

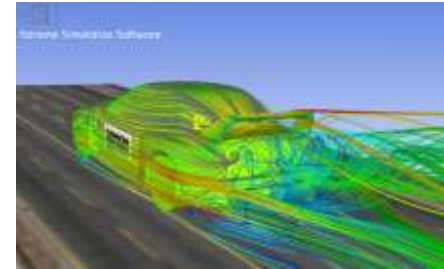
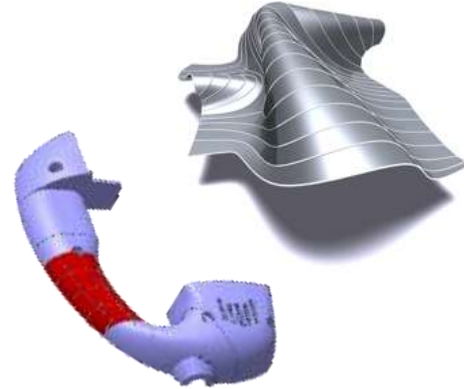
Computeranimation

Appendix A – Meshes

Surface Representations

Why do we need surfaces at all

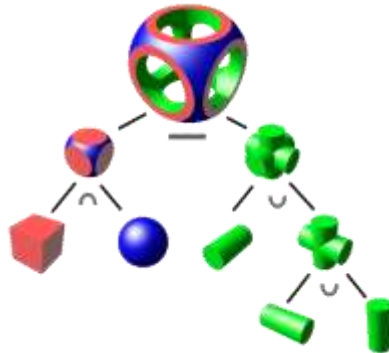
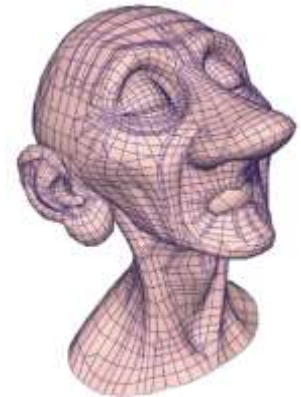
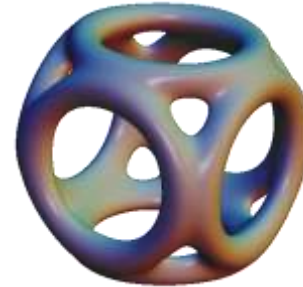
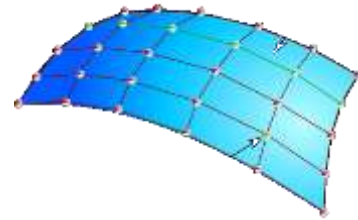
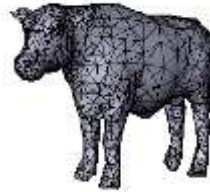
- Design (CAD)
- Rendering
- Simulation
- Documentation Preservation
- Quality Control
- Custom Fitting



Surface Representations

Description of Objects' Surfaces

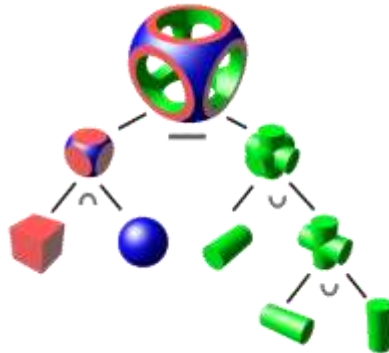
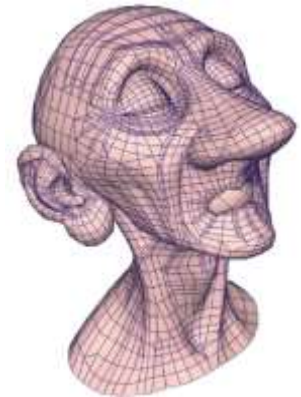
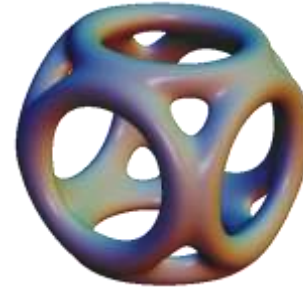
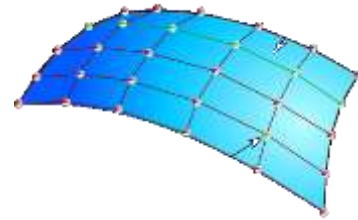
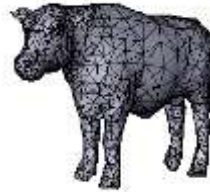
- Polygonal Meshes
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry
- Subdivision Surfaces



