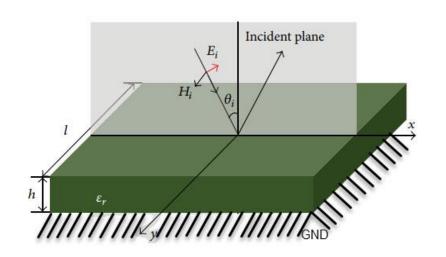
بسم الله الرحمن الرحيم



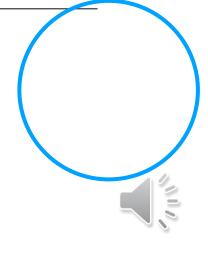


Plane-Wave Reflection from a Grounded Slab of Complex Media

Supervisor: Dr. Seyed Mohammad Hashemi

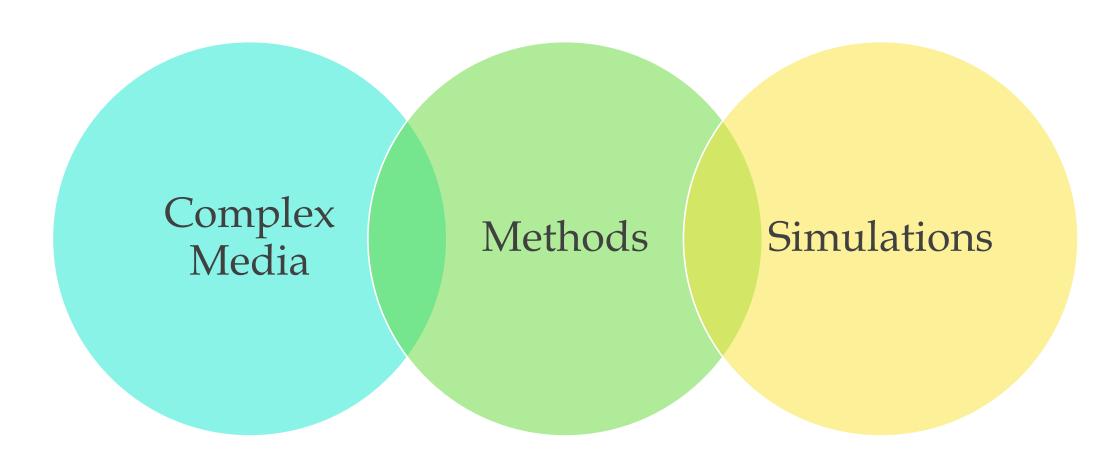
Student: Mohammad Reza Farhadi Nia

March 2021

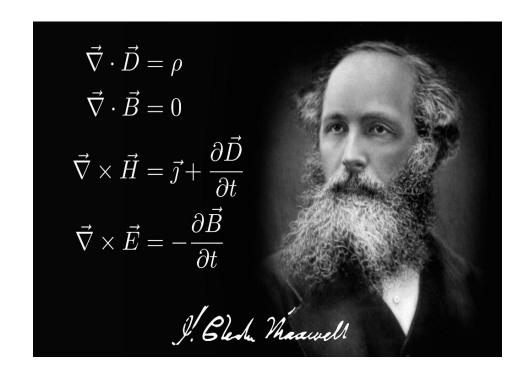


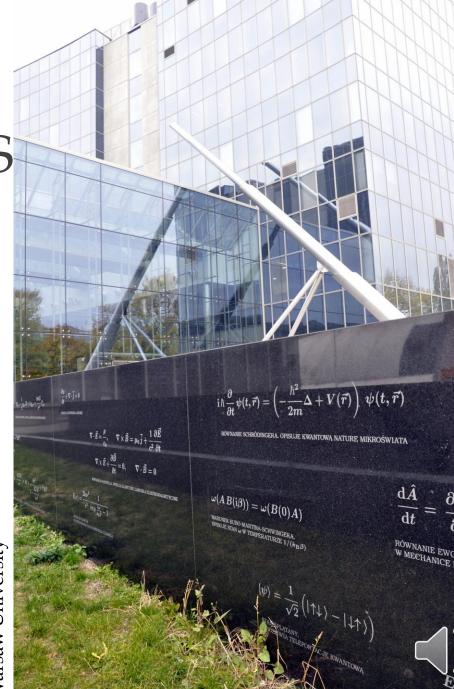










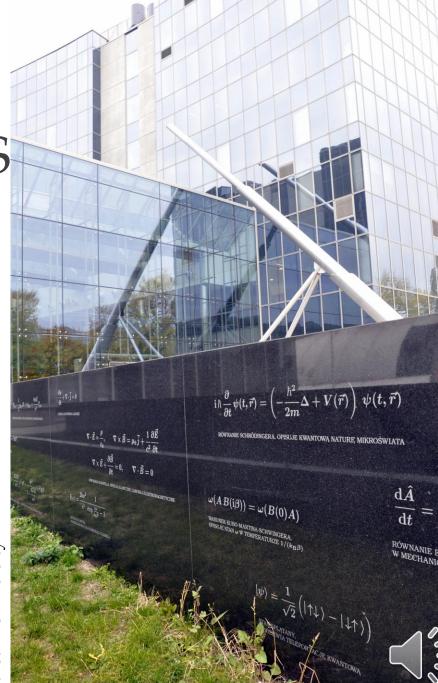


$$\nabla \times \underline{\tilde{H}}(\underline{r},t) - \frac{\partial}{\partial t} \underline{\tilde{D}}(\underline{r},t) = \underline{\tilde{J}}_{e}(\underline{r},t)$$

$$\nabla \times \underline{\tilde{E}}(\underline{r},t) + \frac{\partial}{\partial t} \underline{\tilde{B}}(\underline{r},t) = -\underline{\tilde{J}}_{m}(\underline{r},t)$$

$$\nabla \cdot \underline{\tilde{D}}(\underline{r},t) = \underline{\tilde{\rho}}_e(\underline{r},t)$$

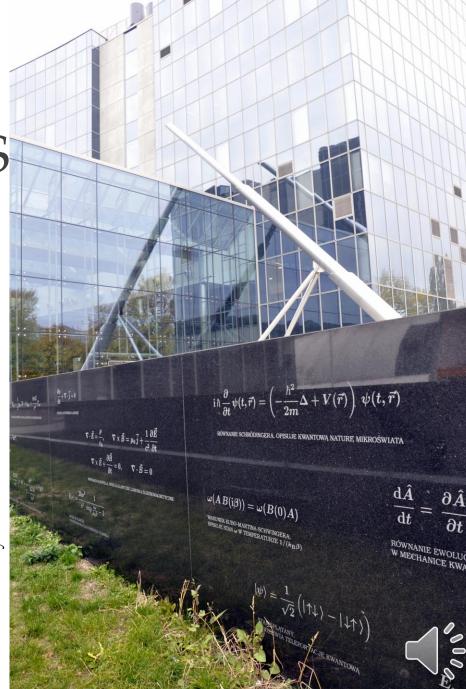
$$\nabla \cdot \underline{\tilde{B}}(\underline{r},t) = -\underline{\tilde{\rho}}_m(\underline{r},t)$$





