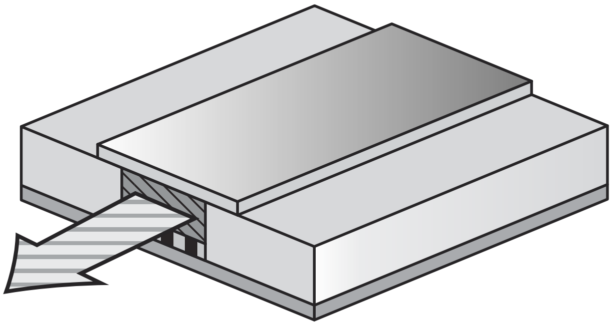
Semiconductor Optoelectronic Devices
Homework III



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Coldren's book

Problem 2.6

$\lambda = 1.55\,\mu\mathrm{m}\,, \qquad \mathrm{L} = 400\,\mathrm{nm}\,, \qquad \eta_\mathrm{i} = 80\%\,, \qquad \mathrm{R} = 0.32$

$\mathrm{B} = 10^{-10}\,\mathrm{cm}^3/\mathrm{s}\,, \qquad \mathrm{u_g} = \dfrac{\mathrm{c}}{\bar{\mathrm{n}} - \lambda\frac{\partial \bar{\mathrm{n}}}{\partial \lambda}}\,, \qquad \dfrac{\partial \bar{\mathrm{n}}}{\partial \lambda} = -1\,\mu\mathrm{m}^{-1}$

(a) Modal gain

$$\Gamma \cdot g_\mathrm{th} = a_\mathrm{i} + a_\mathrm{m} = 10 + \frac{1}{L}\ln\left(\frac{1}{R}\right) = 10 + \frac{1}{400 \cdot 10^{-4}}\ln\left(\frac{1}{0.32}\right) \rightarrow$$

$$\boxed{\Gamma \cdot g_\mathrm{th} = 38.486\,\mathrm{cm}^{-1}} \qquad (1) \quad \boxed{\texttt{eq:modalgain}}$$

(b) Differential efficiency

$$\eta_\mathrm{d} = \frac{\eta_\mathrm{i} \cdot \ln\left(\dfrac{1}{R}\right)}{L \cdot a_\mathrm{i} + \ln\left(\dfrac{1}{R}\right)} = \frac{\eta_\mathrm{i} a_\mathrm{m}}{a_\mathrm{i} + a_\mathrm{i}} = \frac{\eta_\mathrm{i} a_\mathrm{m}}{\Gamma g_\mathrm{th}} = 0.8 \cdot \frac{28.486}{38.486} \rightarrow$$

$$\boxed{\eta_\mathrm{d} = 0.592132} \qquad (2) \quad \boxed{\texttt{eq:dif\_eff}}$$

(c) Axial mode spacing

$$\Delta\lambda = \frac{\lambda^2}{2\bar{n}_g L} = \frac{(1.55)^2\ (\mu\mathrm{m})^2}{2 \cdot 3.8 \cdot 400\,\mu\mathrm{m}} \rightarrow \boxed{\Delta\lambda = 7.9029\,\mathrm{nm}} \qquad (4) \quad \boxed{\texttt{eq:ax\_mod\_space}}$$

Problem 2.7

Cleaved GaAs, $h = 0.1\,\mu\mathrm{m}\,, \qquad \mathrm{L} = 300\,\mu\mathrm{m}\,, \qquad \mathrm{J_{th}} = 1\mathrm{kA/cm}^2\,,$

$\eta_\mathrm{i} = 1\,, \qquad \mathrm{a_i} = 10\,\mathrm{cm}^{-1}\,, \qquad \Gamma = 0.1\,, \qquad \mathrm{R} = 0.32$

(a) Threshold carrier density

$$\Gamma \cdot g_\mathrm{th} = a_\mathrm{i} + a_\mathrm{m} = 10 + 37.9811 = 47.9811 \rightarrow$$

$$\boxed{g_{th} = 479.811\,\mathrm{cm}^{-1}} \qquad (5)$$

From Table 4.4 at page 224 we find the values for GaAs

$$N_{tr} = 1.85 \cdot 10^{18}\,\mathrm{cm}^{-3} \qquad \& \qquad g_{ON} = 1500\,\mathrm{cm}^{-1}\,.$$

