Photonics – 2020

Second Project: Bit Rate in SMFs

In this project we study dispersion-induced limitations in SMFs (single-mode fibers) operated in the linear regime. Namely, the main goal is to calculate the maximum value for the **bit rate** under the assumption that we have **chirped Gaussian pulses** propagating along the fiber. So, at the input, the pulse envelope takes the form ($C \in \mathbb{R}$ is the dimensionless **chirp** parameter)

$$\boxed{z=0} \quad \rightarrow \quad \mathsf{A}(0,t) = \mathsf{A}_0 \exp\left[-\frac{1+iC}{2}\left(\frac{t}{T_0}\right)^2\right].$$

For Gaussian pulses, the RMS (root mean square) value of the pulse width is

$$\sigma_0 = \frac{T_0}{\sqrt{2}} .$$

Furthermore, if one takes into account the GVD (group-velocity dispersion) parameter β_2 as well as β_3, one gets $(0 \le z \le L)$

$$\left(\frac{\sigma}{\sigma_0}\right)^2 = \left(1 + C\,\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + \left(\frac{\beta_2 z}{2\sigma_0^2}\right)^2 + \left(1+C^2\right)^2\left(\frac{\beta_3 z}{4\sqrt{2}\,\sigma_0^3}\right)^2$$

for optical sources with small spectral width (i.e., for optical sources whose spectral width is much smaller than the bit rate). If one introduces

$$\tau_0 = \sqrt{|\beta_2|\,L}$$

and

$$p = \frac{1}{4}\left(1+C^2\right) \ge \frac{1}{4} ,$$

then we get, if $\beta_2 \neq 0$,

$$\left(\frac{\sigma}{\tau_0}\right)^2 = \left(\frac{\sigma_0}{\tau_0}\right)^2 + \operatorname{sgn}(\beta_2)\, C\left(\frac{z}{L}\right) + p\left(\frac{\tau_0}{\sigma_0}\right)^2\left(\frac{z}{L}\right)^2 + \frac{p^2}{2}\left(\frac{\tau_0}{\sigma_0}\right)^4\left(\frac{\beta_3^2}{|\beta_2|^3 L}\right)\left(\frac{z}{L}\right)^2.$$

Let us define a normalized distance along the fiber as

$$\xi = \frac{z}{L}, \quad 0 \le \xi \le 1,$$

and, also, a normalized square width χ, such that

$$\chi = \left(\frac{\sigma}{\tau_0}\right)^2.$$

Obviously, one has $\chi = \chi(\xi)$. Accordingly, we may write

$$\boxed{\chi(\xi) = \chi_0 + \operatorname{sgn}(\beta_2)\, C\,\xi + \frac{p}{\chi_0}\left(1 + \frac{a\,p}{2\,\chi_0}\right)\xi^2}$$

if one defines the following dimensionless parameter

$$a = \frac{\beta_3^2}{|\beta_2|^3 L}.$$

At the input, where $\xi = 0$, we get $\chi_0 = \chi(0)$. At the output, where $\xi = 1$, we get instead $\chi_1 = \chi(1)$ with

$$\chi_1 = \chi_0 + \operatorname{sgn}(\beta_2)\, C + \frac{p}{\chi_0}\left(1 + \frac{a\,p}{2\,\chi_0}\right).$$

In Fig. 1 we present the variation of χ_1 with χ_0 for $C = 0$ (hence, $p = 0.25$) and $a = 1$.

This last equation shows that χ_1 depends on χ_0. The minimum value for χ_1 is reached when $\chi_0 = \chi_0^{\mathrm{opt}}(C)$ which is a solution for

$$\frac{d\,\chi_1}{d\,\chi_0}=0 \quad \Rightarrow \quad \boxed{\chi_0^3-p\,\chi_0-q=0}\,,$$

