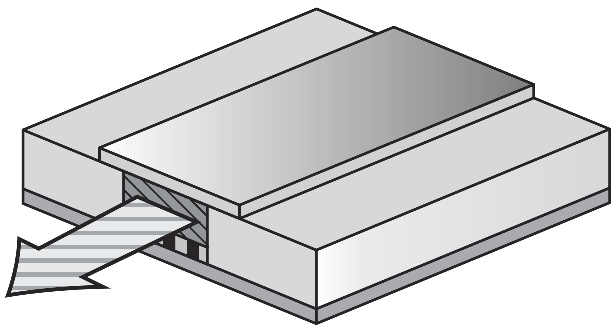
Optoelectronics and Lasers
Homework II



Filippos Tzimkas-Dakis
MSc. Photonics & Nanoelectronics
Faculty of Physics, University of Crete

October 30, 2020

Asymmetric Planar Dielectric Waveguide

In the present report we write the electromagnetic analysis for an Asymmetric Dielectric Planar Waveguide (ADPWG) in order to find the waveguided modes.Modes which are transverse electric (TE) and transverse magnetic (TM) with regards to their electric and magnetic components. Firstly, we follow the analysis of the waveguide depicted in Fig.1 and we end up to two equations, one for TE and one for TM, that shall be solved to find the effective transmission parameter β of every mode. Our notation and analysis is slightly different from the corresponding analysis in the Pochi Yeh's book, but is complete and fully understandable. Secondly, we implement our results on a computational example for specific values. Thirdly, in contrast to the Symmetric DPWG in ADPWG there is a cut-off wavelength for the TE_0 mode. Therefore, we make a thickness sweep over the waveguiding layer and we present the way that effective refractive index changes with respect to the specified thickness. At the end of the report, we prove the orthogonality between the TE_0 eigenmodes.

(a) Electromagnetic analysis

We write Maxwell's equations in a place without sources, or sources are at infinity, and assuming harmonic oscillation with $\exp(+j\omega t)$ in frequency domain as

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \tag{1a}$$
$$\nabla \times \mathbf{B} = j\omega \mathbf{D} \tag{1b}$$
$$\nabla \cdot \mathbf{D} = 0 \tag{1c}$$
$$\nabla \cdot \mathbf{B} = 0\,. \tag{1d}$$

The above equations are accompanied with the relations which connects the electric field with dielectric displacement and magnetic field with magnetic induction

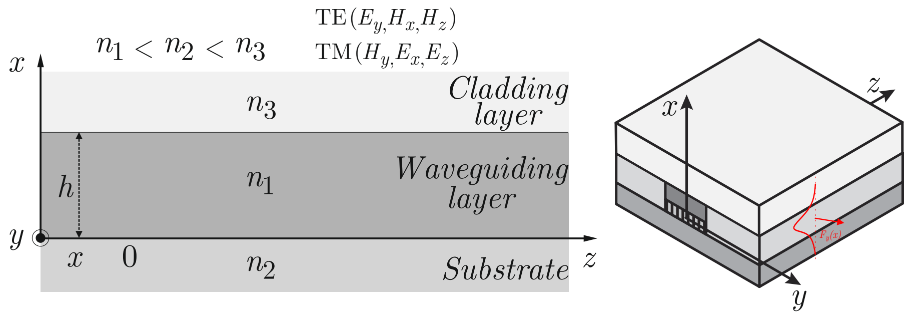
$$\mathbf{D} = \varepsilon \mathbf{E} \tag{2a}$$
$$\mathbf{B} = \mu \mathbf{H}\,. \tag{2b}$$

Also, it is known that dielectric constant $\varepsilon = \varepsilon_r \varepsilon_0$ is related to refractive index as $\varepsilon_r = n^2$. Regarding the magnetic constant $\mu = \mu_r \mu_0$, in our problem we can assume that is constant and $\mu_r = 1$ and it is actually consistent with the experimental results.

