Path Tracking and Obstacle Avoidance for Autonomous Mobile Robots

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1 INTRODUCTION

Autonomous Mobile Robots (AMRs) are required to move around to perform some tasks, this makes the ability to navigate effectively and efficiently an important measure of success for any such robot. For navigation, an AMR is required to plan a path leading to its goal point, generate a trajectory along the planned path, and appropriately track such generated trajectory.

Path planning involves searching for an optimal collision-free path from an initial position to some desired goal position which conforms to its physical constraints of the AMR in question [3]. Path planning is divided into: global path planning in which the environment known completely; and the local path planning in which only some section of the environment is known. Common global path planning techniques include A* heuristic search, visibility graph method, generalized Voronoi diagram, ant colony algorithm, genetic algorithm, and the artificial potential field method [9, 15].

The Artificial Potential Field (APF) method draws inspiration from classical physics and was first proposed in 1986 by Khatib [12]. With this method, an AMR seems to move towards its goal position and avoid obstacles instinctively. This is because repulsive potentials are generated to represent obstacles, while attractive potentials are generated to represent the goal position, effectively transforming the path planning problem into an optimization problem [6].

APF approach gives room for the possibility of real-time online path planning. Globally planned paths can be updated with local information from robot sensors-e.g. new location of a moving obstacle, thereby making it possible to plan paths that avoid dynamic obstacles. This, and its mathematical conciseness [15] are some of the reasons why APF-based approaches are widely used in path planning for AMRs. However, if special care is not taken in formulating the potential func-

tions, the robot might get stuck at a local minimum and thereby never reaching its goal position. To prevent this, several methods of formulating the potential functions have been proposed. These include gaussian-shaped repulsive functions [8], the super-quadratic potential function [16], simulated annealing technique [17], and methods that utilize search techniques with the capability of escaping local minima [2].

A trajectory can be generated from a planned path by time parametrization [14]. To enable collision-free navigation, the generated trajectory must respect the dynamical and kinematical constraints of the AMR. Due to nolinear dynamics, path tracking of AMRs is generally performed using sliding mode control [7], robust control [11], fuzzy logic control [1], or model predictive control (MPC) [6].

In recent years, a lot of research efforts have been poured into implementing MPC algorithms for navigation. Götte et al. in [5] propose a model predictive planning and control (MPPC) approach which handles both trajectory planning and tracking. In [10], Nolte et al. present a generalized approach for path and trajectory planning with model predictive frameworks. A contrained linear time-varying MPC was implemented by Gutjahr et al. in [4] for path tracking and trajectory optimization. While a multiconstrained MPC was presented by Ji et al. in [6] solely for the purpose of path-tracking.

In the following, path tracking and obstacle avoidance for an AMR is investigated. Avoidance of static obstacles in a completely known environment is considered, while taking the robot's kinematic limitations (such as its size, shape, and its steering constraints) into account. An APF approach is used for path planning and trajectory generation, while a multi-constrained model predictive control is used for tracking the generated trajectory.

2 PATH PLANNING AND TRAJECTORY GENERATION

This section focuses on path planning and trajectory generation based on Artifical Potential Fields (APFs). While many researchers have proposed a vast array of path planning techniques, APFs provide an intuitive formulation that allow real-time modifications of planned trajectories. A feature that becomes quite essential in configuration spaces with dynamic obstacles.

The following assumptions are made in this study to simplify the mathematical representation of collision-free trajectories:

- A configuration space of 50m x 50m.

- Static obstacles with rounded geometry of known sizes and positions.

- Constant longitudinal velocity of 1m/s.

2.1 PATH PLANNING

Two coordinate systems are considered. The Autonomous Mobile Robot's (AMR's) body coordinate system $o - xy$, which is centered on the vehicle's center of mass with the x-axis is along the AMR's longitudinal axis, while the y-axis is along it's lateral axis; and a fixed earth coordinate system $O - XY$, which is defined to be colinear with the vehicle body coordinate system at the instant of path planning. At any point in time, the angle of rotation between both corrdinate systems is the AMR's yaw angle ψ. The position of the AMR in the fixed-earth coordinate is given as $\vec{X}_r$, the position of the goal point as $\vec{X}_g$, and position of obstacles as $\vec{X}_o$.

