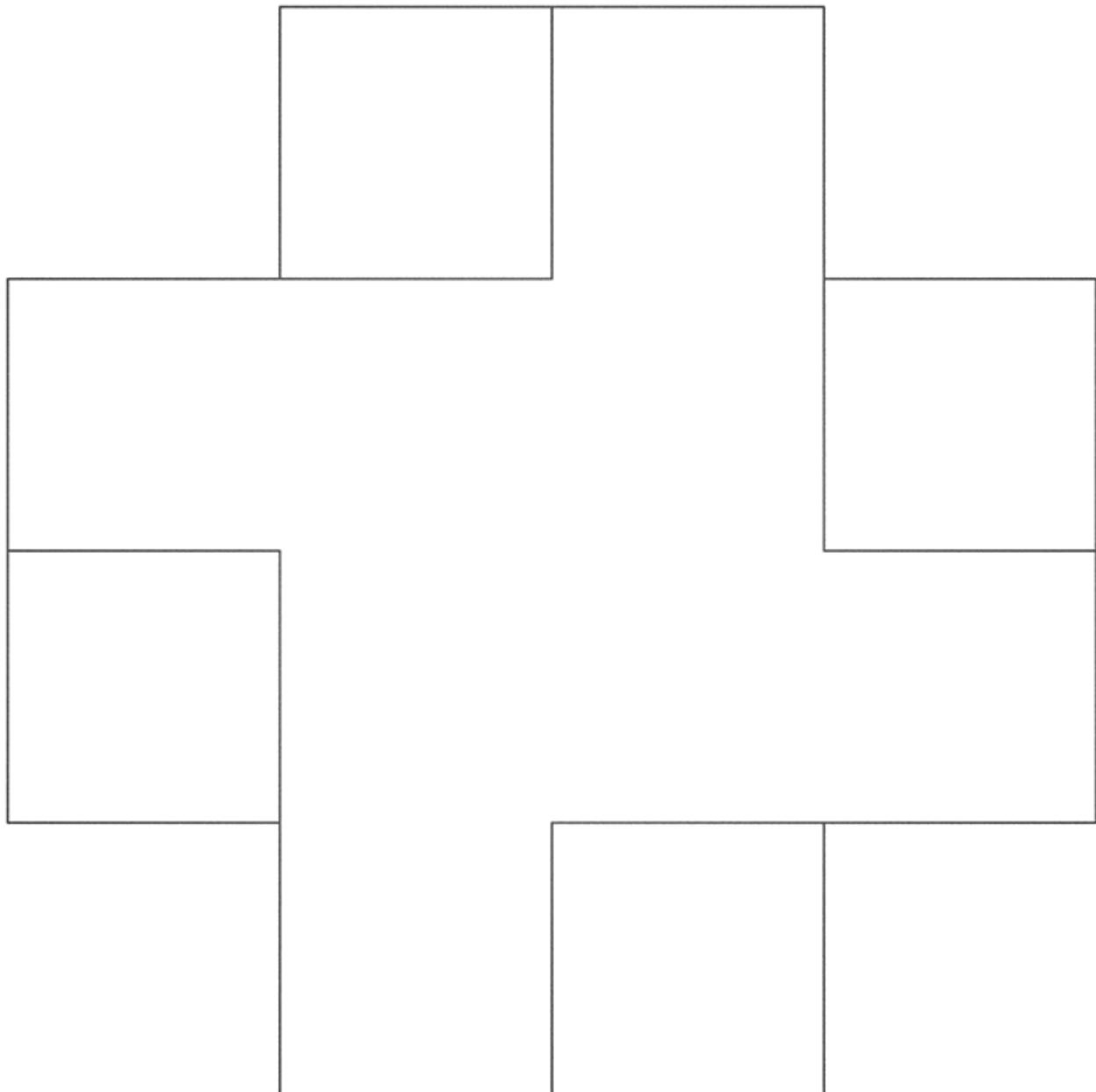
R i n g s

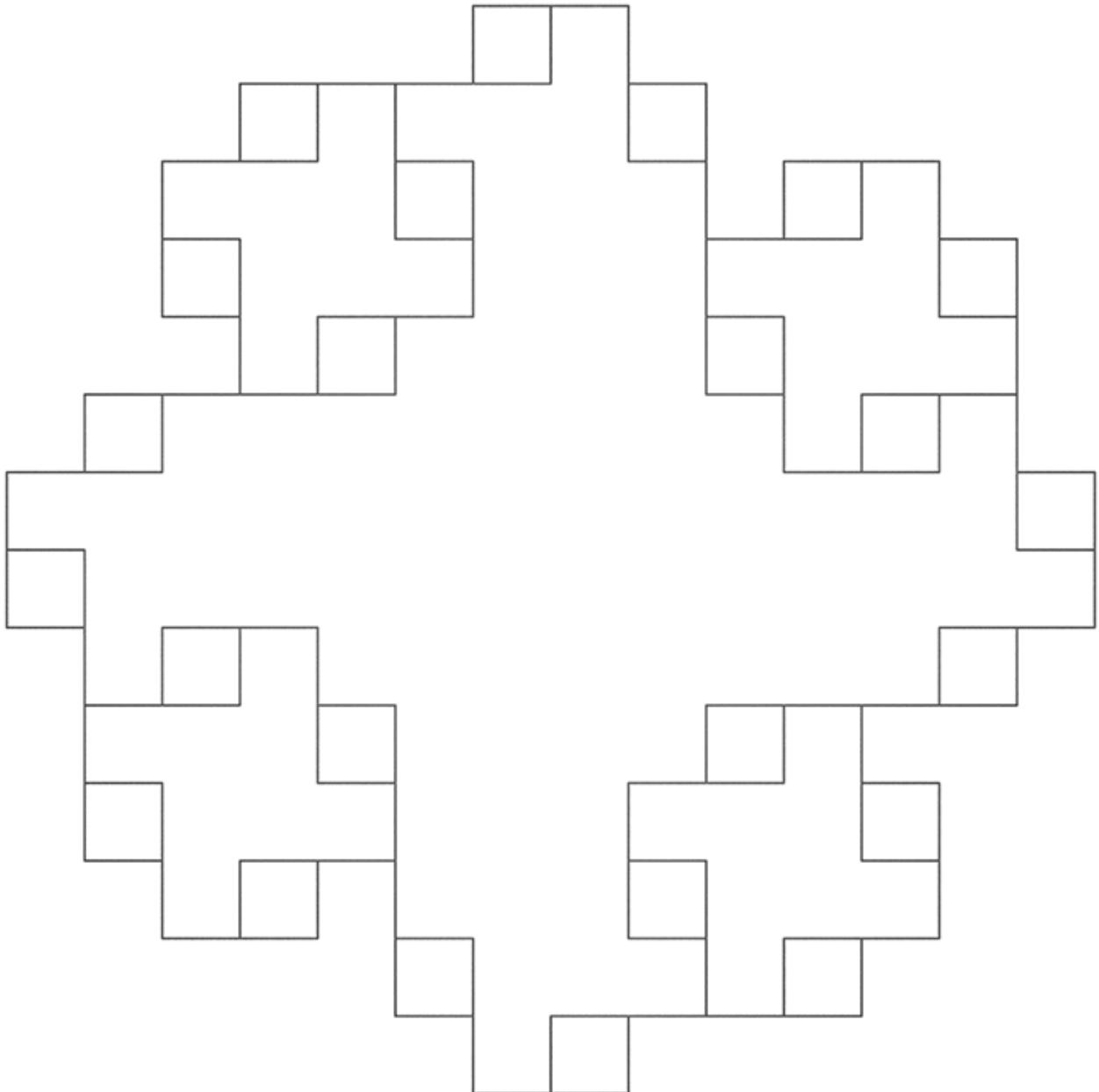
Written by [Paul Bourke](#)

