Part III Chapter 2

Pricing federal public securities

by Márcia Fernanda Tapajós Tavares Ronnie Gonzaga Tavares

1 Introduction

Pricing a security is not merely a matter of mathematical criteria and market conventions. Rather, the way it is priced affects its negotiation and can either stimulate or reduce its liquidity in the secondary market. Consequently, pricing a security affects its cost.

As the National Treasury is aware that prices strongly impact a bond offering's success, it, with the collaboration of market participants, has gradually moved to simplify its instruments. These measures include reducing their number, adjusting interest coupon payments to make them fungible, and standardizing the interest rate convention¹ so securities can be more easily compared with other investments.

This chapter describes the characteristics of the main public debt securities, methodologies by which they are priced, and the inputs needed to price them correctly.

Following this introduction, Section 2 reviews the characteristics of domestic and international debt securities such as interest rates, cash flows, indexes, and the day-counting convention. Section 3 discusses calculation formulas. Section 4 describes the way prices are formed and the inputs used as references (for pricing) in both local and international markets.

2 Instruments

While many instruments exist to finance the Federal public debt, the ones described here are the most important and account for more than 90% of the total.

2.1 Domestic Securities

Domestic public securities are detailed in Decree No. 3,859/2001,² which establishes the main characteristics of Domestic Federal Public Debt (DFPD). This chapter analyzes the main ones currently issued through public offerings (auctions) or those which, while no longer part of the Treasury financing strategy, were important in the recent past.³

¹ Where securities are denominated in reais, they are expressed on the basis of business days/252; dollar-denominated securities follow the external 30/360 standard.

² For further detail see the decree summary table in Annex 5.

³ NTN-Cs, for example.

Letras do Tesouro Nacional (LTNs) are the simplest securities in the domestic market in terms of pricing, since they pay no interest coupons and have a single principal flow on the date of maturity (zero-coupon bond or discount bond). The value of the principal to be paid is always R\$1,000, regardless of the issuance or redemption date.

Notas do Tesouro Nacional, F series (NTN-Fs) are fixed-rate securities that pay semi-annual, compound interest coupons (10%/year) and have one principal flow on the date of maturity (plain-vanilla bond). Like LTNs, the amount of the principal to be paid on maturity is always R\$1,000.

Notas do Tesouro Nacional, B and C series (NTN-Bs and NTN-Cs) are inflation-linked securities that pay semi-annual coupons and have one principal payment on the date of maturity, similar to NTN-Fs. However, cash flow (principal and interest coupons) amounts are updated from the reference date, according to the security's index, which is the IPCA (Brazilian Consumer Price Index - CPI) for NTN-Bs and IGP-M (Brazilian Wholesale Price Index) for NTN-Cs. On maturity, they pay R\$1,000, adjusted according to the index from the reference to the redemption date.

Letras Financeiras do Tesouro (LFTs) are floating-rate securities whose structure is similar to that of LTNs, as they pay no interest coupons and have one principal flow on the date of maturity. However, the principal amount is adjusted according to the SELIC rate⁴ accumulated in the period, i.e., the amount of R\$1,000 is adjusted based on the index previously indicated, from the reference to the redemption date.

Issuance Day counting maturity Security Index Interest Principal convention5 (general rule) Fixed rate 6, 12 and 24 months BD/252 LTN At maturity None NTN-F Fixed rate BD/252 3. 5 and 10 At maturity 10%/year paid semi-annually years NTN-B **IPCA** BD/252 3, 5, 10, 20, 30 and At maturity 6%/year paid semi-annually 40 years NTN-C IGP-M 6%/year paid BD/252 No longer issued At maturity semi-annually6 Selic 3 and 5 years At maturity BD/252 LFT None

Table 1. Main DFPD securities

As previously mentioned, this section presents only those securities that constitute National Treasury financing instruments, and does not cover all local debt instruments; these are described in Decree No. 3,859 of July 2001.

⁴ This is the average weighted rate of one-day repo operations with public securities registered in the SELIC system.

⁵ Details on the day-counting convention are in Annex 1.

⁶ This applies, except for NTN-C 2031, whose coupon is 12%/year and paid semi-annually.

2.2 External debt securities

Brazil's external debt currently includes *global* bonds and *euro* bonds whose definition differs from that which characterizes private Brazilian bonds issued in the international market.

According to the classification, eurobonds are all securities issued in a market whose bond currency differs from that of the issuing market; global bonds are those that can be issued in any market and, unlike eurobonds, can be issued in the same currency as that of the country where they are offered. Thus, public debt securities denominated in reais but issued in international markets can be classified either as global bonds or eurobonds.

According to External Federal Public Debt (EFPD) security classifications, global bonds can only be traded in the US market. However, their currency is not necessarily the US dollar; since 2005, the Treasury has issued global bonds in reais. For their part, eurobonds (securities traded in the euromarket), are usually denominated in euros. The samurai market (Japanese market – bonds denominated in yens), which had been one of Brazil's financing alternatives, has not been accessed since 2001.

2.2.1 Global USD bonds and Global BRL bonds

Global US dollar bonds and Global BRL bonds are the Treasury's main external financing securities. The former are issued in the global market, i.e., they can be traded in different markets and have been an integral part of the Treasury's action strategy for many years. BRLs, which are fixed-rate securities in reais and also defined as global bonds, were issued for the first time in September 2005, with the BRL 2016. Since then, they have been issued routinely. With regard to interest payments, both securities pay semi-annual coupons and have a single principal flow at maturity, similar to NTN-Fs.

2.2.2 Eurobonds

Brazilian eurobonds are securities issued in euros that pay annual interest coupons and have a single principal flow at maturity. Since they are issued in euros, there is no nominal value adjustment. Securities issued more recently can also be traded in the global market, and are thus also global bonds. However, to simplify the definition, they are referred to as eurobonds, as are all other securities issued in euros.

2.2.3 A-Bond

When it was first issued, the A-Bond was Brazil's most liquid security, second only to the Global 2040. Unlike other securities, it is not a traditional financing instrument; rather, it was issued in an exchange operation, in which it replaced the Brady C-Bond. The latter had an embedded *call* option, which gave the issuer (the Treasury) the right to repurchase it at par value, on or after October 15, 2005. In order to reduce the large sum that would be disbursed when it exercised its option, the Treasury exchanged part of the C-Bonds for similar securities (the A-Bonds) which had longer maturities and no embedded call option. For the few investors that did not participate, the Treasury repurchased their C-Bonds on the date the option was exercised (April 15, 2006).

