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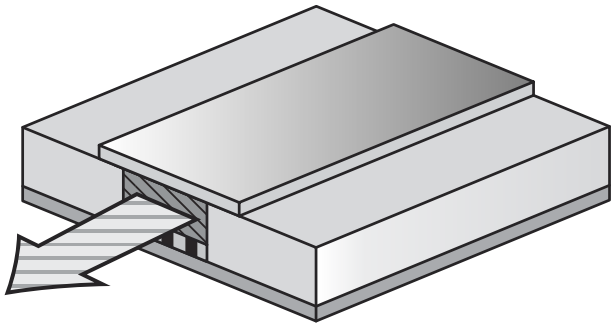
Semiconductor Optoelectronic Devices
Final Exam



January 5, 2021

Exercise 1

Tunable diode laser

A tunable three-section DBR as in Fig. 1 is constructed to operate near 1.55 µm from InGaAsP/InP materials. Above threshold, the wavelength is tuned by changing the effective indices in the phase and DBR passive sections by injecting current. For no current injection, the operating wavelength is $1.57\,\mu$m, the effective index in all sections is 3.4, $\partial\bar{n}/\partial N = 10^{-21}\,\mathrm{cm}^3$, $\eta_i = 70\%$, and the carrier lifetime is independent of carrier density and equals 3 ns in all sections. The waveguide cross section in all regions is $0.2 \times 3\mu\mathrm{m}^2$; the gain, phase shift, and grating regions are each $200\,\mu$m long; and the grating has a reflectivity per unit length of 100 cm^{-1}. The other mirror is a cleaved facet. Plot the wavelength vs. current to the grating:

(a) Assume no current is applied to the phase shift region and show at least three axial mode jumps.

(b) Repeat for a phase shift current sufficient to maintain operation at the grating's Bragg wavelength.

(c) In (b) also plot the required phase shift current on the opposite axis. Stop plots when any current reaches 50 mA.

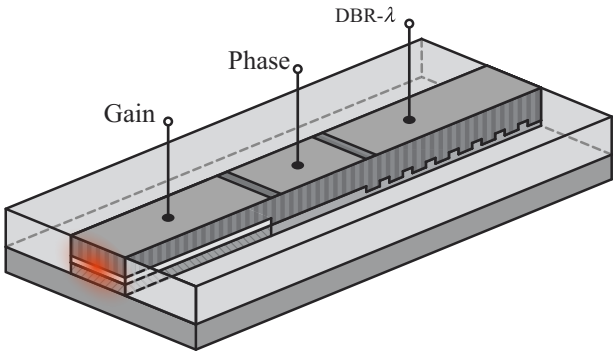


Figure 1: Schematic of a tunable single-frequency three-section DBR laser. The laser consists of a gain, a phase and a DBR mirror section. Each section has its electrode with which one can apply three different currents.

Solution

As a first step we shall calculate the effective length L_{eff} of the Bragg reflector at the one end,

$$L_{eff} = \frac{1}{2\kappa}\tanh\left(\kappa L_g\right) = \frac{0.964}{2 \cdot 100\,\mathrm{cm}^{-1}} = 48.2\,\mu\mathrm{m}\,. \tag{1}$$

Secondly, now can calculate the spacing of the modes $\mathrm{d}\lambda$, using the equation below

$$\mathrm{d}\lambda = \frac{\lambda_0^2}{2\bar{n}_g L} = \frac{\lambda_0^2}{2\bar{n}_g \left(L_a + L_p + L_{eff}\right)} = \frac{1.570^2}{2 \cdot 3.8 \cdot (200 + 200 + 48.2)} \rightarrow$$
$$\mathrm{d}\lambda = 0.723\,\mathrm{nm}\,, \tag{2}$$

where L_a, L_p, L_{eff} are the lengths of gain, passive and Bragg sections. The reader may mentioned a factor of 2 in the denominator which comes from the fact that a full round trip, inside a linear cavity, is the distance that a photon has to propagate starting from the one end and back to it. If we had a ring laser the total round trip would have been equal to its perimeter, thus without the factor of 2. Next, we write the way that the effective index is affected due to the applied current at each electrode,

$$\Delta\bar{n} = \frac{\partial\bar{n}}{\partial N}\frac{\eta_i \tau}{qV} I\,, \tag{3}$$

where η_i the quantum efficiency, τ the carrier lifetime, q the electron charge, V the physical volume of the section and I the applied current. Using our parameters we get

$$\Delta\bar{n} = 109.2 \cdot I\,, \tag{4}$$

with I in mA units. Now we are ready to proceed and answer the questions.

(a)
For this case we need the shift of the center wavelength of the grating, which moves in direct proportion to the index according to Eq.(4). More specifically,

$$\Delta\lambda_{\mathrm{g}} = \lambda_{\mathrm{g}}\frac{\Delta\bar{n}_{\mathrm{DBR}}}{\bar{n}_{\mathrm{DBR}}} = \frac{109.2 \cdot 1570}{3.4} I_{\mathrm{g}}(\mathrm{mA})\,, \tag{5}$$

where $\Delta\lambda_g$ is given in nm and I_{g} the electric current applied to Bragg grating. Also, the cavity modes will shift due to the applied current in the DBR section and this shift is determined via

$$\Delta\lambda_m = \frac{\lambda_m \left(\Delta\bar{n}_a L_a + \Delta\bar{n}_p L_p + \Delta\bar{n}_{\mathrm{DBR}} L_{eff}\right)}{\bar{n}_{ga} L_a + \bar{n}_{gp} L_p + \bar{n}_{g\mathrm{DBR}} L_{eff}}\,. \tag{6}$$

However, in our case we apply current only in the Bragg section, thus the above equation reduces to

$$\Delta\lambda_m = \frac{\lambda_m \Delta\bar{n}_{\mathrm{DBR}} L_{eff}}{\bar{n}_{ga} L_a + \bar{n}_{gp} L_p + \bar{n}_{g\mathrm{DBR}} L_{eff}} = \frac{\lambda_m \Delta\bar{n}_{\mathrm{DBR}} L_{eff}}{3.8\left(L_a + L_p + L_{eff}\right)}\,, \tag{7}$$

where $\lambda_m = 1570\,\text{nm}$ and Δn_{DBR} given by Eq.(4). After all the above calculations we can finally find the lasing mode via the equation below

$$\lambda_{\text{lasing}} = \lambda_0 + \Delta\lambda_m + \left[\frac{\Delta\lambda_{\text{g}} - \Delta\lambda_m}{\text{d}\lambda}\right]\text{d}\lambda\,, \tag{8}$$

where $\Delta\lambda_m$ is given from Eq.(7) and the square brackets $[\cdot]$ denote rounding of the inside number to the nearest integer. We choose a reasonable range for the grating current, like $I_{\text{g}} \in [0, 60]\,\text{mA}$, and we depict the result of Eq.(8) in Fig. 2(a). As we expected, there is a mode hopping due to the shift of the Bragg grating reflectivity. There is also a slight slope in the "steps" because there is a small shift in the cavity modes as stated in Eq.(7). Furthermore, in Fig. 2(b) we show the way that $\Delta\lambda_{\text{g}}$ and $\Delta\lambda_m$ evolve with the applied current. Mode hopping occurs because the wavelength shift of the Bragg grating is much greater than the shift in the cavity modes. This plot will also help us understand what we shall change in order to achieve tunability without mode hopping asked in question (b).

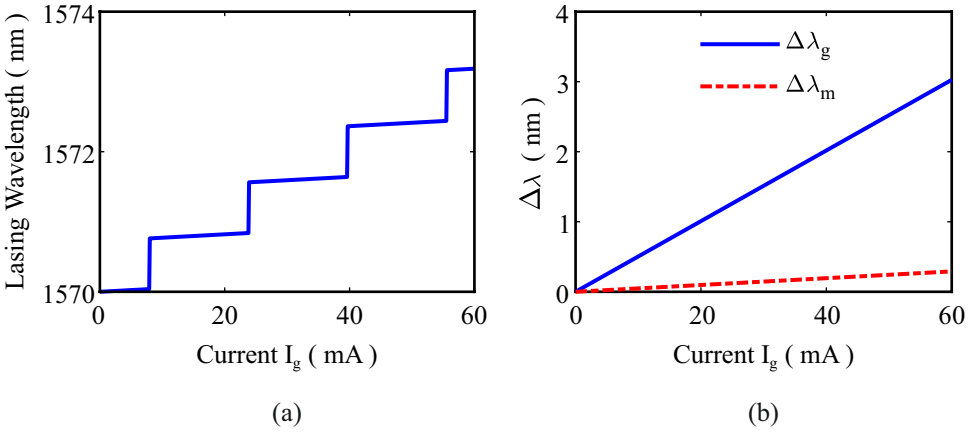


Figure 2: (a) Lasing wavelength of the versus the induced current I_{g} revealing the expected mode hoping between the cavity modes. (b) Wavelength shift of Bragg grating (blue) and cavity modes (red) due to the applied current I_{g}.

