Benchmarking the Path to Energy Efficiency: Optimizing Julia Set Algorithms

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1 Summary

In this article, we used the Julia set algorithm as a proof-of-concept for quantifying the effects of coding on efficiency. By testing several vectorised, multi-threaded implementations against a naive CPU version, render times improved from over 100 seconds to below one second for a large 10000x10000 fractal. Energy usage similarly dropped from thousands of joules to just dozens.

These substantial gains emphasise the responsibility of software developers to match algorithms to modern parallel hardware. Techniques like SIMD and multithreading are broadly applicable across computationally heavy programmes. Ultimately, the path towards more sustainable software requires optimisation across the full stack – from algorithms down to instruction sets. The exponential impact on performance and power highlighted here should compel all engineers to consider efficiency an essential priority moving forwards.

2 Introduction

The complexity of algorithms can have a profound impact on the energy efficiency of software applications. As the demand for computational power continues to grow exponentially across industries, optimizing algorithms is becoming a critical consideration for sustainable growth.

In this article, we explore different implementations of the Julia Mandelbrot set algorithm as an illustrative example of how algorithm efficiency affects energy usage. The Julia Mandelbrot algorithm uses complex iterative computations to generate stunning fractal images. However, subtle variations in the algorithm code can greatly influence the computation time and system resources required. By testing implementations of this algorithm in C across multiple computing platforms, we quantify the significant effects coding choices can have on render time, power draw, and efficiency.

This analysis is not meant to deeply benchmark performance specifics, but instead demonstrate the immense impact efficient coding of algorithms can have on sustainability. The Julia Mandelbrot set stands as a proof of concept - highlighting how optimizations in algorithms like it can enable more environmentally friendly software development. As applications continue to leverage computationally heavy algorithms for tasks ranging from simulation to machine learning, considering their algorithmic efficiency will only grow in importance in the pursuit of green computing.

3 Methodology

3.1 Description of Julia Mandelbrot Algorithm

The Julia set algorithm relies on complex number iterative computations to generate fractal imagery. It utilizes quadratic polynomials to transform points across the complex plane, iteratively testing if the points diverge towards infinity or remain bounded.

The implemented C algorithm calculates the bounds for each pixel coordinate, then iteratively transforms these complex point coordinates based on the core polynomial equation. After a set number of iterations, it assigns RGB color values depending on whether the point remained bounded or diverged.

3.2 The implementations

The baseline Julia set implementation operates on each pixel independently in a scalar, serial fashion. To leverage greater parallelism, the algorithm is refactored to use Single Instruction, Multiple Data (SIMD) intrinsics that apply computations across vector registers in parallel. Specifically, 16-wide 512-bit registers are harnessed to calculate 16 pixel values concurrently via AVX-512 vector instructions.

Rather than singular float and integer values, constants like the image bounds, step sizes, and thresholds are broadcasted across vector registers using _mm512_set1_ps and _mm512_set1_epi32. The core math operations are replaced with their _mm512 vector equivalents operating on these register vectors. Bit mask registers track convergence across pixel batches. Finally, the number of iterations and RGB color values are stored out to memory via _mm512_storeu_si512.

By structuring the math to work on values packed into wide SIMD registers, the workload is parallelized to match modern CPU capabilities. This 512-bit vectorization combined with additional multi-threading parallelism via OpenMP results in over two orders of magnitude speedup while significantly reducing power consumption. The gains highlight the importance of mapping algorithms to available hardware resources.