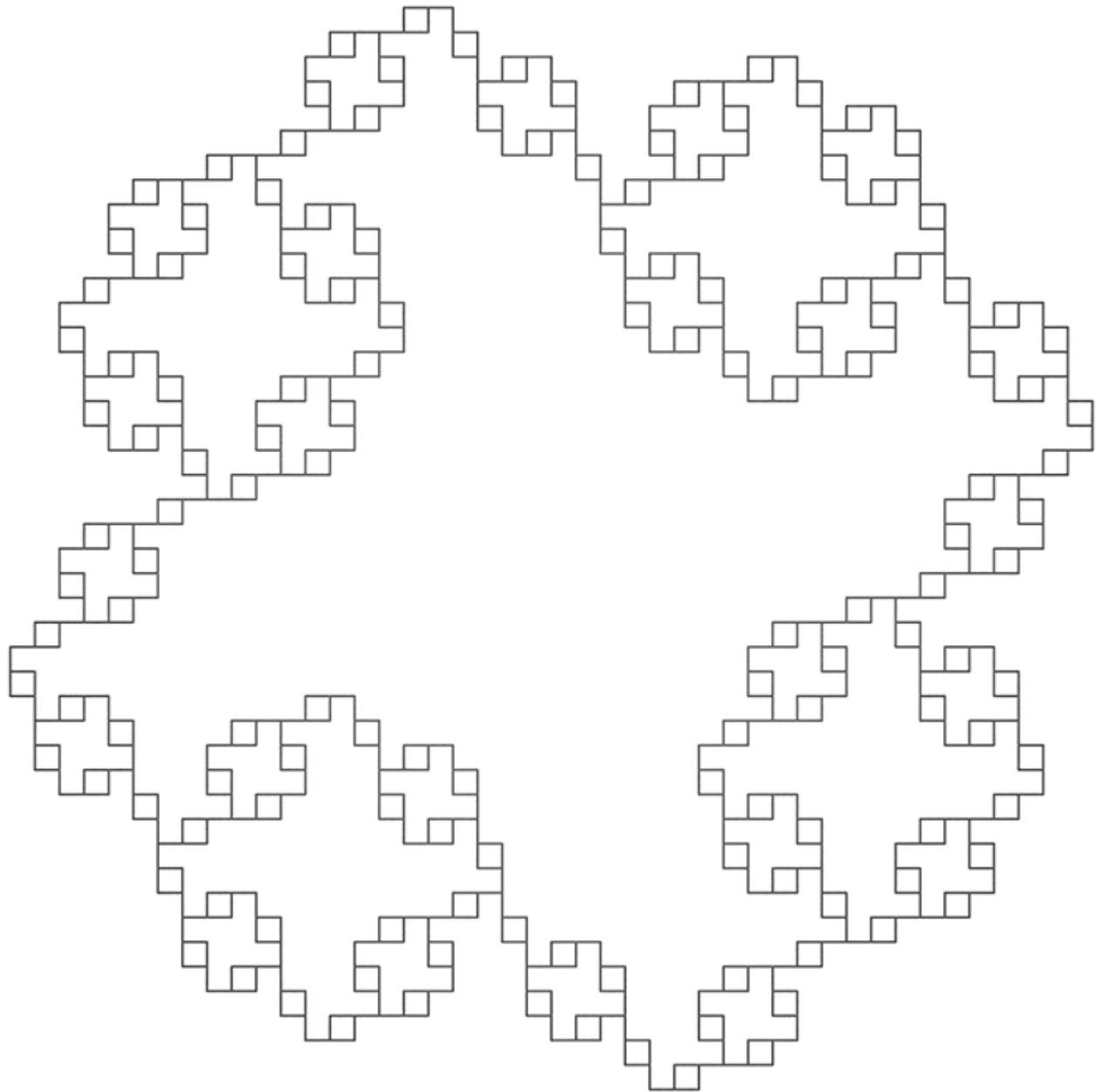
June 1990

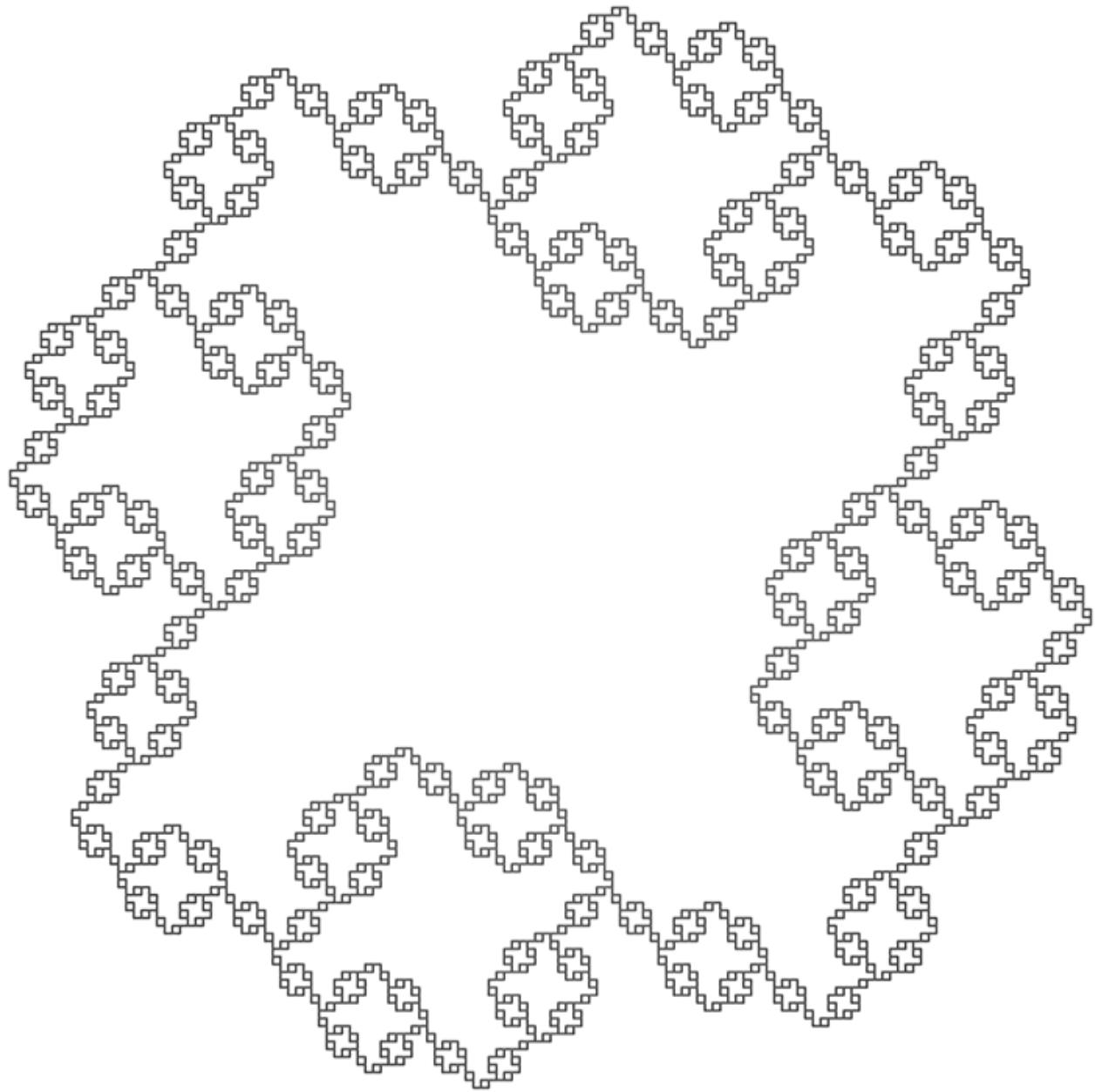
axiom = F+F+F+F
F -> FF+F+F+F+F+F-F

angle = 90









Dragon Curve

Written by [Paul Bourke](#)