Surface Representations

Description of Objects' Surfaces

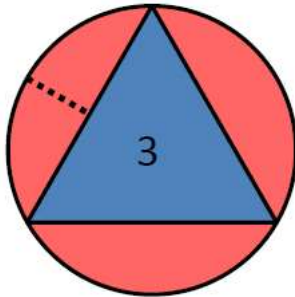
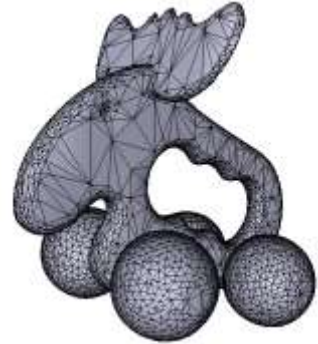
- Polygonal Meshes ← in this course
- Parametric Surfaces
- Implicit Surfaces
- Constructive Solid Geometry
- Subdivision Surfaces



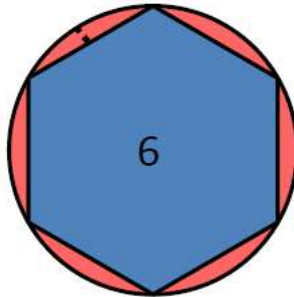
Polygonal Meshes

Piecewise Linear Approximation

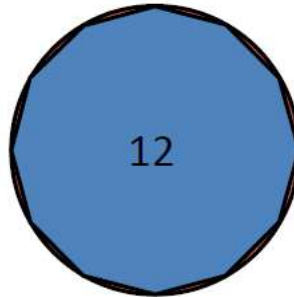
- Error $O(h^2)$



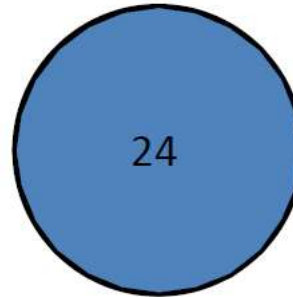
25%



6.5%



1.7%

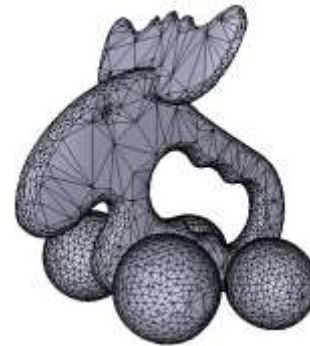


0.4%

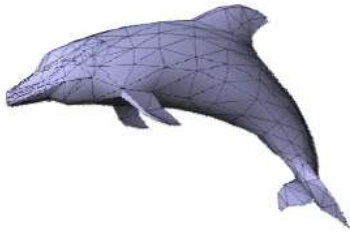
Polygonal Meshes

Polygonal Meshes are Good!

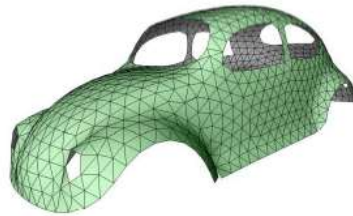
- Approximation $O(h^2)$
- Arbitrary Topology
- Piecewise smooth Surfaces
- Adaptive Refinement
- Efficient Rendering



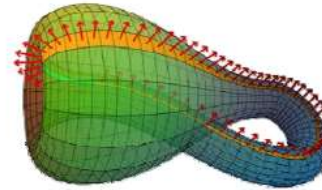
Mesh Zoo



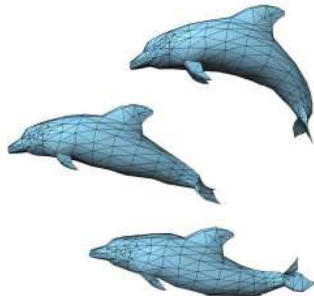
Single component
closed, triangular,
orientable manifold