The universal potential (U) which guides the AMR to it's goal position is composed of the attractive potential (U_A) and the repulsive potential (U_R).

The attractive potential (U_A) is formulated such that it's minumum point is at the goal position. A mathematical expression for the attractive potential is given as:

$$U_A(\vec{X}_r, \vec{X}_g) := \frac{1}{2} K_{att} \left\| \vec{X}_r - \vec{X}_g \right\|^2$$

where
K_{att} is a constant.
An attractive potential field with AMR initial position $(0,0)$ and goal position $(50,30)$ and $K_{att} = 0.01$ is shown in Figure 1.
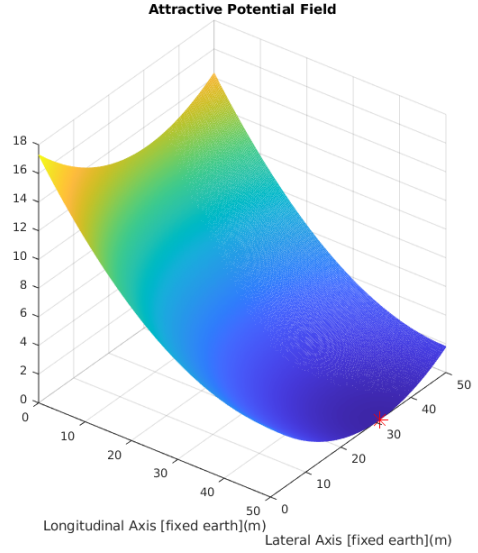

Figure 1: Attractive Potential Field

The second component of the universal potential is the repulsive potential which results from superimposing the repulsive potential of each obstacle. The repulsive potential of an obstacle has it's peak at the obstacle's position and reduces as the distance from the obstacle increases. It can be expressed as:

$$U_R(\vec{X}_r, \vec{X}_o) := \begin{cases} \frac{1}{2} K_{rep} \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right)^2; & \text{if } \rho < \rho_0 \\ 0; & \text{otherwise} \end{cases}$$

where
$\rho := \left\| \vec{X}_r - \vec{X}_o \right\|$;
K_{rep} is a constant;
ρ_0 is a constant that dictates the maximum distance from which the influence of an obstacle can be felt. A suitable selection according to kinetic theory is $\rho_0 \geq V_{MAX}/2A_{MAX}$ where V_{MAX} is the maximum speed of the AMR and A_{MAX} is the maximum decceleration of the AMR [13].

Figure 2 shows the universal potential obtained by fusing the attractive potential from Figure 1 with repulsive potentials obtained from circular obstacles of radius $1m$ at coordinates (14.87, 33.28), (10, 8), (26, 12), (19, 19), and (34, 23) with $K_{rep} = 10$.

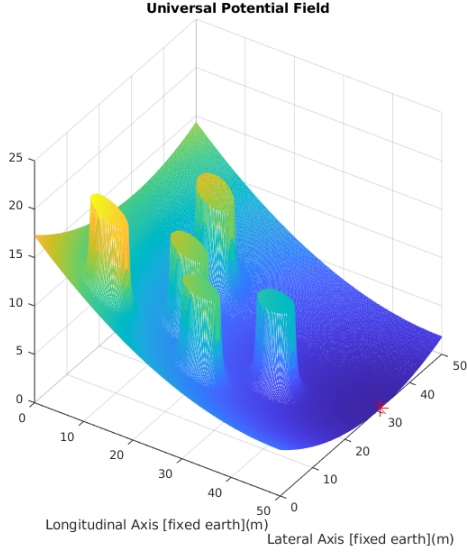After setting up the universal potential, a simple gradient descent along this potential yields a "force" that

