TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

LRS

# MPC based Path Tracking Using Potential Field for Autonomous Mobile Robot

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# Motivation

- Navigation is essential for the success of any Mobile Robot
- Ability to avoid obstacles is paramount for any AMR
- Navigation entails path planning, trajectory generation, and successful tracking of planned trajectory
- Planned trajectories should be modifiable online to accommodate unknown and dynamic obstacles
- An Artificial Potential Field (APF) based approach is a gentle and intuitive introduction to path planning and trajectory generation
- Model Predictive Control is drawing a lot of research attention

# Aims & Objectives

## Aims

- Investigate Artificial Potential Field based path planning in presence of known static obstacles
- Explore how Model Predictive Control can be used for path tracking for AMRs

## Objectives

- Generate collision-free trajectory from initial point to goal point using APF-based approach
- Design a Model Predictive Controller based on a SISO linearized model
- Design a Model Predictive Controller based on a SIMO linearized model
- Tune a PID controller based on the simulation model
- Compare the performance of the 3 controllers in different simulation scenarios

# Assumptions

The following assumptions were made about the AMR in this study:

- 3 degree-of-freedom bicycle model- 1 front wheel and 2 rear wheels
- Constant forward velocity of 1m/s
- Front wheel controlled
- Motion in x-y plane only (ignore rolling and pitching motions)
- Constant model parameters
- Indoor environment with smooth and flat surface (ignore gravitational and aerodynamic side forces)
- Configuration space of 50m x 50m
- Rounded obstacles of known sizes and positions

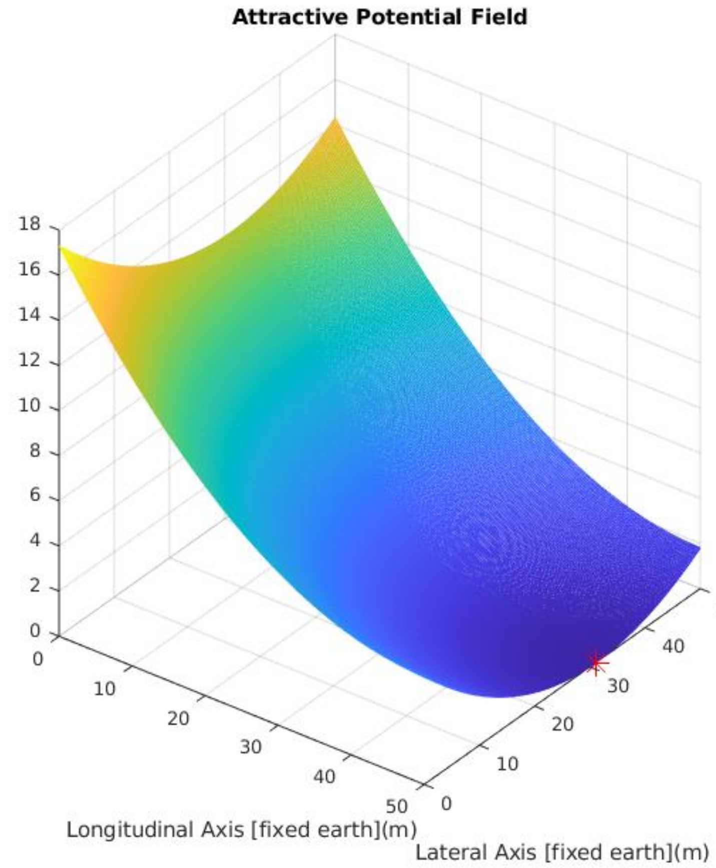
# Artificial Potential Fields

- Attractive Potential

$$U_A = \frac{1}{2} K_{att} \left\| \vec{X} - \vec{X}_{goal} \right\|^2$$

- Attractive Force

$$F_A = -K_{att}(\vec{X} - \vec{X}_{goal})$$





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# Artificial Potential Fields

- Repulsive Potential

$$U_R = \frac{1}{2} K_{rep} \left( \frac{1}{\|\vec{X} - \vec{X}_{obs}\|} - \frac{1}{\rho_0} \right)^2 \qquad ; \|\vec{X} - \vec{X}_{obs}\| <= \rho_0$$

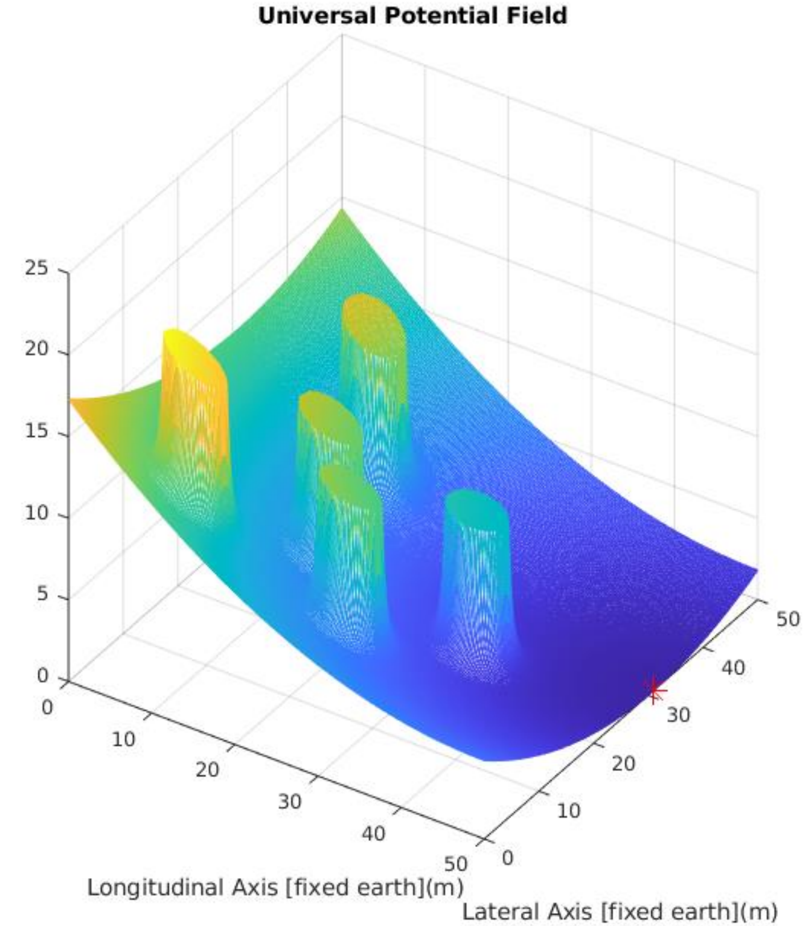
$$U_R = 0 \qquad ; \text{otherwise}$$

- Repulsive Force

$$F_R = K_{rep} \left( \frac{1}{\|\vec{X} - \vec{X}_{obs}\|} - \frac{1}{\rho_0} \right) \frac{1}{\|\vec{X} - \vec{X}_{obs}\|^3} (\vec{X} - \vec{X}_{obs})$$

- Obstacle influence region

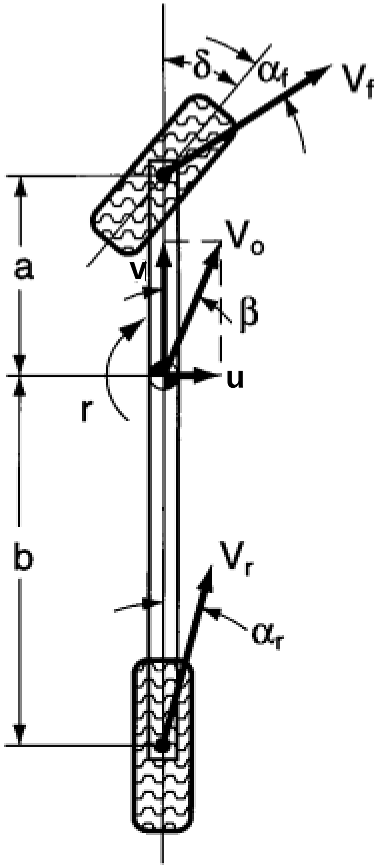
$$\rho_0 = \frac{V_{MAX}}{2 * A_{MAX}}$$