(b)

Now we shall try to cancel this mode hopping by applying a current at the phase section, as depicted in Fig. 1. In order to achieve this we must insert a current I_p such that the third order in Eq.(8) rounds to zero. In other words, the two lines in Fig. 2(b) have to be identical. In this regime Bragg shifting $\Delta\lambda_{\text{g}}$ remains the same, as in Eq.(5), while the cavity mode shift takes the form

$$\Delta\lambda'_m = \frac{\lambda_m\left(\Delta\bar{n}_p L_p + \Delta\bar{n}_{\text{DBR}} L_{eff}\right)}{\bar{n}_{ga} L_a + \bar{n}_{gp} L_p + \bar{n}_{g\text{DBR}} L_{eff}}\,, \tag{9}$$

where $\Delta\bar{n}_p$ is given by Eq.(4) times a factor α, which is about to be calculated right now. To calculate this coefficient we have to solve the equation below

$$\frac{\Delta\lambda_{\rm g} - \Delta\lambda'_m}{{\rm d}\lambda} = 0 \rightarrow \Delta\lambda_{\rm g} - \Delta\lambda'_m = 0 \xrightarrow[(5)]{(9)}$$
$$I_p = \left(\frac{n_g}{nL_p}\left(L_{eff} + L_p + L_a\right) - \frac{L_{eff}}{L_p}\right) I_{\rm g} \rightarrow$$
$$I_p = 2.2636 \cdot I_{\rm g}\,. \tag{10}$$

Hence, the new lasing wavelength is given by the equation

$$\lambda'_{\rm lasing} = \lambda_0 + \Delta\lambda'_m\,. \tag{11}$$

The resulting graph is given in Fig. 3(a) and reveals that the second current I_p helps the cavity modes to follow the pace of the Bragg shift. As we stated above the two shifts, $\Delta\lambda_{\rm g}$ and $\Delta\lambda'_m$, should be identical and this is shown in Fig. 3(b). Consequently, canceling the diference between the two shifts we can achieve a tunable DBR laser without mode hopping.

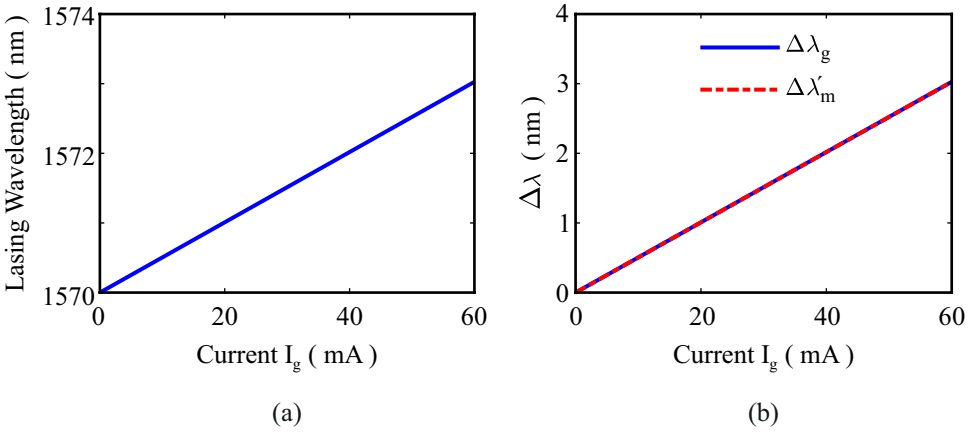


Figure 3: (a) Lasing wavelength of the versus the induced current $I_{\rm g}$ revealing linear tunability without mode hopping. (b) Wavelength shift of Bragg grating (blue) and the new shift of cavity modes (red) due to the applied current I_p. Although $I_{\rm g}$ is on x-axis, $\Delta\lambda'_m$ evolves with I_p as given in Eq.(10).

(c)

At this point we shall plot (a) and (b) questions accompanied by the current I_p. Fig. 4 depicts the three curves and stops when phase current reaches the 50 mA. The last figure mentions the great significance of the phase current, because except for it the laser would have not be able to emit light in any wavelength.

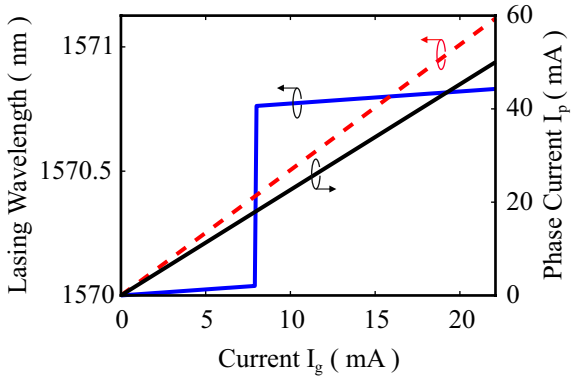


Figure 4: The two types of lasing(left axis), mode hopping (blue) and tunable (red), and the phase current I_p (black-right axis) versus Bragg current I_g. The graph stops when phase current reaches 50 mA.

Exercise 2

Reflectivity of a DBR mirror

Consider a dielectric DBR mirror consisting of Al_2O_3 (n = 1.7) and HfO_2 ($n = 2.1$) quarter-wave stacks, designed to be centered at 600nm. Estimate the number of periods needed to reach reflectivity values of 99% at 600 nm. For this number of periods, calculate and plot the reflectivity spectrum versus wavelength, neglecting absorption effects, and showing at least two minima on either side of the central maximum.

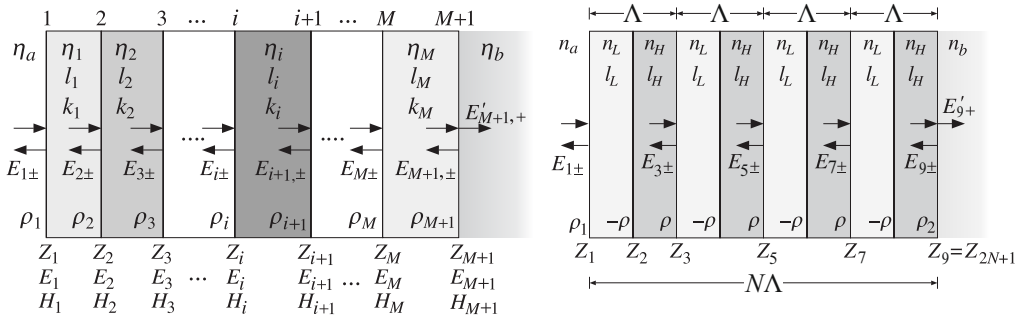


Figure 5: (a) General multilayer dielectric slab structure with different characteristics. (b) Eight-layer, or four-period, dielectric Bragg reflector. In both cases we assumed material with refractive index n_a from the left side of the mirror and material with n_b from the right side with infinite width.

Solution

Before we proceed to solution, it seems reasonable to develop some of the formulae which are about to be used afterwards. However, we shall clarify that we will not focus on strict proofs because this is not the meaning of this exercise. As it is shown in Fig. 5(a) every dielectric layer has its characteristics such as: characteristic impedance Z_i, refractive index n_i, width l_i, wave-number k_i, etc. Thus, if we have a stack of dielectric layers one after the other we should define some other coefficients. The elementary reflection coefficients ρ_i from the left of each interface are defined in terms of the characteristic impedances or refractive indices as:

$$\rho_i = \frac{\eta_i - \eta_{i-1}}{\eta_i + \eta_{i-1}} = \frac{n_{i-1} - n_i}{n_{i-1} + n_i}\,, \qquad i = 1, 2, 3, \ldots, M+1\,, \tag{12}$$

where $\eta_i = \eta_0/n_i$, and we use the convention $n_0 = n_a$ and $n_{M+1} = n_b$. We can also write the forward E_{i+} and the backward E_{i-} electric fields, but there is no need for this in this exercise. As we know from the applied electromagnetism we can define the reflection responses, assuming propagation from left to right, at every layer as

$$\Gamma_i = \frac{E_{i-}}{E_{i+}} \rightarrow \Gamma_i = \frac{\rho_i + \Gamma_{i+1}\mathrm{e}^{-2jk_il_i}}{1 + \rho_i\Gamma_{i+1}\mathrm{e}^{-2jk_il_i}} \qquad i = M, M-1, \ldots, 1\,, \tag{13}$$

where $\Gamma_{M+1}\rho_{M+1}$ and in order to get the second expression we used the transfer, or ABCD, matrices and some algebra. We can also define the impedances $Z_i = E_i/H_i$ at any layer as

$$Z_i = \eta_i \frac{Z_{i+1} + j\eta_i \tan(k_il_i)}{\eta_i + jZ_{i+1}\tan(k_il_i)} \qquad i = M, M-1, \ldots, 1\,, \tag{14}$$

and initialized by $Z_{M+1} = \eta_b$. The objective of all these recursions is to obtain the overall reflection response Γ_1 into medium η_a. A very well known technique in microwave technology and transmission lines.