Taking the curl of the eq.(1a-b) and using the equations (2b) one shall create the wave equation in vector form (also known as Helmholtz equation) for the electric and magnetic field as

$$\nabla^2 \times \mathbf{E} - \omega^2 \mu\varepsilon \mathbf{E} = 0 \tag{3a}$$
$$\nabla^2 \times \mathbf{H} - \omega^2 \mu\varepsilon \mathbf{H} = 0\,, \tag{3b}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ the Laplacian operator. We shall solve these equations for the case of a planar dielectric waveguide with different refractive indices at the substrate and cladding layer, see Fig.1 .



(a) (b)

Figure 1: (a) Two-dimensional schematic of the Asymmetric Planar Dielectric Waveguide supporting TE and TM modes with transmission along z axis. (b) Three-dimensional representation of the APDWG with a typical feature of the localized TE_0 mode.

Transverse Electric modes (TE)

In this subsection we examine the case in which the electric field has only one component and especially along axis y, i.e. $\mathbf{E} = (0, E_y, 0)$ with $E_y \neq 0$. However, the afore mentioned component is a fucnction of x, $E_y = E_y(x)$, and that because there is a variation of the refractive index along this axis. Hence, from eq.(3a) we write

$$\nabla^2 E_{yi}\,(x,z) + n_i^2 k_0^2 E_{yi}\,(x,z) = 0\,, \quad i = 1,2,3 \qquad (4)$$
$$k_0 = \omega\sqrt{\mu_0 \varepsilon_0}.$$

We follow the well known method to assume solutions in the form of

$$E_{yi}(x,y) = E_i(x)\exp\left(-j\beta z\right), \qquad (5)$$

where total solution is a product of two coefficients which are functions of x and z, respectively. Specifically, an exponential with the transmission constant β which describes the change of phase while the field is transmitted along z axis and a function $E_i(x)$ (generally $E_i(x,y)$) which depicts the modes' profile.
We substitute eq.(5) to eq.(4) and we get

$$\frac{\partial^2 E_i(x)}{\partial x^2} + \left(n_i^2 k_0^2 - \beta^2\right) E_i(x) = 0\,, \; i = 1,2,3\,. \qquad (6)$$

At this point we shall assume that the field under discussion is retained mainly in the waveguiding layer with the mode profile we are about to reveal, while outside of this layer is decaying exponentially, i.e. in substrate and cladding layer. Hence, we write $E_{yi}(x)$ for every region along x axis

$$\text{WG layer } E_1(x) = A_1 \cos(\kappa x - \phi), \qquad \kappa = \sqrt{n_1^2 k_0^2 - \beta^2} \tag{7a}$$

$$\text{Substrate } E_2(x) = A_2 \exp(\gamma x), \qquad \gamma = \sqrt{\beta^2 - n_2^2 k_0^2} \tag{7b}$$

$$\text{Cladding } E_3(x) = A_3 \exp(-\delta(x-h)), \quad \delta = \sqrt{\beta^2 - n_3^2 k_0^2}. \tag{7c}$$

To determine the coefficients A_1, A_2, A_3, ϕ but also κ, γ, δ one needs to implement the boundary conditions. We demand the continuity of the tangental components of the electric and magnetic field at the borderlines $x = 0$ and $x = h$. From the eq.(1a) we get

$$\nabla \times \mathbf{E} = -j\omega\mu\,(\hat{x}H_x + \hat{z}H_z) \Longrightarrow$$

$$H_x = -\frac{\beta}{\omega\mu}E_y \tag{8a}$$

$$H_z = -j\frac{1}{\omega\mu}\frac{\partial E_y}{\partial x}\,. \tag{8b}$$

One can easily see that $H_x \propto E_y$ and $H_z \propto \partial E_y/\partial x$, thus the boundary conditions demand the continuity of E_y and $\partial E/\partial x$ at $x = 0,\, h$.