# AMR Model for MPC Design

**2 DOF bicycle model**

- $\alpha_f$- front tyre slip angle
- $\alpha_r$- rear tyre slip angle
- $\delta$- front wheel angle
- $\beta$- side-slip angle
- r- yaw velocity
- a- distance of front wheel from center of mass (CoM)
- b- distance of rear wheel from CoM
- $V_0$- velocity of CoM
- $V_f$- velocity of front tyre
- $V_r$- velocity of rear tyres



# AMR Model for MPC Design

Equations of motion:

$$F_{xf}cos(\delta) + F_{xr} = m(\dot{u} - \dot{\psi}v)$$

$$aF_{xf}cos(\delta) - bF_{xr} = I_{zz}\dot{r}$$

Lateral tyre forces:

$$F_{xf} = C_f\gamma_f$$

$$F_{xr} = 2C_r\gamma_r$$

Tyre angles:

$$\gamma_f = atan\left(\frac{u + a\dot{\psi}}{v}\right) - \delta$$

$$\gamma_r = atan\left(\frac{u - b\dot{\psi}}{v}\right)$$

With small angles approximations,

$$\beta = \frac{u}{v} \qquad \gamma_f = \frac{u + a\dot{\psi}}{v} - \delta = \beta + \frac{a\dot{\psi}}{v} + \delta \qquad \gamma_r = \frac{u - b\dot{\psi}}{v} = \beta - \frac{b\dot{\psi}}{v}$$

The equations of motion then yeild the following differential equations,

$$\dot{\beta} = -\frac{\beta}{mv}\left(C_f + 2C_f\right) + r\left[\frac{1}{mv^2}\left(2bC_r - aC_f\right)\right] + \delta\frac{C_f}{mv}$$

$$\dot{r} = -\frac{\beta}{I_{zz}}\left(aC_f - 2bC_r\right) - r\left[\frac{1}{I_{zz}v}\left(a^2C_f + 2b^2C_r\right)\right] + \delta\frac{aC_f}{I_{zz}}$$

# AMR Model for MPC Design

With the input as the front wheel angle (δ), selecting state variables as:

- lateral displacement of CoM, $x_c$
- side-slip angle, β
- yaw angle, ψ
- yaw velocity, r

$$\begin{bmatrix} \dot{x}_c \\ \dot{\beta} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & v & v & 0 \\ 0 & -\frac{C_f+2C_r}{mv} & 0 & \frac{2bC_r-aC_f}{mv^2}-1 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2bC_r-aC_f}{I_{zz}} & 0 & -\frac{\left(2b^2C_r+a^2C_f\right)}{I_{zz}v} \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_f}{mv} \\ 0 \\ \frac{aC_f}{I_{zz}} \end{bmatrix} \delta$$

SISO MPC, selecting the output as:

- lateral displacement of CoM, $x_c$

$$x_c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix}$$

SIMO MPC, selecting the outputs as:

- lateral displacement of CoM, $x_c$
- yaw angle, $\psi$

$$\begin{bmatrix} x_c \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix}$$

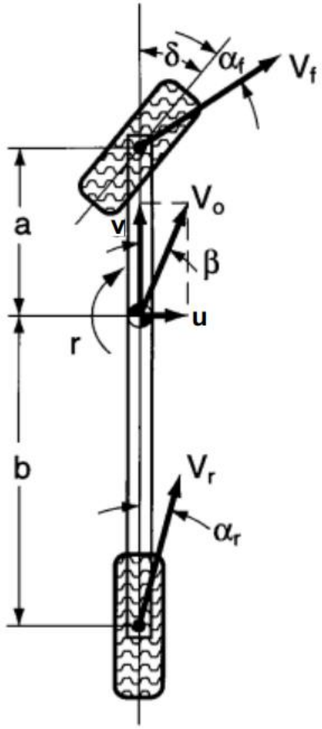
# Simulation

- Simulations were carried out on MATLAB and Simulink
- Vehicle Body 3DOF Single Track model from Simulink's Vehicle Dynamics Blockset was used for simulations
- Perfect tracking of the AMR's longitudinal position is assumed since this study focuses on lateral displacement control
- Controllers:
  - PID controller- pidTuner (MATLAB)
  - SISO MPC
  - SIMO MPC
- Performance of the three controllers were compared on a basis of a scaled error norm

# Simulation

## AMR Parameters

- Mass (kg): 505
- Yaw mass moment of inertia (kg.m^2): 808.5
- Distance of front wheel from CoM, a(m): 0.35
- Distance of rear wheels from CoM, b(m): 0.4125




## Path Planning Parameters

- $K_{att}$: 0.01
- $K_{rep}$: 10
- AMR size allowance (m): 0.35
- Obstacle radius (m): 1
- AMR initial position: (0, 0)
- Goal position: (50, 31)
- Obstacle locations: (14.87, 33.28); (10, 8); (26, 12); (19, 19); and (34, 23)