August 1990

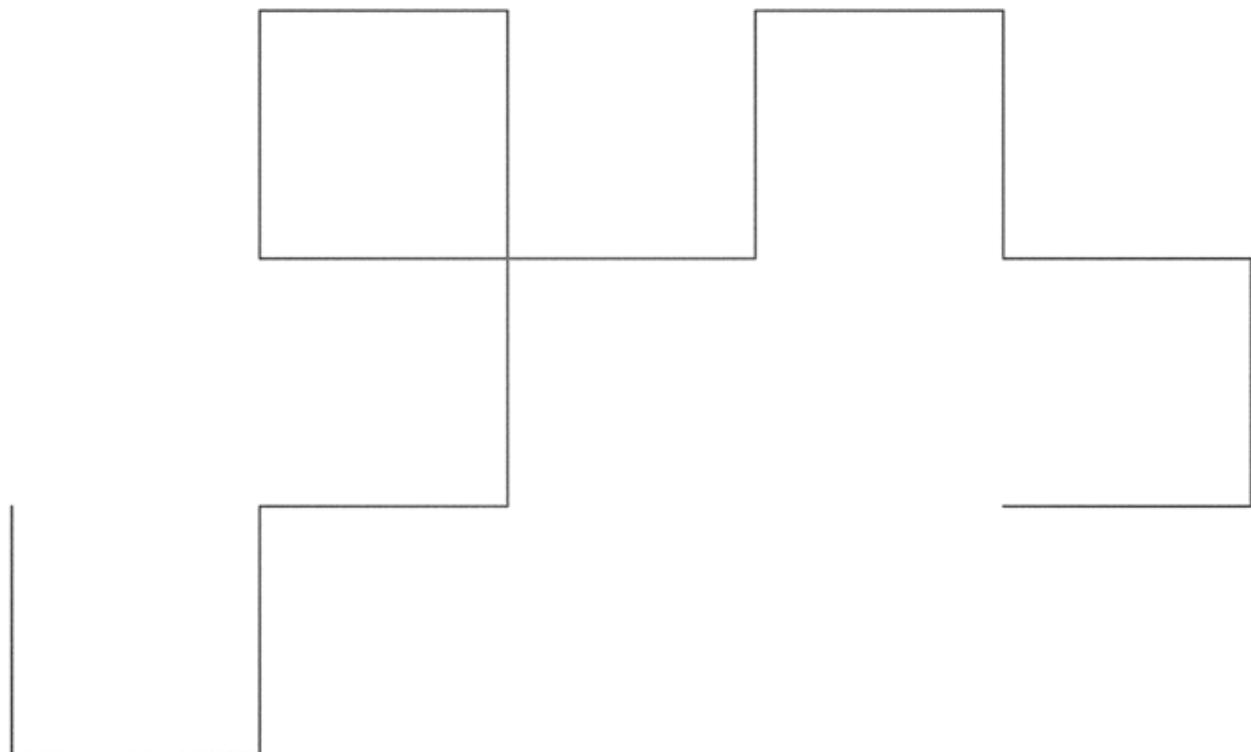
Attributed to David and Knuth, 1970

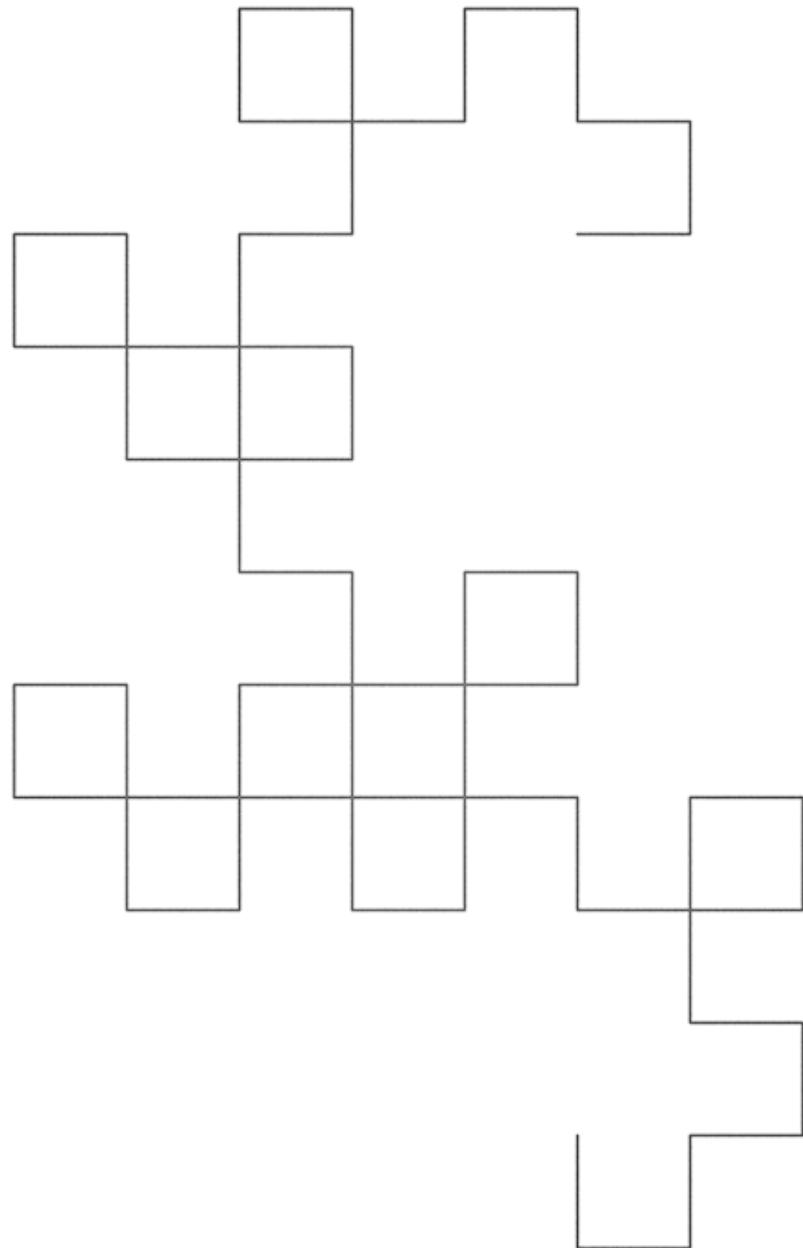
axiom = FX

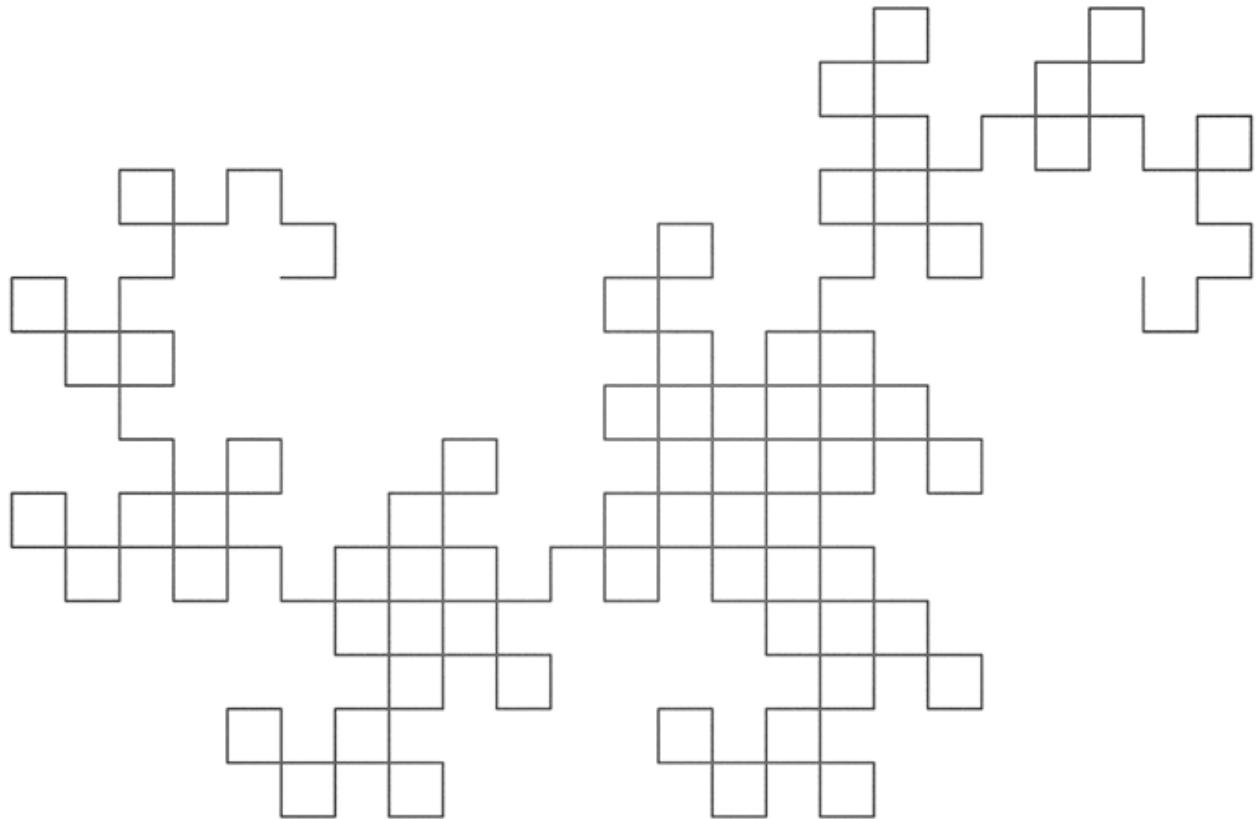