<u>Continuity at $x = 0$.</u>

$$\left.\begin{aligned} A_2 &= A_1 \cos(\phi) \\ \gamma A_2 &= \kappa A_1 \sin(\phi) \end{aligned}\right\} \Rightarrow \tan(\phi) = \frac{\gamma}{\kappa}\,. \tag{9}$$

<u>Continuity at $x = h$.</u>

$$\left.\begin{aligned} A_3 &= A_1 \cos(k \cdot h - \phi) \\ \delta A_2 &= \kappa A_1 \sin(\kappa \cdot h - \phi) \end{aligned}\right\} \Rightarrow \tan(\kappa \cdot h - \phi) = \frac{\delta}{\kappa}\,. \tag{10}$$

Combining the two equations above with the identity $\tan(a \pm b) = \frac{\tan(a) \pm \tan(b)}{1 \mp \tan(a)\tan(b)}$ we get

$$\tan(\kappa \cdot h) = \frac{\kappa(\gamma + \delta)}{\kappa^2 - \gamma\delta}. \tag{11}$$

Given that κ, γ, δ are functions of β, it is obvious that eq.(11) is an equation that engages transmission constant β, angular frequency ω and the waveguide characteristics (thickness h, and refractive indices). Also, eq.(11) is identical to equation 11.2-5 that we where asked to prove.

Transverse Magnetic modes (TM)

In this subsection we shall follow almost the same steps for the magnetic field and the eq.(3b), but we shall mention any differences when we meet them. The scheme now is that we magnetic field has only one component and it is parallel to axis y. Hence, we write eq.(3b) in the form

$$\nabla^2 H_{yi}(x,z) + n_i^2 k_0^2 H_{yi}(x,z) = 0\,, \;\; i = 1,2,3 \qquad (12)$$
$$k_0 = \omega\sqrt{\mu_0 \varepsilon_0}.$$

As in the previous case, we assume a product of two coefficients to be a solution in the form of

$$H_{yi}(x,y) = H_i(x)\exp(-j\beta z). \qquad (13)$$

Hence, eq.(12) takes the form

$$\frac{\partial^2 H_i(x)}{\partial x^2} + \left(n_i^2 k_0^2 - \beta^2\right) H_i(x) = 0\,, \;\; i = 1,2,3\,. \qquad (14)$$

For the same reasons we mentioned in the previous section, we want the field to be guided in the the waveguiding layer. Thus, we write the solution for each layer as

$$\text{WG layer } H_1(x) = A_1 \cos(\kappa x - \phi)\,, \qquad \kappa = \sqrt{n_1^2 k_0^2 - \beta^2} \qquad (15a)$$

$$\text{Substrate } H_2(x) = A_2 \exp(\gamma x), \qquad \gamma = \sqrt{\beta^2 - n_2^2 k_0^2} \qquad (15b)$$

$$\text{Cladding } H_3(x) = A_3 \exp(-\delta(x-h)), \quad \delta = \sqrt{\beta^2 - n_3^2 k_0^2}, \qquad (15c)$$

where A_1, A_2, A_3, are different from the former ones, but unless someone wants to calculate the profiles of the modes these amplitudes will not concern us. With regards to boundary conditions, the tangential components of electric E_z and magnetic field H_y must be continuous at $x = 0$ and $x = h$. From eq.(1b) we get

$$\nabla \times \mathbf{B} = -j\omega\varepsilon\left(\hat{x}E_x + \hat{z}E_z\right) \Longrightarrow \qquad (16a)$$

$$E_{xi}(x) = \frac{\beta}{\omega\varepsilon_0}\frac{1}{n_i^2} H_y(x) \qquad (16b)$$

$$E_{zi}(x) = j\frac{1}{\omega\varepsilon_0 n_i^2}\frac{\partial H_y}{\partial x}. \qquad (16c)$$

Therefore, we demand the continuity of $E_{zi} \propto \frac{1}{n_i^2}\partial H_y/\partial x$ and $H \propto H_y$ at points $x = 0$ and $x = h$.