where

$$q=a\,p^2=\left(1+C^2\right)^2\frac{a}{16}\,.$$

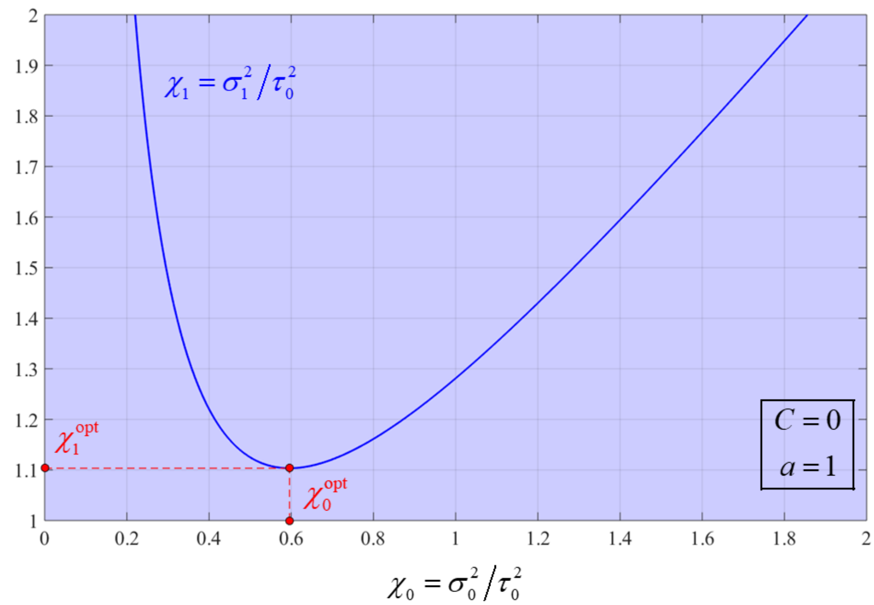

Figure 1

After having determined $\chi_0^{\mathrm{opt}}(C)$, we can define

$$\chi^{\mathrm{opt}}(\xi,C)=\chi_0^{\mathrm{opt}}(C)+\mathrm{sgn}(\beta_2)\,C\,\xi+\frac{p}{\chi_0^{\mathrm{opt}}(C)}\left[1+\frac{a}{\chi_0^{\mathrm{opt}}(C)}\right]\xi^2\,,$$

so that

$$\chi_1^{\mathrm{opt}}(C)=\chi^{\mathrm{opt}}(\xi=1,C)=\chi_0^{\mathrm{opt}}(C)+\mathrm{sgn}(\beta_2)\,C+\frac{p}{\chi_0^{\mathrm{opt}}(C)}\left[1+\frac{a}{\chi_0^{\mathrm{opt}}(C)}\right].$$

Moreover, you can also define the especially important function

$$\boxed{\mu(\xi, C) = \frac{\chi^{\text{opt}}(\xi, C)}{\chi_0^{\text{opt}}(C)}}.$$

Of course, we always have

$$\mu(\xi = 0, C) = 1.$$

For $a = 0$, we just obtain

$$\boxed{a = 0} \quad \mapsto \quad \mu(\xi, C) = 1 + \operatorname{sgn}(\beta_2) \frac{C}{\chi_0^{\text{opt}}(C)} \xi + \frac{1 + C^2}{4\left[\chi_0^{\text{opt}}(C)\right]^2} \xi^2.$$

This function is important to justify the final choice of the C parameter to calculate the bit rate. You should notice that, for $\beta_2 C < 0$, we may get a value $C = C_0$, such that

$$\boxed{C_0} \mapsto \boxed{\mu(\xi = 1, C_0) = 1}.$$

Let us define

$$\boxed{\mu_0(\xi) \equiv \mu(\xi, C_0)}.$$

By this definition we always have $\mu_0(0) = \mu_0(1) = 1$.

When $\beta_3 = 0$, the optimum value $\chi_0 = \chi_0^{\text{opt}}$ is simply given by $(q = 0)$

$$\chi_0^{\text{opt}} = \sqrt{p}.$$

So, then, we have

$$\left\| \begin{array}{l} \beta_2 \neq 0 \\ \beta_3 = 0 \end{array} \right. \quad \Rightarrow \quad \left\| \begin{array}{l} \chi_0^{\text{opt}}(C) = \dfrac{1}{2}\sqrt{1 + C^2} \\ \chi_1^{\text{opt}}(C) = \operatorname{sgn}(\beta_2) C + \sqrt{1 + C^2} \end{array} \right.$$

$$\therefore \quad \boxed{\mu(\xi, C) = 1 + \frac{2\,\mathrm{sgn}(\beta_2)\,C}{\sqrt{1+C^2}}\,\xi + \xi^2}\,.$$