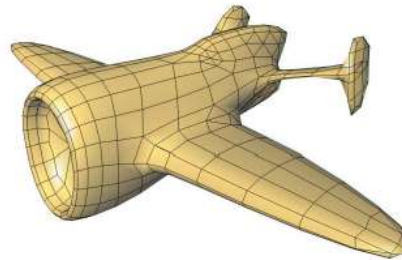
With boundaries



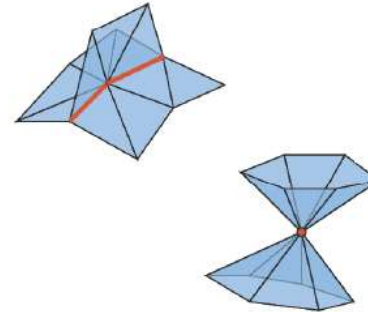
Non-orientable



Many components



Triangles and Quads

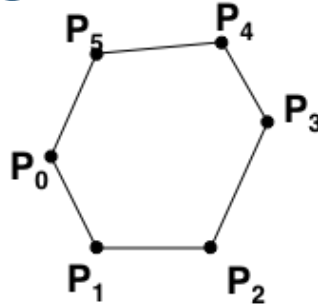


Non-manifold

Polygonal Meshes

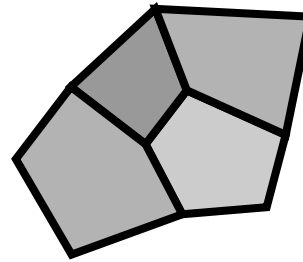
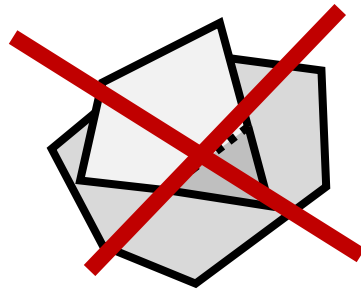
Polygon

- Ordered set of vertices



Polygon mesh

- Collection of Polygons satisfying certain restrictions



Polygonal Meshes

Most Often Used Polygonal Meshes in Computer Graphics

- Triangle meshes
- Quad meshes

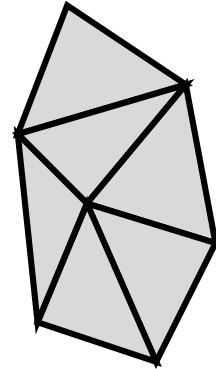
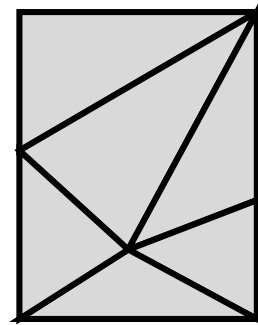
Topology vs. Geometry

- **Geometry**: The shape of an object: Basically the vertex position
- **Topology**: connectivity / neighbourhood relation

Polygonal Meshes

Topologically Equivalent

- Mesh only differs in vertex Positions



Geometrically (almost) equivalent

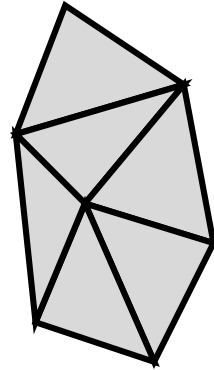
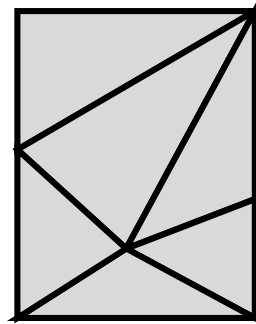
- Same shape, different connectivity of mesh-elements



Polygonal Meshes

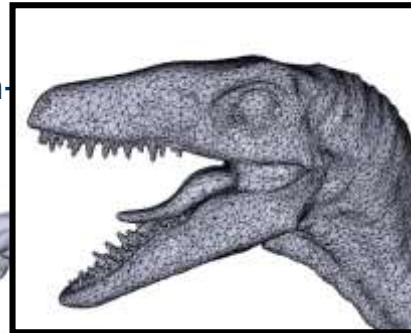
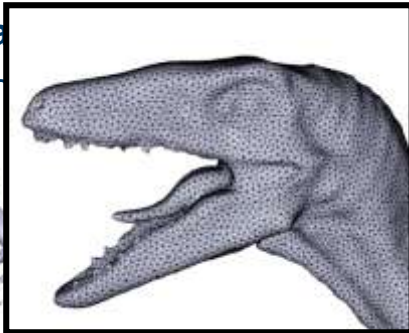
Topologically Equivalent