# Simulation

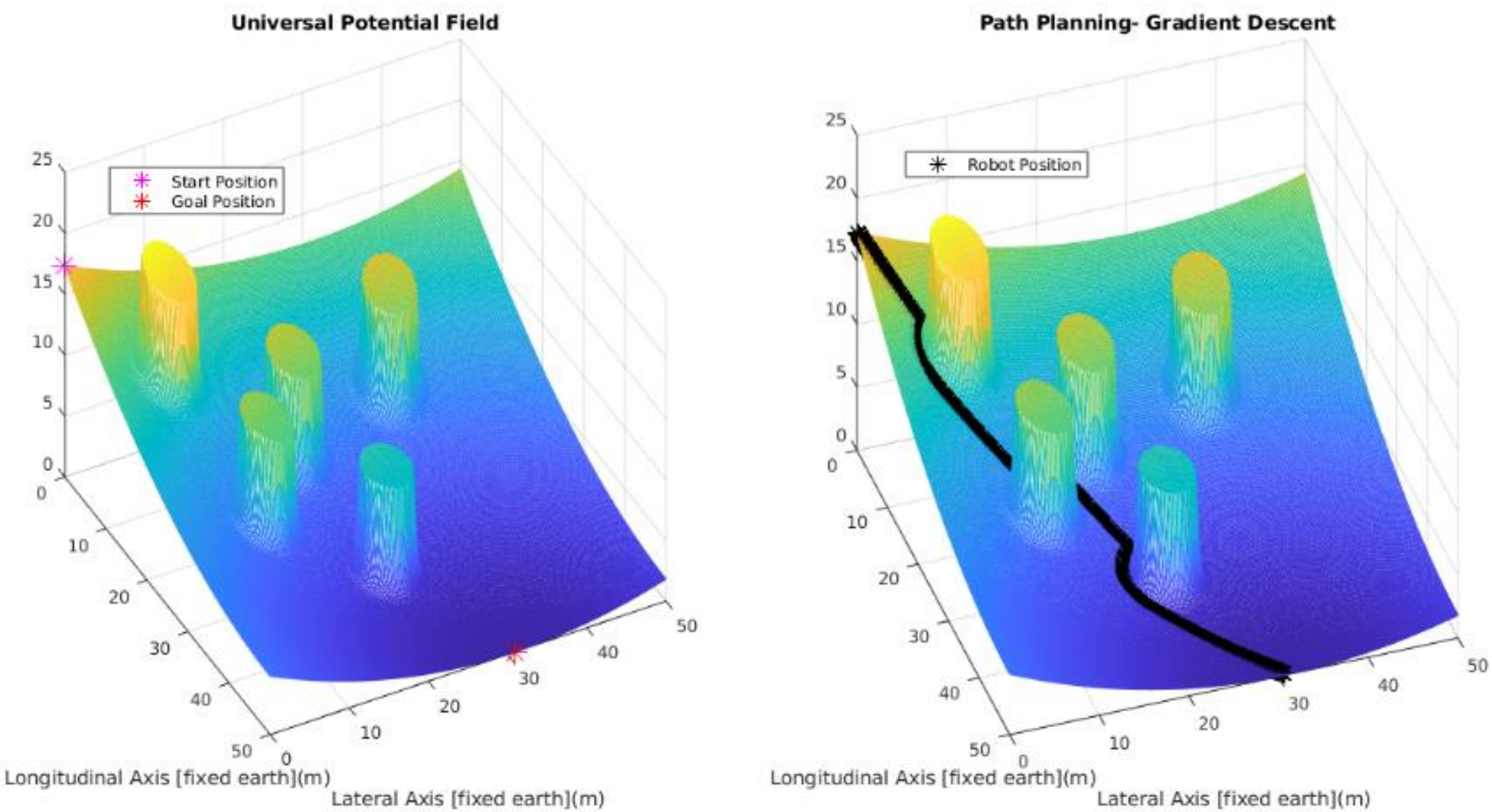
**PID Controller**

- Simulation sampling time (seconds): 0.01

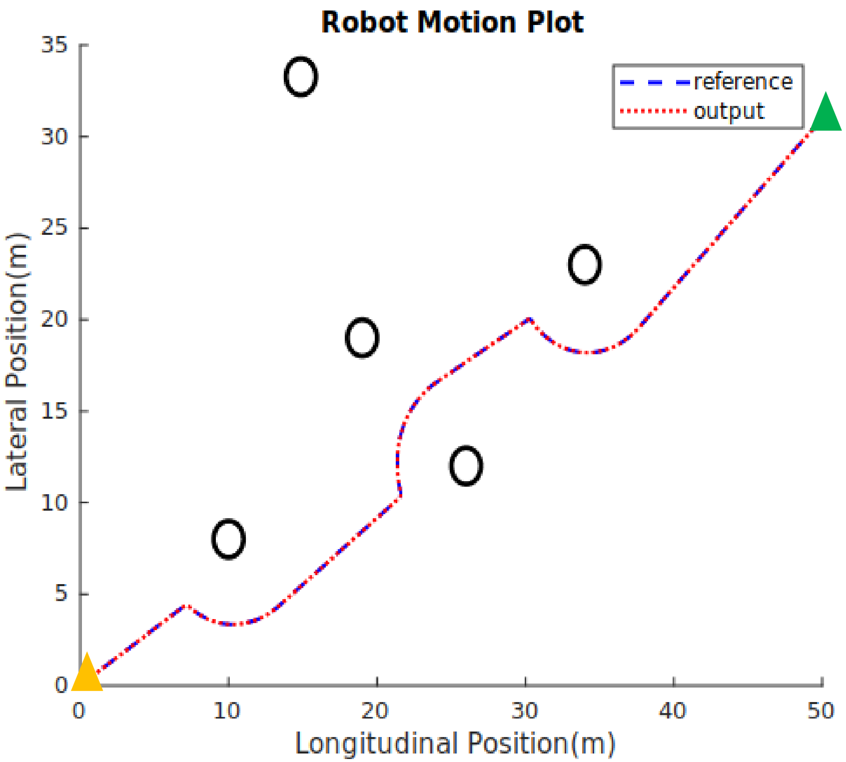
**MPC Controllers**

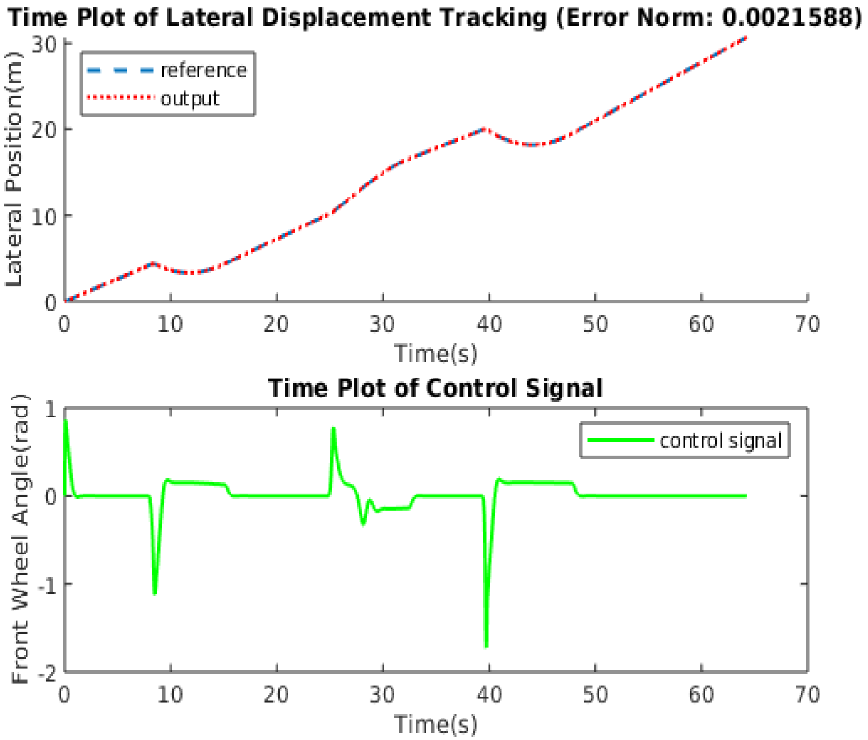
- Simulation sampling time (seconds): 0.05
- Prediction horizon (time-steps): 25
- Control horizon (time-steps): 4
- Maximum front wheel angle: ±40 degrees
- Maximum front wheel angular velocity: ±30 degrees/second

# Scenario 1

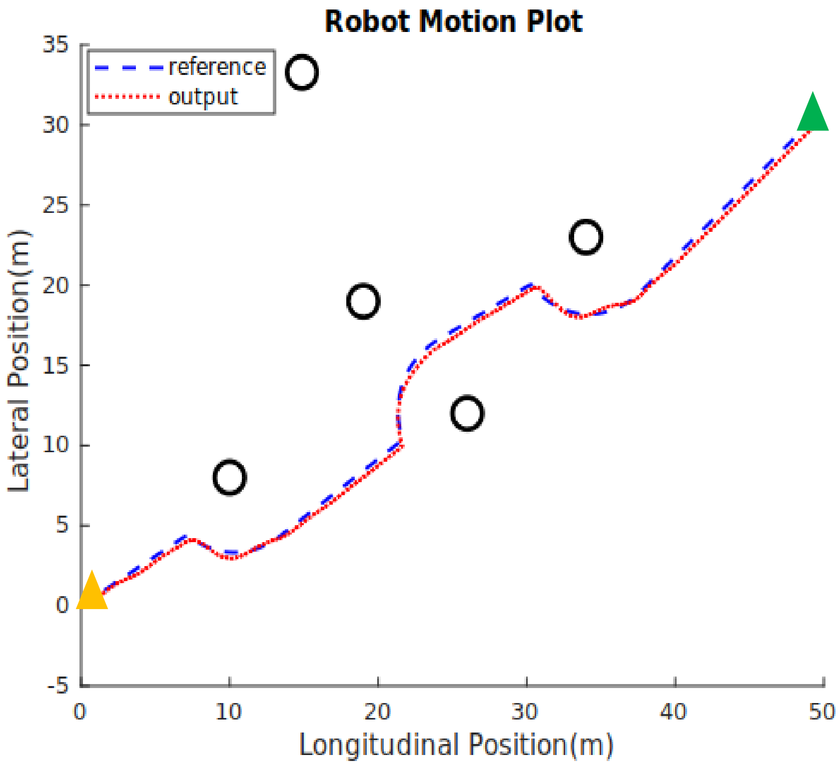

Universal Potential Field
Start Position
Goal Position
Longitudinal Axis [fixed earth](m)
Lateral Axis [fixed earth](m)
Path Planning- Gradient Descent
Robot Position
Longitudinal Axis [fixed earth](m)
Lateral Axis [fixed earth](m)