⁷ See Part III, Chapter 4 for more details on the exchange operation.

⁸ The call option could only be exercised on two dates each year - April 15 and October 15 - and securities selected for the call

Table 2. The main external debt securities

Security	Index	lssuance maturity (general rule)	Principal	Interest	Day counting convention
Global	Dollar	10 and 30 years	At maturity	Varies according to maturity	30/360
BRL	Real	10, 15 and 20 years	At maturity	Varies according to maturity	30/360
Euro	Euro	10 years	At maturity	Varies according to maturity	Act/Act
A-bond	Dollar	Maturity in 2018	In 18 equal tranches	8%/year paid semi-annually	30/360

Besides these securities, the Treasury issued a single floating-rate security in the market in dollars (Global FRN 2009⁹), in June 2004. At present, no new ones are being considered; thus, this instrument was not described in this chapter.

3 Pricing

Pricing a bond involves more than discounting a cash flow at a given rate of return for a selected date. When discounted at a single rate, this is called Yield-to-Maturity (YTM) and represents the yield of an investment on that date. However, a security with periodic interest coupons and/or amortization payments can also be priced by discounting each of the flows at a rate valid for that period. It should be noted that after calculating the price with this method, a reverse calculation can be made to determine the YTM equivalent to the price found: By calculating the security price at different rates or at the equivalent YTM, the price will always be the same.

Public security trading can vary according to where it is conducted, whether in Brazil or another country. ¹¹ Negotiations in the domestic market involve YTMs, which are expressed on an annual basis and follow a day-counting convention (see Tables 1 and 2). In the international market, operations are made according to the clean price of securities. ¹²

In the domestic market, the financial settlement of public securities normally occurs on T+1, i.e., the first business day after the Treasury's auctions or trade in the secondary market. In the international market, however, financial settlement occurs in T+3 (business days) for trades in the secondary market and T+5 (business days) for new issues.

option had to be repurchased in full.

⁹ For details on these and other public debt instruments, visit the Treasury's website at: http://www.tesouro.fazenda.gov.br/divida_publica/.

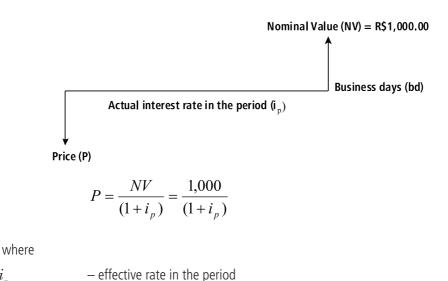
¹⁰ The YTM has a limitation: If the security involves periodic interest and/or amortization payments, its use presupposes that the cash flows received are re-invested at precisely the same YTM rate.

¹¹ For each YTM of a specific security, there is a single equivalent price and vice-versa.

¹² Box 1 in this chapter explains the difference between a security's *clean* and *dirty* price.

3.1 Domestic Debt Securities¹³

Letras do Tesouro Nacional (LTNs) are called zero coupon or discount bonds. These are purchased in both the primary and secondary markets and their yield is directly linked to the negotiated discount. The LTN cash flow is shown below:



Since negotiations are carried out based on the security's annual rates, the applicable formula is:

 i_{α}

Notas do Tesouro Nacional, F Series (NTN-Fs) are fixed-rate securities that pay semi-annual interest coupons. The date on which interest coupons are paid is calculated by counting back six months from a security's maturity or at the last determined interest coupon. The coupon value is fixed (even when the first payment is in fewer than six months), from the date on which the security was issued; this convention guarantees that all NTN-Fs with the same maturity are fungible.

NTN-Fs can be purchased with a premium or discount in the primary and secondary markets, depending on the interest coupon and the yield the investor wants. Since they have an interest flow, the securities can be priced at a single rate (YTM) or specific discount rates for each maturity. The result, as noted earlier, is the same regardless of the criterion. In general, this security is calculated on the basis of a market curve.

¹³ All calculations presented here have a decimal place rounding and truncation rules. These rules, as well as examples of calculations are detailed in Annex 3.

Interest coupon values are calculated exponentially, i.e., the value of the coupon follows the formula below:

$$c_s = (1 + c_a)^{\frac{1}{2}} - 1$$

where:

 c_{\circ} — semi-annual coupon

 c_a — annual coupon

The NTN-F cash flow is shown below:

where:

C – interest coupon payment (semi-annual)

NV - R\$1,000

i — effective annual rate (truncated at the 4th decimal place)

 t_n — business days in the period

Notas do Tesouro Nacional, B and C series (NTN-Bs e NTN-Cs), like NTN-Fs, which are IPCA-linked and IGP-M-linked, respectively, have interest payment dates calculated retroactively every six months from the dates the bonds mature. Since they are subject to monetary adjustment, the interest value is fixed on a percentage basis of the adjusted nominal value.

NTN-B, NTN-C, NTN-D and LFT post-fixed securities cannot be calculated like the NTN-F, which is a fixed-rate security. Where securities are indexed, it is not possible to know the values of future interest coupons and the principal on the trading date. Thus, an intermediate calculation (with all cash flows in 100 basis points or percentage points) is required to determine the bond price, which is multiplied by the adjusted bond's nominal value.

Interest coupon values are calculated exponentially, i.e., the value of the coupon follows the formula below:

$$c_s = (1 + c_a)^{\frac{1}{2}} - 1$$

where

 $c_{\rm s}$ — semi-annual coupon

 c_a — annual coupon

Unlike the NTN-Fs, the pricing of NTN-Bs and NTN-Cs is generally not based on market rates, as no sufficient liquid yield curves exist in the IPCA or IGP-M for this purpose. However, pricing can follow the same method as with the NTN-F, by using the NTN-B¹⁴ and C zero-curve.

The NTN-B/NTN-C cash flow is shown below:

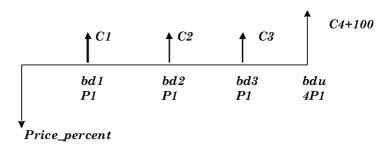
P — ANV x price percent

where

P — price

ANV – present nominal value

Price_percent - price calculated in percentage points



Price_percent =
$$\frac{C1}{(1+i)^{t_1/252}} + \frac{C2}{(1+i)^{t_2/252}} + \dots + \frac{Cn}{(1+i)^{t_n/252}} + \frac{100}{(1+i)^{t_n/252}}$$

Price_percent =
$$\sum_{t=1}^{n} \frac{C_t}{(1+i)^t} + \frac{100}{(1+i)^t}$$

where

C — interest coupon payment considering the bond price as 100% (semi-annual)

i — effective annual rate

 t_n — business days in the period

¹⁴ NTN-Bs 2015 and 2024 (principal), securities offered in the Treasury Direct (TD) Program, are priced by the NTN-B.