- Mesh only differs in vertex Positions



Geometrically

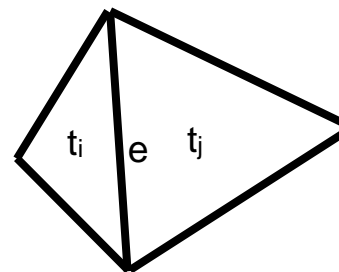
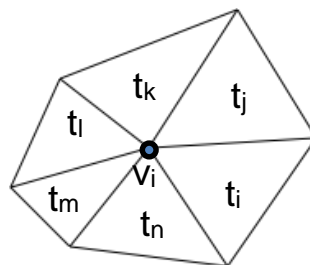
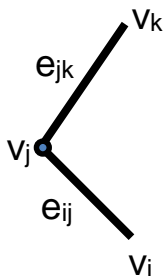
- Same shape and position of mesh



Triangle Meshes

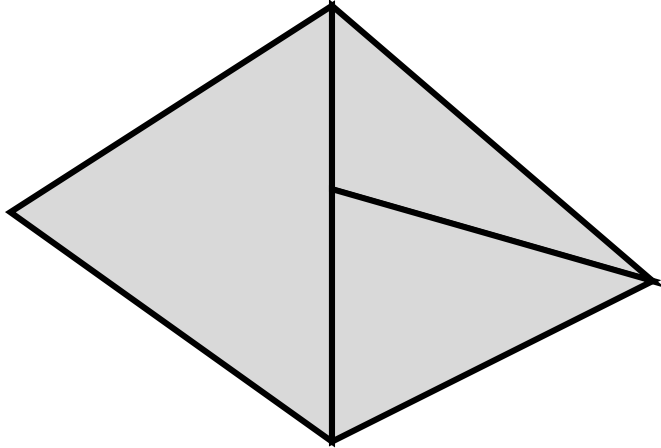
Definition: A triangulation of a set of points consists of:

- Vertices $v_i, \{v_i, i=1, \dots, N\} \subseteq \mathbb{R}^3$
- Edges $\subseteq \{\overline{v_i v_j}, 1 \leq i, j \leq N\}$
- Triangles $\subseteq \{\Delta(v_i v_j v_k), 1 \leq i, j, k \leq N\}$
- The triangulation can also have attributes (normals, texture-coordinates, etc...)
- Edges sharing a vertex called adjacent
- Triangles sharing a vertex or an edge are called adjacent



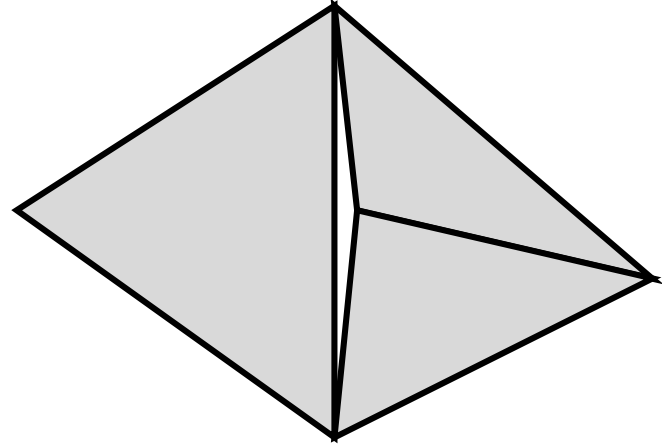
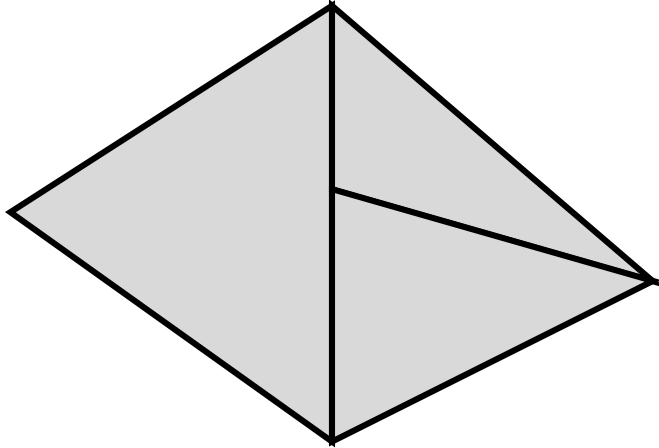
Triangle Meshes

Is this a valid Triangle Mesh?



Triangle Meshes

Is this a valid Triangle Mesh?