The above process was general and we only need to use these formulae in a specific case where we only have two different materials with refractive indices n_H, n_L and widths $\lambda/4$, respectively. In this special case of $\lambda/4$ widths the impedances in the center wavelength λ_0 take the form

$$Z_1 = \frac{\eta_L^2}{Z_2} = \frac{\eta_L^2}{\eta_H^2/Z_3} = \frac{\eta_L^2}{\eta_H^2}Z_3 = \ldots \left(\frac{\eta_L^2}{\eta_H^2}\right)^M Z_{M+1} = \left(\frac{\eta_L^2}{\eta_H^2}\right)^{2N}\eta_b\,, \tag{15}$$

where M the number of layers and N the number of periods. Thus, now we calculate the reflection response at $\lambda 0$ as

$$\Gamma_1 = \frac{Z_1 - \eta_a}{Z_1 + \eta_a} = \frac{n_a\left(\frac{n_H}{n_L}\right)^{2N} - n_b}{n_a\left(\frac{n_H}{n_L}\right)^{2N} + n_b}\,, \tag{16}$$

where it is obvious that for $N \to \infty$ we get $\Gamma_1 = 1$ and thus 100% reflectivity. If we had chosen a bilayer with first the layer with high index n_H we would have gotten $\Gamma_1 \to -1$, something that does not make any difference at all. However, we shall mention that Γ is connected with the electric field, so the reflectivity will be given by the relation

$$R = |r|^2 \equiv |\Gamma_1|^2 \,. \tag{17}$$

In our exercise the least reflectivity in λ_0 is $R \geq 0.99$, so from Eq.(16) we find that

$$M \geq 15 \qquad \text{or} \qquad N \geq 30 \,, \tag{18}$$

where we assumed that the reflector is enclosed in air, hence $n_a = n_b = 1$. We continue by calculating the recursive relation Eq.(14) for range of different wavelengths around $\lambda_0 = 600\,\text{nm}$. Then we calculate Γ_1 from the well-known relation

$$\Gamma_1 = \frac{Z_1(\lambda) - \eta_a}{Z_1(\lambda) + \eta_a} \,, \tag{19}$$

with $Z_1(\lambda)$ being a function of wavelength (or frequency). The results of the last equation are depicted in Fig. 6. We have also calculated an extra case of materials $n_a = 1.5$ and $n_b = 3.1$ whose results are shown in Fig. 7. The last figure reveals the fact that the reclectivity in the other wavelengths depends a lot on the materials η_a and η_b, something that is purely logical and expected, but we will not discuss it here.

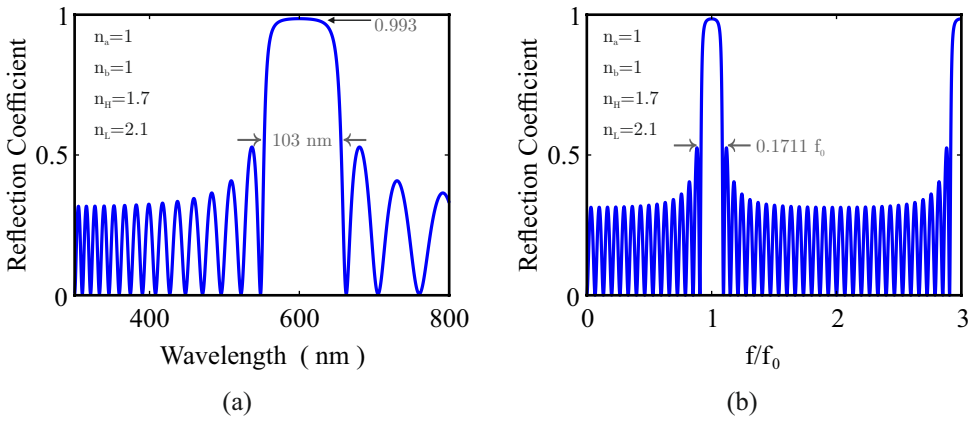


Figure 6: Graphs (a) and (b) depict the Bragg reflector in wavelength and frequency respectively. We assumed the following refractive indices: $n_a = 1\,, n_b = 1\,, n_L = 1.7\,, n_H = 2.1\,$. Also, for reflectivity over 99% we used $M = 15$ periods and thus obtaining $R_{max} = 99.3\,\%$.As it is expected the Bragg mirror is periodic in frequency (and wavelength) and has the same bandwidth. This periodicity is obvious in graph (b), where it could be also shown in (a) but the figure would become even smaller.

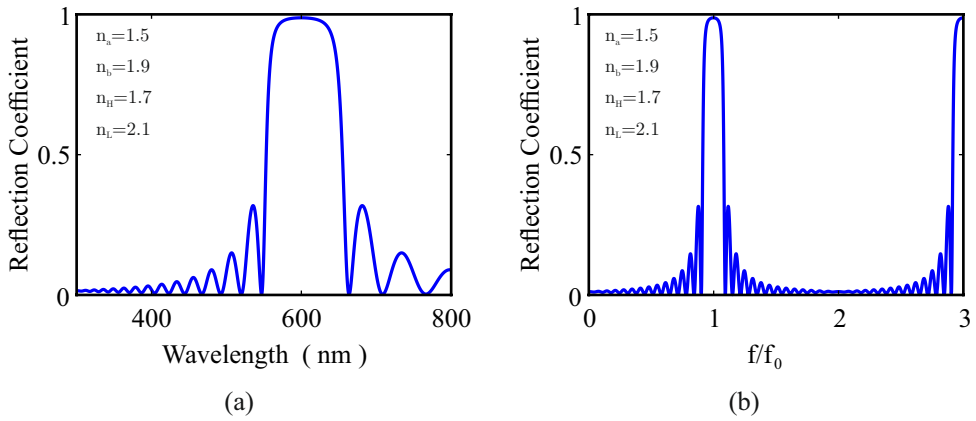


Figure 7: Graphs (a) and (b) depict the Bragg reflector in wavelength and frequency respectively. We assumed the following refractive indices: $n_a = 1.5\,, n_b = 1.9\,, n_L = 1.7\,, n_H = 2.1\,.$ Also, for reflectivity over 99% we used $M = 15$ periods and thus obtaining $R_{max} = 99.1\,\%$. As it was expected, reflectivity away from the central wavelength depends also on the materials η_a ans η_b and more specifically on their differences with the bi-layer materials η_H and η_L.

(* We would like to mention that the above results have also been tested for their validity with online software. For more information check Prof. Sophocles J. Orphanidis.*)

Exercise 3

Gain of a quantum well

Calculate and plot the TE and TM optical gain spectra of a 14 nm GaAs quantum well with infinite barriers, for five different values of sheet carrier densities between 10^{12} and $10^{13} cm^{-2}$. For each carrier density, estimate the respective quasi-Fermi levels. Consider only the first two electron, heavy-hole and light-hole levels. Produce two plots, one for TE and one for TM polarization. In the plots, make sure to indicate the $\pm g_{max}(E)$ curves in dashed lines and mark by arrows the relevant optical transitions. Operating temperature T=300K. Ignore line-shape broadening.