$$\underline{\tilde{D}}(\underline{r},t) = F\left\{\underline{\tilde{E}}(\underline{r},t),\underline{\tilde{B}}(\underline{r},t)\right\}$$

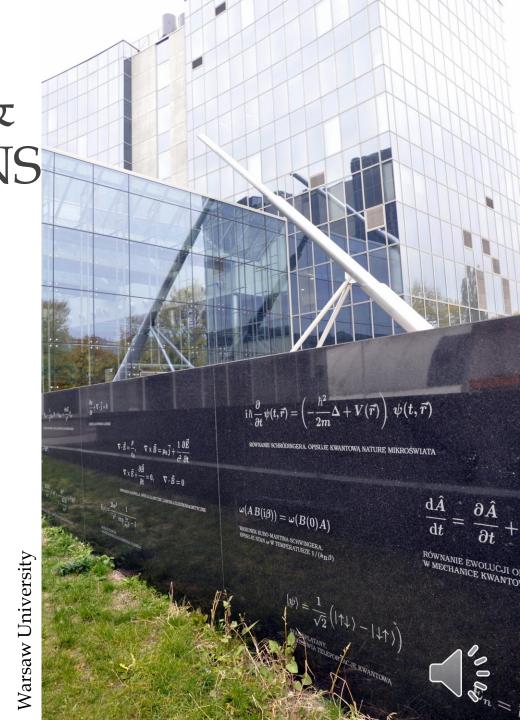
$$\underline{\tilde{H}}(\underline{r},t) = G\left\{\underline{\tilde{E}}(\underline{r},t),\underline{\tilde{B}}(\underline{r},t)\right\}$$



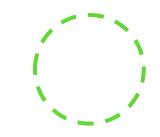
$$\underline{\underline{D}}(\underline{r},\omega) = \underline{\underline{\varepsilon}}_{EH}(\underline{r},\omega) \cdot \underline{\underline{E}}(\underline{r},\omega) + \underline{\underline{\xi}}_{EH}(\underline{r},\omega) \cdot \underline{\underline{H}}(\underline{r},\omega)$$

$$\underline{B}(\underline{r},\omega) = \underline{\zeta}_{EH}(\underline{r},\omega) \cdot \underline{E}(\underline{r},\omega) + \underline{\mu}_{EH}(\underline{r},\omega) \cdot \underline{H}(\underline{r},\omega)$$

$$\begin{bmatrix} \underline{D}(\underline{r}, \omega) \\ \underline{B}(\underline{r}, \omega) \end{bmatrix} = \underline{\underline{K}}_{bi}(\omega) \cdot \begin{bmatrix} \underline{E}(\underline{r}, \omega) \\ \underline{H}(\underline{r}, \omega) \end{bmatrix}$$



