Functions and Options in APART Package

- `ApartAll[expr, {x,y,z,⋯}]`

  We change the old function `$Apart` to `ApartAll`, the syntax is the same as [1], *i.e.*, `expr` is a product of terms which are linear composition of `{x,y,z,⋯}`, `ApartAll` will perform the reduction or decomposition on `expr` to the irreducible forms, which are expressed with function `ApartIR`.

- `ApartIR[pcs,cs,np,vars]`

  `ApartIR` is the new version of `$ApartIR` function in the old version, but the syntax is totally different from each other. The arguments: `pcs, cs, np` are corresponding to the inner representation in the new version, the conversion to the normal format is defined as:

  $$\texttt{ApartIR[pcs,cs,np,vars]} = \texttt{Times@@(((\#1.vars\&)/@pcs+cs))\^{}np)}$$

  for example

  $$\texttt{ApartIR}[\{\{0,1\},\{1,0\}\},\{\texttt{b},\texttt{a}\},\{-1,-1\},\{\texttt{x},\texttt{y}\}] = \frac{1}{(\texttt{a}+\texttt{x})(\texttt{b}+\texttt{y})}$$

  This inner format is more efficient than the old one, and can be very useful for the conversion to other format, *e.g.*, FIRE or FIESTA format.

- `ApartComplete[expr]`

  `ApartComplete` function is provided to facilitate the input to FIRE package, in which we need to provide enough linear independent propagators, `ApartComplete` function will automatically add linear independent terms with 0 exponent, *e.g.*, there are two external momenta p_1 and p_2 at one loop level with k as the loop momentum, so generally, we need **3** linear independent propagators while using FIRE package, we can use APART to facilitate the input process:

  $$\texttt{ApartComplete}[\texttt{ApartAll}[\frac{1}{\texttt{k}^2}\frac{1}{(\texttt{k}+\texttt{p}_1)^2-\texttt{m}^2},\{\texttt{k}^2,\texttt{k}\cdot\texttt{p}_1,\texttt{k}\cdot\texttt{p}_2\}]]\to\frac{1}{\texttt{k}^2}\frac{1}{(\texttt{k}+\texttt{p}_1)^2-\texttt{m}^2}\frac{1}{(\texttt{k}\cdot\texttt{p}_2)^0}$$

  where the extra eikonal propagator $k\cdot p_2$ is added.

- `ApartVars` Options: `SignVars` and `VarsSign`

  Usually, the propagator $1/(k^2-m^2)$ may be transformed to $1/(m^2-k^2)$ in the MATHEMATICA, *i.e.*, the sign of k^2 is changed, generally the sign of k^2 can not be guaranteed during the MATHEMATICA evaluation, if we want to keep the sign of some variables always positive or negative, we need the `ApartVars` options: `SignVars` and `VarsSign`, for example:

  $$\texttt{expr} = \frac{1}{\texttt{k}^2-\texttt{m}^2}\frac{1}{(\texttt{k}+\texttt{p})^2}\frac{1}{(\texttt{k}-\texttt{p})^2}$$
  $$\texttt{ApartAll}[\texttt{expr},\{\texttt{k}^2,\texttt{k}\cdot\texttt{p}\}]$$

  will result in

  $$-\frac{\left\|\frac{1}{(m^2-k^2)(k^2+2k\cdot p+p^2)}\right\|}{2\left(m^2+p^2\right)}-\frac{\left\|\frac{1}{(k^2-2k\cdot p+p^2)(k^2+2k\cdot p+p^2)}\right\|}{m^2+p^2}-\frac{\left\|\frac{1}{(m^2-k^2)(k^2-2k\cdot p+p^2)}\right\|}{2\left(m^2+p^2\right)}$$

  where we can see the sign of k^2 can be negative or positive, to make the sign of k^2 always positive, we just add the following `SetOptions` before calling `ApartAll`

  $$\texttt{SetOptions}[\texttt{ApartVars},\texttt{SignVars}\to\{\texttt{k}^2\},\texttt{VarsSing}\to 1]$$

  the output looks like

  $$\frac{\left\|\frac{1}{(k^2-m^2)(k^2+2k\cdot p+p^2)}\right\|}{2\left(m^2+p^2\right)}+\frac{\left\|\frac{1}{(k^2-m^2)(k^2-2k\cdot p+p^2)}\right\|}{2\left(m^2+p^2\right)}-\frac{\left\|\frac{1}{(k^2-2k\cdot p+p^2)(k^2+2k\cdot p+p^2)}\right\|}{m^2+p^2}$$

while to keep the sign of k^2 always negative, one needs the following options:

$$\texttt{SetOptions}[\texttt{ApartVars}, \texttt{SignVars} \to \{\texttt{k}^2\}, \texttt{VarsSing} \to -1]$$

and the corresponding output is

$$\frac{\left\|\frac{1}{(m^2-k^2)(-k^2+2k\cdot p-p^2)}\right\|}{2\left(m^2+p^2\right)} + \frac{\left\|\frac{1}{(m^2-k^2)(-k^2-2k\cdot p-p^2)}\right\|}{2\left(m^2+p^2\right)} - \frac{\left\|\frac{1}{(-k^2-2k\cdot p-p^2)(-k^2+2k\cdot p-p^2)}\right\|}{m^2+p^2}$$

[1] F. Feng, $Apart: A Generalized Mathematica Apart Function, Comput. Phys. Commun. **183**, 2158 (2012) [arXiv:1204.2314 [hep-ph]].