<u>Continuity at $x = 0$.</u>

$$\left.(E_{z2}{=}E_{z1})\right|_{x=0} \quad \Rightarrow \quad \left.\begin{aligned} A_2 &= A_1\cos(\phi) \\ \frac{\gamma}{n_2^2}A_2 &= \frac{\kappa}{n_1^2}A_1\sin(\phi) \end{aligned}\right\} \Rightarrow \tan(\phi) = \frac{\gamma'}{\kappa}\,. \qquad (17)$$

Continuity at $x = h$.

$$(E_{z3}{=}E_{z1})\big|_{x=h} \quad \Rightarrow \quad \left.\begin{aligned} A_3 &= A_1 \cos(k \cdot h - \phi) \\ \frac{\delta}{n_3^2} A_2 &= \frac{\kappa}{n_1^2} A_1 \sin(\kappa \cdot h - \phi) \end{aligned}\right\} \Rightarrow \tan(\kappa \cdot h - \phi) = \frac{\delta'}{\kappa}\,, \tag{18}$$

where $\gamma' = \gamma(n_1^2/n_2^2)$ and $\delta' = \delta(n_1^2/n_3^2)$. Now, we use the same identity for $\tan(x)$ and we get

$$\tan(\kappa \cdot h) = \frac{\kappa(\gamma' + \delta')}{\kappa^2 - \gamma'\delta'}. \tag{19}$$

The solutions of the above equation returns the effective transmission parameters for every TM mode. Also, we have ended up to the relation we had to prove.

(b)

Next we shall solve the transcendental equation of the TE modes, i.e. (11), for an APDWG with characteristics (see Fig.(1)):

- wavelength $\lambda_0 = 350\,\text{nm}$
- thickness $h = 500\,\text{nm}$
- refractive indices $n_1 = 2.6$, $n_2 = 2.4$, $n_3 = 1$

We solved this equation graphically and we reveal the results in Fig.(2) and in Table 1 for better precision. Specifically, we calculated the effective transmission constant β_{eff} and also the effective refractive index n_{eff}. Moreover, we found out that for the given parameters we have three TE modes which one feels different transmission constant. Obviously, we see that $\beta_{TE_0} > \beta_{TE_1} > \beta_{TE_2}$.

Table 1: Gathered results for TE modes of APDWG. This table contains all the information needed to answer (b) and (c).

Figure 2(a)	Transmission constant (rad/μm)	Effective Refractive index	Minimum thickness (nm)
TE_0	46.569	2.5829	64
TE_1	45.714	2.5301	242
TE_2	43.953	2.4442	413

For the sake sake of completeness, in Fig.3 we present the profiles of the three guided modes. More specifically, we see three different types of profiles with $(i+1),\ i = 0, 1, 2$ nodes and the way they decay outside of the waveguiding