Thus,

$$N_{th} = 1.85 \cdot 10^{18} \exp\left(\frac{479.811}{1500}\right) \rightarrow \boxed{N_{th} = 2.54736 \cdot 10^{18}\,\mathrm{cm}^{-3}}. \qquad (6)$$

(b) Output power per micrometer

$$\eta_d = \eta_i \frac{a_m}{a_i + a_m} = 0.7915\,,$$

$$J_{th} = \frac{\eta_i I_{th}}{wL} \rightarrow I_{th} = J_{th} \cdot wL \rightarrow \boxed{I_{th} = 0.3\,\mathrm{mA}}$$

with the same way we find $\boxed{I = 0.6\,\mathrm{mA}}$.
Thus,

$$P_0 = \eta_d \frac{hv}{q}\left(I - I + th\right) = 0.7915 \frac{hv}{q} 0.3 \xrightarrow[\lambda=908\mathrm{nm}]{for\,GaAs}$$
$$P_0 = 0.7915 \cdot 1.3654 \cdot 0.3\,\mathrm{mW} = 0.32412\,\mathrm{mW}\,.$$

The above power is referred to both facets,so for the one we get

$$\boxed{\mathcal{P}_{01} = \frac{P_0/2}{1\,\mu\mathrm{m}} = 0.16206\ \mathrm{kW}/\mu\mathrm{m}}\,. \tag{7}$$

(c) Photon and carrier density when $J = 2\,\mathrm{kA/cm^2} > J_{th}$. In this case, both densities will be equal to threshold carrier density because we are in steady-state and every carrier produces one photon. Thus,

$$\boxed{N = N_p = N_{th} = 2.54736 \cdot 10^{18}\,\mathrm{cm^{-3}}} \tag{8}$$

Problem 2.9
Material: InGaAsP/InP $\quad \lambda = 1.3\,\mu\mathrm{m} \quad R = 0.32\,.$

- $L = 200\,\mu\mathrm{m} \quad J_{th} = 3\,\mathrm{kA/cm^2} \quad \eta_d = 60\%$
- $L \ = 400\,\mu\mathrm{m} \quad J'_{th} = 2\,\mathrm{kA/cm^2} \quad \eta'_d = 50\%$

$$a_i = \frac{\eta'_d - \eta_d}{L \cdot \eta_d - L'\eta'_d} \ln\left(\frac{1}{R}\right) = \frac{0.5 - 0.6}{0.6 * 200 - 0.5 * 400} \ln\left(\frac{1}{0.32}\right) \rightarrow \tag{9}$$
$$\boxed{\langle a_i \rangle = 14.245\,cm^{-1}}\,. \tag{10}$$

Problem
What is the photon lifetime in a $\mathbf{500}\,\mathbf{\mu m}$ cleaved-mirror InP cavity with internal losses $\langle \mathbf{a_i} \rangle = \mathbf{10\,cm^{-1}}$
$L = 500\,\mu\mathrm{m}, \quad R = 0.32, \quad a_i = 10\,\mathrm{cm^{-1}}, \quad B = 10^{-10}\,\mathrm{cm^3/s},$
$u_g = \dfrac{c}{\bar{n} - \lambda\frac{\partial \bar{n}}{\partial \lambda}}, \quad \dfrac{\partial \bar{n}}{\partial \lambda} = -1\,\mu\mathrm{m}^{-1}$

(c) Fnd the photon lifetime.
From Table 1.1 at page 14, InP is a material which is used at $\lambda_1 = 1.55\mu$m and $\lambda_2 = 1.3\,\mu$m. So, we will calculate photon lifetime in both cases.