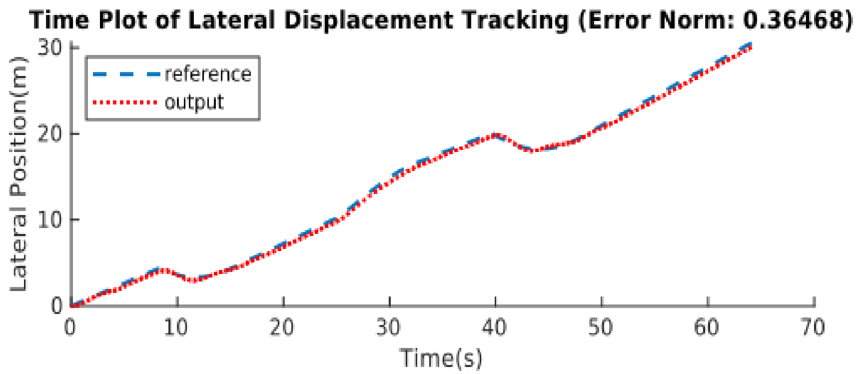
# Scenario 1 PID

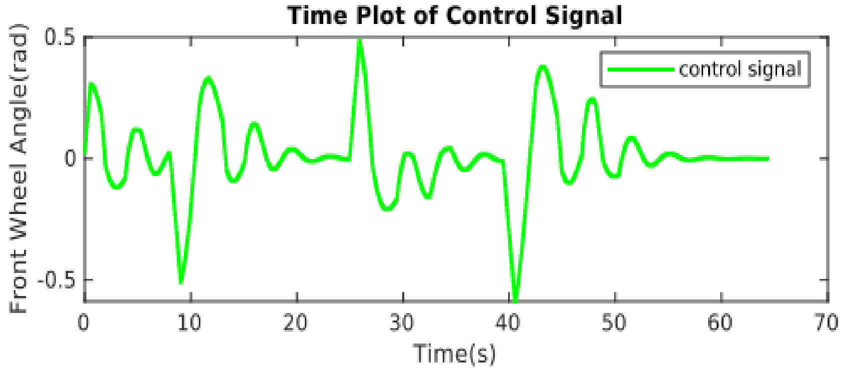



# Scenario 1 SIMO MPC



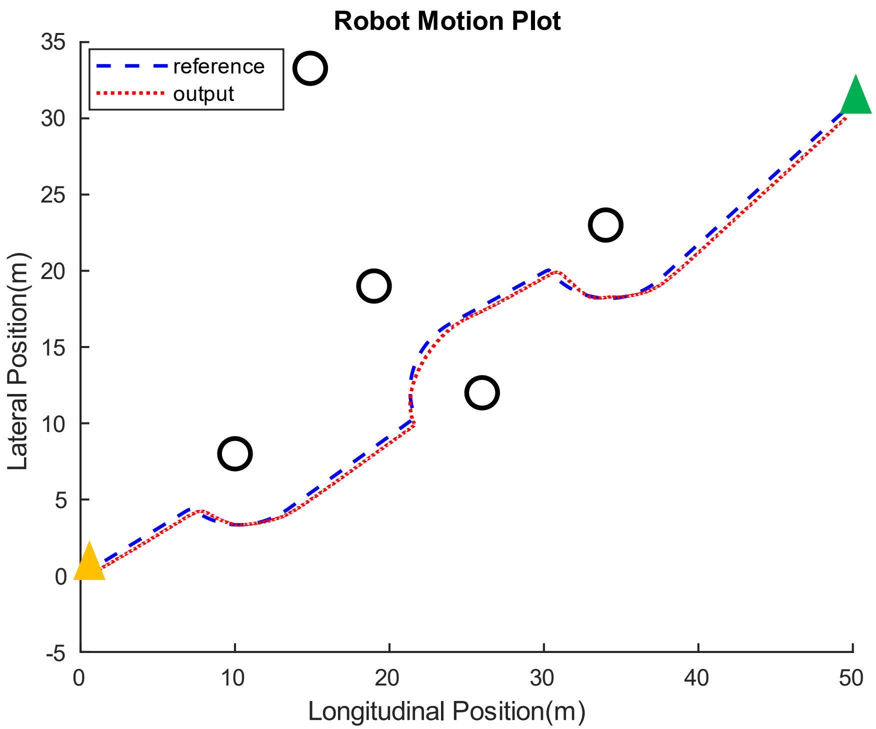



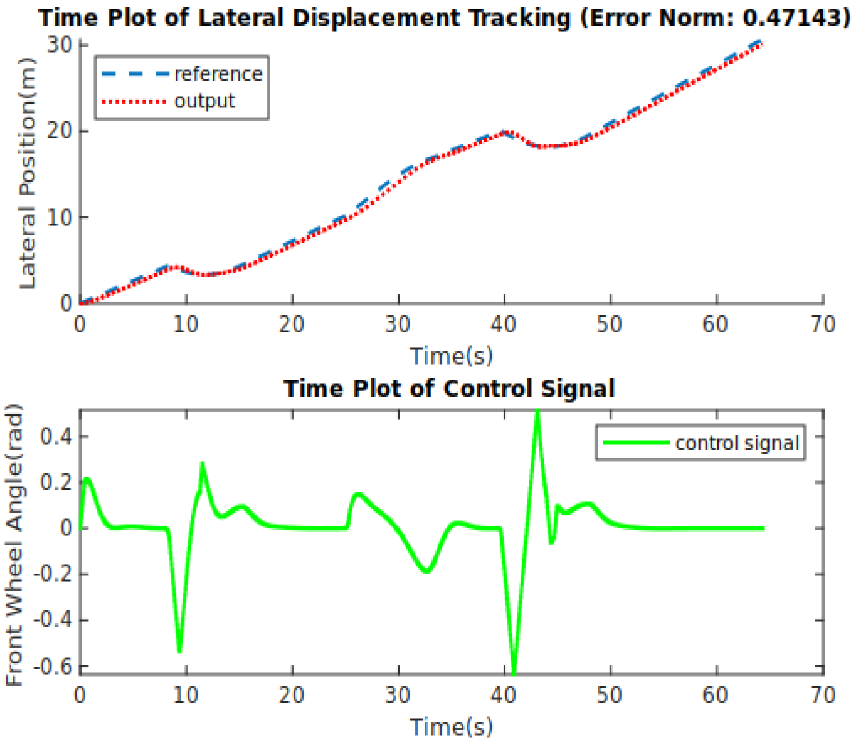




Start point

End point

# Scenario 1 SISO MPC

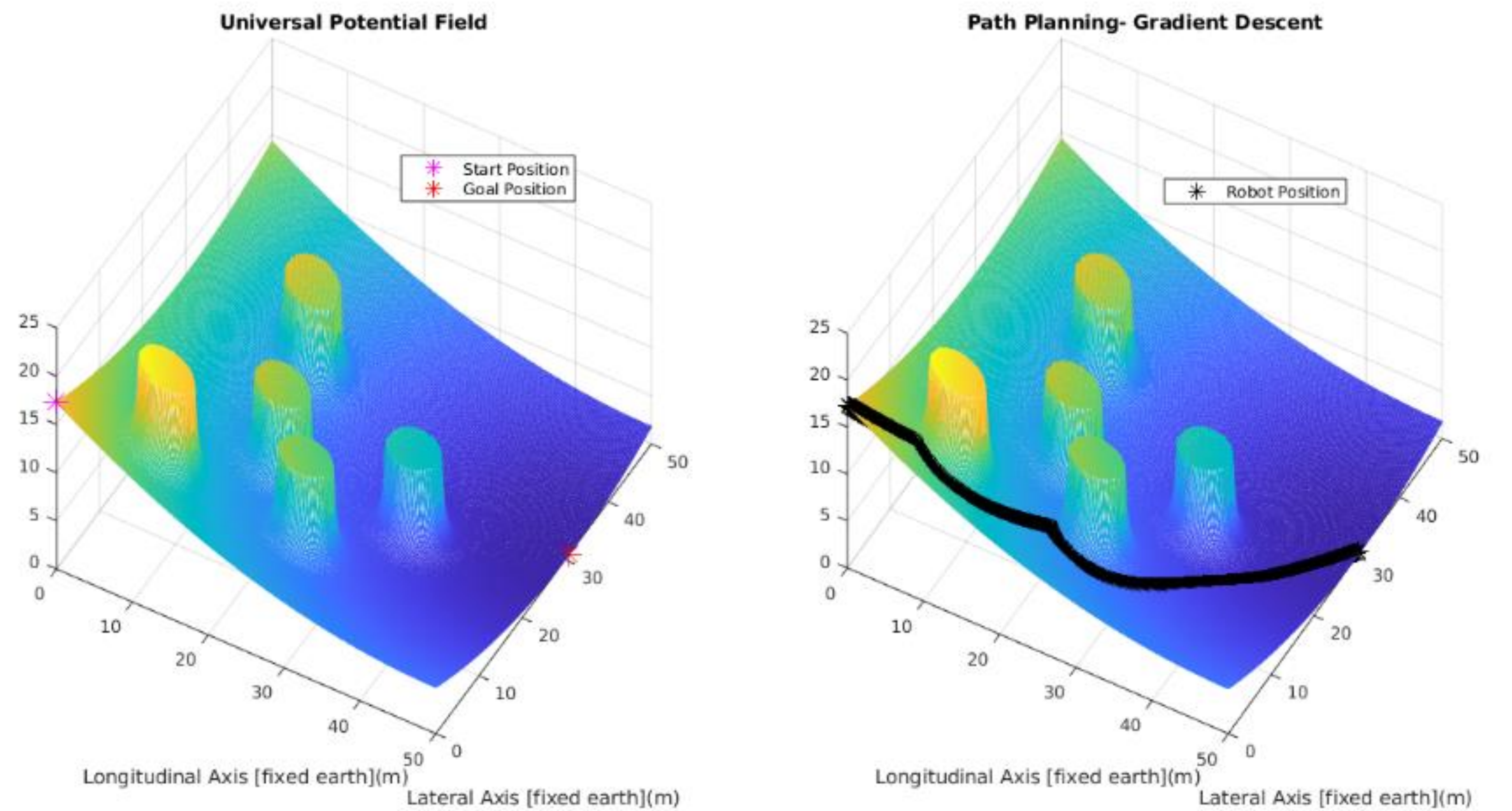


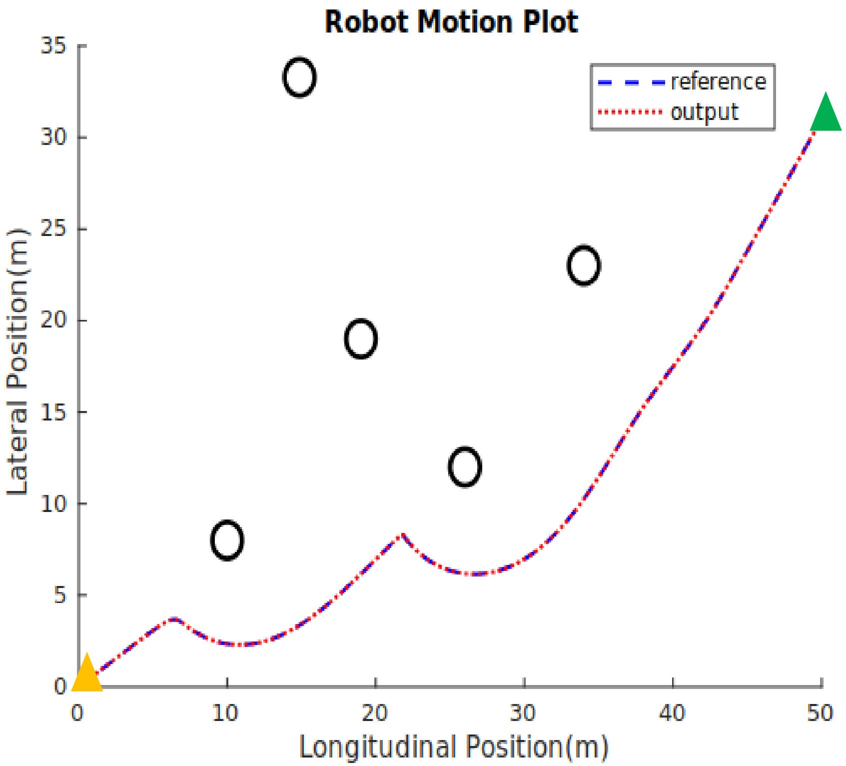