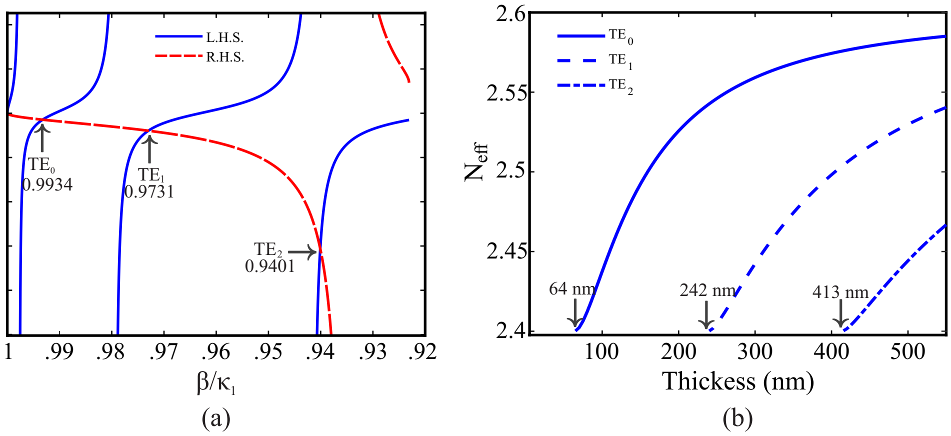


Figure 2: (a) Graphic solution for the eq.(11), where with blue (solid) line is the L.H.S. and with red (dashed) line is the R.H.S. In the x-axis is the normalized quantity β/κ_1. Also, the poionts of intersction are lebeled inside the figure. (b) Representation of the effective refractive index for every guided mode versus different thickness h. In contrast to the SPDWG, TE$_0$ in APDWG generates for thickness greater than 64 nm.

layer. The higher the mode is the slower it decays outside the guiding layer.

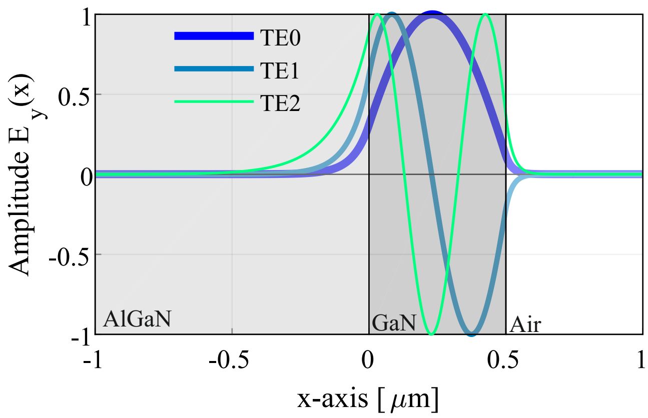


Figure 3: Mode profiles for the asymmetric planar/slab dielectric waveguide. Modes depicted: TE$_0$ blue(bold) line, TE$_1$ light-blue (solid) line and TE$_2$ cyan (thin) line. Lower ordered modes are more localized in the waveguiding material, i.e. the material with the higher refractive index.

(c)

Although we calculated, numerically the minimum thickness needed for at least one mode at this point we find the lower thickness via the analytical way. As we know, effective transmission constant β_{eff} satisfies the inequality

$$\beta_2 \leqslant \beta_{\text{eff}} \leqslant \beta_1\,, \tag{20}$$

where $\beta_{1,2}$ the transmission constants in materials with the two higher refractive indices, and a mode can be guided at the limit where $\beta_{\text{eff}} = \beta_2$. This is the lower limit, because for $\beta\text{eff} < \beta_1$ we have an imaginary part which forces the mode to decay exponentially.
Hence,

$$\beta = \beta_2 \Rightarrow \tan\left(k_0 h\sqrt{n_1^2 - n_2^2}\right) = \frac{k_0^2\sqrt{n_1^2 - n_2^2}\sqrt{n_2^2 - n_3^2}}{k_0^2\sqrt{n_1^2 - n_2^2}} \Rightarrow$$

$$h = \frac{1}{k_0\sqrt{n_1^2 - n_2^2}}\left(m\pi + \tan^{-1}\left(\frac{n_2^2 - n_3^2}{n_1^2 - n_2^2}\right)\right). \tag{21}$$

Therefore, for our parameters we get

$$h = 63.58\,\text{nm}\,. \tag{22}$$

A result that verifies the thickness we found through the computational way, see Table 1.

