

Linear Algebra

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Concepts

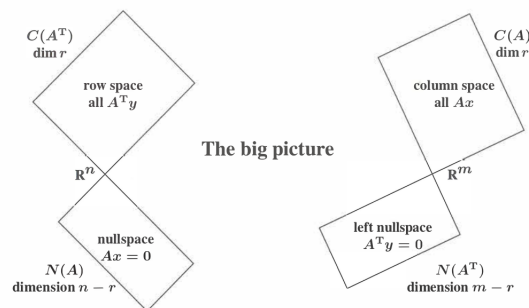
Determinant - $\det(A)$

Singular matrix - $\det(A) = 0$

Vector Spaces and Subspaces

Four Subspaces

Let A a m by n matrix

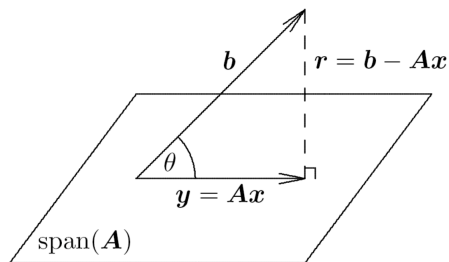


- $r = \text{rank}(A) = \dim(C(A)) = \text{Number of pivots}$
- $N(A) = \text{Null space of } A = \text{Ker}(A) = (\text{FR}) \text{ Noyau de } A$
- $C(A) = \text{Column Space of } A = \text{Im}(A) = (\text{FR}) \text{ Image de } A$

Fundamental Theorem of Linear Algebra

- The column space and row space both have dimension r .
- The nullspaces have dimension $n - r$ and $m - r$
- $N(A)$ is the orthogonal complement of the row space $C(A^T)$ (in) \mathbb{R}^n
- $N(A^T)$ is the orthogonal complement of the column space $C(A)$ (in) \mathbb{R}^m

Least Squares



- $\text{span}(A) \perp r$
- $\text{span}(A) = C(A) = \text{Im}(A)$
- $y \in \text{span}(A)$

$$\Rightarrow \text{span}(A) \perp r$$

because each column of A is perpendicular to r

$$\Leftrightarrow A^T r = \vec{0}$$

$$\Leftrightarrow A^T(b - Ax) = \vec{0}$$

$$\Leftrightarrow x = (A^T A)^{-1} A^T b$$

$$\Leftrightarrow y = A(A^T A)^{-1} A^T b$$

$$\Leftrightarrow \rho = A(A^T A)^{-1} A^T$$

ρ being the projection matrix

Matrix factorizations

1. $A = LU$
2. $A = LDU$
3. $PA = LU$
4. $EA = R$
5. $S = C^T C$
6. $A = QR$ - orthonormal columns in Q , upper triangular R

Requirements

7. $A = X \Lambda X^{-1}$ - Eigenvectors in X , eigenvalues in Λ
8. $A = Q \Lambda Q^T$
9. $A = B J B^{-1}$
10. $A = U \Sigma V^T$ - SVD: (orthogonal U is $m \times m$)($m \times n$ singular value matrix $\sigma_1, \dots, \sigma_r$ on its diagonal)(orthogonal V is $n \times n$)
11. $A^+ = V \Sigma^+ U^T$ - Pseudoinverse
12. $A = QS$
13. $A = U \Sigma U^{-1}$
14. $A = QTQ^{-1}$