$$a_m = \frac{1}{L}\ln\left(\frac{1}{R}\right) = \frac{1}{500\cdot 10^{-4}}\ln\left(\frac{1}{0.32}\right) \rightarrow \boxed{a_m = 22.78\,\mathrm{cm}^{-1}} \tag{11}$$

$$\Gamma\cdot g_{th} = 22.78 + 10 = 32.788\,\mathrm{cm}^{-1} \tag{12}$$

Group velocities,

$$u_g = \frac{c}{\bar{n} - \lambda\frac{\partial\bar{n}}{\partial\lambda}} \rightarrow$$

$$\boxed{u_{g1} = 6.356\cdot 10^7\ \mathrm{m/s}}\,, \tag{13}$$

$$\boxed{u_{g2} = 6.652\cdot 10^7\ \mathrm{m/s}}\,. \tag{14}$$

Therefore, photon lifetime shall be calculated as

$$\tau_p = \frac{1}{\Gamma\cdot g_{th}\cdot u_g} \rightarrow$$

$$\boxed{\tau_{p1} = \frac{1}{6.356\cdot 10^9\cdot 32.788} = 4.798\,\mathrm{ps}} \tag{15}$$

$$\boxed{\tau_{p2} = \frac{1}{6.652\cdot 10^9\cdot 32.788} = 4.585\,\mathrm{ps}} \tag{16}$$

Problem
One Quantume Well: $\lambda_1 = 1\,\mu m$ $\quad L = 500\,\mu m\,,$ $\quad n_{\text{eff}} = 3\,,$
$\partial n_{\text{eff}}/\partial\lambda = -1\,\mu m^{-1}\,,$ $\quad g(N) = g_0\ln(N/N_0)\,,$ $\quad r_1 = r_2 = 0.3\,,$
$\Gamma_1 = 0.1\,,$ $\quad g_0 = 100\,\mathrm{cm}^{-1}.$
(a) Threshold current

$$\boxed{N_{th} = N_{tr}\exp\left(\frac{g_{th}}{g_0}\right)} \quad \& \quad \boxed{g_{th} = \frac{a_i + a_m}{\Gamma_1}}\,. \tag{17}$$

$$I_{th} = e\cdot V_1\cdot B\cdot(N_{th})^2 \rightarrow$$

$$\boxed{I_{th}\,(V_1,\Gamma_1) = e\cdot V_1\cdot B\cdot(N_{tr})^2\cdot\exp\left(2\frac{\langle a_i\rangle + a_m}{\Gamma_1 g_0}\right)} \tag{18}$$

(b) Find the number M of quantum wells that minimizes I_{th}.

$$I_{thMQW} = Z \cdot M \cdot \exp\left(2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right),$$

where $Z = e \cdot V_1 \cdot B \cdot (N_{tr})^2$ is independent of number of quantum wells. In order to minimize I_{thMQW} we take the derivative with respect to M,

$$\begin{aligned}
\frac{\partial I_{thMQW}}{\partial M} &= Z \cdot \exp\left(2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right) \\
&\qquad + \frac{\partial}{\partial M}\left(2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right) \cdot Z \cdot M \cdot \exp\left(2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right) \rightarrow \\
\frac{\partial I_{thMQW}}{\partial M} &= Z \cdot \exp\left(2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right)\left(1 - 2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 g_0}\right) = 0 \rightarrow \\
&\left(1 - 2\frac{\langle a_i\rangle + a_m}{M \cdot \Gamma_1 \cdot g_0}\right) = 0 \rightarrow \boxed{M = 2\frac{\langle a_i\rangle + a_m}{\Gamma_1 \cdot g_0}}.
\end{aligned} \tag{19}$$

Losses

$$\langle a_i\rangle + a_m = 10 + \frac{1}{500 \cdot 10^{-4}} \ln\left(\frac{1}{0.3^2}\right) = 58.1589\,\mathrm{cm}^{-1}. \tag{20}$$

We find

$$\boxed{M = 2\frac{58.1589}{0.1 \cdot 100} = 11.6318}. \tag{21}$$

M takes discrete values, so in order to be sure we shall check both values, 11 and 12. We obtain,

$$\begin{aligned}
I_{thMQW}^{M=11} &= 31.6687 \cdot Z\,, \\
\boxed{I_{thMQW}^{M=12} = 31.6337 \cdot Z}&.
\end{aligned}$$

Therefore, we conclude that the number of quantum well should be $M = 12$.