(d)
Last but not least we have to prove the orthogonality between the eigenmodes of the above mentioned eigenvalue problem. We begin from the general form of eq.(4) for two different TE modes we write

$$\nabla_t^2 \mathbf{E}_1 + \left(k_0^2 n^2(x,y) - \beta_1^2\right)\mathbf{E}_1 = 0 \tag{23a}$$
$$\nabla_t^2 \mathbf{E}_2 + \left(k_0^2 n^2(x,y) - \beta_2^2\right)\mathbf{E}_2 = 0\,, \tag{23b}$$

where $\beta_{1,2}$ the transmission constant of each mode and $\nabla_t^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ the Laplacian operator on the transverse plane. We have to mention that field vectors refer to the transverse plane. We multiply the above equations with $\mathbf{E}_2$ and $\mathbf{E}_1$ respectively and we subtract them to get

$$\mathbf{E}_2\nabla_t^2\mathbf{E}_1 - \mathbf{E}_1\nabla_t^2\mathbf{E}_2 = \left(\beta_1^2 - \beta_2^2\right)\mathbf{E}_1\cdot\mathbf{E}_2 \Rightarrow$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}_2\nabla_t^2\mathbf{E}_1 - \mathbf{E}_1\nabla_t^2\mathbf{E}_2\,\mathrm{d}x\mathrm{d}y = \left(\beta_1^2 - \beta_2^2\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}_1\cdot\mathbf{E}_2\,\mathrm{d}x\mathrm{d}y \overset{\star}{\rightarrow}$$
$$\lim_{C\to\infty}\oint_C\left(\mathbf{E}_2\cdot\nabla_t\right)\mathbf{E}_1 - \left(\mathbf{E}_1\cdot\nabla_t\mathbf{E}_2\right)\mathrm{d}\bar{\ell} = \left(\beta_1^2 - \beta_2^2\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}_1\cdot\mathbf{E}_2\,\mathrm{d}x\mathrm{d}y \rightarrow$$
$$\left(\beta_1^2 - \beta_2^2\right)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{E}_1\cdot\mathbf{E}_2\,\mathrm{d}x\mathrm{d}y = 0\,. \tag{24}$$

At the last step we used the fact that the fields are localized inside the waveguide layer, as we shown above, and they decays exponentially at infinity. Thus the contour integral is equal to zero. Also, at $\star$ we used the relation below accompanied with Gauss' theorem,

$$\mathbf{E}_2 \nabla_t^2 \mathbf{E}_1 - \mathbf{E}_1 \nabla_t^2 \mathbf{E}_2 = \nabla_t \left(\left(\mathbf{E}_1 \cdot \nabla_t \right) \mathbf{E}_2 - \left(\mathbf{E}_2 \cdot \nabla_t \right) \mathbf{E}_1 \right) , \tag{25}$$

because as we can see from eq.(1b)

$$\nabla_t \cdot \mathbf{E} = \nabla \cdot \nabla \times \mathbf{H} = 0, \tag{26}$$

because in TE modes electric field $\mathbf{E}$ does not have a componet along z axis. Gathering all the above information we shall conclude that eq.(24) leads to

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{E}_1 \cdot \mathbf{E}_2 \, \mathrm{d}x \mathrm{d} = 0, \quad \text{when } \beta_1 \neq \beta_2. \tag{27}$$

Consequently, two different eigenmodes are orthogonal to each other and, thus, if we normalize them they can form and orthonormal basis in the Hilbert space. Obviously, if $\beta_1 = \beta_2$ we get the zero result from the parenthesis outside the integral, while the integral returns a finite value.