X -> X+YF+

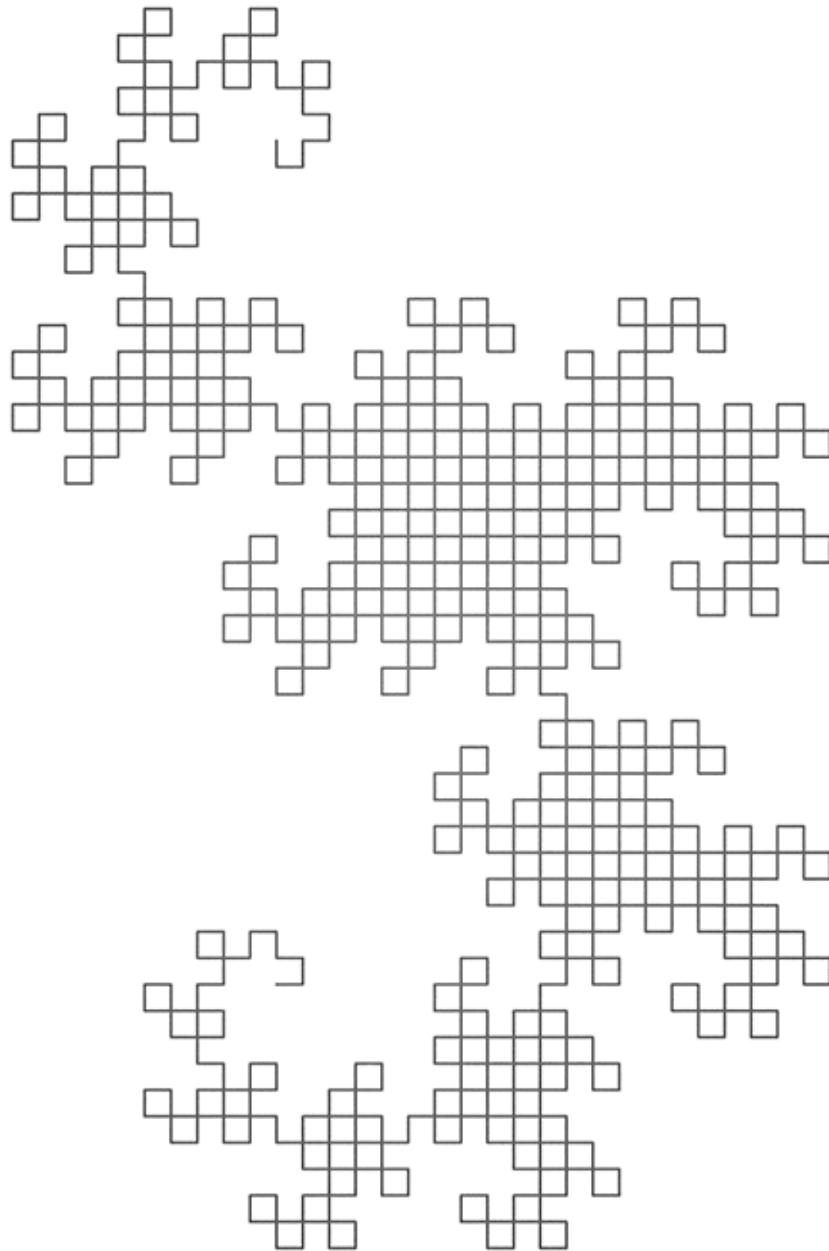
Y -> -FX-Y

angle = 90

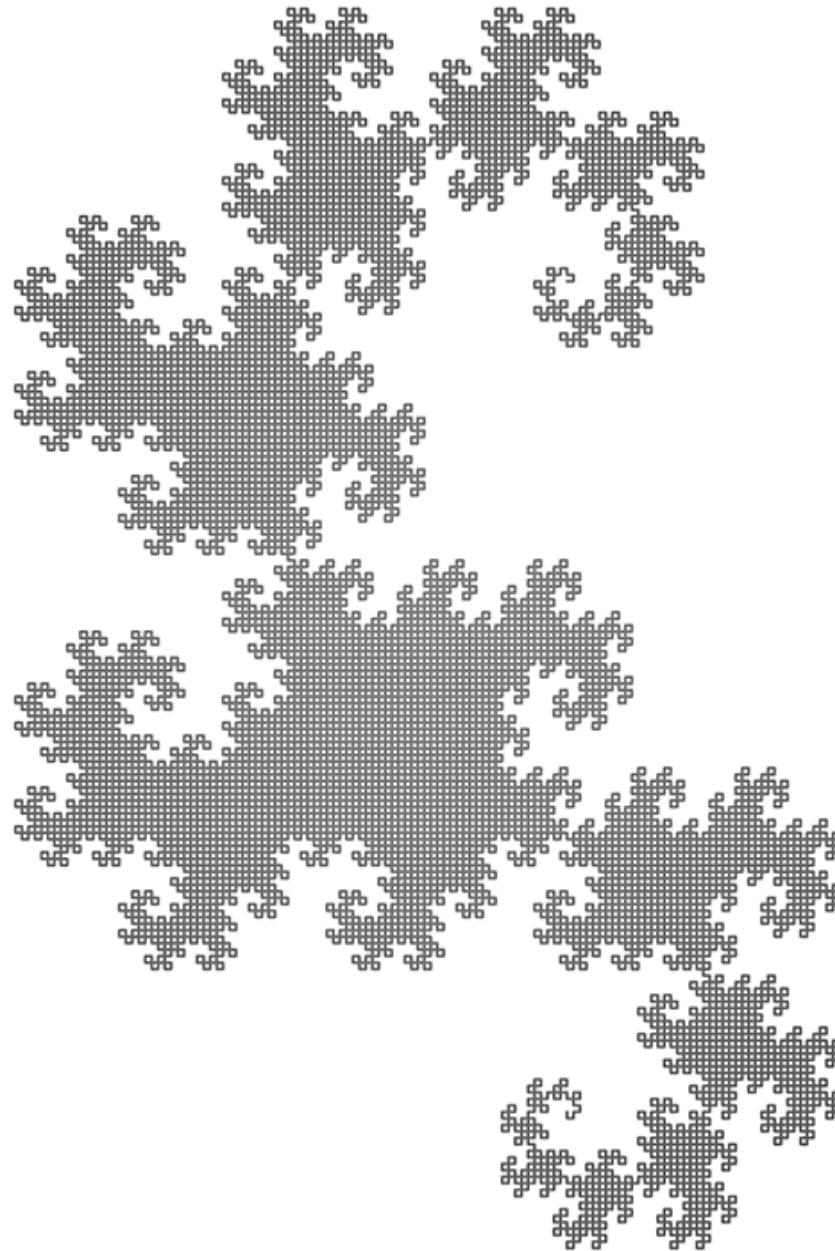












Hexagonal Gosper

Written by [Paul Bourke](#)