COMPLEX MEDIA

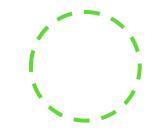


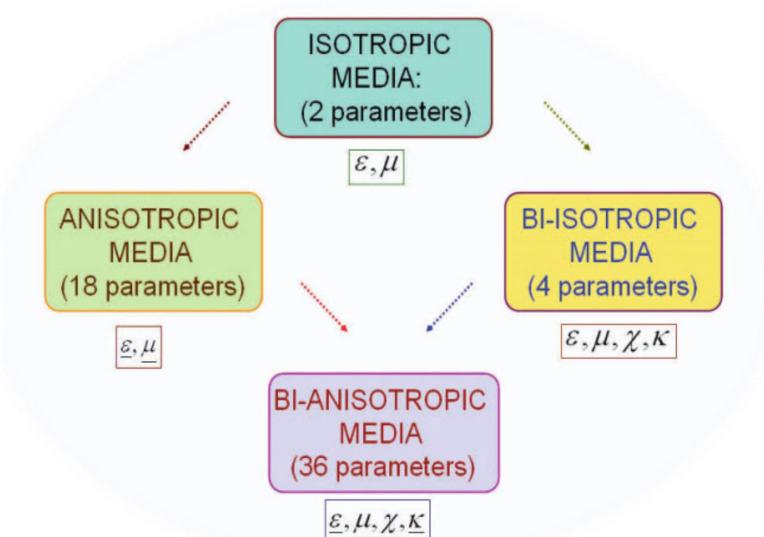
$$\begin{bmatrix} \underline{D}(\underline{r}, \omega) \\ \underline{B}(\underline{r}, \omega) \end{bmatrix} = \underline{\underline{K}}_{bi}(\omega) \cdot \begin{bmatrix} \underline{E}(\underline{r}, \omega) \\ \underline{H}(\underline{r}, \omega) \end{bmatrix}$$

$$\underline{K}_{bi}(\omega) = \begin{bmatrix} \underline{\varepsilon}_{bi}(\omega) & \underline{\xi}_{bi}(\omega) \\ \underline{\varepsilon}_{a}(\omega) & \underline{\varepsilon}_{y}(\omega) & \varepsilon_{y}(\omega) \\ \underline{\zeta}_{bi}(\omega) & \underline{\mu}_{bi}(\omega) \end{bmatrix} = \begin{bmatrix} \varepsilon_{x}(\omega) & \varepsilon_{\alpha}(\omega) & \varepsilon_{\beta}(\omega) \\ \varepsilon_{\alpha}(\omega) & \varepsilon_{y}(\omega) & \varepsilon_{y}(\omega) \\ \varepsilon_{\beta}(\omega) & \varepsilon_{y}(\omega) & \varepsilon_{z}(\omega) \end{bmatrix} \begin{pmatrix} \xi_{x}(\omega) & \xi_{\alpha}(\omega) & \xi_{\beta}(\omega) \\ \xi_{\alpha}(\omega) & \xi_{y}(\omega) & \xi_{y}(\omega) \\ \xi_{\beta}(\omega) & \xi_{y}(\omega) & \xi_{z}(\omega) \end{pmatrix} \begin{pmatrix} \zeta_{x}(\omega) & \zeta_{\alpha}(\omega) & \zeta_{\beta}(\omega) \\ \zeta_{x}(\omega) & \zeta_{x}(\omega) & \zeta_{y}(\omega) & \zeta_{y}(\omega) \\ \zeta_{\alpha}(\omega) & \zeta_{y}(\omega) & \zeta_{y}(\omega) & \zeta_{y}(\omega) \\ \zeta_{\beta}(\omega) & \zeta_{y}(\omega) & \zeta_{z}(\omega) \end{pmatrix} \begin{pmatrix} \mu_{x}(\omega) & \mu_{\alpha}(\omega) & \mu_{\beta}(\omega) \\ \mu_{\alpha}(\omega) & \mu_{y}(\omega) & \mu_{y}(\omega) \\ \mu_{\beta}(\omega) & \mu_{y}(\omega) & \mu_{z}(\omega) \end{pmatrix}$$