Solution

If we assume the initial state $|i\rangle$ in the valence band and the final state $|f\rangle$

in the conduction band we can write them as

$$|i\rangle = u_v\left(\vec{r}\right)\frac{\exp\left(j\vec{k}_h\vec{r}\right)}{\sqrt{L_xL_y}}\phi_n\left(z\right) \tag{20a}$$

$$|f\rangle = u_c\left(\vec{r}\right)\frac{\exp\left(j\vec{k}_e\vec{r}\right)}{\sqrt{L_xL_y}}\psi_m\left(z\right)\,, \tag{20b}$$

where $u_{v,c}(\vec{r})$ if the periodic Bloch function, $\vec{k}_{h,e}$ the momentum in the $x-y$ plane, $\vec{r}$ the 2D position vector and $\phi_n\,,\psi_m$ the eigenfunctions of the infinite quantum well for holes and electrons, respectively. The last functions are given

$$\phi_n(z) = \sqrt{\frac{2}{L_z}}\sin\left(\frac{n\pi z}{L_z}\right) \tag{21a}$$

$$\psi_m(z) = \sqrt{\frac{2}{L_z}}\sin\left(\frac{m\pi z}{L_z}\right). \tag{21b}$$

These two equations reveals the transitions that can take place in our problem. Due to the orthogonality between them the only transitions we will take care of are those who satisfy $n = m$. So, there will be two (2) transitions between electrons and heavy holes and 2 (two) transitions for electrons and light holes. We continue by writing the corresponding energies of the above mentioned wavefunctions,

$$E_i = E_{h,n} - \frac{\hbar^2k_h^2}{2m_h^*} \tag{22a}$$

$$E_f = E_{e,m} + E_g + \frac{\hbar^2k_e^2}{2m_e^*}\,, \tag{22b}$$

where $E_{h,n}\,,E_{e,m}$ are the energy eigenvalues of the infinite quantum wells

$$E_{h,n} = -\frac{\hbar^2\pi^2n^2}{2m_h^*L_z^2} \tag{23a}$$

$$E_{e,m} = \frac{\hbar^2\pi^2m^2}{2m_e^*L_z^2}\,, \tag{23b}$$

where $m_e^* = 0.067m_0$, $m_{hh}^* = 0.45m_0$, $m_{lh}^* = 0.082m_0$ and $L_z = 14\,\text{nm}$ the wells width. In the above equations we have taken the zero point energy at the valence band, thus $E_v = 0$, $E_c = E_g$, with E_g being the energy gap between the two band, for our case $E_g = 1.42\,\text{eV}$ (GaAs energy bandgap). At this point we define the reduced densities of states in 2 dimensions, which

we are about to use soon, as

$$\rho_{rh}^{2D} = \frac{m_{rh}}{\pi\hbar L_z}, \quad \text{with} \quad m_{rh} = \frac{m_e m_{hh}}{m_e + m_{hh}} \tag{24a}$$

$$\rho_{rl}^{2D} = \frac{m_{rl}}{\pi\hbar L_z}, \quad \text{with} \quad m_{rl} = \frac{m_e m_{lh}}{m_e + m_{lh}} \tag{24b}$$

$$\tag{24c}$$

where the first one refers to the e-hh and the second to the e-lh transition.Also, m_{rh} and m_{rl} are the respective reduced masses. At this point we can write the general function which predicts the gain of the quantum well

$$\begin{aligned} g\left(E_{21}\right) &= \sum\sum g_{21}^{\text{sub}}(n_c, n_v) = \\ &= \sum\sum g_{max}\left(E_{21}\right)\left(f_c - f_v\right)) = \\ &= \sum\sum \frac{\pi q^2\hbar}{n\epsilon_0 c m_0^2}\frac{1}{E_{21}}|M_T\left(E_{21}\right)|^2\rho_{rj}\left(E_{21}\right)\Theta\left(E_{21} - E_{e,m}^{h,n}\right)\left(f_c - f_v\right) \\ &= \sum\sum \frac{\pi q^2\hbar}{2n\epsilon_0 c m_0^2}\frac{2|M_0|^2}{m_0}\frac{|M_T\left(E_{21}\right)|^2}{|M_0|^2}\frac{1}{E_{21}}\rho_{rj}\left(E_{21}\right)\times \\ &\qquad \times\Theta\left(E_{21} - E_{e,m}^{h,n}\right)\left(f_c - f_v\right), \end{aligned} \tag{25}$$

where q the electron charge, $\hbar$ the reduced Planck's constant, $n = 3.52$(not to be confused with the quantum number n of the states and energies in Valance band) the refractive index of GaAs, $c = 3 \cdot 10^8$ the speed of light in vacuum, ϵ_0 the permitivity of vacuum and m_0 the static mass of electron. Additionally, the second order is a constant of every material and in GaAs is given as $2|M_0|^2/m_0 \simeq 29\,\text{eV}$, while the third order depends on the transition (e-hh or e-lh) and on the polarization of the incoming electromagnetic wave (TE & TM). The values of this parameter are given in Table 1. Also, Θ represents the multiple heaviside function and its roots are discussed few lines below. Our first goal is to find the g_{max} curve which is somewhat the

Table 1: Magnitude of $|M_T|^2/|M_0|^2$ in Quantum well ($k_t \sim 0$) for Different Transitions and Polarizations

Polarization	C–HH	C–LH
TE	1/2	1/6
TM	0	2/3

envelope of the possible gains. The only thing left is to find the "roots" of this function, these roots sign the spots where g_{max} has to change value in a abrupt way, like a step function. These "roots" are gathered in Table 2. Now we have anything we need to calculate the maximum gain for both TE

and TM polarizations. The results of these calculations are depicted in Fig. 8, in graphs (a) and (b) respectively.

The last order of Eq.(25) represents the difference between the two Fermi-

Table 2: Energies for transitions C-HH and C-LH (GaAs: $E_g = 1.42$ eV)

Energy "Root"	C–HH	C–LH
(1)	1.453013 eV	1.472146 eV
(2)	1.551708 eV	1.628242 eV

Dirac distributions, which have occurred due to the injected carries N. These carriers (and holes) have been displaced from equilibrium and thus creating two quasi-Fermi levels, one for each band. These two functions have the forms

$$f_c\left(E_e\right) = \frac{1}{1+\mathrm{e}^{\frac{E_e - F_c}{kT_B}}} \tag{26a}$$

$$f_v\left(E_h\right) = \frac{1}{1+\mathrm{e}^{\frac{E_h - F_v}{kT_B}}}\,, \tag{26b}$$

where F_c, F_v are the quasi-Fermi energy levels of each band, k_B the Boltzmann's constant and T the temperature. In our discussion we assume room temperature and thus $k_B T = 0.0259\, eV$. In order to proceed we shall change the variables such that

$$E_e\,, E_h \longrightarrow E = \frac{\hbar^2 k^2}{2m_r^*}\,, \tag{27}$$

and if we further assume that there is no broadening shaping (we have a delta function) we can write

$$E = E_{21} - E_{e,m}^{h,n} = \frac{\hbar^2 k^2}{2m_r^*} \longrightarrow k = \sqrt{\frac{2m_r^*}{\hbar^2}\left(E_{21} - E_{e,m}^{h,n}\right)}\,. \tag{28}$$

After these intermediate steps we shall write the energies $E_e\,, E_h$ in a more practical (for our calculations) form

$$E_h = E_{h,n} - \frac{\hbar^2 k^2}{2m_h^*} = E_{h,n} - \left(E_{21} - E_{e,m}^{h,n}\right)\frac{m_r^*}{m_h^*} \tag{29a}$$

$$E_e = E_g + E_{e,m} + \frac{\hbar^2 k^2}{2m_e^*} = E_g + E_{e,m} + \left(E_{21} - E_{e,m}^{h,n}\right)\frac{m_r^*}{m_e^*}\,, \tag{29b}$$

where $E_{h,n}\,, E_{e,m}$ are given by Eq.(23b). Finally, we can write the Fermi distribution we are about to use in our calculations

$$f_v\left(E_{21}\right) = \frac{1}{1+\exp\left(\dfrac{E_{h,n} - \left(E_{21} - E_{e,m}^{h,n}\right)\frac{m_r^*}{m_h^*} - F_v}{k_B T}\right)} \tag{30a}$$

$$f_c\left(E_{21}\right) = \frac{1}{1+\exp\left(\dfrac{E_g + E_{e,m} - \left(E_{21} - E_{e,m}^{h,n}\right)\frac{m_r^*}{m_e^*} - F_c}{k_B T}\right)}\,. \tag{30b}$$

To find quasi-Fermi energy level for conduction band we have to solve this equation

$$N = \int \mathrm{d}E \rho_e^{2D} \Theta\left(E - E_{e,m}\right) f_c^m\left(E\right)\,, \tag{31}$$

which can be transformed into[1]

$$N = \rho_e^{2D} k_B T \left[\ln\left(\mathrm{e}^{\frac{\mathrm{E_{e,1}-F_c}}{\mathrm{k_B T}}} + 1\right) - \frac{E_{e,1} - F_c}{k_B T} + \ln\left(\mathrm{e}^{\frac{\mathrm{E_{e,2}-F_c}}{\mathrm{k_B T}}} + 1\right) - \frac{E_{e,2} - F_c}{k_B T}\right]. \tag{32}$$

With the same way we can calculate the quasi-Fermi level for the valence band with the only difference that there will be 4 (four) terms (one logarithmic and one fraction in each term) and two (2) density of states (one for heavy and one for light holes). This is due to the fact that we have heavy holes and light holes and we take into account 2 states of the quantum well. More specifically one has to calculate the quasi fermi levels as many times as the number of different injected carriers they want to test. We take 6 (six) different values for carriers N in the range $N \in [1, 11] \cdot 10^{12}\,\mathrm{cm}^{-2}$ and the results are gathered in Table 3.

