CMPT 404
Homework 0

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1. For the function $g(x) = -3x^2 + 24x - 30$, find the value for x that maximizes $g(x)$.

In order to maximize the function, first we take to derivative of the function and get that $\frac{d}{dx}g(x) = g'(x) = -6x + 24$. Next, we set this equal to zero to find the critical points and find that $x = 4$. This is a maximum with $g(4) = 18$.

2. Differentiate $f(x) = 3x_0^3 - 2x_0x_1^2 + 4x_1 - 8$ with respect to x_0 and x_1.

$$\frac{d}{dx_0}f(x) = 9x_0^2 - 2x_1^2$$

$$\frac{d}{dx_1}f(x) = -4x_0x_1 + 4$$

3. Consider the matrix $A = \begin{bmatrix} 1 & 4 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 & 5 \\ 0 & -1 & 4 \end{bmatrix}$.

a) Can you multiply the two matices?

You cannot multiply these matrices as the dimentions are incompatible.

b) Multiply A^T and B and give its *rank*.

$$A^T B = \begin{bmatrix} -2 & -2 & 13 \\ -8 & 1 & 16 \\ 6 & -3 & -3 \end{bmatrix}$$

This has a rank of 2.

4. Definitions

The Simple Gaussian Distribution is a symmetric, bell shaped probability curve.

The multivariate Gaussian Distribution is a multivariate analog to the Simple Gaussian Distribution in which each variable creates a Simple Gaussian Distribution when the other variables are held constant.

The Bernoulli Distribution is a distribution that gives 1 with a given probability, and 0 otherwise.

The Binomial Distribution describes the probability of a number of succeses in a series of Bernoulli trials.

The Exponential Distribution is a constantly decreasing probability curve given by a negative exponential function.

6. Take the random variable $X\ N(2,3)$. What is its expected value?

The expected value is the mean, which is 2.

7. a) What is x^* if $y = 1.1$ and $Z = \mathbb{N}$, where $\mathbb{N}$ is the set of natural numbers?

Since 1 is the closest natural number to 1.1, $x^* = 1$.

7. b) Locate x^* in the following picture:

8. Suppose that random variable Y has the following distribution:

$$p(y) = \begin{cases} e^{-y} & \text{if } y \geq 0 \\ 0 & \text{if } y < 0 \end{cases}$$

a) Verify that $\int\limits_{-\infty}^{\infty} p(y)dy = 1$.

$$\int\limits_{-\infty}^{\infty} p(y)dy = \int\limits_{0}^{\infty} e^{-y}dy = -e^{-y}|_0^\infty = 0 - (-1) = 1$$

b) What is $\mu_Y = E[Y] = \int\limits_{-\infty}^{\infty} p(y)y$?

$$\int\limits_{-\infty}^{\infty} p(y)ydy = \int\limits_{0}^{\infty} e^{-y}dy = -e^{-y}y|_0^\infty + \int\limits_{0}^{\infty} e^{-y}dy = (0-0) + 1 = 1$$

c) What is $\sigma_Y = \text{Var}[Y] = \int\limits_{-\infty}^{\infty} p(y)(y-\mu_Y)^2dy$?

$$\begin{aligned}
\int\limits_{-\infty}^{\infty} p(y)(y-\mu_Y)^2dy &= \int\limits_{0}^{\infty} e^{-y}(y-1)^2dy \\
&= -(y-1)^2e^{-y}|_0^\infty + 2\int\limits_{0}^{\infty} e^{-y}(y-1)dy \\
&= 1 + 2(\int\limits_{0}^{\infty} e^{-y}dy - \int\limits_{0}^{\infty} e^{-y}dy) \\
&= 1 + 2(1-1) = 1
\end{aligned}$$

d) What is $E[Y|Y \geq 10]$?

$$E[Y|Y \geq 10] = \frac{\int\limits_{10}^{\infty} e^{-y} y dy}{\int\limits_{10}^{\infty} e^{-y} dy} = \frac{-e^{-y}(y+1)|_{10}^{\infty}}{-e^{-y}|_{10}^{\infty}} = \frac{11e^{-10}}{e^{-10}} = 11$$