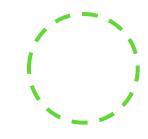
COMPLEX MEDIA

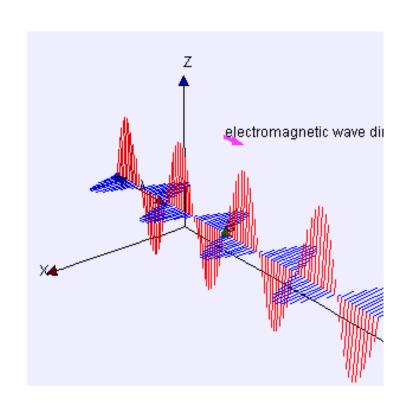


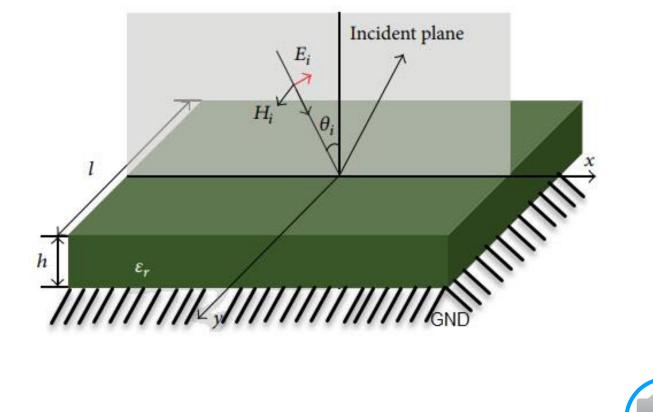












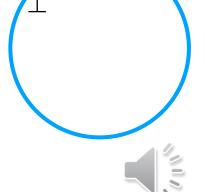


TRANSFORM MATRIX

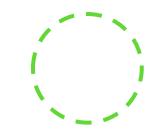


$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{Z_{2}e^{\gamma_{2}d} - Z_{1}e^{\gamma_{1}d}}{Z_{2} - Z_{1}} & -Z_{1}Z_{2}\frac{e^{\gamma_{2}d} - e^{\gamma_{1}d}}{Z_{2} - Z_{1}} \\ -\frac{e^{\gamma_{2}d} - e^{\gamma_{1}d}}{Z_{1} - Z_{2}} & \frac{Z_{1}e^{\gamma_{2}d} - Z_{2}e^{\gamma_{1}d}}{Z_{1} - Z_{2}} \end{pmatrix} \quad Z_{1,2} = -\frac{1}{2}$$

$$\det\left[\underline{\underline{L}}(jk) + j\omega\underline{\underline{K}}_{EH}(\omega)\right] = 0$$



REFLECTION



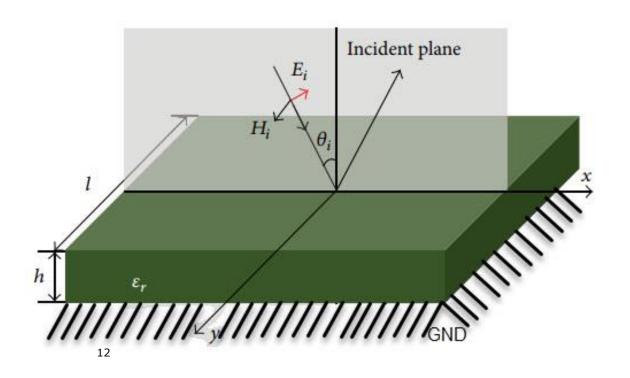
$$\Gamma = s_{11} - \frac{s_{12}s_{21}\Gamma_L}{1 + s_{22}\Gamma_L} = \frac{\left(-D^2 + \left(A - CZ_0\right)D - 2AD + 2BC\right)Z_0^2 + B\left(A - CZ_0\right)Z_0 + B^2}{\left(\left(A + D + CZ_0\right)Z_0 + B\right)\left(DZ_0 + B\right)}$$

$$Z_{0TE} = \eta_0 \, \frac{k_0}{\sqrt{k_0 - k_x^2}},$$

$$Z_{0TM} = -\eta_0 \, rac{\sqrt{k_0 - k_x^2}}{k_0} \, ,$$

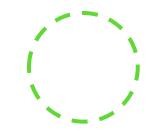
$$k_x = k_0 \sin(\theta_i)$$

$$\Gamma_L = -1$$









STAGE 01

k

STAGE 03

$$T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

Q.E.D

STAGE 02

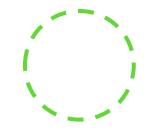
STAGE 04

$$Z_{1,2}$$

$$\Gamma = s_{11} - \frac{s_{12}s_{21}}{1 + s_{22}}$$





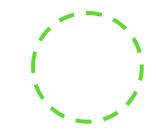


$$Z_{TM1} = \frac{E_{1 \angle}}{H_{1 \perp}} = \frac{E_x}{H_y} = \frac{-(k_x + \omega \zeta_{23})(+k_z - \omega \xi_{12}) - \omega^2 \mu_{22} \varepsilon_{13}}{(k_x + \omega \zeta_{23})(-\omega \varepsilon_{11}) - (\omega \varepsilon_{13})(k_z - \omega \zeta_{21})}$$

$$Z_{TE1} = -\frac{E_{1\perp}}{H_{1\perp}} = -\frac{E_{y}}{H_{x}} = -\frac{(k_{x} - \omega \xi_{23})(\omega \mu_{11}) + (\omega \mu_{13})(k_{z} + \omega \xi_{21})}{(k_{x} - \omega \xi_{23})(+k_{z} + \omega \xi_{12}) + \omega^{2} \varepsilon_{22} \mu_{13}}$$