Problem

We have the following rate equations for carrier and photon density

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\eta_i I}{qV} - \frac{N}{\tau} - u_g N_p g\,, \tag{22a}$$

$$\frac{\mathrm{d}N_p}{\mathrm{d}t} = \Gamma u_g N_p g + \Gamma \beta_{sp} R_{sp} - \frac{N_p}{\tau_p}\,. \tag{22b}$$

When laser reaches steady-state condition we have $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\mathrm{d}N_p}{\mathrm{d}t} = 0$. Thus, firstly, we solve Eq.(22b) with respect to N_p and we then we substitute the result to Eq.(22a). We write

$$(22\mathrm{b}) \rightarrow N_p \left(\frac{1}{\tau_p} - \Gamma g u_g \right) = \Gamma \beta_{sp} R_{sp} \rightarrow \boxed{N_p = \frac{\tau_p \Gamma \beta_{sp} R_{sp}}{1 - \tau_p \Gamma g u_g}}. \tag{23}$$

$$(22\mathrm{a}) \rightarrow \frac{N}{\tau} = \frac{\eta_i I}{qV} - g u_g N_p \rightarrow \frac{N}{\tau} = \frac{\eta_i I}{qV} - g u_g \frac{\tau_p \Gamma \beta_{sp} R_{sp}}{1 - \tau_p \Gamma g u_g} \rightarrow$$

$$\xrightarrow[A=C=0]{N/\tau = AN + BN^2 + CN^3} \boxed{N^2 = \frac{\eta_i I}{qV} - u_g \frac{\tau_p \Gamma \beta_{sp} a \left(N^3 - N^2 N_{tr} \right)}{1 - \tau_p \Gamma u_g a \left(N - N_{tr} \right)}}. \tag{24}$$

eq:2besolved

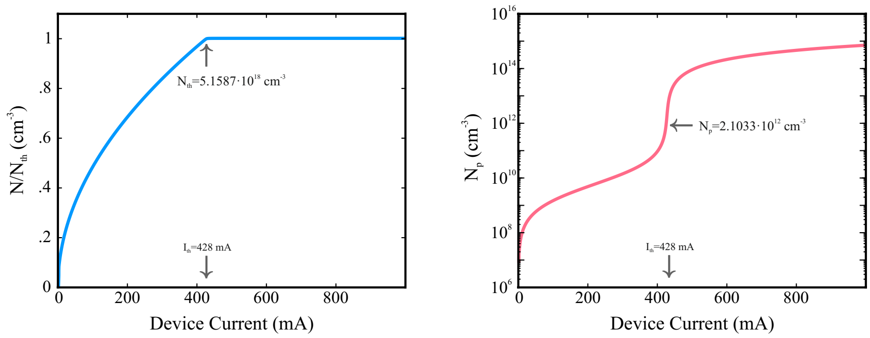
Where we used the relations $R_{sp} = BN^2$, $g = a\left(N - N_{tr}\right)$ and also that $A = C = 0$ because we neglect all non-radiative recombination and other parasitic phenomena. Now, we can solve the latter equation by giving values to current I and finding the roots of current density, N, and after that we can calculate the photon density, N_p. Table 1 gathers all the parameters that have been used to produce the graphs in Figure 1.

Table 1: Parameters for the numerical solution of Eq. (24).

Parameter	Value
Transparency carrier density	$N_{tr} = 2 \cdot 10^{18}\,\mathrm{cm}^{-3}$
Gain constant	$a = 2.5 \cdot 10^{-16}\,\mathrm{cm}^2$
Group velocity	$u_g = 6.3158 \cdot 10^9\,\mathrm{cm}$
Confinement	$\Gamma = 0.2$
Length	$L = 500\,\mu\mathrm{m}$
Width	$W = 50\,\mu\mathrm{m}$
Thickness	$H = 0.2\,\mu\mathrm{m}$
Wavelength	$\lambda = 1.55\,\mu\mathrm{m}$
njection efficiency	$\eta_i = 1$
Refractive index	$n = 3.2 + 1.55 = 4.75$
Temperature	$T = 300\,\mathrm{K}$
Radiative recombination coefficient	$B = 10^{-10}\,\mathrm{cm}^{-3}/\mathrm{s}$
Spontaneous emission factor	$\beta_{sp} = 10^{-5}$
Photon lifetime	$\tau_p = 10^{-12}$

table:tab1

In Figure 1a we depict the asked densities. More specifically Fig. 1a depicts the carrier density versus driving current I, while 1b depicts the photon density in a logarithmic scale. As we expected, both curves occur a steep point which in our case reveals the threshold current I_{th}. Therefore, when the driving current is greater than threshold carrier density is nearly constant. Furthermore, it is remarkable that when the driving current is near the threshold photon density reacts with a "step" in its curve. Last but not least, although carrier density do not grow further, photon density actually does grow with a slower rate than in lower currents.