The results for different energies E_{21} (or optical wavelengths) are given in Fig. 8, both for TE and TM modes. We shall mention that TM modes have only C-LH transitions because the coefficient $|M_T|^2/|M_0|^2$ is zero for the transition C-HH as pointed in Table 1. Furthermore, we shall mention that all the gain curves are between the expected envelope which denotes the maximum gain, staircase like curve over zero, and the maximum losses, staircase like curve under zero. Also, for wavelengths longer than 1.4486 ev ($E_g = 1.42\,\mathrm{eV}$) the medium is transparent as we expected

[1] we used that

$$\int \mathrm{d}x \frac{1}{1+\mathrm{e}^x} = -\ln\left(1+\mathrm{e}^{-x}\right)$$

Table 3: Quasi-Fermi energies for Conduction and Valence bands

N (P) 10^{12} (cm^{-2})	F_c (eV)	F_v (eV)
1	1.474	0.050
3	1.536	0.018
5	1.579	0.002
7	1.616	-0.009
9	1.652	-0.018
11	1.688	-0.026

from the theory. The steps in TE modes gain have the following sequence E_{hh1} , E_{lh1} , E_{hh2} , E_{lh2} while for TM modes E_{lh1} , E_{lh2} (see Table 2). Fig. 8 reveals that there should be a minimum density of carriers in order for the quantum well to produce gain. For example, if $N = 1 \cdot 10^{12}$ there is only loss for all the wavelengths, while for higher values of N we see that there is a range of wavelengths that can feel gain. Last but not least, maximum gain envelope is projected on every curve showing with that way that if we increase a lot the injected carriers we will reach this delicate curve.

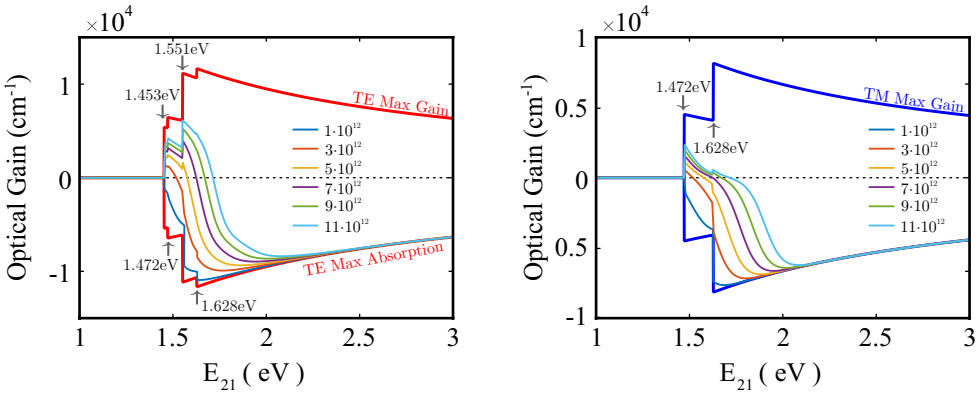


Figure 8: These graphs depict the gain spectra for a GaAs quantum well with width $L_z = 14\,\text{nm}$. (a) Depicts the gain for TE modes and (b) the gain for TM modes. In (a) the maximum gain curve (red) has four steps because of the involvement of both the HH and LH, while in (b) the maximum gain curve (blue) has two steps because only the LH contibute to this transition.The higher the carries density is the bigger range of wavenlengths can be amplified.

(* Although we can not "debug" or check the consistency of our code with benchmark problems, the obtained results seem to be in logical agreement with similar cases found in bibliography. *)

Code

In this section we attach the MATLAB code that has been developed to produce the above mentioned results. For comments, mistakes, clarifications etc. please send a message to dakisfilippos@gmail.com.

```
% Filippos Tzimkas-Dakis  UoC December 2020
% Final Project ------- Optoelectronics and Lasers
% Exercise 1 Tunable DBR Laser
%%
close all
clear all
clc
tic

lambda0 = 1570*10^-9;  % center WL (meters)
n       = 3.4;         % refractive index
ng      = 3.8;         % group refractive index (Coldrens Table 2.1 pg 59)
hi      = 0.7;         % quantum efficiency
tau     = 3*10^(-9);   % carrier life time (sec)
WG_cs   = 0.2*3*10^(-12); % waveguide cross section (m2);
L_a     = 200*10^(-6);    % active medium length (m)
L_p     = 200*10^(-6);    % passive medium length (m)
L_g     = 200*10^(-6);    % Bragg grating length (m)
k       = 100/(10^-2);    % reflectivity/(m)
m2      = (ng -1)/(ng+1); % cleaved facet reflectvity
n_H     = 3.40;           % reflective index of InGaAsP @ 1.550
n_L     = 3.17;           % reflective index of InP @ 1.550
q       = 1.602*10^(-19); % electron charge Coulomb
dndN    = 10^(-21);       % cm^3
dndN    = dndN *10^(-6);  % m^3

% r     = (n_H - n_L)/(n_H + n_L); % reflectivity between two layers
LAMDA = lambda0/(2*n);          % n*LAMDA = lambda0 !
m     = L_g/LAMDA;              % number of periods
Leff  = (1/2/k)*tanh(k*L_g);    % effective WL Coldrens (3.63)
% Leff_ = LAMDA/4/r;            % Approximation for high reflectivity
dl0   = (lambda0^2)/(2*ng*(L_a + L_p + Leff)); % mode spacing (m)
Zg    = dndN * hi*tau/(q*WG_cs*L_g);  % everything from (3.77) except from I
                                      % 1/Ampere
Zp    = dndN * hi*tau/(q*WG_cs*L_p);  % same here