Letras Financeiras do Tesouro (LFTs) are also called zero coupon bonds or discount bonds, and can be purchased with a premium or discount, depending on the yield the investor wants to obtain. Although they have no interest flow, these securities are linked to an index (the SELIC rate), which means it is impossible to determine beforehand the value of the principal at maturity. Thus, pricing follows a formula similar to that used for NTNs but without the interest flow. The flow is calculated in 100 basis points or percentage points to generate an amount that will be multiplied by the adjusted nominal value of the security, to determine the price.

The LFT cash flow is shown below:

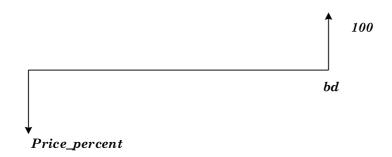
P – ANV x price percent

where

P — price

ANV — present nominal value

Price_percent – price calculated in percentage points



$$Price_percent = \frac{100}{(1+i_p)}$$

where

 $i_{_{D}}$ — effective rate in the period.

Since trades are based on the security's annual rates, the formula is:

$$(1+i_p) = (1+i_a)^{\frac{bd}{252}}$$

where

 i_a — effective annual rate

 ${m bd}$ — business days in the period

3.2 External Debt Securities

Some aspects of external debt security pricing include the following:

- Interest coupons are calculated on a linear basis. The discount, however, is calculated on an exponential basis;
- Operations in the secondary market are settled in T+3 business days while in the primary market, the standard is T+5;
- Present value is calculated according to the security day-counting criterion;¹⁵
- The *clean* price is used in trades, while the *dirty* price is used to settle negotiations (see Box 1 for details);
- The YTM determined for calculation purposes is expressed in a nominal annual rate ¹⁶ and should always be changed into an effective annual rate;
- The issuance price of an external debt security is normally close to par. Interest coupons are determined on the evening of an operation, based on the YTM projected for the security. This differs from what occurs in domestic issuances, where interest coupons are predetermined, allowing securities to be fungible, and the issuance price is seldom close to par.

Box 1. Clean and dirty prices

The dirty price of an operation is the calculation of a security's present value (price) multiplied by the adjusted nominal value (ANV). The clean price is based on the dirty price, minus the interest due between the last coupon payment and the settlement date of the operation. This is the value the security seller will be paid once the transaction is complete.

Although the screen price¹⁷ is the one negotiated by market participants, it does not represent the price investors will pay or be paid in the operation. The rationale is that interest accrued through the settlement date belongs to the security seller – who held it until that moment – and not to the security buyer.

Following a diagnosis of these opportunities and challenges, alternative financing strategies for public debt management as well as existing constraints and trade-offs should be evaluated.

Clean price = dirty price - pro rata interest¹⁸

Another aspect to be considered when calculating the clean price is whether the security involves amortization or capitalization. If one or both apply, the principal of the security will change over its life. Thus, once the price is calculated, the result must be adjusted to indicate the percentage of the security value, ¹⁹ based on the formula below.

Dirty PV adjusted =
$$\frac{Quotation}{\left(\frac{OB_{t-1}}{100}\right)}$$

where

OBt-1 — outstanding balance of the security in t-1.

3.2.1 Global US bonds and Global BRL bonds

The calculation of global US and BRL bond prices uses the 30/360 business days standard, i.e., each month has 30 days and each year 360 days (by convention). The values of interest coupons are calculated in a linear form as follows:

$$c_s = \frac{c_a}{2}$$

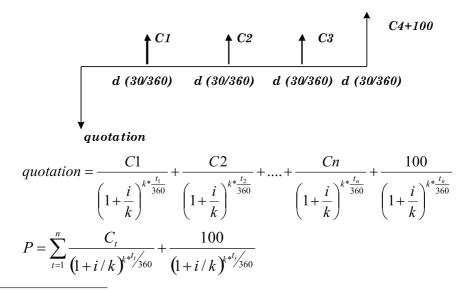
where

 c_{\circ} — semi-annual coupon

 c_a — annual coupon

The price of external debt securities can be based on many different discount rates. To that effect, a zero external curve in the currency of reference should be calculated (using the bootstrapping²⁰ method) and applied.

The global or BRL cash flow is shown below:



¹⁵ For details, see Annex 1 in this chapter.

¹⁶ The nominal rate is simply a rate expression format. Therefore, any calculation requires determining the security capitalization criterion so the nominal rate can be converted into an effective rate. The exception is the eurobond, which has an annual interest flow (annual capitalization). The rate determined on the screens is also an annual rate and as such does not require adjustments.

¹⁷ For historical reasons, the screen price of a security is often called the clean price. However, the screen price is nothing more than the security price minus the pro rata interest in the period (in 100 basis points or percentage points). In traditional securities, whose principal is a bullet (total payment at maturity date), the nominal value is equal to \$1,000 (dollars, euros or yens). As a result, the security's clean price is the screen price (in percentage points) multiplied by 1,000.

¹⁸ Interest accrued from the last interest payment to the date the operation was settled.

¹⁹ This adjustment will be shown in the calculation of the A-bond, a security with amortization.

²⁰ The bootstrapping technique determines the flow of a security as several single flows separately, i.e., each coupon payment is treated as if it were a zero-coupon bond.

where

C — interest payment considering the security price at 100%

i — nominal annual rate

k — payment frequency of the security coupon each year (as this security pays semi-annual coupons, k = 2, i.e., two interest coupons are paid each)

t – number of days in the 30/360 standard

With regard to eurobonds, the convention used to calculate the price differs from that applied to global bonds. Day counting is expressed in actual/actual terms and the value of coupons need not be calculated, since these are paid annually.