Code

In this section we attach the MATLAB code that has been used to produce the above mentioned results. For comments, mistakes, clarifications etc. please send a message to dakisfilippos@gmail.com.

```
% Filippos Tzimkas-Dakis  UoC October 2020
% Homework 2 ------- Optoelectronics and Lasers
% This is a script to calculate the waveguided eigenmodes TM of
    a field in
% a dielectric slab waveguide.
% Also, this script calculates all the transmision constants \
   beta for
% specific characteristics (refractive indexes n1,n2,n3, width/
   thickness
%h, frequency or wave length f,\lambda).
%% Solving for TE modes
close all
clear all
clc
c  = 3*10^8; % speed of light in vacuum
n1 = 2.6;     % refractive index of the waveguide layer / core
n2 = 2.4;       % refractive inde of the substrate
n3 = 1;     % refractive inde of the cladding layer

d       = 500*1e-9;     % thickness of the eaveguide layer (
   meters)
```

```
lamda0 = 350*1e-9;      % wavelength in vacuum
k0     = 2*pi/lamda0;   % wave-number in vacuum
k1     = n1 *k0;        % wave-number in core
k2     = n2 *k0;        % wave-number in substrate
k3     = n3 *k0;        % wave-number in cladding layer

xi = (n2^2-n3^2)/(n1^2-n2^2); % APDWG "asymmetry" factor (0== SPDWG)
V = k0*d/2*sqrt( n1^2-n2^2 ); % V-number, same definition as for SPDWG
VasosTE = atan( sqrt(xi) )/2; % V-number "asymmetry-offset" for TE-modes
TE_modes = ceil( (V-VasosTE)/(pi/2) ); % Number of modes expected

%========== The waveguide looks like this ==========
%|                                       |  y?
%|      n3 , k3                          |
% ---------------------------------------  x?--> z
%|      n1, k1 , thickness d             |
%|                                       |
% ---------------------------------------
%|      n2 , k2                          |
%|                                       |
%===================================================

b = linspace(k1,max(k2,k3),2000); % creates the \beta vector
% this loop calculates only the cases where the R.H.S. is real

kapa  = sqrt(k1^2 - b.^2); % variable of the Charecteristic Equation
gama  = sqrt(b.^2 - k2^2); % variabla of the Ch.Eq.
delta = sqrt(b.^2 - k3^2); % variabla of the Ch.Eq.

% L.H.S of equation (15)
f1 = tan(kapa*d);
% R.H.S of equation (15)
f2 = (kapa.*(gama + delta))./(kapa.^2 - delta.*gama);
%% Finding the TE modes
% Plots the two parts of the Ch.Eq. in order to find the specific points of
% intersection
figure
hold on
plot(b/k1,f1/max(f2),'Color','blue','Linewidth',1.5) % L.H.S.
plot(b/k1,f2/max(f2),'Color','red','Linewidth',1.5)  % R.H.S.
xlabel('\beta /\kappa_1')
ylabel('A.U.')
legend('L.H.S.','R.H.S.')
title('TE modes')
set(gca, 'XDir','reverse')
%%
% solver's options
options = optimset('Display','off');
```

```
% Characteristic equation
fun1 = @(t)tan(sqrt(k1^2 - t^2)*d) - (sqrt(k1^2 - t^2)*...
    (sqrt(t^2 - k2^2) + sqrt(t^2 - k3^2)))/(k1^2 - t^2 - sqrt(t^2 - k3^2)*sqrt(t^2 - k2^2));
neff = NaN*ones(1,TE_modes);
% Calculates the infinities of Tan(kappa*d)
for  n = 0:TE_modes-1
    k(n+1) = sqrt(k1^2 - ((pi/2/d)^2)*(2*n + 1)^2);
    neff(n+1) = fsolve(fun1,k(n+1)*(0.99999),options)/k0;
end
% Prints the results. Effective Refr. Index
% and transmission constant
fprintf('\nThe results from the above figure are \n \n')
for i = 1:TE_modes
    fprintf('****TE%d n_eff = %6.4f   beta = %6.2f (rad/um)****\n',...
        i-1,neff(i),neff(i)*k0*10^-6);
end
fprintf('\n')

%% Calculates N_eff of every guided mode