```

```
Ig        = (0:0.0001:0.06);   % Bragg current (Ampere)
Delta_lg = lambda0*Zg*Ig/n;   % Bragg WL shift (nm)
%-------------------------------------------------
% mode shifting due to el. current in Bragg area
Delta_lm = (Delta_lg*n)*Leff/(ng *(L_a + L_p + Leff));
% New WL of lasing (only bragg current)
Delta_lasing1 = Delta_lm + round((Delta_lg - Delta_lm)/dl0)*dl0
    ;
% -------------------------------------------------
%-------------------------------------------------
%%
Ip             = Ig*2.2636;         % Phase current (Ampere)
Delta_lp       = lambda0*Zp*Ip/n;   % Phase WL shift (nm)
% mode shifting due to el. current in Bragg and Passive areas
Delta_lm2      = (Delta_lp*n*L_p +Delta_lg*n*Leff)/(ng *(L_a +
    L_p + Leff));
% New WL of lasing (Bragg and Phase current)
Delta_lasing2 = Delta_lm2 + round((Delta_lg - Delta_lm2)/dl0)*
    dl0;
%-------------------------------------------------
toc
%% Plotting results
figure
subplot(2,2,1)
plot(Ig*10^3,(lambda0 + Delta_lasing1)*10^9,'LineWidth',1.5,'
    Color','blue')
xlabel('Current I_g ( mA )')
ylabel('Lasing Wavelength ( nm )')
title('Only I_g')
hold off

subplot(2,2,2)
plot(Ig*10^3,Delta_lg*10^9,'LineWidth',1.5,'Color','blue')
hold on
plot(Ig*10^3,Delta_lm*10^9,'LineWidth',1.5,...
    'LineStyle','-.','Color','r')
xlabel('Current I_g ( mA )')
ylabel('\Delta _\lambda ( nm )')
legend('\Delta\lambda _g','\Delta\lambda_m')
title('Only I_g')
hold off

subplot(2,2,3)
plot(Ig*10^3,(lambda0 + Delta_lasing2)*10^9,'LineWidth',1.5,'
    Color','blue')
xlabel('Current I_g ( mA )')
ylabel('Lasing Wavelength ( nm )')
title('I_g and I_p')
hold off

subplot(2,2,4)
plot(Ig*10^3,Delta_lg*10^9,'LineWidth',1.5,'Color','blue')
ylabel('\Delta _\lambda ( nm )')
xlabel('Current I_g ( mA )')
```

```
hold on
plot(Ig*10^3,Delta_lm2*10^9,'LineWidth',1.5,...
    'LineStyle','-.','Color','r')
legend('\Delta\lambda _g','\Delta\lambda_m')
title('I_g and I_p')
% ax1 = gca; % current axes
% ax1_pos = ax1.Position; % position of first axes
% ax2 = axes('Position',ax1_pos,...
%     'XAxisLocation','top',...
%     'YAxisLocation','right',...
%     'Color','none');
% line(Ip*10^3,Delta_lm2*10^9,'LineWidth',1.5,...
%     'Parent',ax2,'LineStyle','-','Color','r')
% xlim([0 max(Ip)*10^3])6.592 "
%xlabel('Current I_g ( mA )')
%%
figure
plot(Ig*10^3,(lambda0 + Delta_lasing1)*10^9,'LineWidth',1.5,'
    Color','blue')
hold on
plot(Ig*10^3,(lambda0 + Delta_lasing2)*10^9,'LineWidth',1.5,'
    Color','red')
xlabel('Current I_g ( mA )')
ylabel('Lasing Wavelength ( nm )')
hold off
%% Plotting both cases (a) and (b) in common graph
[~,ind] = min(abs(Ip-0.05));
figure
subplot(2,2,1)
yyaxis left
plot(Ig(1:ind)*10^3,(lambda0 + Delta_lasing1(1:ind))*10^9,...
                        'LineWidth',1.5,'Color','blue')
hold on
plot(Ig(1:ind)*10^3,(lambda0 + Delta_lasing2(1:ind))*10^9,...
                       'LineWidth',1.5,'Color','red')
xlabel('Current I_g ( mA )')
ylabel('Lasing Wavelength ( nm )')
ylim((lambda0+[0,Delta_lasing2(ind)*1.01])*10^9)
xlim([0 Ig(ind)]*10^3)
yyaxis right
plot(Ig(1:ind)*10^3, Ip(1:ind)*10^3,'LineWidth',1.5,'Color','k'
    )
ylabel('Phase Current ( mA )')
ylim([0 60])
xlabel('Bragg Current I_p ( mA )')
plt = gca;
plt.YAxis(1).Color = 'k';
plt.YAxis(2).Color = 'k';
%% ================================================
% Filippos Tzimkas-Dakis  UoC December 2020
% Final Homework ------- Optoelectronics and Lasers
% Exercise 2 Bragg reflector
%%
close all
```

```
clear all
clc
tic

n_h = 2.1;  % High Refractive index
n_l = 1.7;  % Low Refractive index
n_a = 1.5;  % Refr. Index from the left
n_b = 1.9;  % Refr. Index from the right
%%
Threshold = 0.99;     % reflectivity at center WL
Gamma     = 0;        % Reflection coefficient Gamma (transmission lines, microwaves etc)
                      % it is connected with r, the reflection coefficient of the EM fiel
R         = Gamma^2; % Power Reflection Coefficient R^2
N         = 1;        % intialize N
% Calculating the periodes needed for a specified reflectivity
while (R<= Threshold )
    Gamma = (n_a*(n_h/n_l)^(2*N)-n_b)/(n_a*(n_h/n_l)^(2*N)+n_b);
    R     = Gamma^2;
    N = N +1;
end
N = N-1; % Total number of periods needed
fprintf('\n The periods needed are %d \n',N)
fprintf(' The Reflectivity is R = %f \n \n',R)
%%
c       = 3*10^8;               % speed of light (m/s)
lambda0 = 600*10^-9;            % center wavelength (meters)
k0      = 2*pi/lambda0;         % wave-number at center wavelength
lambda  = (300:0.1:800)*10^-9;  % Wavelength span for sweep
d_lambda = lambda(2) - lambda(1);
freq    = linspace(0,3/lambda0,2000);% frequebcy span for sweep
df      = freq(2) - freq(1);
k       = 2*pi./lambda;         % Wavevector in free space
d_h     = (1/n_h)*(lambda0/4);  % physical Q-length for High-index layer
d_l     = (1/n_l)*(lambda0/4);  % physical Q-length for Low-index layer
k_h     = k*n_h;                % Wavevector in High index layer
k_l     = k*n_l;                % Wavevector in Low indez layer
heta    = 377;                  % free space characteristic impedance
Z_h     = heta/n_h;             % High-index characteristic impedance
Z_l     = heta/n_l;             % Low-index characteristic impedance
M       = 2*N;
Z       = 0*ones(M+1,length(lambda));

Zf(M+1,1:length(freq)) = (heta/n_b)*ones(1,length(freq));    % Char. Impedance at the Right End (material b) FREQ
```

```
Z(M+1,:)  = heta/n_b;    % Char. Impedance at the Right End (material b) WVLG
Z_a       = heta/n_a;    % Char. Impedance at the Left End (material a)
% Setting the parameters for the periodic stack
for i = M:-2:2
    ZZ(i)    = Z_h;
    d(i)     = d_h;
    kk(i,:)  = k_h;           % k-number in High index layer for wavelengthe sweep
    kkf(i,:) = 2*pi*freq*n_h; % k-number in High index layer for frequency sweep

    ZZ(i-1)    = Z_l;
    d(i-1)     = d_l;
    kk(i-1,:)  = k_l;           % k-number in Low index layer for wavelengthe sweep
    kkf(i-1,:) = 2*pi*freq*n_l; % k-number in Low index layer for frequency sweep
end
%%
for q = M+1:-1:2
     % calculating characteristic impedance for wavelength sweep
     Z(q-1,:) = ZZ(q-1)*(Z(q,:) + 1i*ZZ(q-1)*tan(kk(q-1,:)*d(q-1)))./(ZZ(q-1) + 1i*Z(q,:).*tan(kk(q-1,:)*d(q-1)));
     % calculating characteristic impedance for frequency sweep
     Zf(q-1,:)=ZZ(q-1)*(Zf(q,:) + 1i*ZZ(q-1)*tan(kkf(q-1,:)*d(q-1)))./(ZZ(q-1) + 1i*Zf(q,:).*tan(kkf(q-1,:)*d(q-1)));
end
%%
% Reflection coefficient Gamma (transmission lines, microwaves etc)
% it is connected with r, the reflection coefficient of the EM field
G  = (Z(1,:)- Z_a)./(Z(1,:) + Z_a);
% Reflection coefficient Gamma for frequency sweep
Gf = (Zf(1,:)- Z_a)./(Zf(1,:) + Z_a);
% Power Reflection Coefficient R^2
RR = abs(G).^2;
% Power Transmission Coefficient T^2
TT = 1 - RR;
% Power Reflection Coefficient R^2, frequency sweep
RRf = abs(Gf).^2;
% Power Transmission Coefficient T^2, frequency sweep
TTf = 1-RRf;
RRf_dB = 10*log10(RRf);
RR_dB = 10*log10(RR);
% Calculating the 3-dB Bandwidth of the Reflector/Mirror
BW_f  = fwhm_dakis(RRf')*df*c;                  % BW in frequency
BW_wl = fwhm_dakis(RR')*d_lambda*c/(lambda0^2); % BW in wavelength