▲ Start point

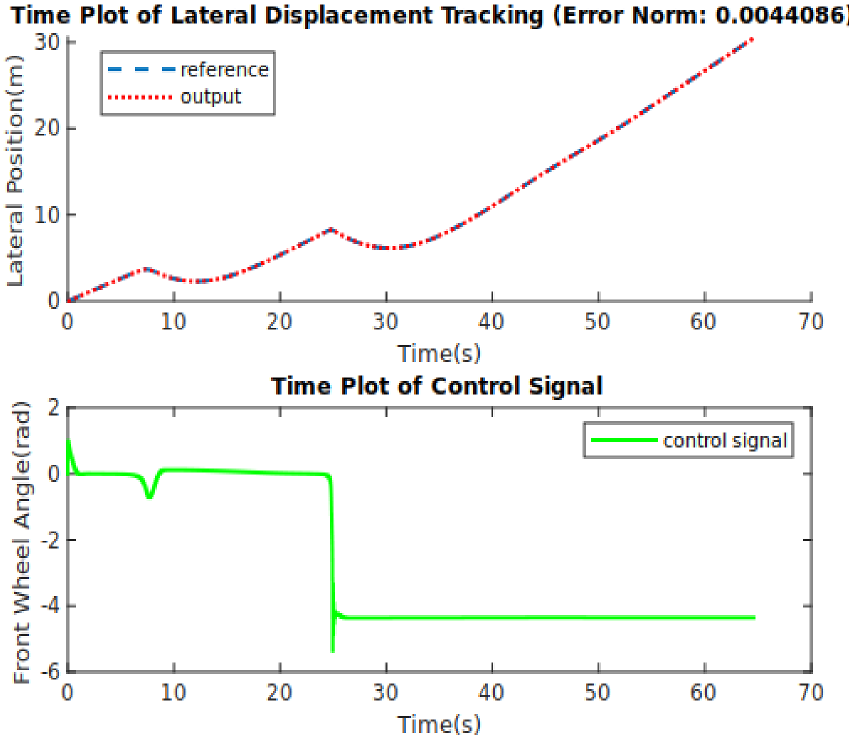
▲ End point

# Scenario 2


Universal Potential Field
Start Position
Goal Position
Longitudinal Axis [fixed earth](m)
Lateral Axis [fixed earth](m)
Path Planning- Gradient Descent
Robot Position
Longitudinal Axis [fixed earth](m)
Lateral Axis [fixed earth](m)

