

1 Multiple Systems

The general idea is to view multiple systems together as a formation of a single, compound system. This concept leads to a description of how quantum states, measurements, and operations work for multiple systems.

1.1 Classical Information

1.1.1 Classical States via the Cartesian Product

For simplicity, let's first begin by introducing two systems then generalizing to more than two systems. Let X be a system whose classical state set is Σ and Y be a second system with classical state set Γ . Reasonably assume that Σ and Γ are two finite and nonempty sets that do not necessarily equal one another. Now imagine the two systems X and Y , placed side-by-side to one another, with X on the left and Y on the right. We can view the two systems as a single system denoted by (X, Y) or simply XY . The set of classical states of (X, Y) is the *Cartesian Product* of Σ and Γ , defined as:

$$\Sigma \times \Gamma = \{(a, b) : a \in \Sigma \text{ and } b \in \Gamma\}.$$

Essentially, the Cartesian Product is the mathematical notion that captures the idea of viewing an element of one set and an element of another set together to form a single element of a single (combined) set. To say (X, Y) is in the classical state $(a, b) \in \Sigma \times \Gamma$ means that X is in the classical state $a \in \Sigma$ and Y is in the classical state $b \in \Gamma$. The classical state of the joint system (X, Y) is (a, b) . For more than two systems we can generalize classical states of the joint systems