```

```
toc
%% Plotting  Results
figure
subplot(2,2,1)
plot(lambda*10^9,RR,'LineWidth',1.2)
hold on
xlim([min(lambda) max(lambda)]*10^9);
set(gca,'FontSize',14);
ylabel(' Power Reflection Coefficient ')
xlabel(' Wavelength  ( nm ) ')
hold off

subplot(2,2,3)
plot(lambda*10^9,RR_dB,'LineWidth',1.2,'Color','blue')
xlim([min(lambda) max(lambda)]*10^9);
set(gca,'FontSize',14);
ylim([-30,0.5])
ylabel(' Power Reflection Coef. ( dB ) ')
xlabel(' Wavelength  ( nm ) ')
hold off

subplot(2,2,2)
plot(freq*lambda0,RRf,'LineWidth',1.2)
xlim([min(freq*lambda0) max(freq*lambda0)]);
set(gca,'FontSize',14);
ylabel(' Power Reflection Coefficient ')
xlabel(' f/f_0 ')
hold off

subplot(2,2,4)
plot(freq*lambda0,RRf_dB,'LineWidth',1.2,'Color','b')
xlim([min(freq*lambda0) max(freq*lambda0)]);
ylim([-30 0.5])
set(gca,'FontSize',14);
ylabel(' Power Reflection Coefficient ')
xlabel(' f/f_0 ')
hold off
%% Online code Check
na = n_a;
nb = n_b;
nH = n_h;
nL = n_l; % refractive indices

LH = d_h*n_h/lambda0;
LL = d_l*n_l/lambda0; % optical thicknesses in units of ?0
la0 = 600;            % central WL in units of nm
rho = (nH-nL)/(nH+nL);                % reflection coefficient
    ?
la2 = pi*(LL+LH)*1/acos(rho) * la0;   % right bandedge
la1 = pi*(LL+LH)*1/acos(-rho) * la0;  % left bandedge
Dla = la2-la1;  % bandwidth
N = M/2;        % number of bilayers
n = [na, repmat([nL,nH], 1, N), nb]; % indices for the layers A
    |H(LH)N|G
```

```
L = [repmat([LL,LH], 1, N)];          % lengths of the layers H(
    LH)N
la = linspace(100,1000,2001);         % plotting range is 300 ?
    ? ? 800 nm
Gla = abs(multidiel(n,L,la/la0)).^2; % reflectance as a
    function of ?
figure; plot(la,Gla);
%================================================
% Filippos Tzimkas-Dakis  UoC December 2020
% Final Project ------- Optoelectronics and Lasers
% Exercise 3 Quantum Well Gain
%%
tic
close all
clear all
clc

m0   = 9.11*10^(-31);      % electron mass (kg)
me   = 0.067*m0;           % e-mass in conduction band (kg)
mhh  = 0.45*m0;            % hh-mass (kg)
mlh  = 0.082*m0;           % lh-mass (kg)
mr_h = me*mhh/(me + mhh); % ehh reduced mass (kg)
mr_l = me*mlh/(me + mlh); % elh reduced mass (kg)

ev2joul = 1.60218e-19;     % converts eV to Joule
joul2ev = 6.242e+18;       % converts Joule to eV

kT_ = 0.026;               % (eV) Room temperature T=300K
kTT= kT_*ev2joul;          % (joule) Room temperature T=300K
q  = 1.6*10^(-19);         % electric charge (Coulomb)
h  = 6.62607015*10^(-34); % Plancks Constant (J*s)
h_bar = h/2/pi;            % h-bar (J*s)

L  = 14 *10^(-9);          % QWell width (m) <<<<<--------------

e0 = 8.85418*10^(-12);     % vacuum permitivity (Farad/m)
c  = 3*10^8;               % speed of light (m/s)
n  = 3.52;                 % ref. index GaAs

Eg_  = 1.42;               % Energy bandgap (ev)
Ev_  = 0;
Eg   = Eg_*ev2joul;        % Energy gap (joule)
Ee   =   (h_bar^2 *pi^2)/(2*me*L^2)*[1 2^2];  % Carrier QW
    energies (joule)
Ee_  = Ee*joul2ev;                            % Carrier QW
    energies (ev)
Ee__ = Ee_ + Eg_;
Ehh  = - (h_bar^2 *pi^2)/(2*mhh*L^2)*[1 2^2]; % HeavyHole QW
    energies (joule)
Ehh_ = Ehh*joul2ev;                           % HeavyHole QW
    energies (eV)
Elh  = - (h_bar^2 *pi^2)/(2*mlh*L^2)*[1 2^2]; % LightHole QW
    energies (joule)
```

```
318 Elh_ = Elh*joul2ev;                              % HeavyHole QW
    energies (eV)
319
320 const = pi*q^2 *h_bar/(2*n*e0*c*m0); %constant of g_max
321 Mthh  = [1/2 0];            % Mt^2 / M^2  heavy hole [TE TM]
322 Mtlh  = [1/6 2/3];          % Mt^2 / M^2  light hole [TE TM]
323 Mmo_  = 29;                 % (eV) for GaAs 2*(M^2)/m0
324 Mmo   = 29*ev2joul;         % (Joule) for GaAs 2*(M^2)/m0
325
326 E21_ehh = Eg + Ee - Ehh;    % eHH energy levels (joule)
327 E21_elh = Eg + Ee - Elh;    % eLH energy levels (joule)
328
329 r_ehh = mr_h/(pi*h_bar^2 *L)*[1 1]; % reduced density of states
     HH
330 r_elh = mr_l/(pi*h_bar^2 *L)*[1 1]; % reduced density of states
     LH
331
332 En21     = (0:0.001:3)*ev2joul;     % Energy vector (joule)
333
334 % Caltulating Max Gain for TE   --------------------
335 % gain = 1/meters  for 1/cm  divide with 100 !!
336 Gmax_ehh_TE = const* 1 * r_ehh*Mmo *Mthh(1);  % HH TE
    polarization
337 Gmax_elh_TE = const* 1 * r_elh*Mmo *Mtlh(1);  % LH TE
    polarization
338 GMAX_TE     = [Gmax_ehh_TE(1) Gmax_elh_TE(1) Gmax_ehh_TE(2)
    Gmax_elh_TE(2)]; % STEPS
339 EE21_TE     = [E21_ehh(1) E21_elh(1) E21_ehh(2) E21_elh(2)];
       % ROOTS
340 [~,ind1]    = min(abs(En21-E21_ehh(1))); % finding the spots of
     steps
341 [~,ind2]    = min(abs(En21-E21_elh(1))); %
342 [~,ind3]    = min(abs(En21-E21_ehh(2))); %
343 [~,ind4]    = min(abs(En21-E21_elh(2))); %
344 % MaxGain envelope  1/m   TE
345 Y_TE(1:ind1-1)          = 0;                                %
346 Y_TE(ind1:ind2-1)       = GMAX_TE(1);                       %
347 Y_TE(ind2:ind3-1)       = GMAX_TE(1) + GMAX_TE(2);          %
    Creating the Heaveside
348 Y_TE(ind3:ind4-1)       = GMAX_TE(1) + GMAX_TE(2) + GMAX_TE(3);
    % function
349 Y_TE(ind4:length(En21)) = sum(GMAX_TE);                     %
350 Y_TE = (Y_TE/100)./En21;   % gain 1/cm
351 %-------------------------------------------------
352 %-------------------------------------------------
353 % Caltulating Max Gain for TM Polarization ------
354 % gain = 1/meters  for 1/cm  divide with 100 !!
355 Gmax_elh_TM = const* 1 * r_elh*Mmo *Mtlh(2);   % LH TE
    polarization  ONLY
356 GMAX_TM     = [Gmax_elh_TM(1) Gmax_elh_TM(2)]; % STEPS
357 EE21_TM     = [E21_elh(1) E21_elh(2)];         % ROOTS
358 [~,ind1]    = min(abs(En21-E21_elh(1))); % finding the spots of
     steps
359 [~,ind2]    = min(abs(En21-E21_elh(2))); %
```

```
% MaxGain envelope  1/m   TM Polarization
Y_TM(1:ind1-1)          = 0;                        %
Y_TM(ind1:ind2-1)       = GMAX_TM(1);               %
Y_TM(ind2:length(En21)) = GMAX_TM(1) + GMAX_TM(2); % Creating
    the Heaveside
Y_TM = (Y_TM/100)./En21;   % gain 1/cm
% -------------------------------------------------
toc

%% Carriers  and  Quasi-Fermi levels
E21_a   = 1 ;       % Minimun energy
E21_b   = 3;        % Maximum energy
dE21    = 0.01 ;    % step
xLimits = [1 3];    % eV limits for plots

NP = 1:2:11;  % multiplyer of 10^12 carriers and holes
Fc = 0*NP;
Fv = 0*NP;

for i = 1:length(NP)
    % induced carriers and holes
    n_car  = NP(i)*10^12;       % 1/cm^2
    p_car  = n_car;             % 1/cm^2

    rhoe       = me/(pi*h_bar^2) ;    % Density of states 2D J
    ^-1 m^-2
    rhoe_cm    = rhoe*ev2joul/10000 ; % 1/J 1/cm^2

    % density of states for the bound state
    he1        = @(x)rhoe_cm*(0-(-log(exp((Ee__(1) -x)/kT_)+1)*
    kT_ + Ee__(1)-x));
    % density of states for the second state
    he2        = @(x)rhoe_cm*(0-(-log(exp((Ee__(2)-x)/kT_)+1)*
    kT_  + Ee__(2)-x));

    h      = @(x)n_car - (he1(x)+he2(x)); % creates the