# Scenario 2 PID




Start point
End point

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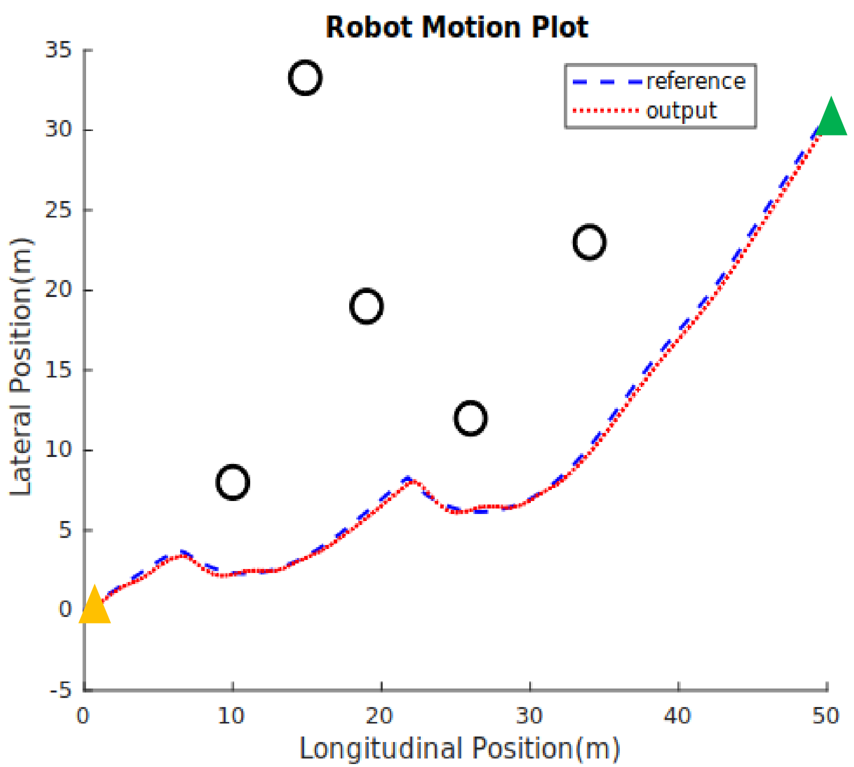
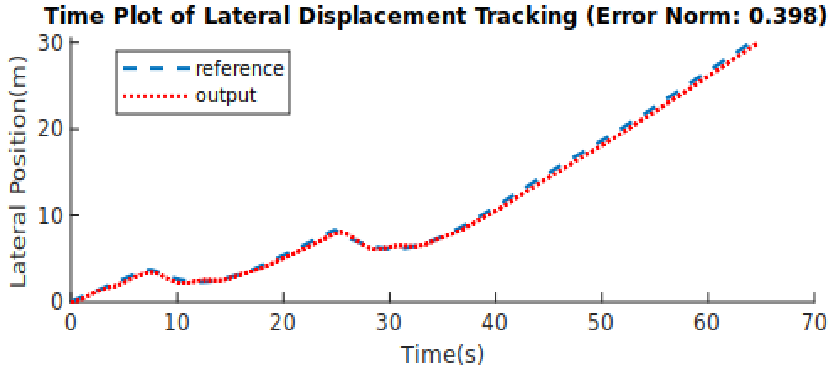
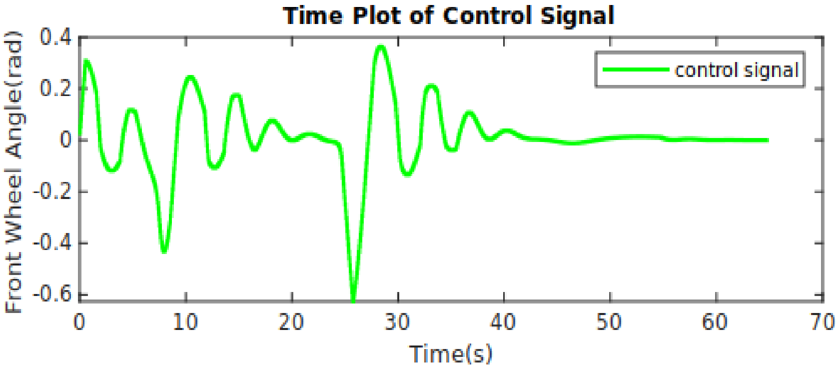
# Scenario 2 SIMO MPC