The euro cash flow is shown below:

$$C1 \qquad C2 \qquad C3$$

$$d(act/act) \qquad d(act/act) \qquad d(act/act) \qquad d(act/act)$$

$$quotation$$

$$quotation = \frac{C1}{\left(1 + \frac{i}{k}\right)^{k+1}} + \frac{C2}{\left(1 + \frac{i}{k}\right)^{k+2}} + \dots + \frac{Cn}{\left(1 + \frac{i}{k}\right)^{k+n}} + \frac{100}{\left(1 + \frac{i}{k}\right)^{k+n}}$$

$$P = \sum_{t=1}^{n} \frac{C_{t}}{\left(1 + i/k\right)^{k+t}} + \frac{100}{\left(1 + i/k\right)^{k+t}}$$

where

C – interest payment considering the security price at 100% (annual)

i — nominal annual rate

k — payment frequency of the security coupon each year (as this security pays annual coupons, k=1)

The A-bond, like other external securities, involves a simple capitalization of interest coupons and the 30/360-day counting standard. However, the A-bond has constant amortizations, which began on July 15, 2009. The table below shows the A-bond flow on December 31, 2008.

Table 3. A-Bond flow on December 31, 2008* (amounts in \$ million)

Dates	Interest coupon	Amortization	Total cash flow (IC + A)	Outstanding balance
01/15/2009	4		4	100
07/14/2009	4	5.56	9.56	94.44
01/15/2010	3.78	5.56	9.33	88.89
07/15/2010	3.56	5.56	9.11	83.33
01/15/2011	3.33	5.56	8.89	77.78
07/15/2011	3.11	5.56	8.67	72.22
01/15/2012	2.89	5.56	8.44	66.67
07/15/2012	2.67	5.56	8.22	61.11
01/15/2013	2.44	5.56	8.00	55.56
15/07/2013	2.22	5.56	7.78	50.00
01/15/2014	2.00	5.56	7.56	44.44
07/15/2014	1.78	5.56	7.33	38.89
01/15/2015	1.56	5.56	7.11	33.33
07/15/2015	1.33	5.56	6.89	27.78
01/15/2016	1.11	5.56	6.67	22.22
07/15/2016	0.89	5.56	6.44	16.67
01/15/2017	0.67	5.56	6.22	11.11
07/15/2017	0.44	5.56	6.00	5.56
01/15/2018	0.22	5.56	5.78	0

^{*} Maturity: 01/15/2018 Interest rate: 8%/year Amortization: 18 equal tranches starting on July 15, 2009

The outstanding balance is constructed according the formula below:

Outstanding Balance, = 100 - amortization

As already noted amortization payments began on July 15, 2009 and are based on the balance outstanding in the semester before they started. From the beginning of amortization to the security maturity date (January 15, 2018), 18 payments will be made. As a result, the amortization value is expressed by:

$$amortization = \frac{100}{18} = 5.56$$

The third column in the table represents the calculation of the A-bond coupon, expressed by the formula below:

$$Coupon = \frac{Interest}{2} * OB_{-1}$$

The fourth column represents the total cash flow the security pays periodically, expressed by the formula below:

$$TCF = amortization + coupon$$

4 Price formation

The pricing of bonds requires several inputs and calculations because local and external debt securities differ. While the general rule for domestic debt is that the value of public securities is based on domestic derivatives' market curves (DI future contracts), the curves of other sovereign issuers are usually the benchmark for Brazil's external debt.

4.1 Domestic debt

Yield rates of Brazilian domestic debt public securities are based on interest rate derivatives (DI future contracts), which is different from what occurs in most countries. In more developed markets, the yield curve of fixed-rate public securities is the benchmark for all other assets. In Brazil, however, this practice is only valid for comparing public bonds with private securities (such as debentures, for example).

In this sense, all nominal domestic rates are derived from the derivatives' markets, mainly DI future contracts, which are used to estimate LTN and NTN-F rates.

Trades of LTNs and NTN-Fs are normally calculated on "basis points" over DI future contracts (similar to SOT — spread over treasury for Global USD bonds). This occurs because of the following features: (a) The DI future contract is, in practice, a swap contract with a daily adjustment, i.e., investment in this asset requires no actual disbursement of resources. Also, no risk exists with the principal (notional value); rather, there is, only the mismatch of flows between the active and passive end of the contract; (b) the degree of liquidity, since contracts are negotiated at the BM&F (Brazilian Mercantile and Futures Exchange)²¹ and their liquidity exceeds that of public securities; (c) the risk factor - as they are negotiated at the BM&F and the counterpart risk is minimized. Due to the guarantees allocated and daily adjustments, the risk is similar to that of a public security which, by definition, is lower than that of private assets.

Defining the premium²² the investor expects is the starting point for calculating securities' yields. Investors know the premium they want to obtain on the DI future yield curve,²³ for example, and based on this estimate, the security rate is calculated and the equivalent price obtained. Market negotiations, however, occur at points over this benchmark asset (DI future). These points are determined by the difference between the yield rate calculated for the security and the DI future contract rate.

²¹ BM&F and the São Paulo Stock Exchange (Bovespa) have merged into BM&FBovespa.

²² Premium calculations are presented in Annex 2.

²³ Currently, fixed rate securities are traded in the form of "points on the ID", as a spread on the swap curve.

On the other hand, inflation-linked securities do not have a reasonable equivalent instrument in the derivatives market. IPCA x DI and IGP-M x DI swap contracts have low liquidity and therefore cannot serve as references for National Treasury bonds. However, shorter-term inflation-linked securities are closely related to the nominal yield curve because of the *break-even inflation*. Since the market calculates inflation based on a non-arbitrage relation, it can be removed from the nominal yield curve (DI future yield curve) and the difference indicates the real rate of inflation-linked securities. However, no benchmark exists for long-term nominal bonds (longer than 10 years). As a result, the price formation of inflation-linked bonds has no relation to any other market instrument.

The rationale presented here, as already mentioned, also applies to NTN-Cs. However, since the Treasury no longer issues these securities, current liquidity is low and this can distort their pricing.

4.1.1 DI future contracts

The One-Day Future Contracts on Interbank Deposit Certificates (DI future contracts) calculated and published by the BM&F, are currently the main assets in the futures market in terms of volume. ²⁴ These contracts use as a reference the average rates of one-day repurchase operations - without collaterals - traded among financial institutions and calculated by Cetip (Over the Counter - OTC Clearing House), known as *CDI Over*.

The object of negotiation (DI future contracts) is the effective interest rate of CDI over contracts, defined as the accumulation of average daily rates calculated by Cetip for the period between the date of the operation in the futures market (inclusive) and the date of the last negotiation (redemption date), exclusive.

