Specific orbital energy is the sum of the kinetic energy and the potential energy:

$$
\begin{aligned}
E_T &= \frac{1}{2}mv^2 + mgr \\
&= m\frac{v^2}{2} - m\frac{\mu}{r}
\end{aligned}
$$

$$\boxed{\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}}$$

Specific orbital energy is conserved; apply this at periapsis and apoapsis.

$$
\begin{aligned}
\mathcal{E} = \frac{v_a^2}{2} - \frac{\mu}{r_a} &= \frac{v_p^2}{2} - \frac{\mu}{r_p} \\
\frac{v_a^2}{2} - \frac{v_p^2}{2} &= \frac{\mu}{r_a} - \frac{\mu}{r_p}
\end{aligned}
$$

Specific Angular Momentum is conserved ($h = r_p v_p = r_a v_a$). apply this at periapsis and apoapsis and substitute $v_p = r_a v_a / r_p$ into the above.

$$
\begin{aligned}
\frac{v_a^2}{2} - \frac{r_a^2 v_a^2}{2r_p^2} &= \frac{\mu}{r_a} - \frac{\mu}{r_p} \\
\frac{v_a^2}{2}\left(\frac{r_p^2 - r_a^2}{r_p^2}\right) &= \frac{\mu}{r_a} - \frac{\mu}{r_p}
\end{aligned}
$$

Isolate the specific kinetic energy at apoapsis.

$$
\begin{aligned}
\frac{v_a^2}{2} &= \left(\frac{\mu}{r_a} - \frac{\mu}{r_p}\right)\left(\frac{r_p^2}{r_p^2 - r_a^2}\right) \\
&= \mu\left(\frac{r_p - r_a}{r_a r_p}\right)\frac{r_p^2}{r_p^2 - r_a^2} \\
&= \mu\frac{r_p}{r_a\left(r_p + r_a\right)}
\end{aligned}
$$

Use semi-major axis $a = \left(r_a + r_p\right)/2$ to eliminate r_p from the above,

$$
\begin{aligned}
\frac{1}{2}v_a^2 &= \mu\frac{2a - r_a}{r_a\left(2a\right)} \\
&= \mu\left(\frac{1}{r_a} - \frac{1}{2a}\right) \\
&= \frac{\mu}{r_a} - \frac{\mu}{2a}
\end{aligned}
$$

Substituting this into the specific orbital energy equation from above gives us another way to compute $\mathcal{E}$,

$$
\begin{aligned}
\mathcal{E} &= \frac{v_a^2}{2} - \frac{\mu}{r_a} \\
&= \left(\frac{\mu}{r_a} - \frac{\mu}{2a}\right) - \frac{\mu}{r_a} \\
&= -\frac{\mu}{2a}
\end{aligned}
$$

Include this term in our preferred form for the specific orbital energy equation:

$$\boxed{\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}}$$

Multiply the above through by $2/\mu$ and rearrange terms to obtain the Vis-Viva equation in a simplified form:

$$\boxed{\frac{v^2}{\mu} = \frac{2}{r} - \frac{1}{a}}$$