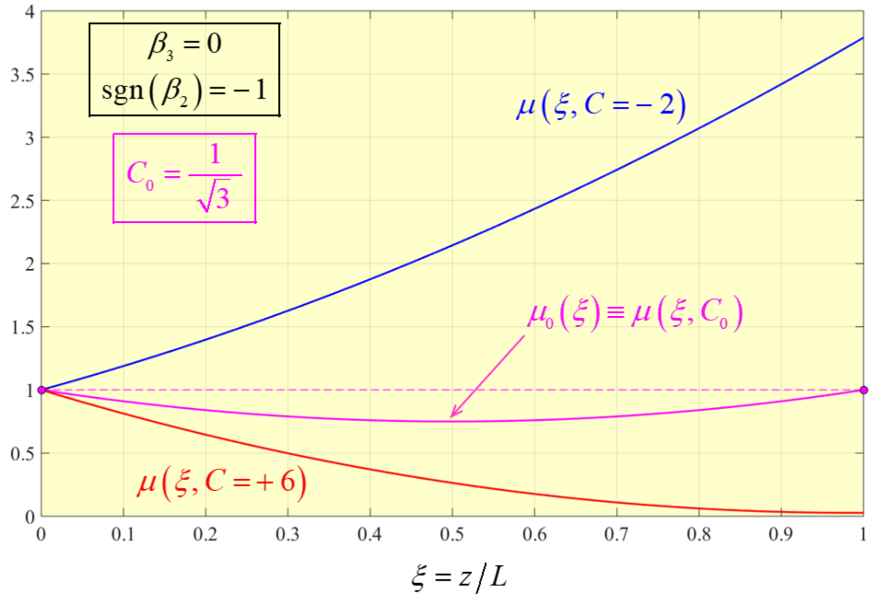

Figure 2

Now, for $\beta_2 C > 0$, it is always $\chi_1^{\mathrm{opt}} > \chi_0^{\mathrm{opt}}$. However, for $\beta_2 C < 0$, we get

$$\chi_1^{\mathrm{opt}}(C) = -|C| + \sqrt{1+C^2}$$

and

$$\mu(\xi, C) = 1 - \frac{2|C|}{\sqrt{1+C^2}}\,\xi + \xi^2 \quad \Rightarrow \quad \mu(1, C) = 2\left[1 - \frac{|C|}{\sqrt{1+C^2}}\right] \quad \Rightarrow \quad \mu(1, \pm\infty) = 0\,.$$

In this case, then, there exists a value $C = C_0$ such that

$$\boxed{C_0 = -\,\mathrm{sgn}(\beta_2)\,\frac{1}{\sqrt{3}}} \quad \Rightarrow \quad \boxed{\chi_1^{\mathrm{opt}}(C_0) = \chi_0^{\mathrm{opt}}(C_0) = \frac{1}{\sqrt{3}}}\,,$$

thereby leading to

$$\boxed{\mu_0(\xi) = 1 - \xi + \xi^2} \quad \mapsto \quad \boxed{\mu_0(1) = 1}.$$

The minimum of this function occurs for $\xi = 1/2$ where $\mu_0 = 3/4$.

In this project we intend to calculate the **maximum** value $B_0\left[\mathrm{Gb}\cdot\mathrm{s}^{-1}\right]$ of the **bit rate** in terms of the fiber length $L\left[\mathrm{km}\right]$. Consider

$$\left[\!\left[\begin{array}{l} L = 1\ \mathrm{km} \\ L = 5\ \mathrm{km} \\ L = 10\ \mathrm{km} \\ L = 50\ \mathrm{km} \\ L = 100\ \mathrm{km} \end{array}\right.\right.$$

You can always adopt the following criterion: For Gaussian pulses, 95% of the pulse energy remains within the bit slot $T_B = 1/B$, where B is the bit rate, as long as $\sigma_{\max} \le T_B/4$. Therefore,

$$B \le B_0 = \frac{1}{4\sigma_{\max}}.$$

You should be careful about this value $\sigma_{\max}$. In fact, we must always have:

$$\sigma_{\max} = \max\left\{\sigma(\xi)\right\}_{0 \le \xi \le 1}.$$

The maximum value $\sigma_{\max}$ does not always occur for $\xi = 1$.

You should consider the following cases:

Case 1 – A fiber with $\beta_2 = -\ 20\ \text{ps}^2/\text{km}$ and $\beta_3 = 0$. You should indicate what value have you chosen for the chirp parameter and why.

Case 2 – A fiber with $\beta_2 = -\ 1\ \text{ps}^2/\text{km}$ and $\beta_3 = 2\ \text{ps}^3/\text{km}$. You should indicate what value have you chosen for the chirp parameter and why.

Case 3 – A fiber with $\beta_2 = -\ 0.1\ \text{ps}^2/\text{km}$ and $\beta_3 = 2\ \text{ps}^3/\text{km}$. You should indicate what value have you chosen for the chirp parameter and why.

Case 4 – A fiber with $\beta_2 = 0$ and $\beta_3 = 2\ \text{ps}^3/\text{km}$. You should indicate what value have you chosen for the chirp parameter and why.

You should always provide full explanations – preferably illustrated with graphical plots – of every choice made along the calculation process. A full characterization of the input pulse (in terms of the chosen values for C and σ_0) are required.

Obviously, **Case 4** was not addressed in this presentation. You should be able to derive that specific case using a similar method as the one herein presented.