di  = (550:-5:1)*1e-9;          % span of thickness to be tested
xi = (n2^2-n3^2)/(n1^2-n2^2);  % APDWG "asymmetry" factor (0==SPDWG)
V  = k0*max(di)/2*sqrt( n1^2-n2^2 ); % V-number, same definition as for SPDWG
VasosTE = atan( sqrt(xi) )/2;  % V-number "asymmetry-offset" for TE-modes
TE_modes = ceil( (V-VasosTE)/(pi/2) ); % Number of modes expected
out = NaN*ones(TE_modes,length(di)); % initialization

for u = 1:length(di)
    d = di(u);         % thickness tested
    V  = k0*d/2*sqrt( n1^2-n2^2 ); % V-number, same definition as for SPDWG
    TEmodes = ceil( (V-VasosTE)/(pi/2) ); % Number of modes expected
    % solver's options
    options = optimset('Display','off');
    % Characteristic equation to be solved for any d
    fun1 = @(t)tan(sqrt(k1^2 - t^2)*d*k1_) - (sqrt(k1^2 - t^2)*...
        (sqrt(t^2 - k2^2) + sqrt(t^2 - k3^2)))/(k1^2 - t^2 - sqrt(t^2 - k3^2)*sqrt(t^2 - k2^2));
    % Calculate the inifities of Tan(kappa * d)
    for  n = 0:TEmodes-1
        k(n+1) = sqrt(k1_^2 - ((pi/2/d)^2)*(2*n + 1)^2)/k1_;
    end
    % Solves the Ch.Eq. and calculates transmission constant \beta for
    % every guided TE mode. Also, a recursive method is used when is needed
    for n = 1:TEmodes
```

```
        ii = -2; % computational parameter
        out(n,u) = fsolve(fun1,k(n)*(0.99999),options);
        if (~isreal(out(n,u)) || out(n,u)<=k2 || out(n,u)>=k1)
            zz  = 1;% computational parameter
            flag = 0;% computational parameter
            while ((out(n,u)<=k2 || out(n,u)>=k1 || flag<1e-3)
    && ii<=-1)
                switch zz
                    case 1,
                        if (u==1)
                            out(n,u) = real(out(n,u));
                        elseif(isreal(out(n,u-1)))
                            out(n,u) = fsolve(fun1,out(n,u-1)
    *0.999,options);
                        else
                            out(n,u) = fsolve(fun1,k(n)
    *(1+0.1*10^ii),options);
                        end
                    case 2,
                        flag = out(n,u);
                        if(isreal(out(n,u)))
                            out(n,u)= (out(n,u) + fsolve(fun1,
    out(n,u)*(1+10*eps),options))/2;
                        else
                            out(n,u)= (out(n,u) + fsolve(fun1,k
    (n)*(1+0.1*10^ii),options))/2;
                        end
                        flag = abs(out(n,u) - flag);
                end
                zz = 2;% computational parameter
                ii= ii + 0.1;% computational parameter
            end
        end
    end
end
% Corrects the computational faults, if something went wrong.
% Ussually, this happens when the thickness step is too small
for n = 1:TE_modes
    for u = 2:length(di)-1
        if( ~isnan(out(n,u-1)*out(n,u+1)))
            if out(n,u)<out(n,u+1)
                mid = real((out(n,u-1) + out(n,u+1)))/2;
                out(n,u) = mid;
            end
        end
    end
end
n_eff = out*k1_/k0; % effective refractive index for every
    guided mode

%% Plot the results    N_eff vs Thickness d
figure
hold on; axis on; box on;
% TE_0
```

```
plot(di*10^9,real(n_eff(1,:)),'Color','blue','LineWidth',2);
% TE_1
plot(di*10^9,real(n_eff(2,:)),'Color','blue','LineWidth',2,'
    LineStyle','--');
% TE_2
plot(di*10^9,real(n_eff(3,:)),'Color','blue','Linewidth',2,'
    LineStyle','-.');

xlabel('Thickess (nm)')
ylabel('N_{eff}')
legend('TE_0','TE_1','TE_2')
ylim([n2*0.999 n1])
xlim([min(di) max(di)]*10^9)
```