Figure 2: Universal Potential Field

Figure 3: Gradient descent along Universal Potential Field
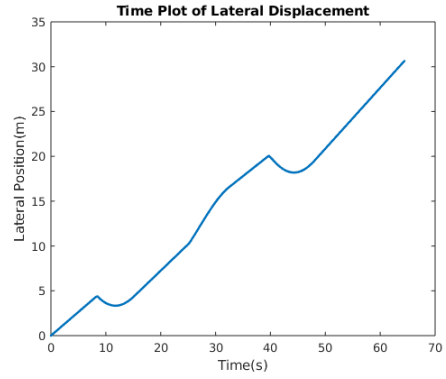
Figure 4: Trajectory- Lateral Displacement

guides the AMR towards the goal position while deflecting it away from the obstacles. This force can be interpreted as the velocity vector of the AMR along the planned path, and can be expressed as:

$$F := F_A + F_R$$

where F_A is the attractive force which is given by:

$$F_A(\vec{X}_r, \vec{X}_o) := -grad(U_A)$$

$$F_A(\vec{X}_r, \vec{X}_o) = -K_{att}\left(\vec{X}_r - \vec{X}_g\right)$$

and F_R is the repulsive force which is given by:

$$F_R(\vec{X}_r, \vec{X}_o) := \sum_{\vec{X}_o \in \Omega} -grad(U_R)$$

$$-grad(U_R) = \frac{K_{rep}}{\rho^3}\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\left(\vec{X}_r - \vec{X}_o\right)$$

where $\rho = \left\|\vec{X}_r - \vec{X}_o\right\|$; Ω is a set of all obstacle coordinates.

Building upon the universal potential depicted in Figure 2, Figure 3 shows the resulting collision-free path from the AMR's initial position $(0, 0)$ to the goal position $(50, 30)$.
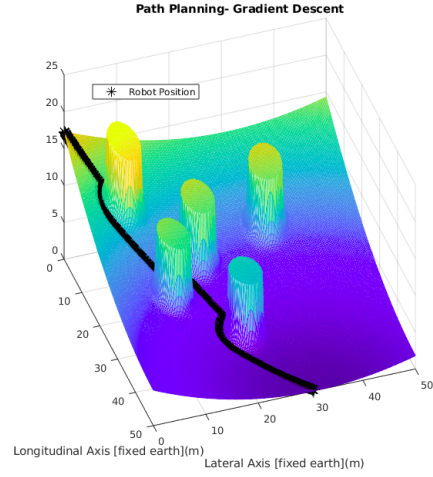
2.2 TRAJECTORY GENERATION

A planned-path can be transformed into a trajectory by time-parametizing it [14]. Following from our earlier assumption of the Autonomous Mobile Robot (AMR) having a constant velocity of $1m/s$, we can generate a trajectory from our planned path by simulating forward in time along the planned path with a velocity vector of magnitude $1m/s$.

Figures 4 and 5 show the trajectory information for the path illustrated in 3 simulated with a sampling time of 0.1 seconds.
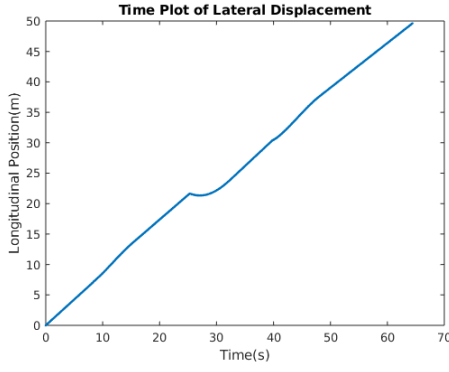
3

Figure 5: Trajectory- Longitudinal Displacement

3 MATHEMATICAL MODEL FOR PATH TRACKING PROBLEM

The mathematical model used for model predictive controller design is described in this section. Since the success of any controller design is highly dependent on the accuracy of the plant model employed, correct modelling of the Autonomous Mobile Robot (AMR) is necessary. While the dynamics of the AMR are nonlinear, MPC allows us to base our design on a linearized model and still get satisfactory results. For the path tracking problem considered in this paper, the following simplifying assumptions are made:

- The AMR is a front wheel controller vehicle with a single front wheel and two real wheels.