July 1990

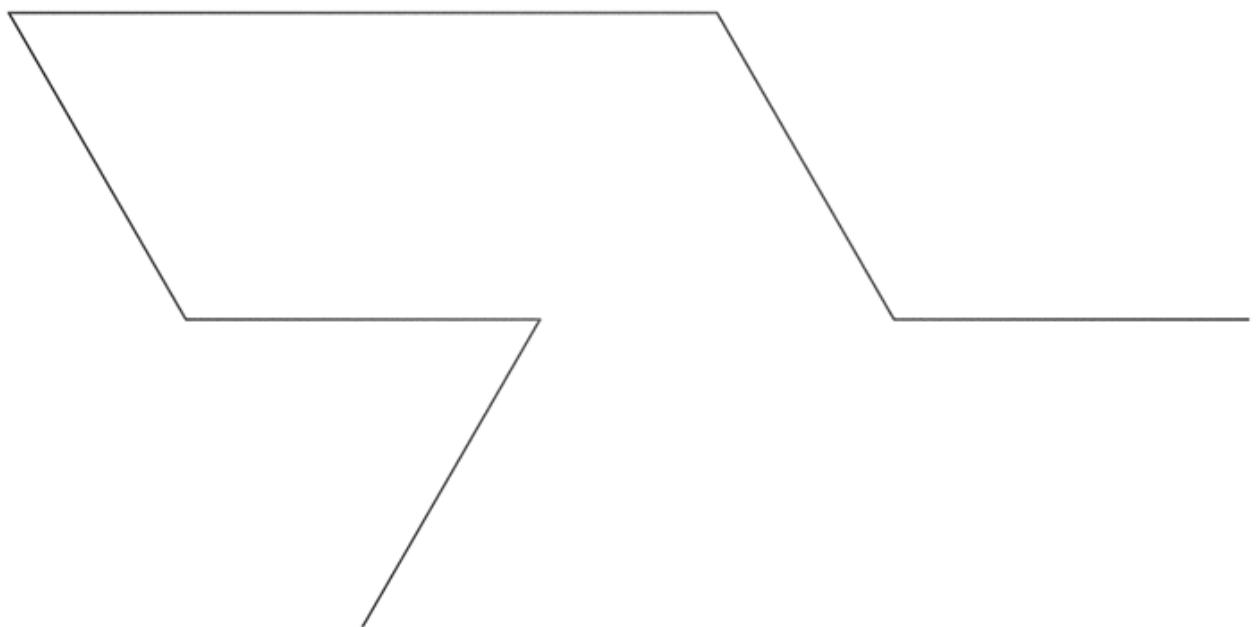
Attributed to Mandelbrot, 1982

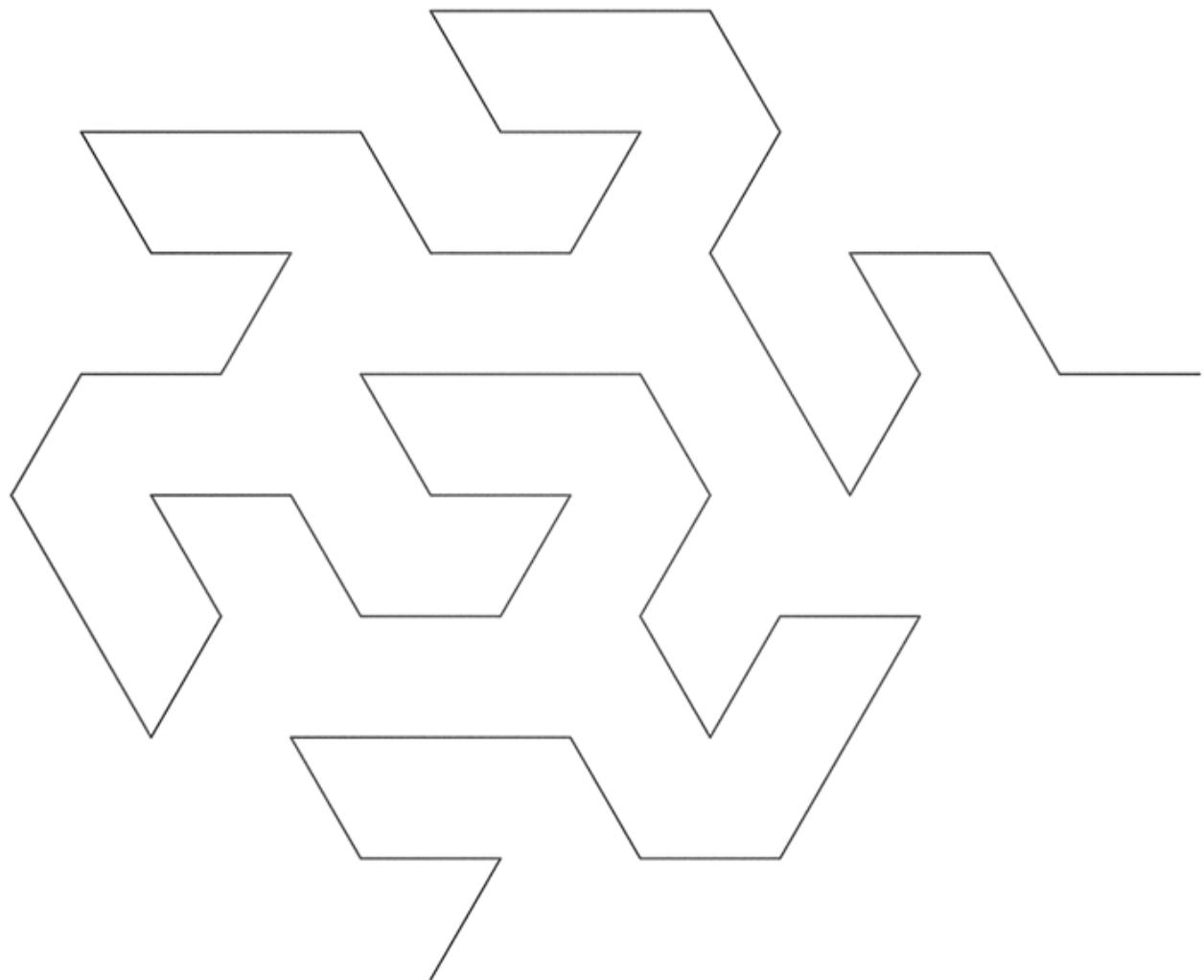
axiom = XF

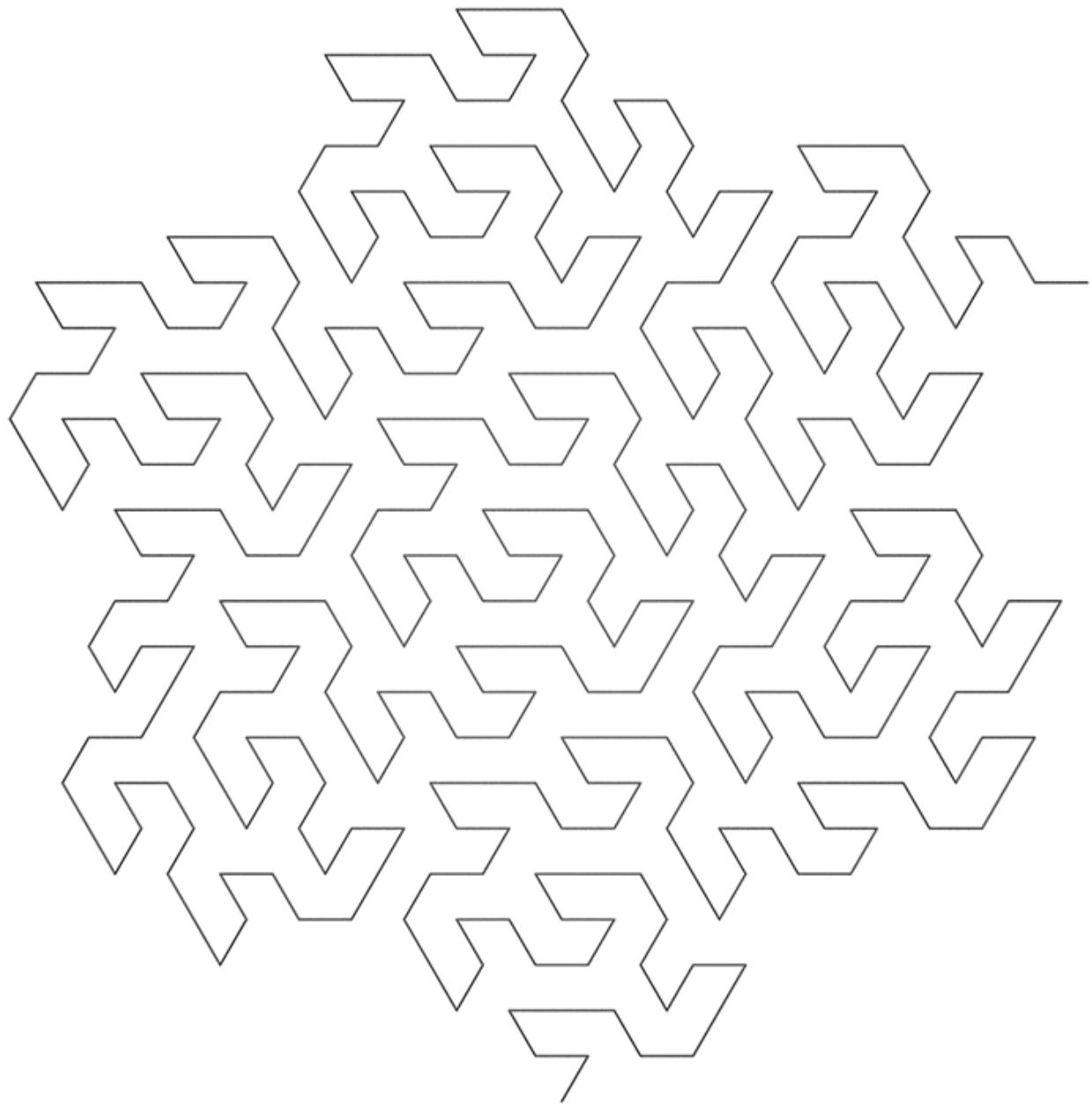
X -> X+YF++YF-FX--FXFX-YF+

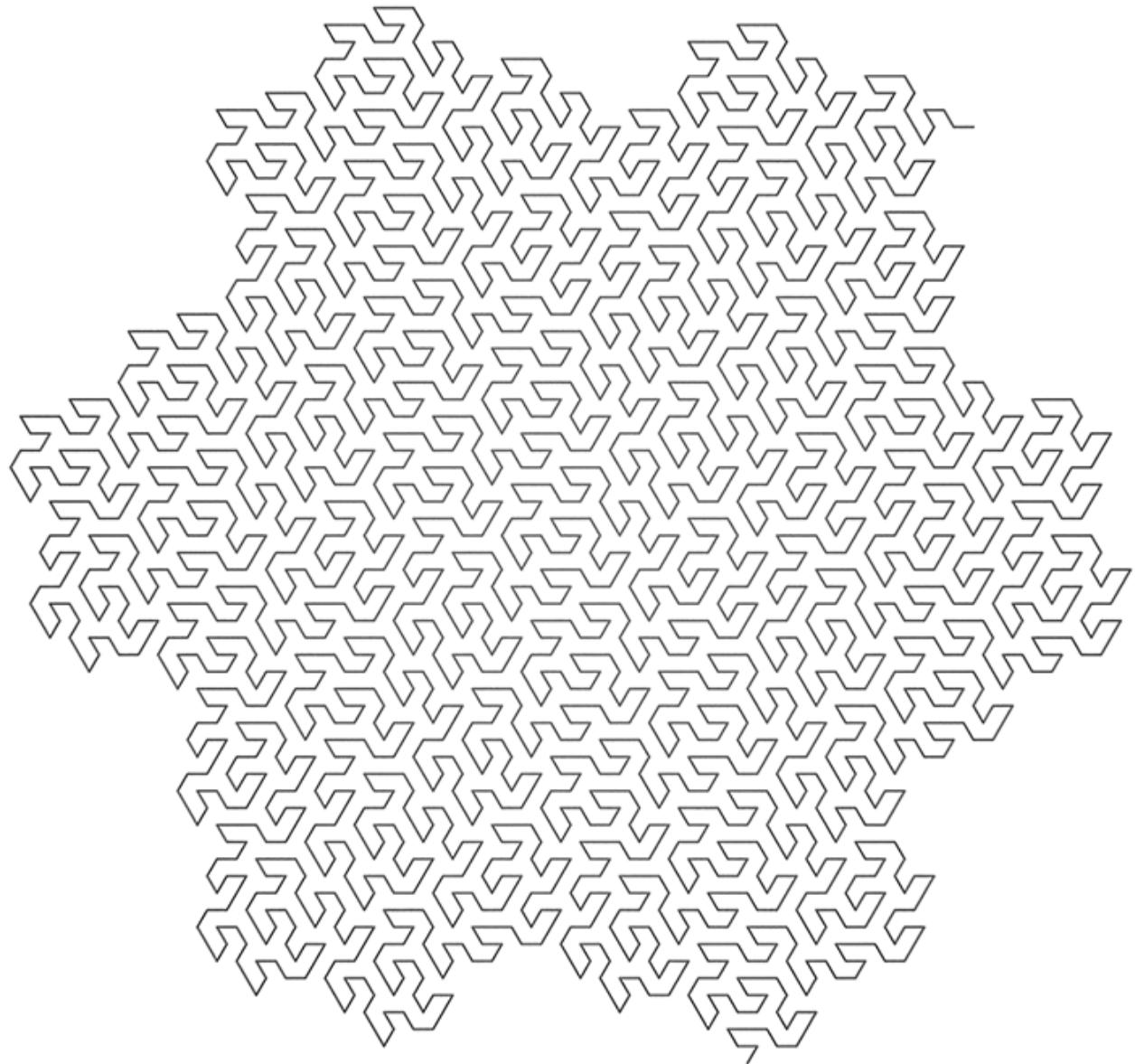