OTHER METHOD

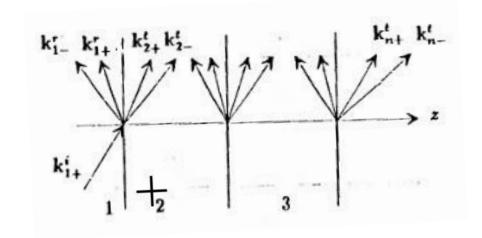


$$Z_{in\pm} = \eta_2 \frac{\eta_{3\pm} \cos(\theta_2 \mp k_2 L \cos(\theta_2) + j \eta_2 \sin(k_2 L \cos(\theta_2)))}{\eta_2 \cos(\theta_2 \pm k_2 L \cos(\theta_2) + j \eta_{3\pm} \sin(k_2 L \cos(\theta_2)))}$$

$$R_{\scriptscriptstyle{\mp\pm}} = \eta_2 \, rac{Z_{\scriptscriptstyle{in\pm}} - \eta_{\scriptscriptstyle{1\pm}}}{Z_{\scriptscriptstyle{in\pm}} + \eta_{\scriptscriptstyle{1\mp}}} e^{\pm 2\, jartheta}, \Gamma = \left| R_{\scriptscriptstyle{\mp\pm}}
ight|$$

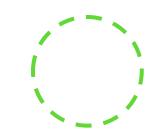
$$R_{co} = \eta_2 \frac{Z_{in+} Z_{in-} \cos(2\theta_1) - \eta_1^2 + j\eta_1 (Z_{in+} - Z_{in-}) \sin(\theta_1)}{Z_{in+} Z_{in-} + \eta_1^2 + \eta_1 Z_{in+} - \eta_1 Z_{in-}}$$

$$R_{cross} = -\frac{jZ_{in+}Z_{in-}\sin(2\theta_1) + j\eta_1(Z_{in+} - Z_{in-})\cos(\theta_1)}{Z_{in+}Z_{in-} + \eta_1^2 + \eta_{1+}Z_{in+} + \eta_{1-}Z_{in-}}$$





OTHER METHOD



$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ \frac{1}{\eta_1} & \frac{1}{\eta_2} & -\frac{1}{\eta_2} & 0 \\ 0 & e^{-\gamma_2 d} & e^{+\gamma_2 d} & -e^{-\gamma_3 d} \\ 0 & \frac{e^{-\gamma_2 d}}{\eta_2} & \frac{e^{+\gamma_2 d}}{\eta_2} & -\frac{e^{-\gamma_3 d}}{\eta_3} \end{bmatrix} \begin{bmatrix} E_{ro} \\ E_2^+ \\ E_2^- \\ E_3^+ \end{bmatrix} = \begin{bmatrix} E_{io} \\ \frac{E_{io}}{\eta_1} \\ 0 \\ 0 \end{bmatrix}$$

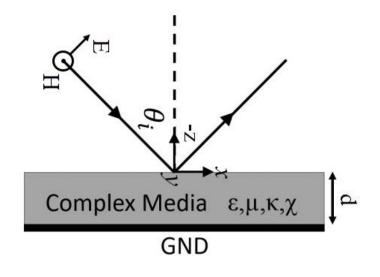
$$\Gamma_{slab} = \frac{E_{ro}}{E_{io}} = \frac{\Gamma_{12} + \Gamma_{23} e^{-2\gamma_2 d}}{1 + \Gamma_{12} \cdot \Gamma_{23} e^{-2\gamma_2 d}}$$

$$\Gamma_{ij} = \frac{\eta_j - \eta_i}{\eta_j + \eta_i}$$





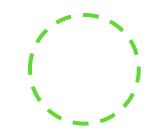
$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ \frac{\cos(\theta_i)}{\eta_1} & \frac{\cos(\theta_j)}{\eta_2} & -\frac{\cos(\theta_j)}{\eta_2} & 0 \\ 0 & e^{-\gamma_2 d \cos(\theta_j)} & e^{+\gamma_2 d \cos(\theta_j)} & -e^{-\gamma_3 d \cos(\theta_k)} \\ 0 & \frac{\cos(\theta_j)}{\eta_2} e^{+\gamma_2 d \cos(\theta_j)} & -\frac{\cos(\theta_j)}{\eta_2} e^{+\gamma_2 d \cos(\theta_j)} & \frac{\cos(\theta_k)}{\eta_3} e^{-\gamma_3 d \cos(\theta_k)} \end{bmatrix} \begin{bmatrix} E_{ro} \\ E_2^+ \\ E_2^- \\ E_3^+ \end{bmatrix} = \begin{bmatrix} E_{io} \\ \cos(\theta_i) \\ E_2^- \\ E_3^+ \end{bmatrix} = \begin{bmatrix} E_{io} \\ \cos(\theta_i) \\ E_3^- \\ 0 \end{bmatrix}$$