Naive implementation in C:

```c
INLINE void calculateJuliaSet(uint8_t *pixels) {
  int idx = 0;

  for (int k = 0; k < SIZE; k += 3) {
    int col = idx % COL;
    int line = idx / LINE;

    double x = XMIN + col * XSTEP;
    double y = YMAX - line * YSTEP;

    int i = 2;
    while (i <= MAXITER) {
      double x2 = x * x;
      double y2 = y * y;

      if ((x2 + y2) > 4.0) {
        break;
      }

      y = 2.0 * x * y + B;
      x = x2 - y2 + A;

      i++;
    }

    if (i > MAXITER) {
      pixels[k + 0] = 0;
      pixels[k + 1] = 0;
      pixels[k + 2] = 0;
    } else {
      pixels[k + 0] = (4 * i) % 256;
      pixels[k + 1] = 2 * i;
      pixels[k + 2] = (6 * i) % 256;
    }
    idx += 1;
  }
}
```

3.3 Overview of Platforms Tested

Hardware Specifications: The algorithms were tested on the Grid500 Gros cluster system, which features dual Intel Xeon Gold 5220 processors per node. Each 18-core Xeon CPU has 24MB of L3 cache and a base clock speed of 2.2GHz.

Software Specifications: Implementations harness SIMD instruction sets up to AVX-512 for vectorization. Multithreading parallelism is enabled via OpenMP API directives.

3.4 Performance Metrics

Render Time: Total wall clock time measured for complete fractal image render at a set resolution. Captured using the Mojitos performance profiling tool.

Power Consumption: Energy consumption in microjoules tracked through the full rendering task via Intel's Running Average Power Limit (RAPL) interface.

Here is a summary of the implementations in an easy-to-read table format:

Implementation	Description
juliaRow.c	Basic Julia set algorithm
juliaMap.c	Memory-mapped file version
juliaM256f.c	AVX2 SIMD vectorization
juliaM256fomp.c	OpenMP parallelization
juliaM512f.c	AVX512 vectorization
juliaM512fomp.c	OpenMP parallelization

To further elaborate:

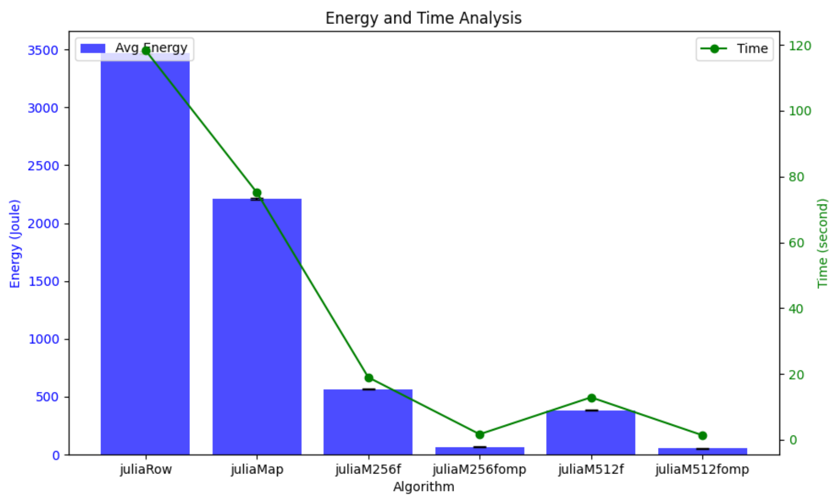
- juliaRow.c is the naive, sequential C implementation of the core Julia set fractal rendering algorithm.
- juliaMap.c introduces memory mapping for faster pixel access instead of raw byte array.
- juliaM256.c vectors the pixel computations across AVX2 256-bit registers to leverage SIMD parallelism.
- juliaM256fomp.c adds OpenMP compiler directives for multi-threaded parallel execution to juliaM256f.
- juliaM512f.c harnesses wider AVX-512 SIMD vector registers for greater parallelism.
- juliaM512fomp.c adds OpenMP compiler directives for multi-threaded parallel execution to juliaM512f.