Y -> -FX+YFYF++YF+FX--FX-Y

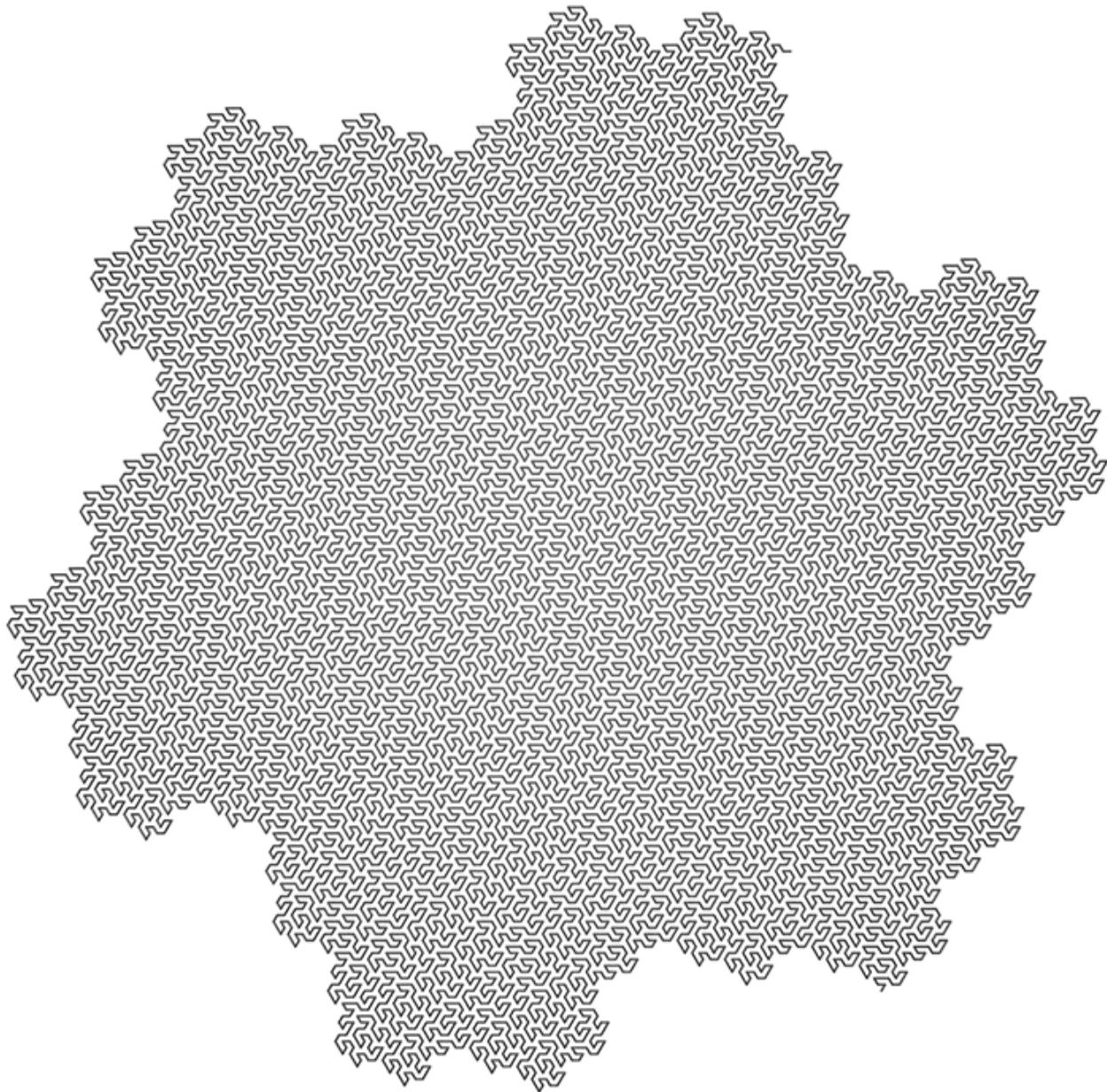
angle = 60











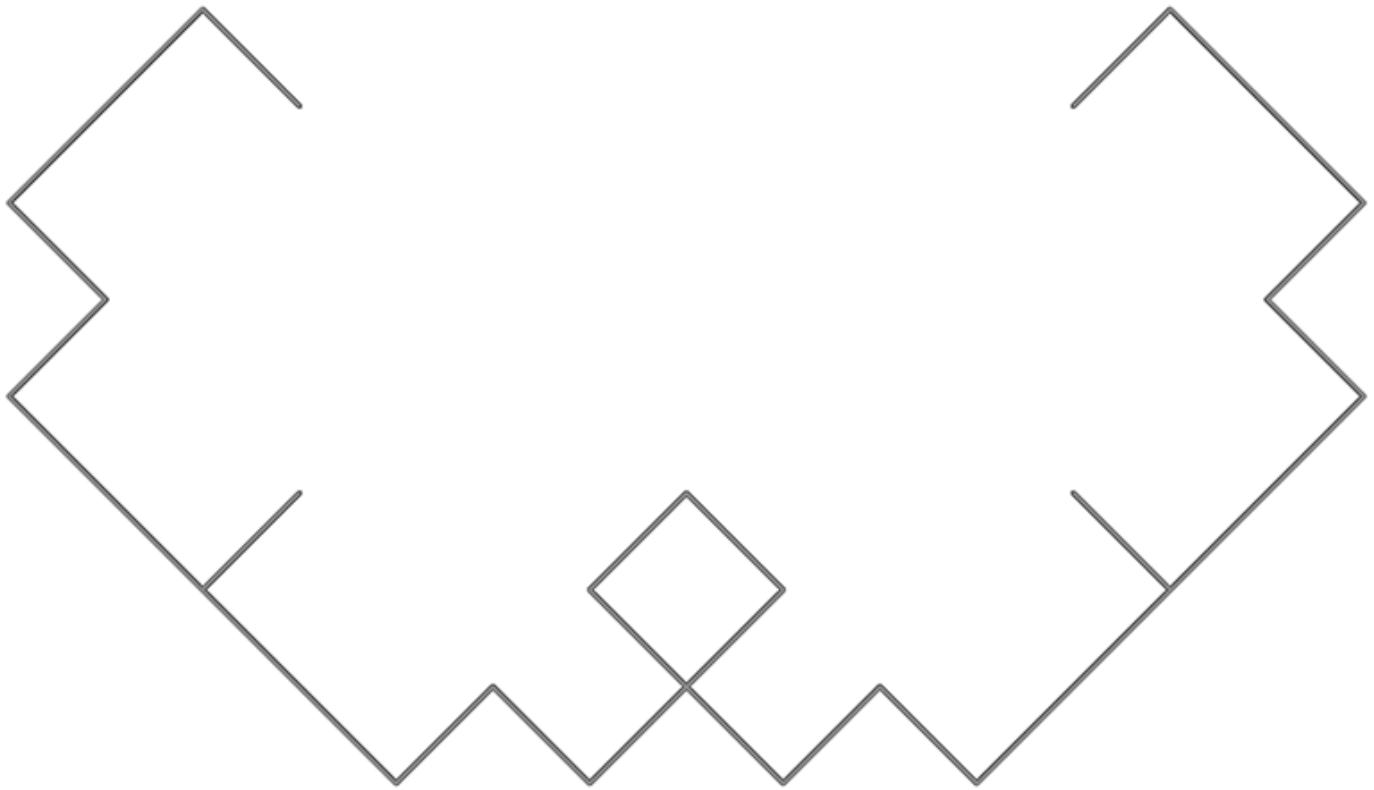
Lévy curve

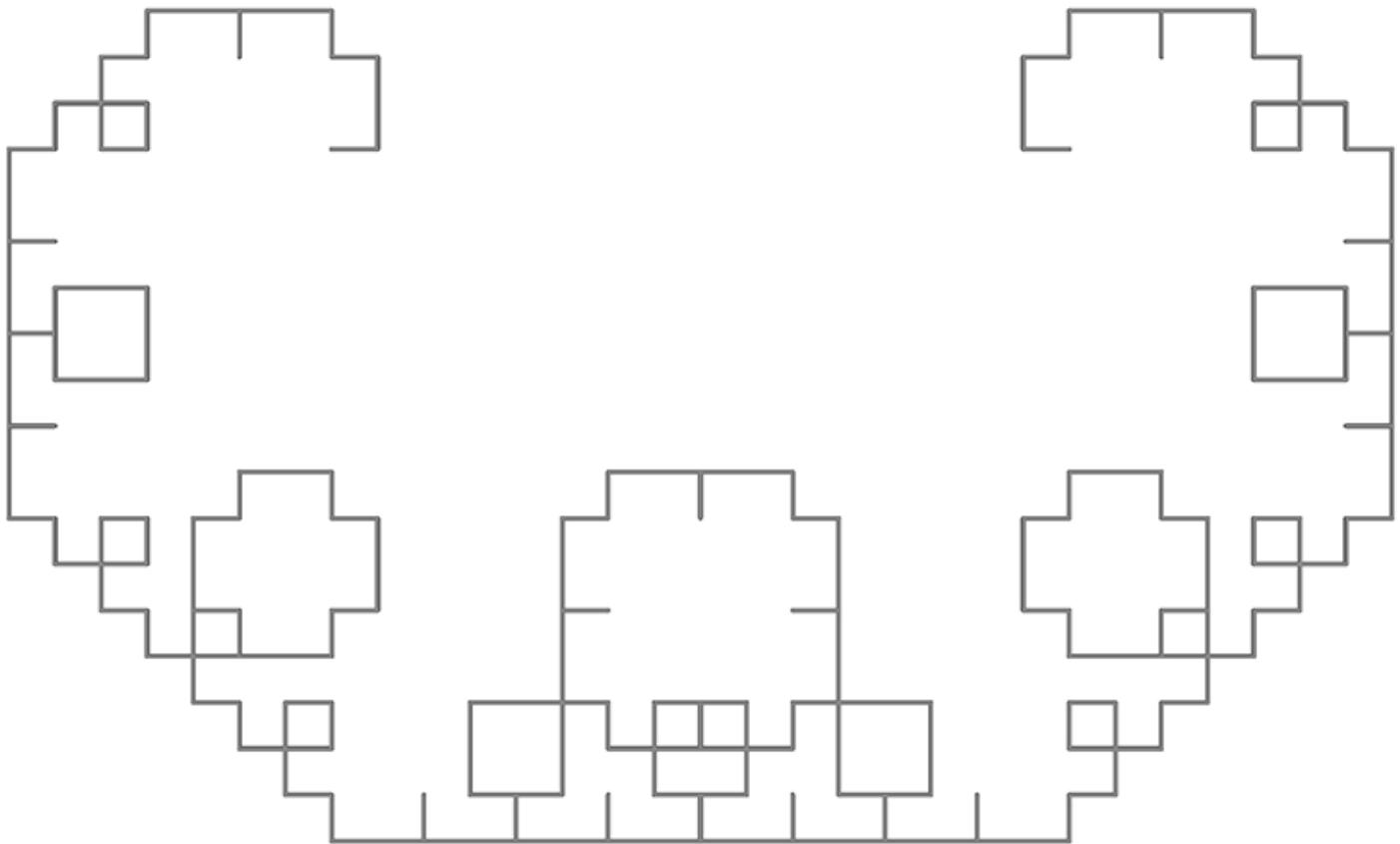
Written by [Paul Bourke](#)