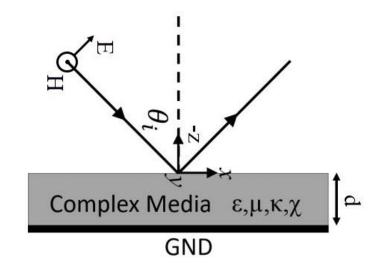
$$\Gamma_{TE,slab} = \frac{E_{ro}}{E_{io}} = \frac{\Gamma_{12} + \Gamma_{23}e^{-2\gamma_2 d}}{1 + \Gamma_{12} \cdot \Gamma_{23}e^{-2\gamma_2 d}} \qquad \eta_{l,TE} = \frac{\eta_l}{\cos(\theta_s)}, d' = d\cos(\theta_j), l = 1, 2, 3, s = i, j, k$$

$$\eta_{l,TE} = \frac{\eta_l}{\cos(\theta_s)}, d' = d\cos(\theta_j), l = 1, 2, 3, s = i, j, k$$





$$\begin{bmatrix} \cos(\theta_i) & \cos(\theta_j) & \cos(\theta_j) & 0 \\ \frac{-1}{\eta_1} & \frac{1}{\eta_2} & \frac{1}{\eta_2} & 0 \\ 0 & \cos(\theta_j)e^{-\gamma_2 d\cos(\theta_j)} & -\cos(\theta_j)e^{+\gamma_2 d\cos(\theta_j)} & -\cos(\theta_k)e^{-\gamma_3 d\cos(\theta_k)} \\ 0 & \frac{e^{+\gamma_2 d\cos(\theta_j)}}{\eta_2} & \frac{e^{+\gamma_2 d\cos(\theta_i)}}{\eta_2} & \frac{e^{-\gamma_3 d\cos(\theta_k)}}{\eta_3} \end{bmatrix} \begin{bmatrix} E_{ro} \\ E_2^+ \\ E_2^- \\ E_3^+ \end{bmatrix} = \begin{bmatrix} E_{io}\cos(\theta_i) \\ \frac{E_{io}}{\eta_1} \\ 0 \\ 0 \end{bmatrix}$$

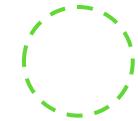


$$\Gamma_{TM,slab} = -\frac{E_{ro}}{E_{io}} = \frac{\Gamma_{12} + \Gamma_{23}e^{-2\gamma_2 d}}{1 + \Gamma_{12} \cdot \Gamma_{23}e^{-2\gamma_2 d}} \qquad \eta_{l,TM} = \eta_l \cos(\theta_s), d' = d\cos(\theta_j), l = 1, 2, 3, s = i, j, k$$

$$\eta_{l,TM} = \eta_l \cos(\theta_s), d' = d \cos(\theta_j), l = 1, 2, 3, s = i, j, k$$



NORMAL INCIDENT



$$d = 10mm$$

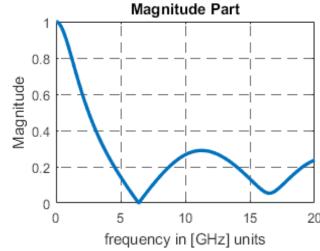
$$1 < \varepsilon_r < 3 \rightarrow \varepsilon_r = 1.42$$

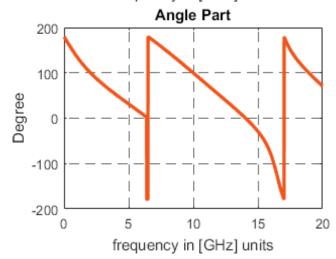
$$1 < \mu_r < 5 \rightarrow \mu_r = 1.97$$

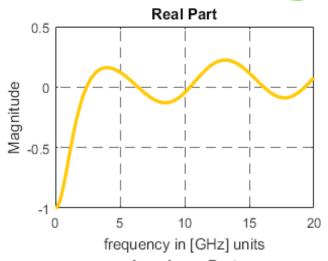
$$0.2 < \sigma[\frac{s}{m}] < 0.5 \rightarrow \sigma = 0.42[\frac{s}{m}]$$