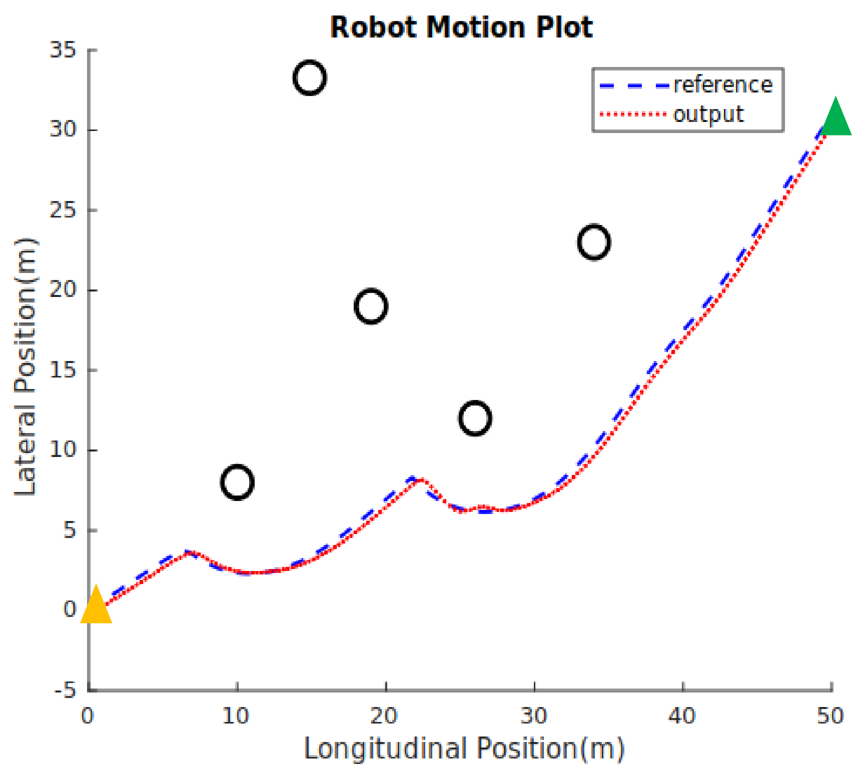


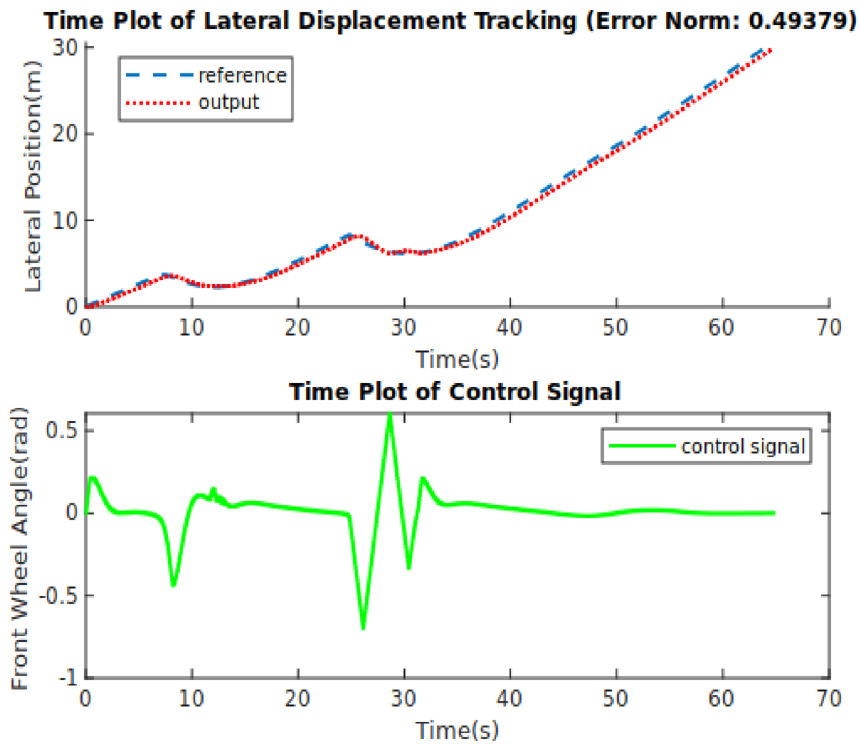




Start point
End point

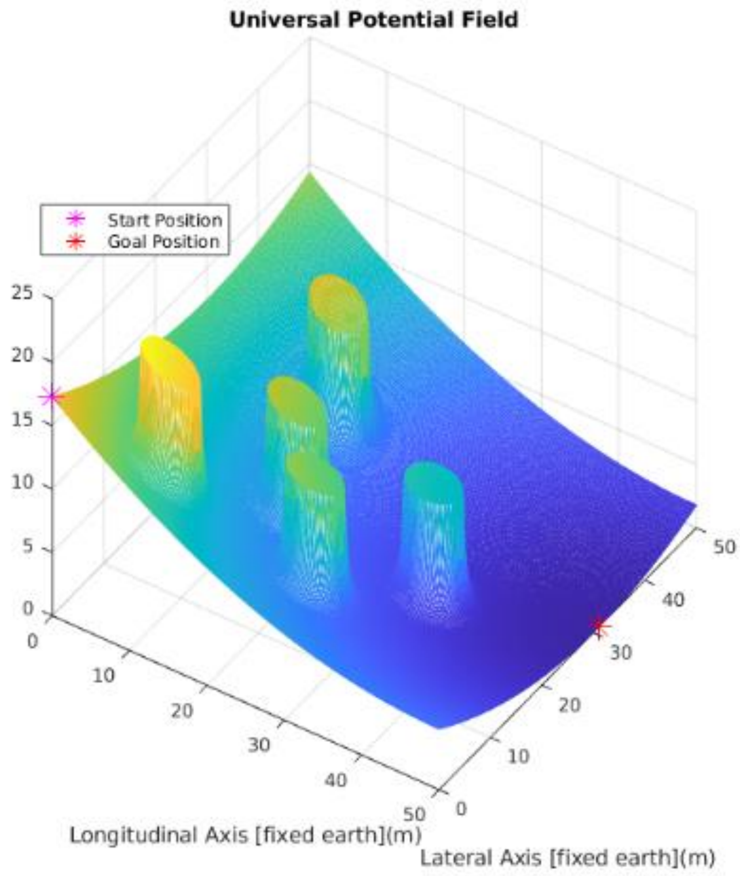
# Scenario 2 SISO MPC

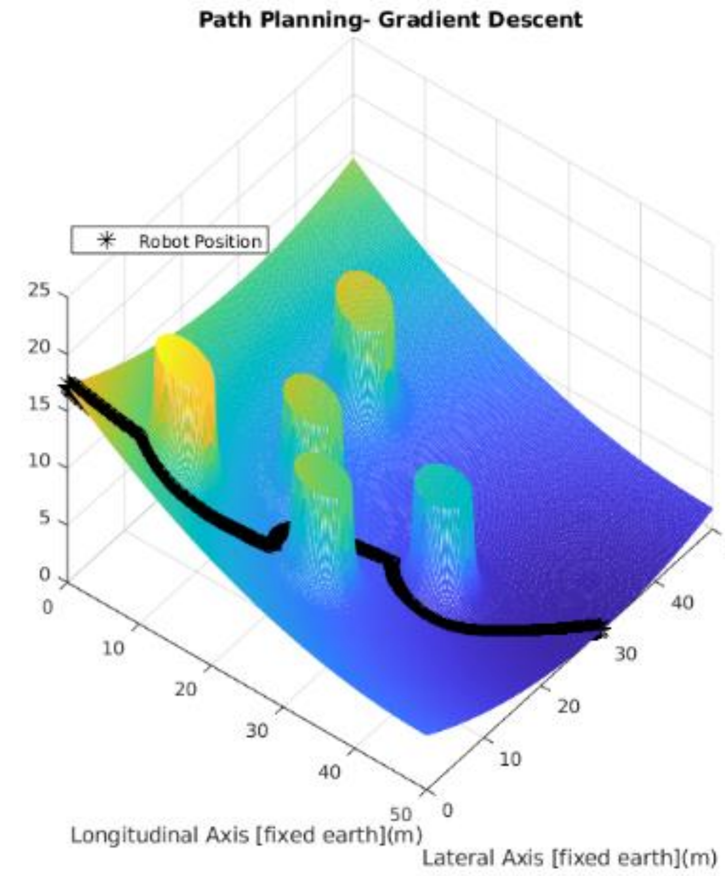


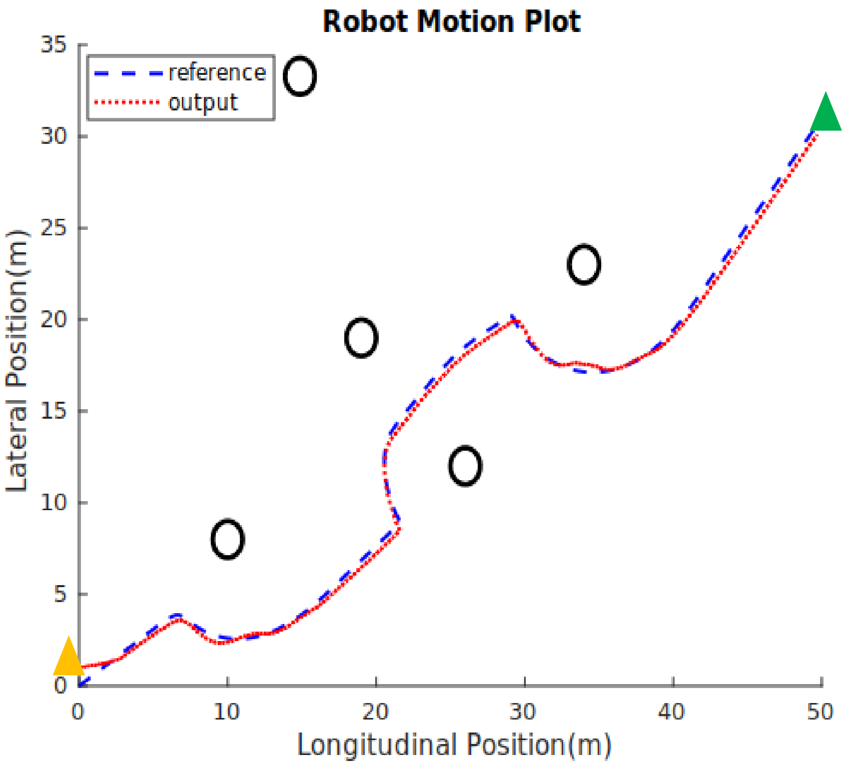

- Start point
- End point

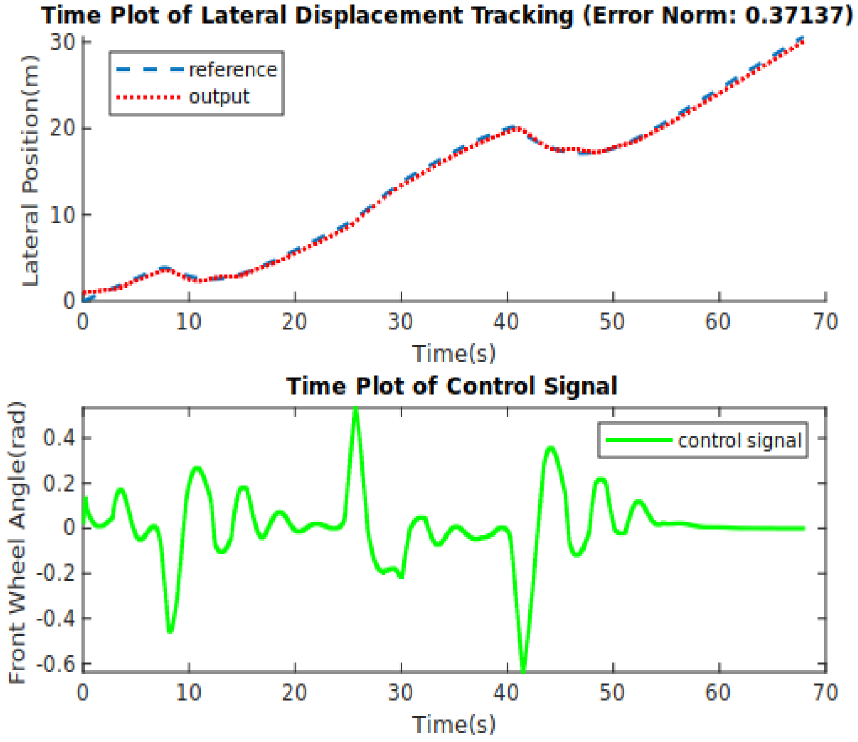
# Scenario 3


Universal Potential Field
Start Position
Goal Position
25
20
15
10
5
0
0
10
20
30
40
50
Longitudinal Axis [fixed earth](m)
0
10
20
30
40
50
Lateral Axis [fixed earth](m)


Path Planning- Gradient Descent
Robot Position
25
20
15
10
5
0
0
10
20
30
40
50
Longitudinal Axis [fixed earth](m)
0
10
20
30
40
50
Lateral Axis [fixed earth](m)