February 2019

axiom = F
F -> -F++F-

angle = 45







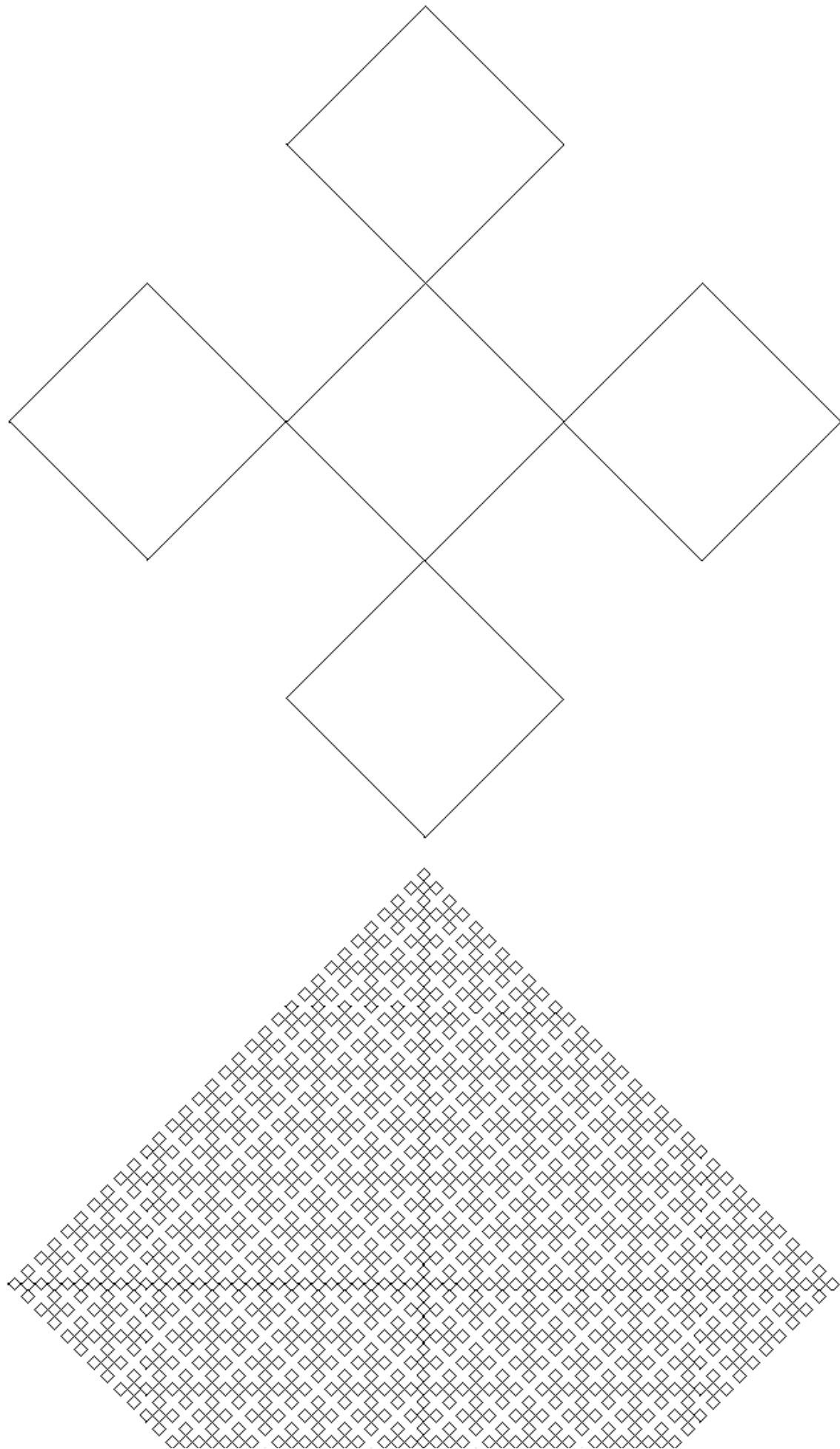
K r i s h n a A n k l e t s

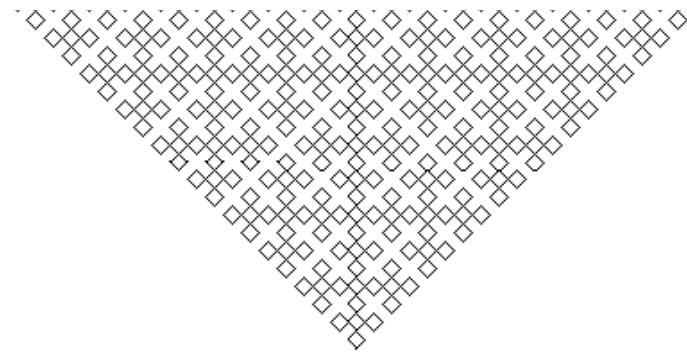
Written by [Paul Bourke](#)

axiom = -X--X

X -> XFX--XFX

angle = 45





M a n g o L e a f

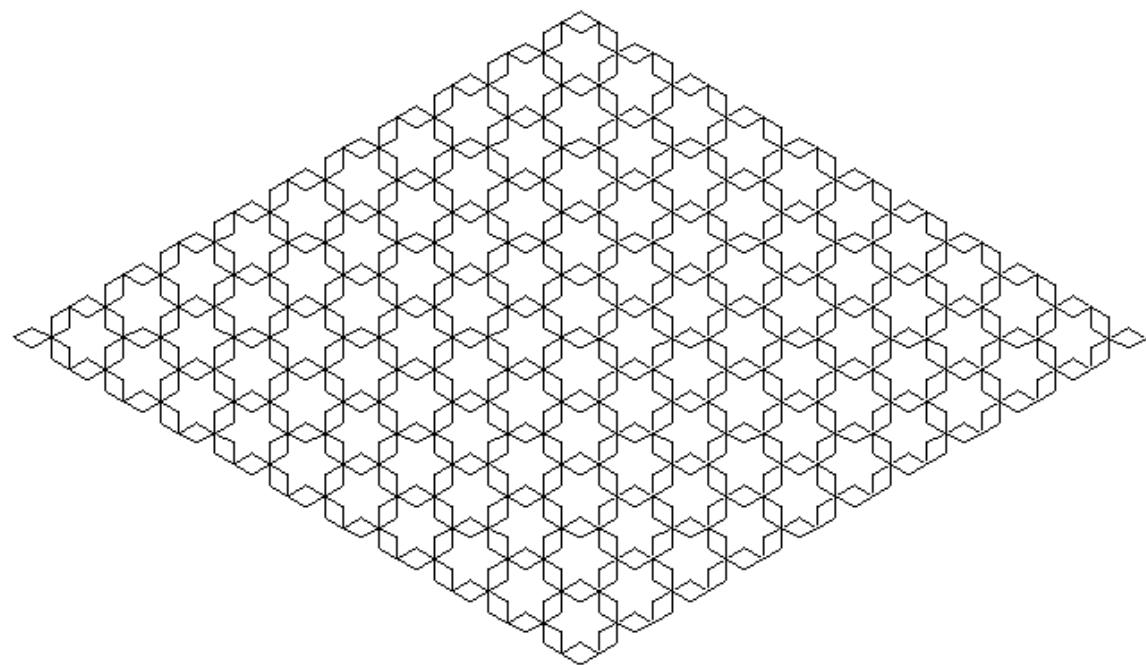
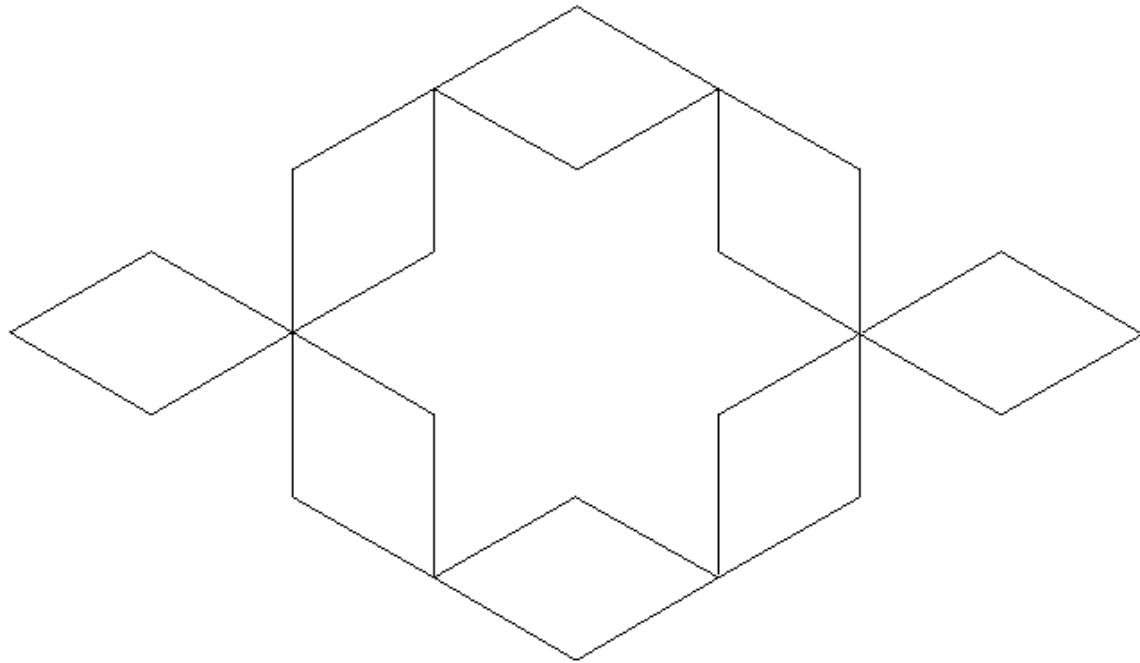
Written by [Paul Bourke](#)