The basic fixed-rate curve of the domestic financial market is calculated on DI future contracts (along all maturities), which are used as the main benchmark for pricing LTNs and NTN-Fs,²⁵ that have been traded with a premium on this curve.

4.2 External debt

External debt securities have as their benchmark the risk-free securities of the market where they are traded. As a result, the price of securities is based on two different pieces of information: The benchmark curve (risk-free)²⁶ of the market where the security was issued and the spread over benchmark, which represents the additional cost paid by Brazilian securities vis-à-vis the risk-free curve.

For these reasons, the different bond indicators (yield to maturity, duration, convexity, etc.) can be separated into these two factors, thus increasing the power to analyze the security, i.e., what derives from changes in the risk-free curve and what was caused by changes in the spread over benchmark.

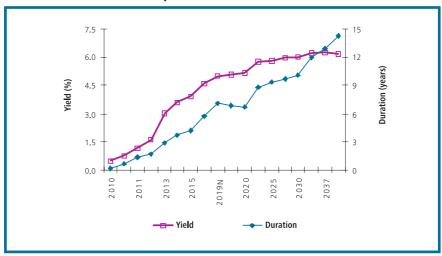
The fact is that over time, the risk-free curve itself begins to include distortions that arise from more recent security placements (on-the-run), which increase liquidity at the respective points on the curve, to the detriment of older securities (off-the-run). The analysis should therefore consider this distortion and possible effects on the yield curve of Brazilian securities. It is very common, for example, to have a spread versus duration factor besides the yield versus duration factor, as shown below.

²⁴ For details on the liquidity of ID contracts see Part III, Chapter 6.

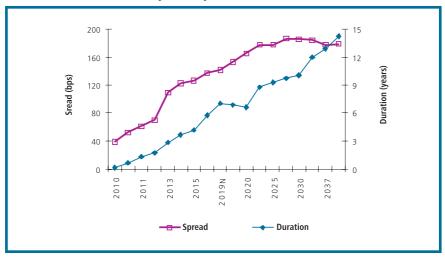
²⁵ Indirectly, NTN-Bs and NTN-Cs also use DI future contracts as a parameter.

²⁶ Exceptions to this rationale are euro-denominated securities. The curves of the German and French treasuries served as bench-

Graph 1. Yield X Duration



Graph 2. Spread X Duration

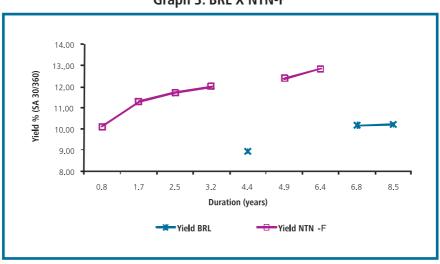


A Brazilian security can also be priced according to the benchmark zero curve, which is calculated using the bootstrapping method, or by applying another scheme. The bonds used to construct the curve are chosen from among the highest liquidity securities, thus eliminating possible distortions. Taking this curve into account, the spread over benchmark is added, thus allowing the Brazilian zero curve to be calculated. Another course is to calculate the Brazilian zero curve based on the most liquid Brazilian external debt securities in the market (and use the bootstrapping method or an alternate one), and then determine the spread over benchmark, based on the benchmark curve.

With regard to real-denominated securities (BRLs), it would appear that their yield should be related to the domestic fixed-rate curve, but until now, this has not occurred, partly due to the difference in the investor

marks for pricing. As time passed, the increasing liquidity of swap contracts in euros in the market turned this instrument into the benchmark for pricing sovereign bonds in that currency.

base: local investors can trade in both local and external markets in *reais* (BRL), while foreign investors may not. Although many already trade in the domestic market, ²⁷ many others, for legal or regulatory issues or even due to transaction costs, do not have access to it. In this regard, a foreign investor willing to invest in Brazil's public securities might prefer to invest in BRL global bonds, aware that the yield is lower than that of Brazilian Treasury bonds issued in the domestic market.



Graph 3. BRL X NTN-F

4.2.1 Spread-Over-Treasury (SOT) of external securities

SOT represents the additional cost paid by Brazilian securities (or of any other country or company) relative to the cost of risk-free bonds. A benchmark curve exists for each of the main markets, represented by the securities of the country offering the lowest risk for that market. For example, with global bonds, the parameter is provided by US Treasury bonds. With those issued in euros, the parameter is based on German Treasury bonds (and, at times, the French Treasury). In the samurai market, Japanese government bonds are termed risk-free.

The SOT can be calculated in different ways. The three most often applied are by the maturity date, duration and zero curve. The most common way is to subtract the YTM from that of a benchmark security maturing on a date closest to that of the security in question. The advantage of this method is its simplicity. However, it is not the most effective, since securities maturing close to each other could have very different durations, due to the value of the coupons and the yield rated.

Example:

The *spread* of the global 2027 can be determined based on the 30-year security of the US Treasury, with the most recent issuance date (on the run), as shown below:

Global 2027 YTM = 6.54% /year US Treasury 30y YTM = 5.20% /year Spread Over Treasury (SOT) = (6.54% - 5.20%)*100 = 134 basis points (bps)

²⁷ In December 2008, while external debt securities accounted for 7.2% of the FPD, the participation of non-residents in domestic debt securities was 6.5%.

A more precise and not too complex way to calculate the spread is based on its duration. The first step is to find the value of the benchmark curve at the point corresponding to that duration and then subtract it from the bond yield.

Table 4. The Global 2027 spread by duration

Maturity	Duration US Treasury bond (years)	US Treasury rate (% / year)
3 months	0.25	0.96
6 months	0.50	1.02
2 years	1.93	1.64
5 years	4.55	3.18
10 years	7.99	4.30
30 years	14.36	5.20

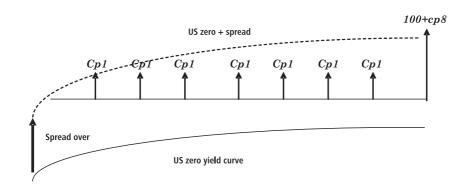
Duration of the global 2027 = 8.03 years.

Calculation of 10 year and 30 year treasuries: Result = 4.31% / year.

Spread = 6.54% - 4.31% = 223 bps.

The most precise, although more complex way to determine the spread of a security involves calculating each of its cash flows compared to the benchmark zero curve.