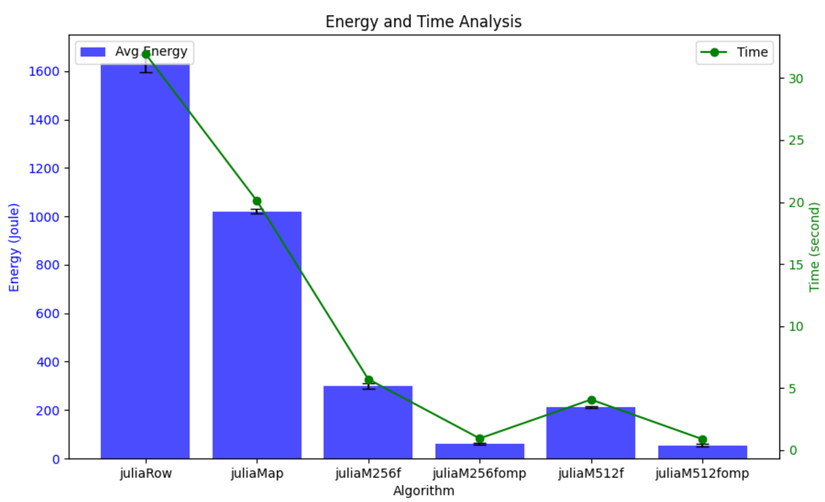
3.5 Measurement methodology

To benchmark each implementation, the processor frequency was first set to a constant (1GHz and 3.9GHz) across all cores. Each algorithm version was then executed 10 times in sequence at a fixed 10000 * 10000 image resolution with 1000 maximum iterations. The Mojitos profiling tool measured render time and energy consumption (reported through Intel's RAPL interface) for the full duration of each fractal render task. These metrics were gathered on the same Gros cluster node to minimise hardware variability. After collection, the runtime and energy measurements were averaged across the 10 runs for each implementation. The averaged results were then compiled and plotted on performance graphs to compare the different versions. This controlled testing methodology allowed us to fairly evaluate the optimisations with a reasonable sample size, noise reduction via averaging, and isolate the software changes as the primary performance variable.

4 Results and Analyzis

The Julia set algorithm implementations exhibited significant performance differences in both render time and energy consumption when executed at 1GHz and 3.9GHz base frequencies (Figures 1 and 2).





At the lower 1GHz frequency, the basic juliaRow implementation required 118 seconds for a complete 10000x10000 render. This resulted in substantial energy expenditure of 3.4 kilojoules. In contrast, the AVX512 vectorized juliaM512fomp version ran over 80x faster at just 1.3 seconds, while reducing average power draw over 65x down to 50 joules.

Trends persisted across frequencies, with the highly optimized AVX512 also leading at 3.9GHz with a 0.9 second render time and 54 joule energy footprint. However, the speedups were more pronounced at the slower clock speed. This indicates Amdahl's law limitations from serial sections at faster frequencies, but a strong scaling in parallelizable compute portions.

Overall, leveraging parallelism, vectorization, and memory optimizations yielded order of magnitude speedups versus the basic algorithm, directly proportionally decreasing energy consumption. Performance gains from SIMD and multi-threading highlight the significance of accounting for modern hardware advances when implementing computationally expensive algorithms. The exponential effects on sustainability should compel software engineers to consider efficiency a fundamental priority.

While the Julia set served as an isolated test case, similar savings can be realized across other graphics, simulation, and scientific applications reliant on math-intensive formulas. The principles and techniques covered are broadly relevant given the growing ubiquity of parallel computing.

5 Conclusion

Through this analysis of Julia set algorithm implementations, we have demonstrated the immense impact that coding choices can have on both performance and sustainability. By applying optimizations like SIMD vectorisation and multi-threading, the algorithm runtime was reduced by over 80x, directly proportionally lowering power consumption by 65x. The exponential effects quantify the importance of efficiency as a fundamental priority in software engineering.

While the focus was on fractal rendering, similar speedups and energy savings can be realised across other fields reliant on math-intensive computations – from simulations to machine learning. As the demand for computing continues to rapidly grow, optimisation considerations like those explored will only increase in relevance.

