



# APART 2: A generalized MATHEMATICA Apart function



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## ARTICLE INFO

### Article history:

Received 4 August 2015

Received in revised form

29 August 2015

Accepted 3 September 2015

Available online 28 September 2015

### Keywords:

Apart

Partial fraction

## ABSTRACT

The MATHEMATICA function `Apart` has been generalized to any dimension in APART package [1], we upgrade this package to the 2nd version, in which the core part has been completely rewritten. The new version is more efficient than the 1st version of APART, and the output is now more compact and more suitable for the input for the FIRE [2] or FIESTA [3].

### New version program summary

*Program title:* APART 2.0

*Catalogue identifier:* AEMK\_v2\_0

*Program summary URL:* [http://cpc.cs.qub.ac.uk/summaries/AEMK\\_v\\_0.html](http://cpc.cs.qub.ac.uk/summaries/AEMK_v_0.html)

*Program obtainable from:* CPC Program Library, Queen's University, Belfast, N. Ireland

*Licensing provisions:* Standard CPC licence, <http://cpc.cs.qub.ac.uk/licence/licence.html>

*No. of lines in distributed program, including test data, etc.:* 96744

*No. of bytes in distributed program, including test data, etc.:* 884975

*Distribution format:* tar.gz

*Programming language:* Mathematica.

*Computer:* Any computer with Mathematica installed.

*Operating system:* Any capable of running Mathematica.

*Classification:* 11.1.

*Catalogue identifier of previous version:* AEMK\_v1\_0

*Journal reference of previous version:* Comput. Phys. Comm. 183(2012)2158

*Does the new version supersede the previous version?:* Yes

*Nature of problem:* As discussed in [1], the general procedure to compute a cross section for a physical process in perturbative quantum field theory involves generating the corresponding amplitude via Feynman diagram and performing the loop integrals in dimensional regularization [7]. The essential part in the computation is to reduce these loop integrals to a small number of standard integrals, which are called master integrals (MI), via the systematic methods of integration by parts (IBP) identities [8, 9] and Lorentz invariance (LI) identities [10]. The basic reduction algorithm is introduced by Laporta [11], which defines an ordering for Feynman integrals, generates IBP identities and solves the corresponding linear equations. Alternative methods to exploit IBP and LI identities for reductions can be found in [12–17]. There are many public computer programs for implementations of different reduction algorithms: AIR [18], FIRE [19] and Reduze [20]. To facilitate the input for Fire [19], Reduze [20], etc. we need to decompose the linear independent propagators to independent ones, this procedure can be done by the APART package [1] which generalizes the Mathematica function `Apart` from one dimension to any  $N$  dimensions.

*Solution method:* We have proven that all linear independent propagators can be decomposed into the summation of linear independent ones in [1], APART is such a Mathematica package that implements such a reduction method and generalizes the Mathematica `Apart` function from 1 to any  $N$  dimensions.

*Reasons for new version:* The Mathematica pattern matching in the last version may become very slow when the number of variables becomes large, this calls for a revised version with a more efficient

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reduction. The feature with all positive or negative sign of some variables is favored in combined usage of FIRE [2] and FIESTA [3].

*Summary of revisions:* We introduce an abstract and compact representation for the linear composition of the independent variables, this results in a more efficient and fast reduction during the APart partial fraction, we also introduce an extra feature to make the sign of some variables always positive or negative during the reduction.

*Running time:* A few seconds or less.

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## Acknowledgments

The author wants to thanks Hai-Rong Dong, Wen-Long Sang and Prof. Yu Jia for many useful discussions. The research is

partially supported by the Fundamental Research Funds for the Central Universities and the National Natural Science Foundation of China under Grants No. 11505285. Finally, The author would like to commemorate his beloved father and mother.