    function to be solved
    Efn    = fzero(h,1);                  % calculating Quasi-
    Fermi for CONDUCTION band
    Fc(i) = Efn;                          % stores the QF of
    Conduction band
    rhohh       = mhh/(pi*h_bar^2);     % Density of states 2D
    J^-1 m^-2
    rhohh_eV_cm = rhohh*ev2joul*10^-4 ; % 1/J 1/cm^2
    % density of states for the bound state
    jhh1        = @(x)rhohh_eV_cm*((log(exp((x-Ehh_(1))/kT_)+1)
    *kT_ + Ehh_(1) - x )-0);
    % density of states for the second state
    jhh2        = @(x)rhohh_eV_cm*((log(exp((x-Ehh_(2))/kT_)+1)
    *kT_ + Ehh_(2) - x )-0);

    rholh       = mlh/(pi*h_bar^2);       % Density of states 2
    D J^-1 m^-2
    rholh_eV_cm = rholh*ev2joul*10^-4;    % 1/J 1/cm^2
```

```
403     % density of states for the bound state
404     jlh1        = @(x)rholh_eV_cm*((log(exp((x-Elh_(1))/kT_)+1)
    *kT_ + Elh_(1) -x)-0);
405     % density of states for the second state
406     jlh2        = @(x)rholh_eV_cm*((log(exp((x-Elh_(2))/kT_)+1)
    *kT_ + Elh_(2)-x)-0);
407
408     % creates the function to be solved
409     j = @(x)p_car-(jhh1(x)+jhh2(x)+jlh1(x)+jlh2(x));
410
411     Efp   = fzero(j,-1); % calculating Quasi-Fermi for Valance
    band
412     Fv(i) = Efp;         % stores the QF of valence band
413     rhorelh = (mr_l/(pi*h_bar^2))/L; % reduced density of
    states for electron and HH J^-1 m^-2
414     rhorehh = (mr_h/(pi*h_bar^2))/L; % reduced density of
    states for electron and LH J^-1 m^-2
415
416     Mlh_TE = 29*ev2joul*m0/(2*6); % se eV->joule 1/6 for TE &
    LH
417     Mhh_TE = 29*ev2joul*m0/(2*2);
418     Mlh_TM = (2/3)*29*ev2joul*m0/2 ;% eV->joule, 2/3 for TM &
    LH
419     Mhh_TM = 0;
420
421     % initializing the functions of gain --------------
422     gain_e_lh_TE = zeros((E21_b-E21_a)/dE21+1,1);
423     gain_e_hh_TE = zeros((E21_b-E21_a)/dE21+1,1);
424     gain_TE(:,i) = zeros((E21_b-E21_a)/dE21+1,1);
425
426     gain_e_lh_TM = zeros((E21_b-E21_a)/dE21+1,1);
427     gain_e_hh_TM = zeros((E21_b-E21_a)/dE21+1,1);
428     gain_TM(:,i) = zeros((E21_b-E21_a)/dE21+1,1);
429     % -----------------------------------------------
430     energy       = zeros((E21_b-E21_a)/dE21+1,1);
431
432     jj = 0;
433     for E21 = E21_a:dE21:E21_b
434         jj = jj+1;
435
436         % calculating the fermi distributions ----------
437         E1lh = Ev_ - (E21-Eg_)*mr_l/mlh;
438         f1elh = 1/(exp((E1lh-Efp)/kT_)+1);
439
440         E2lh  = Eg_+(E21-Eg_)*mr_l/me;
441         f2elh = 1/(exp((E2lh-Efn)/kT_)+1);
442         %-----------------------------------------------
443         % checking in which are we ---------------------
444         if E21 < Ee__(1)-Elh_(1)
445             rhorelh_E21 = 0;
446         elseif ( (E21 >= Ee__(1)-Elh_(1) ) && (E21 < Ee__(2)-
    Elh_(2)) )
447             rhorelh_E21 = rhorelh;
448         else
```

```
449             rhorelh_E21 = 2*rhorelh; % when we go to the second
    state the
450                                      % density of states
    doubles its value
451         end
452         %-----------------------------------------------
453         % Caculating the gain in     1/cm    ----------
454         % TE
455         gain_e_lh_TE(jj) = (pi^1*q^2*h_bar*Mlh_TE*rhorelh_E21*(
    f2elh-f1elh)/(n*e0*c*m0^2*(E21*ev2joul)))/100;
456         % TM
457         gain_e_lh_TM(jj) = (pi^1*q^2*h_bar*Mlh_TM*rhorelh_E21*(
    f2elh-f1elh)/(n*e0*c*m0^2*(E21*ev2joul)))/100;
458         %-----------------------------------------------
459         % calculating the fermi distributions ----------
460         E1hh  = Ev_-(E21-Eg_)*mr_h/mhh;
461         f1ehh = 1/(exp((E1hh-Efp)/kT_)+1);
462
463         E2hh  = Eg_ +(E21-Eg_)*mr_h/me;
464         f2ehh = 1/(exp((E2hh-Efn)/kT_)+1);
465         %-----------------------------------------------
466         % checking in which are we --------------------
467         if E21 < Ee__(1)-Ehh_(1)
468             rhorehh_E21=0;
469         elseif ( (E21 >= Ee__(1)-Ehh_(1)) && (E21 < Ee__(2)-
    Ehh_(2)) )
470             rhorehh_E21=rhorehh;
471         else
472             rhorehh_E21 = 2*rhorehh; % when we go to the second
    state the
473                                      % density of states
    doubles its value
474         end
475         %-----------------------------------------------
476         % Caculating the gain in     1/cm    -----------
477         % TE
478         gain_e_hh_TE(jj) = (pi^1*q^2*h_bar*Mhh_TE*rhorehh_E21*(
    f2ehh-f1ehh)/(n*e0*c*m0^2*(E21*ev2joul)))/100;
479         % TM
480         gain_e_hh_TM(jj) = (pi^1*q^2*h_bar*Mhh_TM*rhorehh_E21*(
    f2ehh-f1ehh)/(n*e0*c*m0^2*(E21*ev2joul)))/100;
481         %-----------------------------------------------
482         % Calculating the TOTAL GAIN  TE and TM  in  1/cm
483         gain_TE(jj,i) = gain_e_lh_TE(jj) + gain_e_hh_TE(jj);
484         gain_TM(jj,i) = gain_e_lh_TM(jj) + gain_e_hh_TM(jj);
485
486         energy(jj) = E21;  % storing for figures
487     end
488
489 end
490
491 %% Prints !
492 fprintf('\n ========== PRINTS ARE HERE ==========')
493 fprintf('\n --------------- ROOTS ----------------\n')
```

```
494 fprintf(' Electron - HEAVY Hole (1) = %f eV \n',E21_ehh(1)*
    joul2ev)
495 fprintf('                        (2) = %f eV \n',E21_ehh(2)*
    joul2ev)
496 fprintf('\n Electron - LIGHT Hole (1) = %f eV \n',E21_elh(1)*
    joul2ev)
497 fprintf('                        (2) = %f eV \n',E21_elh(2)*
    joul2ev)
498 fprintf('\n ----------------------------------------\n')
499 fprintf('\n ------------ Quasi Fermi --------------\n')
500 for i = 1:length(NP)
501     fprintf('\n For N =%d*10^(12), -> FC = %.3f eV \n',NP(i),Fc
    (i))
502     fprintf('                      -> FV = %.3f eV \n', Fv(i))
503 end
504 fprintf('\n ======================================\n')
505 %% PLOTS !
506 % TE Plots
507 subplot(2,2,1)
508 for i = 1:length(NP)
509     plot(energy,gain_TE(:,i),'LineWidth',1.1);
510     hold on
511 end
512 plot(En21*joul2ev,Y_TE,'LineWidth',1.5,'Color','r','LineStyle',
    '-.')
513 plot(En21*joul2ev,-Y_TE,'LineWidth',1.5,'Color','r','LineStyle'
    ,'-.')
514 plot(En21*joul2ev,0*Y_TE,'Color','k','LineStyle','-')
515 xlim([0.5 3]);
516 title('TE gain spectra')
517 xlabel('E_{21} ( eV )')
518 ylabel('Optical Gain (cm^{-1})')
519 xlim(xLimits)
520 hold off
521
522 % TM Plots
523 % figure
524 subplot(2,2,2)
525 for i = 1:length(NP)
526     plot(energy,gain_TM(:,i),'LineWidth',1.1);
527     hold on
528 end
529 plot(En21*joul2ev,Y_TM,'LineWidth',1.5,'Color','r','LineStyle',
    '-.')
530 plot(En21*joul2ev,-Y_TM,'LineWidth',1.5,'Color','r','LineStyle'
    ,'-.')
531 plot(En21*joul2ev,0*Y_TM,'Color','k','LineStyle','-')
532 xlim([0.5 3]);
533 title('TM gain spectra')
534 xlabel('E_{21} ( eV )')
535 ylabel('Optical Gain (cm^{-1})')
536 xlim(xLimits)
537 hold off
538 %=========================================
```