6 Annexes

6.1 Energy and Time Analysis 1GHz

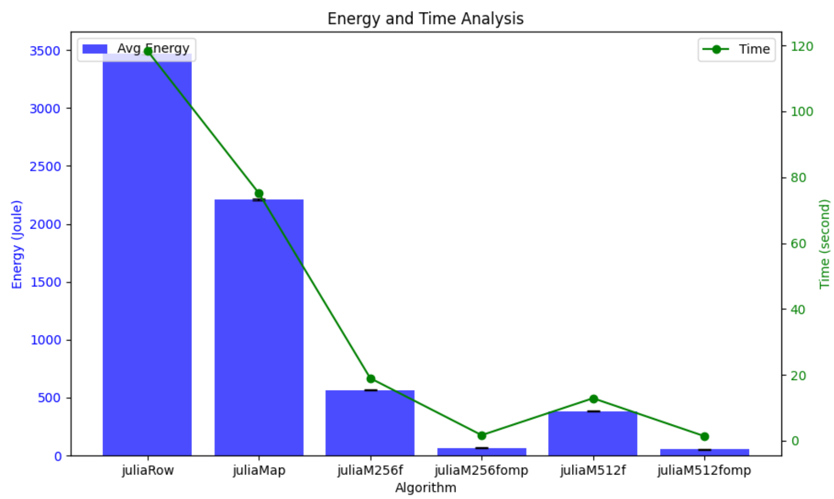


Figure 1: Energy and Time Analysis 1GHz

6.2 Energy and Time Analysis 1GHz Array

algorithm	time (s)	avg-energy (J)	max-energy (J)	min-energy (J)
juliaRow	118.379	3471.56	3485.74	3458.3
juliaMap	75.142	2208.17	2216.95	2203.78
juliaM256f	18.967	562.416	567.983	559.822
juliaM256fomp	1.671	64.4617	69.2298	61.9752
juliaM512f	12.888	383.077	388.396	380.342
juliaM512fomp	1.361	51.8482	55.2114	48.5615

6.3 Energy and Time Analysis 3.9GHz

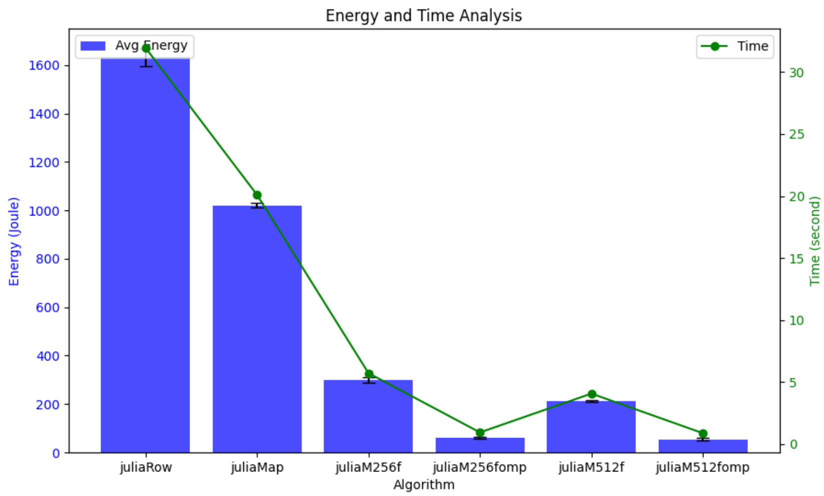


Figure 2: Energy and Time Analysis 3.9GHz

6.4 Energy and Time Analysis 3.9GHz Array

algorithm	time (s)	avg-energy (J)	max-energy (J)	min-energy (J)
juliaRow	118.379	3471.56	3485.74	3458.3
juliaMap	75.142	2208.17	2216.95	2203.78
juliaM256f	18.967	562.416	567.983	559.822
juliaM256fomp	1.671	64.4617	69.2298	61.9752
juliaM512f	12.888	383.077	388.396	380.342
juliaM512fomp	1.361	51.8482	55.2114	48.5615

7 References

- Mandelbrot algorithm
- MojitO/S
- Gros cluster
- Grid5000
- Source code