axiom = Y---Y

X -> {F-F} {F-F}--[--X] {F-F} {F-F}--{F-F} {F-F}--

Y -> f-F+X+F-fY

angle = 60



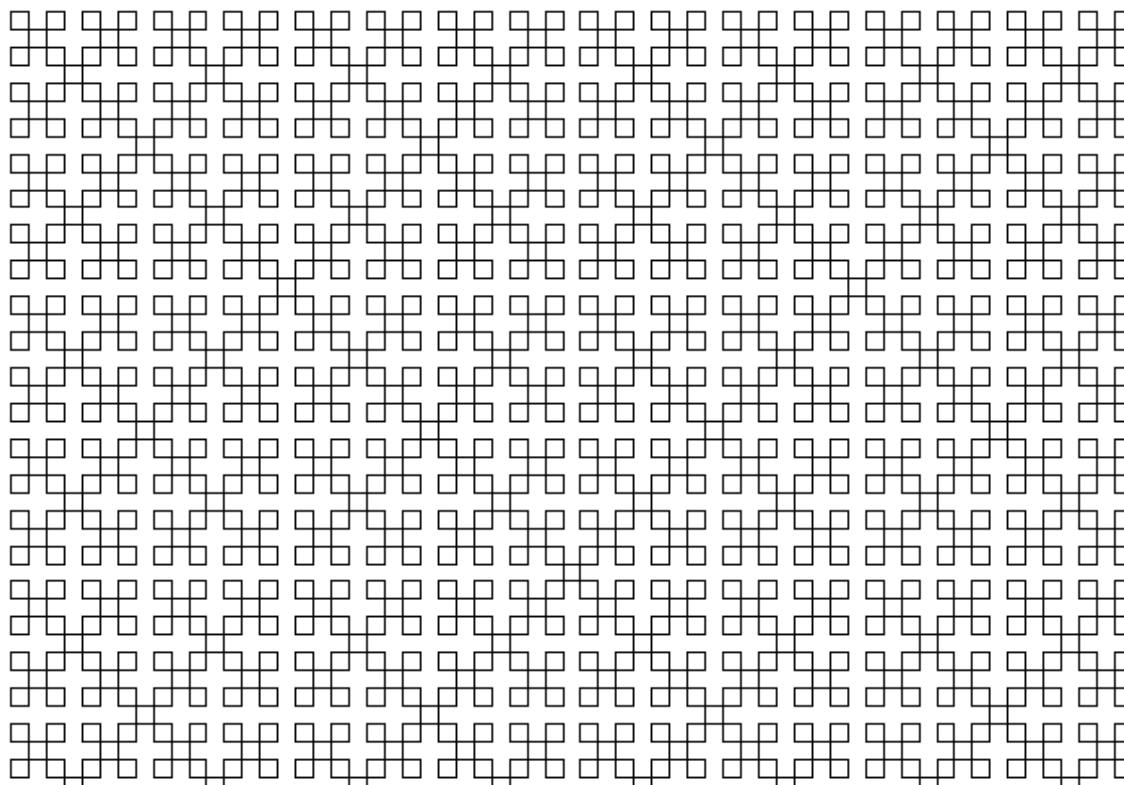
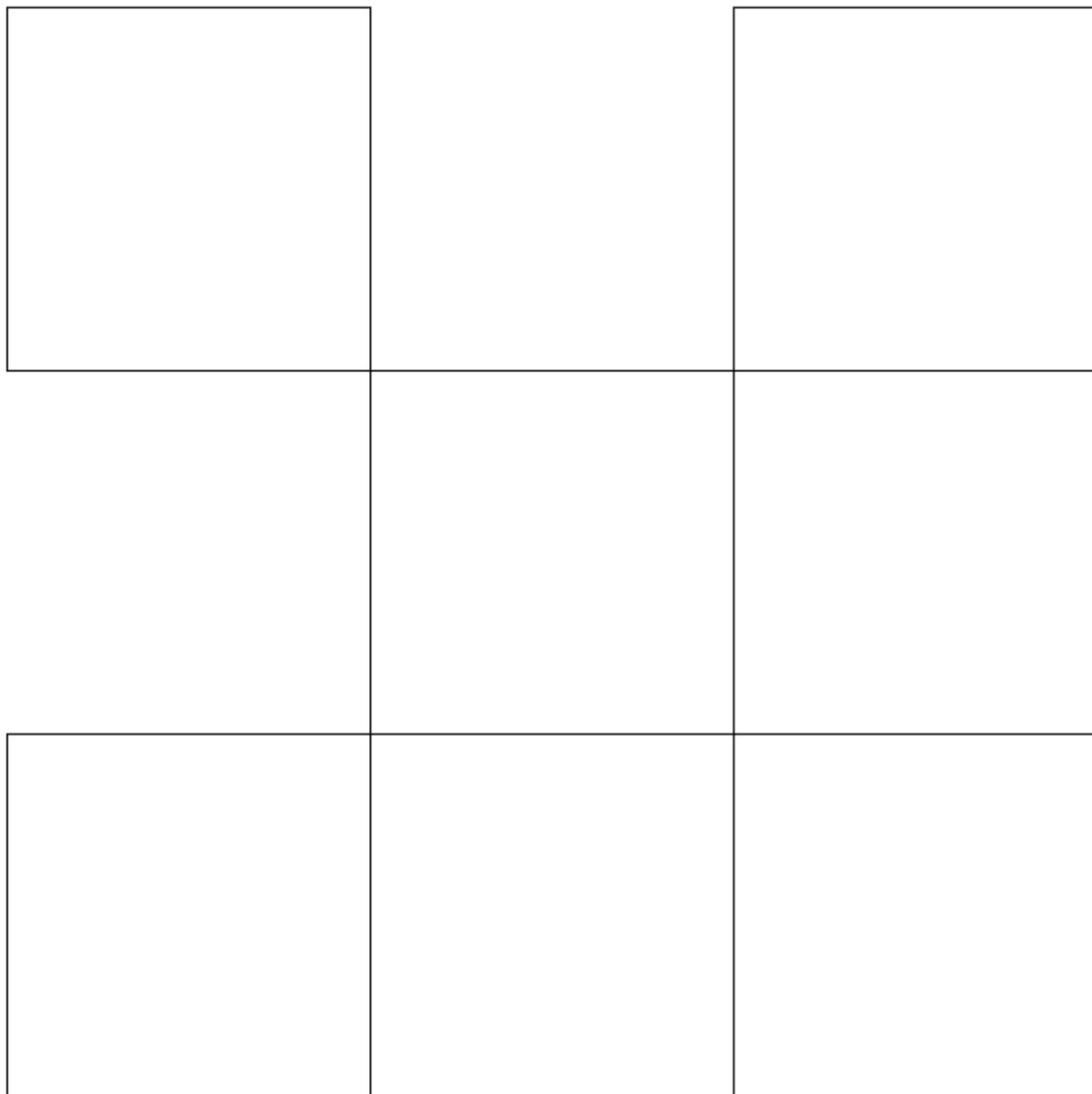
S n a k e K o l a m

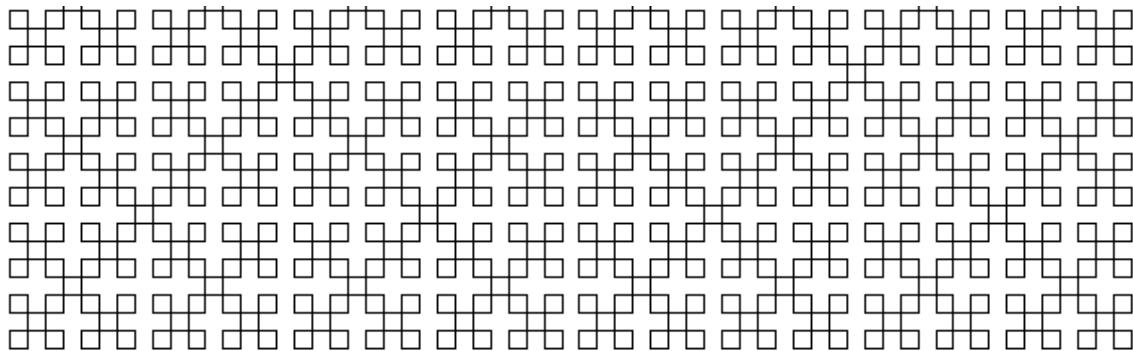
Written by [Paul Bourke](#)

axiom = F+XF+F+XF

X -> X{F-F-F}+XF+F+X{F-F-F}+X

angle = 90





K o l a m

January 2019

axiom = (-D--D)

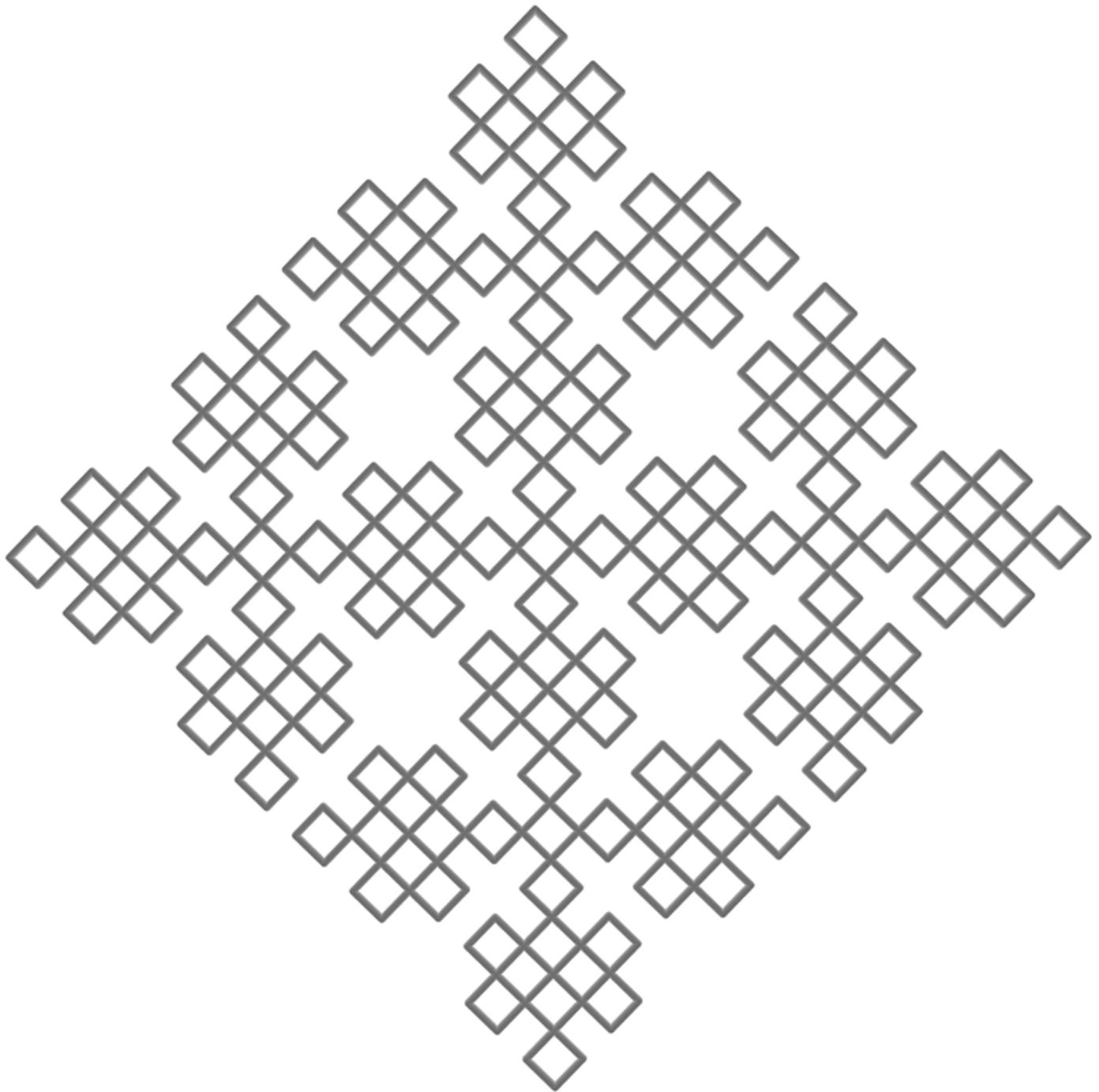
A -> F++FFFF--F--FFFF++F++FFFF--F

B -> F--FFFF++F++FFFF--F--FFFF++F

C -> BFA--BFA

D -> CFC--CFC

angle = 45



Kolams from India

Kolam patterns from the South of India, drawn by rice powder in front of houses every morning before the doorway is used.