In this case, the spread corresponds to the parallel dislocation of the benchmark curve in such a way that the present value of the security (given by this dislocated curve) corresponds to its market value. As a result, each payment is brought to present value by the value of the benchmark zero curve for the corresponding maturity, plus the spread (which is the same for all points on the curve).



5 Conclusion

The chapter described the main instruments used by Federal Public Debt managers, offering details on pricing, characteristics and market conventions associated with these securities.

The methods by which to determine the spread were developed through a management process that sought to improve public debt performance by analyzing the impact that minor changes in calculation methods can have on the demand and liquidity of Treasury bonds. The Treasury, together with various market players, has improved both transparency and investors' safety. However, improvements are an ongoing process, and will continue to help the Treasury best meet its objectives.

Annex 1. Day-counting standards

1) The 30/360 standard considers 12-month years with 30-day months and applies to global bonds, BRLs and A-bonds.

$$d = (Y2 - Y1) x 360 + (M2 - M1) x 30 + (D2 - D1)$$

where

d — number of days between date 1 and date 2

D1 , M1 and Y1 — day, month and year relative to date 1 D2, M2 and Y2 — day, month and year relative to date 2

Where at least one of the two dates falls on the 31st of the month or in the last day of February, the following adjustment is made:

- Where D1 is on the 31st or the last day in February, its value will be 30;
- Where D2 is on the 31st and D1 the 31st, 30th or the last day in February, the D2 value will be 30;
- Where D2 is on the 31st and D1 is not on the 31st, 30th or the last day in February, the D2 value will be 1, M2 value will be that relative to the subsequent month and, if applicable, A2 that of the subsequent year;
- Where D2 is the last day in February, its value will be 30 if D1 is also the last day in February.

Example: Number of days between 07/04/2007-12/24/2007

$$d = (2007-2007) * 360 + (12-7)*30 + (24-04)$$

$$d = 170$$

$$30/360 = 0.472$$

2) The actual/actual standard also considers days and years by counting actual days. This method applies to euro bonds.

Example: number of days between 07/04/2007-12/24/2007

07/04/2007-12/24/2007 = 173 days 12/24/2007-12/24/2006 = 365 days actual/actual = 0,474 3) The actual/365 standard considers actual days and 365-day years. This method applies to samurai²⁸ bonds

Example: Number of days between 07/04/2007-12/24/2007

07/04/2007-12/24/2007 = 173 days

actual/365 = 0.474

4) The actual/360 standard considers actual days and 360-day years. It applies to floating rate securities.²⁹

Example: Number of days between 07/04/2007-12/24/2007

actual/360 = 0.481

5) Business days/252 (bd/252) considers business days to be 252 (excluding Brazil's holidays).

Example: Number of days between 07/04/2007-12/24/2007

07/04/2007-12/24/2007 = 119 business days

bd/252 = 0.472

In Excel, the number of business days can be obtained by using the *totalworkingdays* function, where the initial date is 07/04/2007 and the final date is 12/24/2007, minus one day. The calculation requires a list of holidays in the period.

²⁸ Leap years are not considered when calculating samurai bonds.

²⁹ Currently, Global 2009 is the only floating bond in the outstanding public debt.

Annex 2. Calculation of the public bond premium³⁰

1) LTNs' premium

premium =
$$\left\{ \underbrace{ \begin{bmatrix} 1+y \\ 1+i \end{bmatrix}^{(1/252)} - 1}_{1+i} \right\}$$

where

y — % rate / year of LTN maturing in n

i — % rate / year of future ID with the same maturity as the LTN

Example: What is the premium for the LTN maturing on 01/01/2009 and priced at 10.80% /year?

LTN (01/01/2009) = 10.80 % / year

DI(01/01/2009) = 10.75 % / year

premium =
$$\left\{ \begin{array}{l} (1+0,1080)^{(1/252)} - 1 \\ (1+0,1075)^{(1/252)} - 1 \end{array} \right\}$$

premium = 100.44% of CDI

LTN market negotiation is given in points over the respective maturity of DI future contracts, in this case 5 basis points (equivalent to 10.80% - 10.75%).

2) NTN-Fs' premium

$$\text{premium} = \begin{cases} \boxed{ 1 + NTN - F _rate ^{\binom{1/252}{1-252}} - 1 } \\ \boxed{ 1 + market _rate ^{\binom{1/252}{1-252}} - 1 }$$

where

 $NTN ext{-}F\ rate$ — NTN-F YTM based on the LTN yield curve

 $market\ rate$ — NTN-F YTM based on the DI Future yield curve

$$price = \frac{C1}{1+i} + \frac{C2}{(1+i)^2} + \dots + \frac{Cn}{(1+i)^n} + \frac{NV}{(1+i)^n}$$

$$price = \sum_{t=1}^{n} \frac{C_t}{(1+i)} + \frac{NV}{(1+i)}$$

³⁰ These data are hypothetical and do not reflect daily market rates.

where

C – interest coupon payment (semi-annual)

NV - R\$1,000

i - effective annual rate

To calculate the bond price, the NTN-F rate is based on the LTN yield curve. The market price is obtained when the cash flow is discounted using the DI Future yield curve.

Example: What is the premium for the NTN-F maturing on 01/01/2008, with an interest rate of 10% a year, traded on 07/03/2007, for settlement on 07/04/2007, based on the table below?

Table 5. Interest-rate curves for NTN-F 01/01/2008 premium calculation

Maturity	LTN curve on 07/03/2007 (year)	ID curve on 07/03/2007 (year)	BD (in relation to 07/04/2007)
01/01/08	11.23%	11.20%	124
07/01/08	10.94%	10.89%	247
01/01/09	10.80%	10.76%	378
07/01/09	10.84%	10.78%	500
01/01/10	10.88%	10.78%	628

Based on the table below, these are the UPs:

NTN-F price = 982,858400

 $Market \ price = 984,774676$

Once the prices are determined, it is possible to calculate the YTM:

NTN-F rate = 10.881% / year

 $Market\ rate = 10.786\%\ /\ year$

premium =
$$\left\{ \frac{1 + 0.10881^{(1/252)} - 1}{1 + 0.10786^{(1/252)} - 1} \right\}$$

premium = 100.84% of CDI

NTN-F trades will be 9.5 basis points, resulting from the difference between 10.881% and 10.786%.

3) LFTs premiums

premium =
$$\left\{ \frac{\left[(1+y)^{(1/252)} * (1+i)^{(1/252)} \right] - 1}{\left[(1+i)^{(1/252)} \right] - 1} \right\}$$

where

y - % rate / year of the LFT maturing in n

i — % rate / year of the DI future contracts interpolated to the same maturity date as the LFT.