- Motion is only in the X-Y plane, rolling and pitching motions are ignored.

- Model parameters are constant.

- The AMR is operating in an indoor environment with a smooth and flat surface such that gravitational and aerodynamic side forces can be ignored.

- Constant longitudinal velocity with perfect tracking along the longitudinal axis.

3.1 DYNAMIC MODEL FOR LATERAL PATH TRACKING

With the above assumptions, and also by lumping the two rear wheel into a single wheel, the linear two degree of freedom bicycle model of a conventional vehicle can be employed.

Considering Newton's law of conservation of momentum along the lateral-axis, we have:

$$F_{yf} + F_{yr} = \dot{u} + v\dot{\psi}$$

$$l_f F_{yf} - l_r F_{yr} = I_{zz}(\dot{\psi})$$

Front tyre lateral force

$$F_{xf} cos(\delta) + F_{xr} = m(\dot{u} - \dot{\psi}v)$$

Rear tyre lateral force (2 rear tyres)

$$aF_{xf} cos(\delta) - bF_{xr} = I_{zz}\dot{r}$$

The slip angles are given as:
Front tyre slip angle

$$\gamma_f = atan\left(\frac{u + a\dot{\psi}}{v}\right) - \delta$$

Rear tyre slip angle

$$\gamma_r = atan\left(\frac{u - b\dot{\psi}}{v}\right)$$

The lateral tire forces can be expressed as:

$$F_{xf} = C_f \gamma_f$$

$$F_{xr} = 2C_r \gamma_r$$

The sideslip angle, $\beta = atan\left(\frac{u}{v}\right)$

With small angles approximations,
  Sideslip angle,

$$\beta = \frac{u}{v}$$

Front tyre angle,

$$\gamma_f = \frac{u + a\dot{\psi}}{v} - \delta = \beta + \frac{a\dot{\psi}}{v} + \delta$$

Rear tyre angle,

$$\gamma_r = \frac{u - b\dot{\psi}}{v} = \beta - \frac{b\dot{\psi}}{v}$$

The equations of motion then yeild the following differential equations:

$$\dot{\beta} = -\frac{\beta}{mv}\left(C_f + 2C_f\right) + r\left[\frac{1}{mv^2}\left(2bC_r - aC_f\right)\right] + \delta\frac{C_f}{mv}$$

$$\dot{r} = -\frac{\beta}{I_{zz}}\left(aC_f - 2bC_r\right) - r\left[\frac{1}{I_{zz}v}\left(a^2C_f + 2b^2C_r\right)\right] + \delta\frac{aC_f}{I_{zz}}$$

By selecting the state variables:

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- Lateral displacement of CoM, x_c

- Side-slip angle, β

- Yaw angle, ψ

- Yaw velocity, r

$$\begin{bmatrix} \dot{x}_c \\ \dot{\beta} \\ \dot{\psi} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & v & v & 0 \\ 0 & -\frac{C_f+2C_r}{mv} & 0 & \frac{2bC_r-aC_f}{mv^2}-1 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2bC_r-aC_f}{I_{zz}} & 0 & -\frac{(2b^2C_r+a^2C_f)}{I_{zz}v} \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_f}{mv} \\ 0 \\ \frac{aC_f}{I_{zz}} \end{bmatrix} \delta$$

SISO Model output:

$$x_c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix}$$

SIMO Model output:

$$\begin{bmatrix} x_c \\ \psi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ \beta \\ \psi \\ r \end{bmatrix}$$

REFERENCES

[1] G. Antonelli, S. Chiaverini, and G. Fusco. A fuzzy-logic-based approach for mobile robot path tracking. *IEEE Transactions on Fuzzy Systems*, 15(2):211–221, April 2007.