# Scenario 3 SIMO MPC




▲ Start point

▲ End point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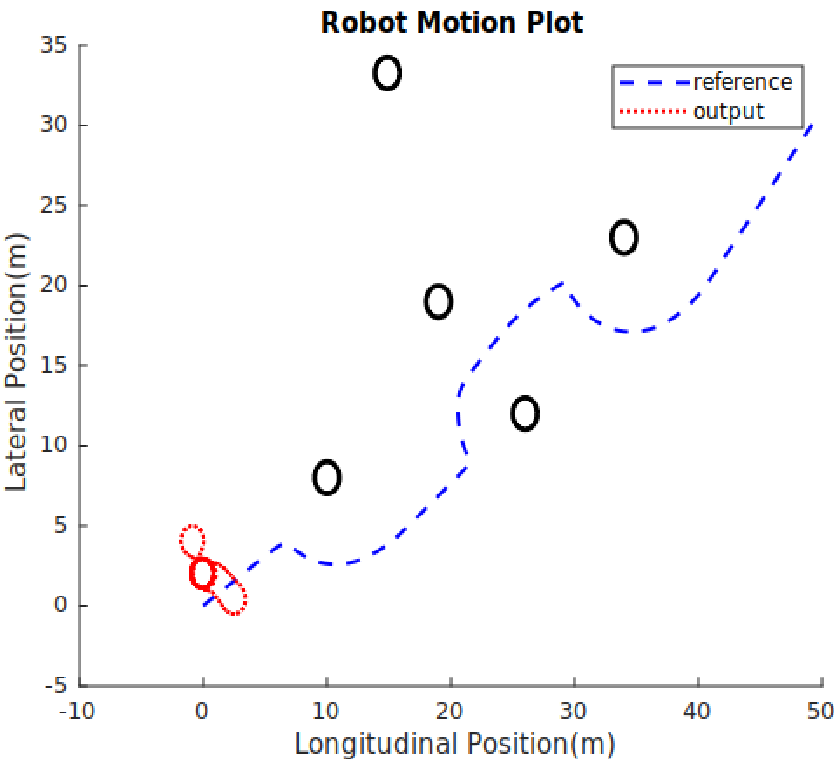
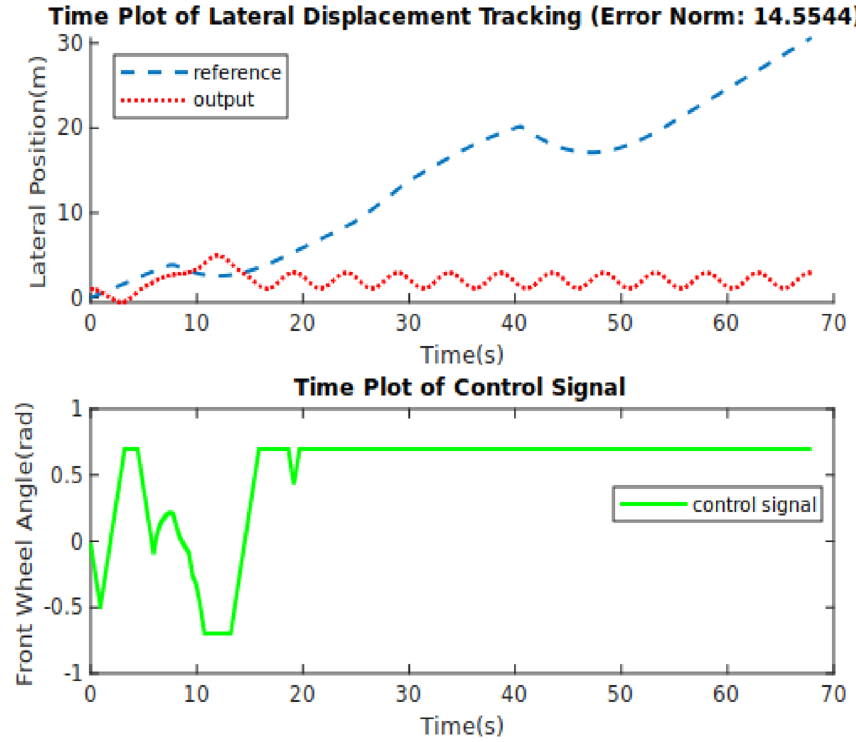
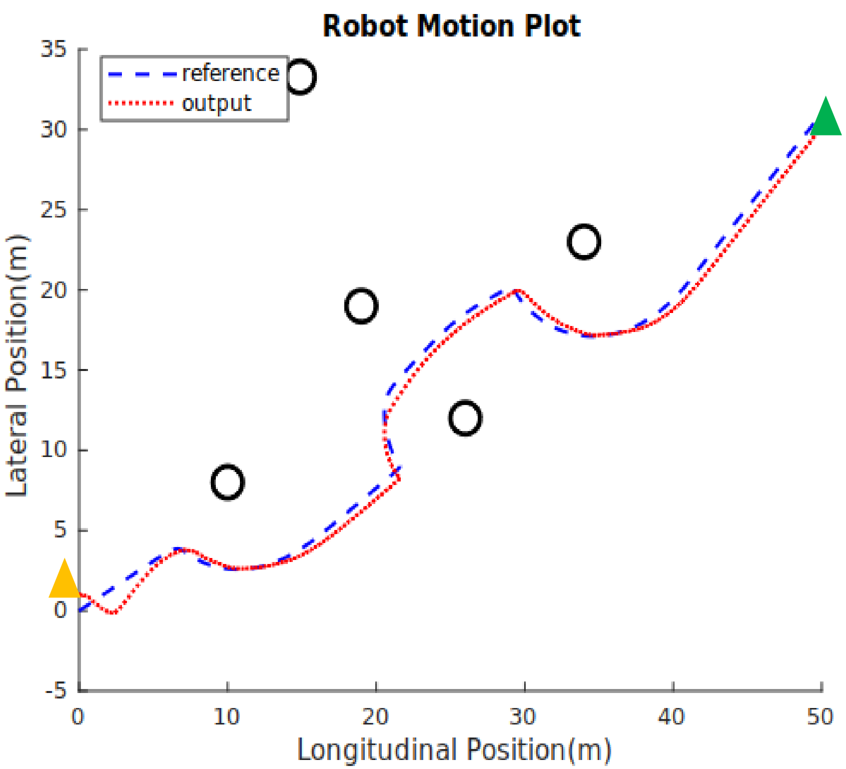
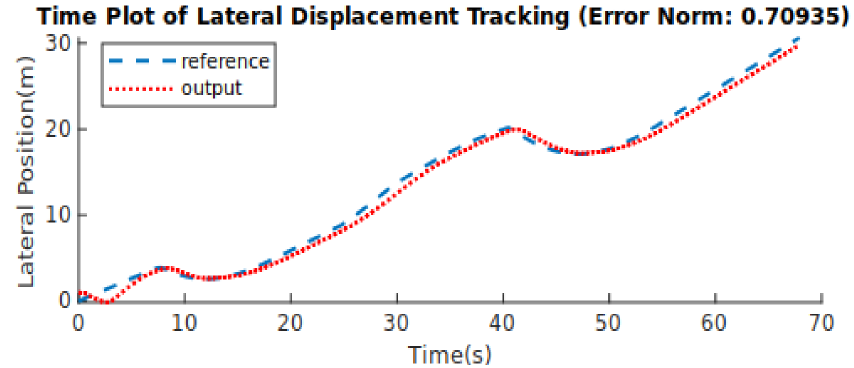
# Scenario 3 SISO MPC- 25

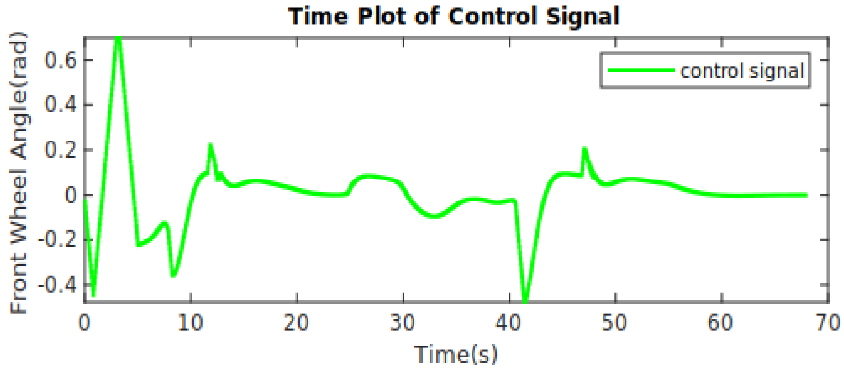


# Scenario 3 SISO MPC- 35




Start point

End point

# Conclusion & Future Works

## Conclusion

In an indoor environment with an accurate mapping and localization scheme, path planning and obstacle avoidance for AMRs moving at relatively slow velocity can be successfully carried out using APFs and MPC.

## Future Works

- Extend obstacle avoidance scheme to include longitudinal position tracking
- Investigate global and local mapping of AMR configuration space
- Study AMR localization techniques based on Ultra Wide Band technologies


Vielen Dank
für Ihre
Aufmerksamkeit