Example: What is the premium for the LFT maturing on 06/07/2010 traded at 0.0006% / year?

$$LFT (06/07/2010) = -0.0006\% / year$$

$$DI(06/07/2010) = 10.80\% / year$$

$$\text{premium} = \begin{cases} \boxed{ (1 - 0.000006)^{(1/252)} * (1 + 0.1080)^{(1/252)} } - 1 \end{cases}$$

premium = 99.99%

Annex 3. Pricing examples

1) LTN

Maturity: 01/01/2009

YTM: 10.8036% (truncated at the 4th decimal place)

Date of trade: 07/03/2007

Business days between 01/01/2009 and 07/04/2007: 378

$$Price = \frac{1,000}{(1+0.108036)^{\frac{378}{252}}} = 857,371797 \text{ (truncated at the 6th decimal place)}$$

2) NTN-F

Maturity: 01/01/2010

Date of trade: 07/03/2007

Table 6. Interest-rate curve for NTN-F 01/01/2010 pricing calculation

LTN curve on 07/03/2007	Maturity	Rate (year)	BD (in relation to 07/04/2007)
LTN	01/01/2008	11.2300%	124
LTN	07/01/2008	10.9400%	247
LTN	01/01/2009	10.8000%	378
LTN	07/01/2009	10.8400%	500
LTN	01/01/2010	10.8800%	628

where

$$C = \underbrace{\left[(1+0.10)^{\frac{1}{2}} - 1 \right]}_{truncated \ at \ 8th \ decimal \ place} *1000 = \underbrace{48.808850}_{truncated \ at \ 6th \ decimal \ place}$$

$$Price = \underbrace{\frac{48.808850}{\left(1 + 0.1123\right)^{\frac{124}{252}}}}_{rounded_at_9th_decimal_place} + \dots + \underbrace{\frac{1,048.808850}{\left(1 + 0.1088\right)^{\frac{628}{252}}}}_{rounded_at_9th_decimal_place} = \underbrace{\frac{982.858400}{982.858400}}_{truncated_at_6th_decimal_place}$$

Given the NTN-F price, it is possible to calculate the YTM of the bond:

$$982.858400 = \frac{48.808850}{(1+YTM)^{\frac{124}{252}}} + \dots + \frac{1,048.808850}{(1+YTM)^{\frac{628}{252}}}$$

YTM: 10.936% / year

Note: Whether an NTN-F cash flow is discounted at different rates or at the equivalent YTM, the price will always be the same. In this case, the difference will be just the present value of each of the cash flows, but the total sum will be identical.

3) NTN-B³¹

Maturity: 01/05/2015

Interest: 6% /year

YTM: 6.5079% (truncated at the 6th decimal place)

Date of trade : 07/03/2007 Accumulated IPCA : 1.651293

$$coupon = \underbrace{\left(1.06\right)^{0.5} - 1}_{rounded_at_8th_decimal_place} *100$$

$$rounded_at_6th_decimal_place$$

$$ANV = 1,000 * \Delta IPCA = 1,651.293$$

$$Price = \underbrace{1.651,293}_{truncado_6^0 decimal} * \underbrace{0,978793}_{truncado_6^0 decimal} = \underbrace{1.616,275009}_{truncado_6^0 decimal}$$

4) LFT

Maturity: 06/07/2010

YTM: -0.0006% (truncated at the 6th decimal place)

Date of trade: 07/03/2007

Business days between 06/07/2010 and 07/04/2007: 763

Accumulated SELIC: 3.14455

³¹ Truncation rules for NTN-Cs and NTN-Ds are identical.

$$quotation = \frac{100}{(1-0.000006)^{\frac{763}{252}}} = 1.0002 \text{ (truncated at the 4th decimal place)}$$

$$ANV = 1,000 * \Delta selic = 3,144.55$$

$$Price = \underbrace{3,144.55}_{truncated_at_6th_decimal_place} * \underbrace{\frac{1.0002}{100}}_{truncate_at_6th_decimal_place} = 3,144.606695 \text{ (truncated at the 6th decimal place)}$$

5) BRL (global bonds follow the same calculation methodology)

Maturity: 01/05/2022

Interest coupon: 12.50%

YTM: 9.00%

Date of trade: 07/10/2007

Date of settlement $(T+3)^{32}$: 07/13/2007 Date last coupon payment: 07/05/2007

where

$$quotation = \frac{6.25}{\left(1 + \frac{0.09}{2}\right)^{2^*t_1}} + \dots + \frac{106.25}{\left(1 + \frac{0.09}{2}\right)^{2^*t_n}} = 128.29\%$$

$$Dirty_price = ANV * price = 1,282.89$$

Clean price = dirty price – pro rata interest rate³³

$$pro_rata$$
 interest = $\left[\frac{\text{int}}{100} * \frac{date\ of\ settlement\ -date\ payment\ last\ coupon}}{360}\right] * 1,000.00$

$$pro_rata \text{ interest} = \left[\frac{12.5}{100} * \frac{07/13/07 - 07/05/07}{360}\right] * 1,000.00 = 2.78$$

Clean price = 1,282.89 - 2.78 = 1,280.11

³² All calculations are based on the settlement date.

³³ Calculations for counting pro rata interest days should follow the bond's criteria. In the case of BRLs, the criterion is 30/360. As a result, in the pro rata formula described above, the difference should be calculated based on 30-day months.

6) Euro³⁴

Maturity: 09/24/2012

Interest coupon: 8.50%

YTM: 8.00%

Data of trade: 07/04/2007

Data of settlement (T+3)35: 07/09/2007

Date of last coupon payment: 09/24/2006

$$quotation = \frac{8.5}{(1+0.08)^{1}} + \dots + \frac{108.5}{(1+0.08)^{n}} = 108.7169\%$$

Dirty
$$price = ANV * price = 1,087.1686$$

$$pro-rata_coupon = \left[\frac{8.5}{100} * \frac{07/09/07 - 09/24/06}{07/09/07 - 07/09/06}\right] * 1,000.00 = 67.0685$$

 $Clean\ price = 1,086.7732 - 67.0685 = 1,020.1001$

A-bond³⁶

YTM: 7,00%

Date of trade: 07/04/2007

Data of settlement (D+3)37: 07/09/2007

Data payment last coupon: 01/15/2007

As mentioned, the A-bond has constant amortizations (18 tranches) starting on 07/15/2009. As a result, price³⁸ is calculated according to the formula below:

$$quotation = \frac{4}{\left(1 + \frac{0.07}{2}\right)^{2^*t_1}} + \frac{9.56}{\left(1 + \frac{0.07}{2}\right)^{2^*t_2}} + \dots + \frac{5.78}{\left(1 + \frac{0.07}{2}\right)^{2^*t_n}} = 1,087.19$$

³⁴ Attention must be paid when calculating the *pro rata* interest. Euro bonds follow the actual/actual criterion.