[2] J. Barraquand, B. Langlois, and J. . Latombe. Numerical potential field techniques for robot path planning. *IEEE Transactions on Systems, Man, and Cybernetics*, 22(2):224–241, March 1992.

[3] Z. X. Cai, H. G. He, and H. Chen. Some issues for mobile robots navigation under unknown environments. In *Proc. IEEE Internation Conference on Decision and Control*, volume 17, pages 385–390, 2002.

[4] B. Gutjahr, L. Gröll, and M. Werling. Lateral vehicle trajectory optimization using constrained linear time-varying mpc. *IEEE Transactions on Intelligent Transportation Systems*, 18(6):1586–1595, June 2017.

[5] C. Götte, M. Keller, C. Rösmann, T. Nattermann, C. Haß, K. Glander, A. Seewald, and T. Bertram. A real-time capable model predictive approach to lateral vehicle guidance. In *2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)*, pages 1908–1913, Nov 2016.

[6] J. Ji, A. Khajepour, W. W. Melek, and Y. Huang. Path planning and tracking for vehicle collision avoidance based on model predictive control with multiconstraints. *IEEE Transactions on Vehicular Technology*, 66(2):952–964, Feb 2017.

[7] Jong-Min Yang and Jong-Hwan Kim. Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots. *IEEE Transactions on Robotics and Automation*, 15(3):578–587, June 1999.

[8] D. Koditschek. Exact robot navigation by means of potential functions: Some topological considerations. In *Proceedings. 1987 IEEE International Conference on Robotics and Automation*, volume 4, pages 1–6, March 1987.

[9] V. Kunchev, L. Jain, V. Ivancevic, and A. Finn. Path planning and obstacle avoidance for autonomous mobile robots: A review. In *Knowledge-Based Intelligent Information and Engineering Systems*, volume 4252, pages 537–544. Springer, Berlin, Heidelberg, 2006.

[10] M. Nolte, M. Rose, T. Stolte, and M. Maurer. Model predictive control based trajectory generation for autonomous vehicles — an architectural approach. In *2017 IEEE Intelligent Vehicles Symposium (IV)*, pages 798–805, June 2017.

[11] Julio E. Normey-Rico, Ismael Alcalá, Juan Gómez-Ortega, and Eduardo F. Camacho. Mobile robot path tracking using a robust pid controller. *Control Engineering Practice*, 9(11):1209 – 1214, 2001. PID Control.

[12] Khatib Oussama. *The Potential Field Approach And Operational Space Formulation In Robot Control*, pages 367–377. Springer US, Boston, MA, 1986.

[13] S. M. H. Rostami, A. K. Sangaiah, J. Wang, and X. Liu. Obstacle avoidance of mobile robots using modified artificial potential field algorithm.

EURASIP Journal on Wireless Communications and Networking, 2019(70), 2019.

[14] Christoph Rösmann, Wendelin Feiten, Thomas Wösch, Frank Hoffmann, and Torsten Bertram. Trajectory modification considering dynamic constraints of autonomous robots. In *Proc. of ROBOTIK 2012- 7th German Conference on Robotics*, pages 74–79, 2012.

[15] P. Shi and Y. W. Zhao. An efficient path planning alogrithm for mobile robot using improved potential field. In *Proc. IEEE Int. Conference ROBIO*, volume 1-4, pages 1704–1708, 2009.

[16] R. Volpe and P. Khosla. Manipulator control with superquadric artificial potential functions: theory and experiments. *IEEE Transactions on Systems, Man, and Cybernetics*, 20(6):1423–1436, Nov 1990.

[17] Q. Zhu, Y. Yan, and Z. Xing. Robot path planning based on artificial potential field approach with simulated annealing. In *Sixth International Conference on Intelligent Systems Design and Applications*, volume 2, pages 622–627, Oct 2006.