Mesh Data Structures

How to represent Triangulations

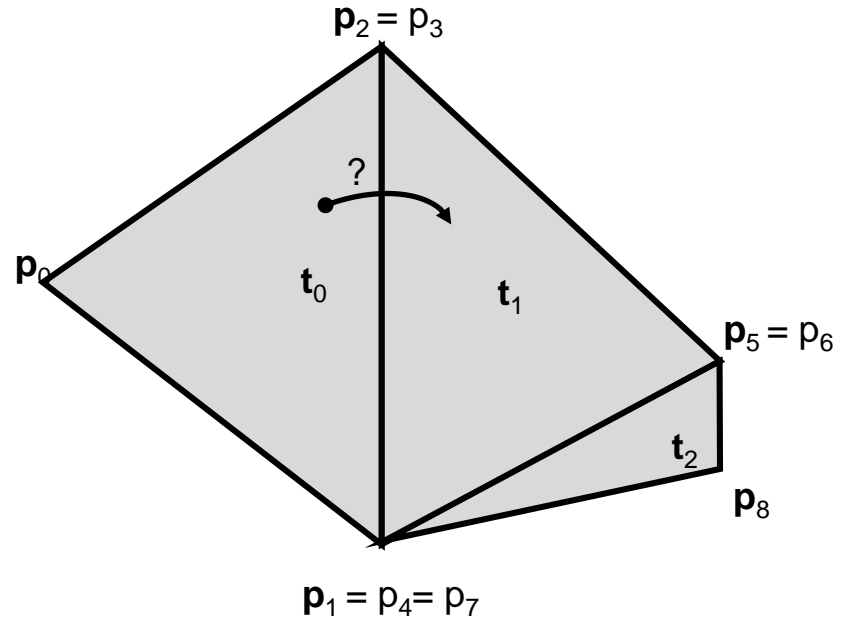
- Considerations: Rendering, Memory usage, complexity of queries
- There is no perfect data structure just a trade off
- Triangle list
- Shared Vertex Structure
- There are many more...

Mesh Data Structures

Triangle List

Triangle list

$p_0: x_0, y_0, z_0;$
 $p_1: x_1, y_1, z_1;$
 $p_2: x_2, y_2, z_2;$ } triangle 0
 $p_3: x_3, y_3, z_3;$
 $p_4: x_4, y_4, z_4;$ } triangle 1
 $p_5: x_5, y_5, z_5;$
 $p_6: x_6, y_6, z_6;$
 $p_7: x_7, y_7, z_7;$
 ...



Mesh Data Structures

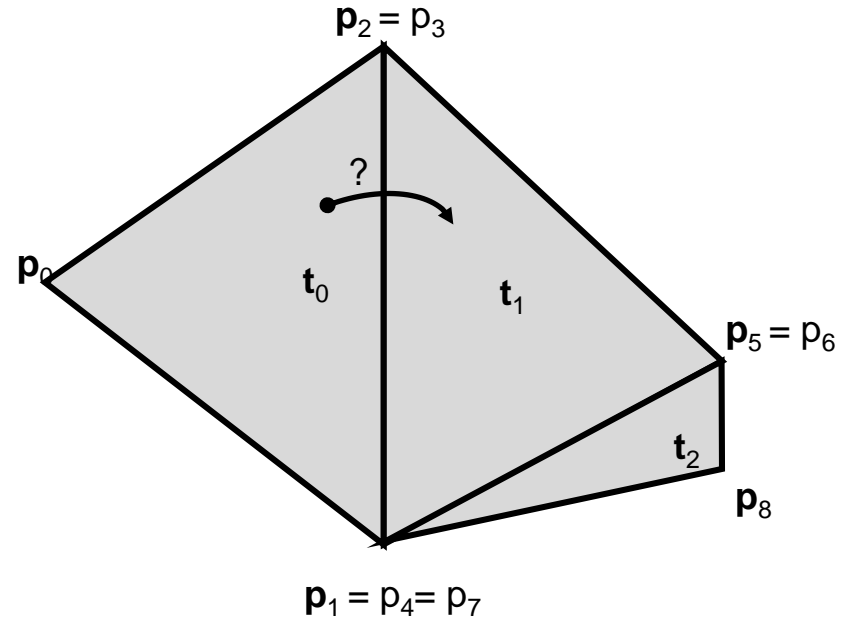
Triangle List

Pros:

- Trivial
- Easy Triangle Removal
- Easy to render

Cons:

- No neighbourhood info (hard to traverse)
- Duplicate Points (waste of storage)



Mesh Data Structures

Shared Vertex Data Structure

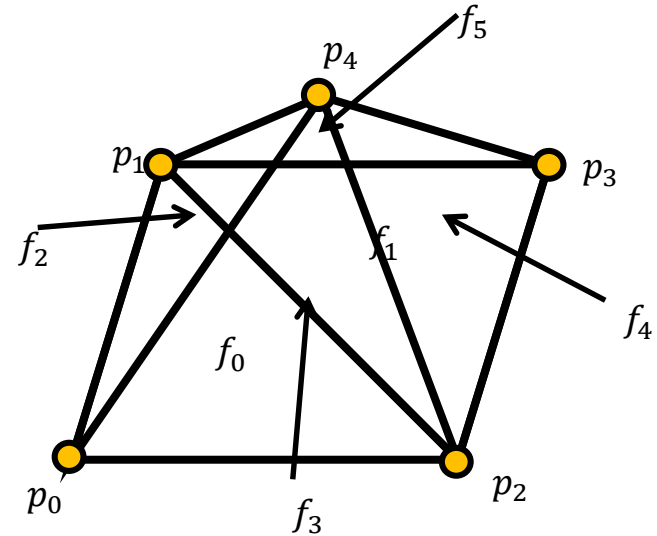
- Also called „indexed face set“
- Consists of two lists

vertex list

p_0 : x_0, y_0, z_0 ;
 p_1 : x_1, y_1, z_1 ;
 p_2 : x_2, y_2, z_2 ;
 p_3 : x_3, y_3, z_3 ;
 p_4 : x_4, y_4, z_4 ;

face list

f_0 : 0, 1, 2;
 f_1 : 2, 1, 3;
 f_2 : 1, 0, 4;
 f_3 : 4, 0, 2;
 f_4 : 4, 2, 3;
 f_5 : 4, 3, 1;



Mesh Data Structures

Shared Vertex Data Structure

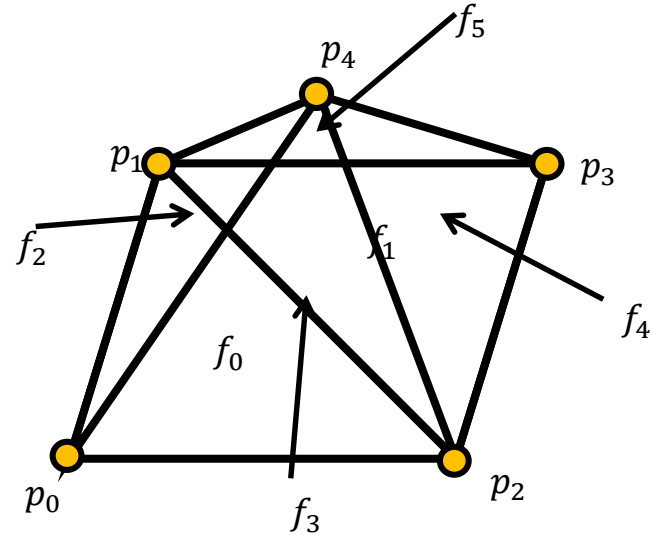
- Remark: geometric queries are difficult
- Extend face list by reference to neighbour faces

vertex list

p_0 : x_0, y_0, z_0 ;
 p_1 : x_1, y_1, z_1 ;
 p_2 : x_2, y_2, z_2 ;
 p_3 : x_3, y_3, z_3 ;
 p_4 : x_4, y_4, z_4 ;

Extended face list

f_0 : 0, 1, 2; 1, 3, 2
 f_1 : 2, 1, 3; 5, 4, 3
 f_2 : 1, 0, 4; ...
 f_3 : 4, 0, 2; ...
 f_4 : 4, 2, 3; ...
 f_5 : 4, 3, 1; ...



Mesh Data Structures

Shared Vertex Data Structure

Pros:

- Easy
- Gold standard for mesh file formats (obj, off)
- Neighbourhood queries cheap (extended version)
- Low storage cost (each vertex stored once)
- Works for Tri/Quad/Poly/Mixed meshes

Cons:

- Neighbourhood queries expensive (non-extended version)
- High Storage cost (extended version)