³⁵ All calculations are based on the settlement date.

³⁶ Attention must be paid when calculating the *pro rata* interest. The A-bond follows the 30/360 rule.

³⁷ All calculations are based on the settlement date.

³⁸ The flows presented in Section 3 can be used to facilitate the calculation.

$$Dirty_price = ANVprice = 1,087.19$$

$$pro-rata_interest = \left[\frac{8}{100} * \frac{07/09/07 - 01/15/07}{360}\right] * 1,000.00 = 38.67$$

 $Clean\ price = 1,087.19 - 38.67 = 1,048.53$

Annex 4. Summary of decree n° 3,859 of July 4, 2001

Table 7. Summary of decree n° 3,859 of July 4, 2001

Secu- rity	Maturity	Interest rate	Form of placement	Nominal value	Nominal value adjustment	Interest payment	Principal payment
NLI	Not specified	ı	Auction or direct issuance	Multiple of R\$1,000	1	1	At maturity
H	Not specified	I	Auction or direct issuance	Multiple of R\$1,000	SELIC Reference date: 07/01/00	1	At maturity
LFT-A	Up to 15 years	I	Direct issuance	Multiple of R\$1,000	SELIC + 0.0245%/ month Reference date: 07/01/00	ı	Up to 180 monthly tranches
LFT-B	Up to 15 years	I	Direct issuance	Multiple of R\$1,000	SELIC Reference date: 07/01/00	ı	At maturity
NTN-A1	Up to 16 years	6% / year	Direct (exchange into BIB)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as BIB
NTN-A3	Up to 27 years	6% / year	Direct (exchange into Par Bond)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as par bond
NTN-A4	Up to 27 years	Libor + 0.8125% / year.	Direct (exchange into Dis- count Bond)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as discount bond
NTN-A5	Up to 12 years	Libor + 0.8125% / year	Direct (exchange into Flirb)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as Flirb
NTN-A6	Up to 17 years	8% / year	Direct (exchange into C-Bond)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as C-bond
NTN-A7	Up to 15 years	Libor + 0.875% / year	Direct (exchange into DCB)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as DCB
NTN-A8	Up to 12 years	Libor + 0.875% / year	Direct (exchange into NMB)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as NMB
NTN-A9	Up to 9 years	Libor + 0.8125% / year	Direct (exchange into El Bond)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as ei bond
NTN-A10	Up to 9 years	Libor + 0.8125% / year	Direct (exchange into MYDFA)	Multiple of R\$1,000	US dollar	Every 15th of April and October	Same as MYDFA

į			1		N	40000401	
secu- rity	Matunty	וווופופאן ומופ	roilli oi piaceilleilt	Nominal Value	adjustment	payment	payment
NTN-B	Not specified	Not specified (usually 6%/year)	Auction or direct issuance	Multiple of R\$1,000	IPCA (Brazilian CPI) Reference date: 07/01/00	Semi-annually	At maturity
NTN-C	Not specified	Not specified (usually 6%/year)*	Auction or direct issuance	Multiple of R\$1,000	IGP-M (Brazilian Whose-Sale Price Index)	Semi-annually	At maturity
NTN-D	Not specified	Not specified	Auction or direct issuance	Multiple of R\$1,000	Reference date: 07/01/00 US dollar Reference date: 07/01/00	Semi-annually	At maturity
NTN-F	Not specified	Not specified (usually 10%/year)	Auction or direct issuance	Multiple of R\$1,000	ı	Semi-annually	At maturity
H-NLN	Not specified	Not specified	Auction or direct issuance	Multiple of R\$1,000	TR Reference date: 07/01/00	I	At maturity
I-NTN	Not specified	Not specified	Direct issuance (Proex)	Multiple of R\$1,000	US dollar Reference date: 07/01/00	At maturity	At maturity
MTN-M	15 years	Libor + 8.75% / year	Direct (New Money bond)	Multiple of R\$1,000	US dollar Reference date: 07/01/00	Semi-annually	17 semi-annual tranches
NTN-P	15 years minimum	6% / year	Direct issuance (Privatization Program)	Multiple of R\$1,000	TR Reference date: 07/01/00	At maturity	At maturity
NTN-R2	10 years	12% / year	Direct (closed government sponsored social security)	Multiple of R\$1,000	US dollar Reference date: 07/01/00	Monthly	10 annual tranches
CTN	20 years	I	Auction	Multiple of R\$1,000	IGPM Reference date: 07/01/00	I	At maturity
CFT-A	Not specified	Not specified	Direct issuance	Multiple of R\$1,000	IGP-DI	See table below	See table below
CFT-B CFT-C	Not specified Not specified	Not specified Not specified	Direct issuance Direct issuance	Multiple of R\$1,000 Multiple of R\$1,000	TR SELIC	See table below See table below	See table below See table below
CFT-D	Not specified	Not specified	Direct issuance	Multiple of R\$1,000	US dollar Reference date: 07/01/00	See table below	See table below
CFT-E	Not specified	Not specified	Direct issuance	Multiple of R\$1,000	IGP-M	See table below	See table below
CFT-G	Not specified	Not specified	Direct issuance	Multiple of R\$1,000	IPCA	See table below	See table below
CFT-H	Not specified	Not specified	Direct issuance	Multiple of R\$1,000	TJLP	See table below	See table below
CDP Not	Not specified	Not specified	Auction	Multiple of R\$1,000	TR	At maturity	At maturity

*except NTN-C 2031 - 12%/year.

CFTs can be issued in five different subseries, with the following general characteristics:

	CFT Subseries 1	CFT Subseries 2	CFT Subseries 3	CFT Subseries 4	CFT Subseries 5
Interest payment	At maturity	Annually	Semi-annually	Monthly	Periodically – on anniversary
					dates
Principal payment	At maturity	At maturity	At maturity	At maturity	Price table