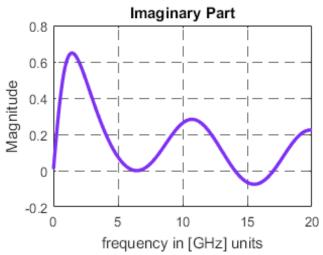
$$\Gamma = 7.4884 \times 10^{-7}$$

$$f = 6.497[GHz]$$



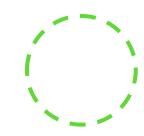








NORMAL INCIDENT



$$d = 10mm$$

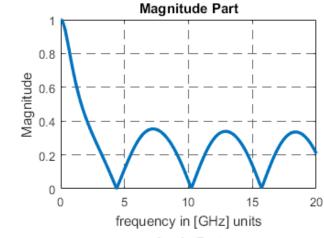
$$\varepsilon_r = 2.010$$

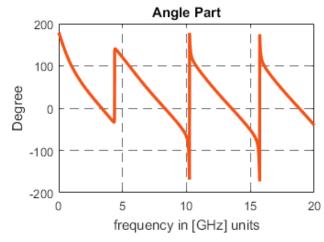
$$\mu_r = 3.924$$

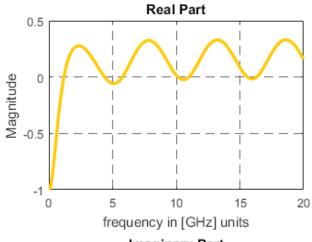
$$\sigma = 0.33 \left[\frac{s}{m}\right]$$

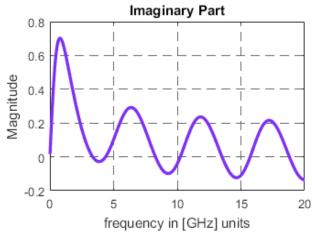
$$\Gamma = 2.0172 \times 10^{-6}$$

$$f = 4.331[GHz]$$













Profile Summary (Total time: 3227.789 s)

Generated 27-Feb-2021 23:02:16 using performance time.

Function Name	Calls	Total Time (s)	Self Time* (s)	Total Time Plot (dark band = self time)	
iteration	1	3227.789	5.802		
iteration>Mininmize	2498631	3221.987	1459.402		
iteration>Mininmize/Coefficient	2498631	949.229	949.229		
iteration>Mininmize/Eta	4997262	407.941	407.941		
iteration>Mininmize/Constant	2498631	291.960	291.960		
iteration>Mininmize/Gamma	2498631	113.454	113.454		

^{*}Self time is the time spent in a function excluding any time spent in child functions. The time includes any overhead time resulting from the profiling process.

Start Profiling Run this code:

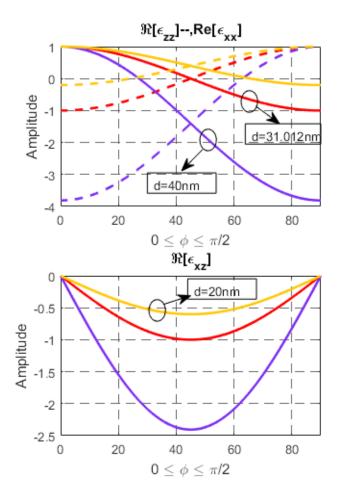
Profile Summary

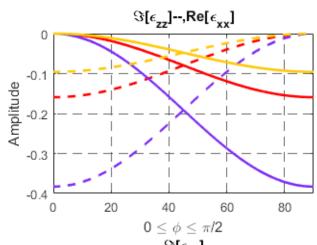
Generated 20-Oct-2020 18:59:09 using performance time.

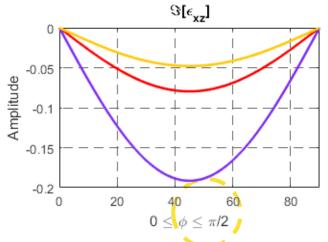
Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
<u>iteration</u>	1	12226.921 s	94.610 s	
iteration>Mininmize	21715539	12152.564 s	3337.686 s	
iteration>Mininmize/Coefficient	21715539	3772.695 s	3772.695 s	
iteration>Mininmize/Eta	43431078	2485.976 s	2485.976 s	
iteration>Mininmize/Constant	21715539	1835.795 s	1835.795 s	
iteration>Mininmize/Gamma	21715539	720.412 s	720.412 s	