(a) (b)

Figure 1: (a) Carrier density normalized to threshold carrier density $N_{th} = 5.1587 \cdot 10^{18\,\mathrm{cm}^{-3}}$ versus driving current. When the driving current becomes equal (or greater) to threshold current, carriers stop growing and the curve reveals a knee. (b) Photon density versus driving current, with y-axis in logarithmic scale. Photon density also feels the difference below and beyond threshold, that is why it becomes extremely abrupt in the a small range around I_{th}.

fig:N_Np

Code

In this section we attach the MATLAB code that has been developed to produce the above mentioned results. For comments, mistakes, clarifications etc. please send a message to dakisfilippos@gmail.com.

```
% Filippos Tzimkas-Dakis  UoC November 2020
% Homework 3 ------- Optoelectronics and Lasers
% Script for the Bonus exercise of homework 3
%%
clc
clear all
close all

c     = 3*10^10;       % speed of light in vacuum (cm/s)
lamda = 1.55*10^(-4);  % um = 10^-6 m = 10^-4 cm
G     = 0.2;           % confinement (clear)
b_sp  = 10^-5;         % clear
```

```
tp     = 1*10^(-12);    % photon lifetime (s)
B      = 10^(-10);      % cm^3/s
H      = 200*10^(-7);   % active region thickness (cm)
L      = 500*10^(-4);   % cavity length (cm)
W      = 50*10^(-4);    % width of metal contacts (cm)
Vp     = H*L*W;         % Cavity Volume (cm^-3)
V      = G*Vp;
hi     = 1;             % injection efficiency (clear)
a      = 2.5*10^(-16); % cm^2
N_tr   = 2*10^(18);     % 1/(cm^3);
n1     = 3.2;           % refractive index (clear)
dn_dl  = -1*10^(+4);    % dn/dlamda = -1/(um)
n      = n1 - lamda*dn_dl; % refractive index for InP at 1.55 um
ug     = c/n;           % group velocity
q      = 1.602*10^-19; % electric charge
T      = 300;           % Temperature (Kelvin)
I      = (0:1:1000)*10^-3; % Ampere
% intialize variables
N1     = NaN*ones(1,length(I));
N2     = N1;
N3     = N2;

% define new parameters to help the solver
K   = G*b_sp*tp*a*ug;
P   = G*ug*tp*a;
Z   = hi*I/q/V;
%%
syms X
for i = 1:length(I)
    % characteristic equation
    EQN = B*(K-P)*X^3 + B*(1 + P*N_tr - K*N_tr)*X^2 + Z(i)*P*X
    - (1+P*N_tr)*Z(i)==0;
    % we use VPAsolve because fsolse couldn't help
    temp = vpasolve(EQN,X);
    N1(i) = temp(1);    % solution separation
    N2(i) = temp(2);    % we are interested in this
    N3(i) = temp(3);
end

[~,index] = min(abs(N2-N3)); % finds threshold index
Ith       = I(index);         % threshold current
Nth       = N2(index);        % thhreshold carriers
N_nrmlzd = N2/Nth;            % normalized carriers density
%%
% photon density
Np = (tp*b_sp*G*B*N2.^2)./(1-tp*G*ug*a*(N2-N_tr));

figure % plot for photon density
semilogy(I*10^3,Np,'LineWidth',2)
xlabel('Device Current (mA)')
ylabel('N_p (cm^{-3}) ')
hold off

figure % plot for carrier density
```

```
plot(I*10^3,N_nrmlzd,'LineWidth',2)
xlabel('Device Current (mA)')
ylabel('N (cm^{-3}) ')
legend('Normalized')
ylim([-0 1.1])
hold off
%%
figure % all solutions together
plot(I*10^3,N1,'LineWidth',2,'Color','b')
hold on
plot(I*10^3,N2,'LineWidth',2,'Color','black')
plot(I*10^3,N3,'LineWidth',2,'Color','red')
xlabel('Device Current (mA)')
ylabel('N (cm^{-3}) ')
legend('N_1','N_2','N_3')
hold off
```