

# Sample Space, Experimental Outcomes, Events, Probabilities

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2020-05-02

## Contents

Experimental Outcomes . . . . .	1
Sample Space and Probabilities . . . . .	1
Union and Intersection and Complements . . . . .	2
Conditional Probability . . . . .	3

## Experimental Outcomes

Go to the [RMD](#), [R](#), [PDF](#), or [HTML](#) version of this file. Go back to [fan's REconTools Package](#), [R Code Examples](#) Repository ([bookdown site](#)), or [Intro Stats with R](#) Repository ([bookdown site](#)).

1. Experiment
  - “We define an experiment as a process that generates well-defined outcomes” (ASWCC P171)
2. Sample Space
  - “The sample space for an experiment is the set of all experimental outcomes” (ASWCC P172)
3. Experimental outcomes
  - “An experimental outcome is also called a sample point to identify it as an element of the sample space.” (ASWCC P172)
4. Events
  - “An Event is a collection of Sample Points” (AWSCC P181), could be just one sample point (one experimental outcome)
5. Probability
  - “Probability is a numerical measure of the likelihood that an event will occur.” (AWSCC P171)

## Sample Space and Probabilities

### Sample Space and Experimental Outcomes

We can use the letter  $S$  or  $\Omega$  to denote sample space. Suppose we have a set of  $n$  experimental outcomes:

$$S = \{E_1, E_2, \dots, E_{n-1}, E_n\}$$

We can call  $S$  a sample space if:

1.  $E_i$  are mutually exclusive:
  - $E_i$  for all  $i$  are separate outcomes that do not overlap.
  - Suppose there are multiple people running for the Presidency, some of these candidates are women, and some of these women are from Texas. If one of the  $E_i$  is that a woman becomes the President, and another  $E_i$  is that someone from Texas becomes the President,  $S$  would not be a sample space, because we could have a woman who is also from Texas win the Presidency.
2.  $E_i$  are jointly exhaustive:
  - For the experiment with well-defined outcomes, the  $E_i$  in  $S$  need to cover all possible outcomes.

- If you are throwing a six sided dice, there are six possible experimental outcomes, if  $S$  only has five of them, it would not be a sample space.

Thinking about the world in terms of sample space is pretty amazing.

### Assigning Probability to Experimental Outcomes

We can assign probabilities to events of a sample space. Since experimental outcomes are themselves events as well, we can assign probabilities to each experimental outcome.

For the mutually exclusive and jointly exhaustive experimental outcomes of the sample space, there are two requirements for assigning probabilities: - Each element of the sample space can not have negative probability of happening, and also can not have more than 1 probability of happening, with  $P$  denotes probability, we have:

$$0 \leq P(E_i) \leq 1$$

- The probabilities of all the mutually exclusive and jointly exhaustive experimental outcomes in the sample space sum up to 1. For an experimental with  $n$  experimental outcomes:

$$\sum_{i=1}^n P(E_i) = 1$$

```
# Load Library
library(tidyverse)

# Define a List of Experimental Outcomes
experimental.outcomes.list <- c('Heavy Rain', 'Light Rain', 'No Rain')

# Probabilities on experimental outcomes
experimental.outcome.prob <- c(0.1, 0.2, 0.7)

# Show these in a Tibble
tibble(tomorrow.experimental.outcomes = experimental.outcomes.list,
       experimental.outcome.prob = experimental.outcome.prob)

# What could happen tomorrow?
# We live in a probabilistic world, from today's perspective, tomorrow is uncertain
# If we draw tomorrow from a hat, given our possible outcomes
# and the probabilities associated with the outcomes
# what are the possible tomorrows?
number.of.tomorrow.to.draw = 20
tomorrow.weather.draws <- sample(experimental.outcomes.list,
                                size = number.of.tomorrow.to.draw,
                                replace = TRUE,
                                prob = experimental.outcome.prob)

# A little tibble to show results
# There are only three unique tomorrows, each of three weather outcomes
# could happen, but the chance of each happening differs by the probabilities
# we set in experimental.outcome.prob
tibble(which.tomorrow = paste0('tomorrow:', 1:number.of.tomorrow.to.draw),
       tomorrow.weather = tomorrow.weather.draws)
```

### Union and Intersection and Complements

Definitions:

1. Complement of Event  $A$ :

- “Given an event  $A$ , the complement of  $A$  is defined to be the event consisting of all sample points that are not in  $A$ . The complement of  $A$  is denoted by  $A^c$ .” (AWSCC P185)
2. The Union of Events  $A$  and  $B$ :
    - “The union of  $A$  and  $B$  is the event containing all sample points belonging to  $A$  or  $B$  or both. The union is denoted by  $A \cup B$ .” (AWSCC P186)
  3. The Intersection of Events  $A$  and  $B$ :
    - “Given two events  $A$  and  $B$ , the intersection of  $A$  and  $B$  is the event containing the sample points belonging to both  $A$  and  $B$ . The intersection is denoted by  $A \cap B$ .” (AWSCC P187)

### Probabilities for Complements and Union

The Probabilities of Complements add up to 1:

$$P(A) + P(A^c) = 1$$

The **Addition Law**:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If two events  $A$  and  $B$  are mutually exclusive, which means they do not share any experimental outcomes (sample points), then:  $P(A \cap B) = 0$ , and  $P(A \cup B) = P(A) + P(B)$ .

The **Multiplication Law for Independent Events**:

$$P(A \cap B) = P(A) \cdot P(B)$$

If the probability of event  $A$  happening does not change the probability of event  $B$  happening, and vice-versa. The two events are independent. Below we arrive this formulation from conditional probability.

### Conditional Probability

We use a straight line  $|$  to denote conditional probability. Given  $A$  happens, what is the probability of  $B$  happening?

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

This says the probability of  $A$  happening given that  $B$  happens is equal to the ratio of the probability that both  $A$  and  $B$  happen divided by the probability of  $B$  happening.

The formula also means that the probability that both  $A$  and  $B$  happens is equal to the probability that  $B$  happens times the probability that  $A$  happens conditional on  $B$  happening:

$$P(A \cap B) = P(A | B) \cdot P(B)$$

If  $A$  and  $B$  are independent, that means the probability of  $A$  happening does not change whether  $B$  happens or not, then,  $P(A | B) = P(A)$ , and:

$$\text{If } A \text{ and } B \text{ are independent: } P(A \cap B) = P(A) \cdot P(B)$$

This is what we wrote down earlier as well.