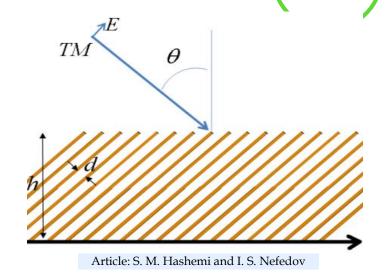


ARTICLE







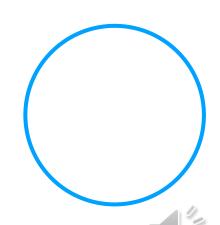


$$d = 31.012nm$$

$$h = 0.1\lambda_0$$

$$\theta = 45^{\circ}$$

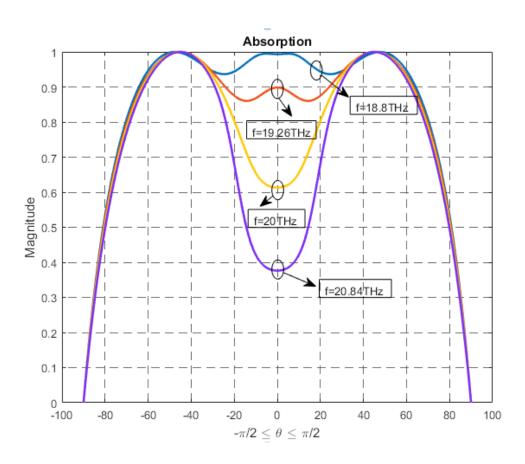
$$\varphi = 45^{\circ}$$







SIMULATION



ARTICLE

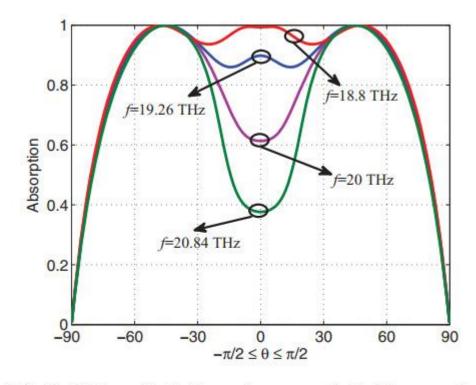
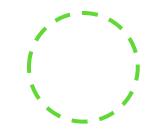


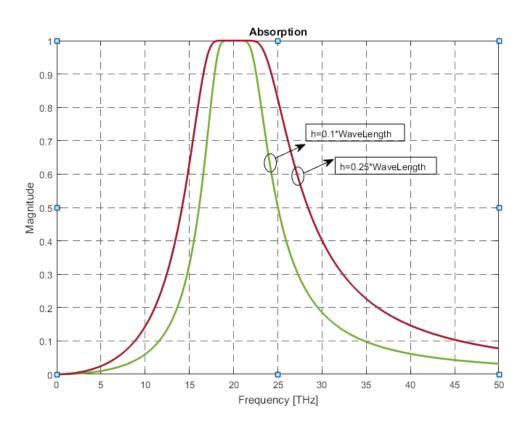
FIG. 9. (Color online) Absorption versus the incidence angle.







SIMULATION



ARTICLE

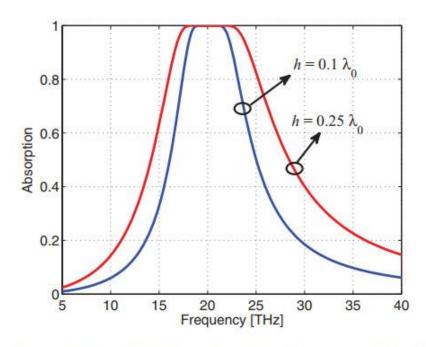
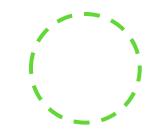
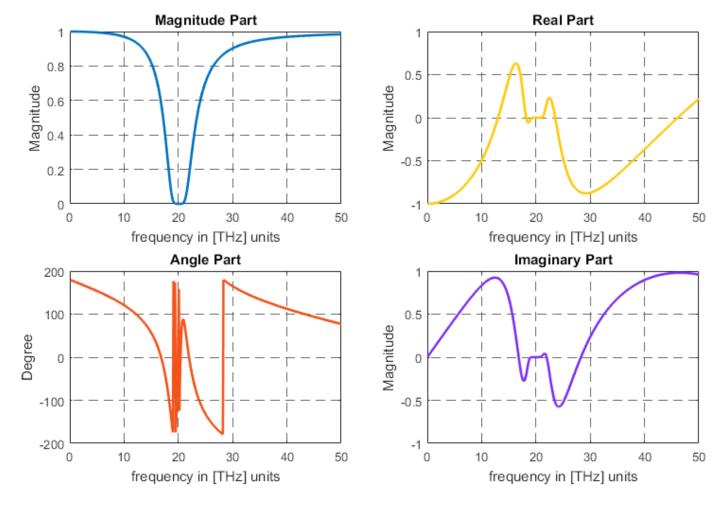


FIG. 8. (Color online) Absorption versus frequency calculated for different thicknesses of the slab. The incidence angle is 45°, the tilt angle is 45°, and λ_0 is the wavelength in free space corresponding to f_0 .















- Absorber
- Array Antenna
- Analysis

...

- Nonhomogeneous
- Nonlinear
- Homogenization

. . .







