# Derivatives - Hull

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## Concepts

**Maturity** - The end of the life of a contract.

**Short** - Short position = Seller position

**Long** - Long position = Buyer position

**Risk factor** - Source of uncertainty.

- IR: Interest Rate
    - Yield Curves: Rate curves given by countries
    - Reference rate curves: EURIBOR, LIBOR, SOFR
    - CDS curves
- FX: Foreign Exchange. ex. EUR/USD
- Equity: ex. BNP equity.
- Commodities: ex. Oil price.

**Spot Price** - The price for immediate delivery.

**OTC Market** - Over-the-counter. A market where traders deal directly with each other or through an interdealer broker. The traders are usually financial institutions, corporations, and fund managers.

**Exchange-traded markets** - A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Contrary to OTC

## Notation

$S$ - Stock price, more generally underlying asset price.

$\Delta$ - Variation

$t$ - Time

$\phi$ - Normal distribution

$\sigma$ - Volatility

$r$ - Interest rate

$c$,$f$ - Price of option

## Assets

Most known:

### - Commodities

### - Real State

### - Currencies

### - Stocks

### - Future contracts

### - Bonds

Types of bonds: Zero coupon bond, Coupon bond, Convertible bond, Callable bond

- Zero coupon bond: Only has one face value payment at maturity.
- Coupon bond: It has multiple coupon payments and one face value payment. Coupon rate = $(k * n)/N, k$ : number of coupons per year.
- Convertible bond: Only convert to stock when market price of the bond ¡ value of the stock.
- Callable bond: Have to return the bonds for the company at a fixed price when the company decides to call it back.

**Pricing:**

- Zero coupon bond:

$$P = \frac{N}{(1 + r_T)^T}$$

- Coupon bond:

⇒ Method 1:

Continuously compounded and compounded once per annum:

$$P = Ne^{-r_T T} + \sum_{i=i_0}^{T} Ce^{-ir_i}, P = \frac{N}{(1 + r_T)^T} + \sum_{i=i_0}^{T} \frac{C}{(1 + r_i)^i}$$

⇒ Method 2 (Yield):

Continuously compounded and compounded once per annum:

$$P = Ne^{-yT} + \sum_{i=i_0}^{T} Ce^{-iy}, P = \frac{N}{(1 + y)^T} + \sum_{i=i_0}^{T} \frac{C}{(1 + y)^i}$$

$P$ : Bond price
$N$ : Principal / face value / value
$C$ : Coupon
$i_0$ : Time for receiving first coupon with years as unit. Ex: 1/2 year, 1 year, etc.
$i$ : Time. The difference between two consecutives $i$ is $i_0$
$r_i$ : Rate interest at time $i$
$T$ : Maturity
$y$ : Yield

## Derivatives

A derivative involves two parties agreeing to a future transaction. Its value depends on (or derives from) the values of other underlying variables.

### Forward contracts

- A contract that obligates the holder to buy or sell an asset for a predetermined delivery price at a predetermined future time.

### Future contracts

- A contract that obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. The contract is settled daily.

### Difference Forward and Future

- Future are traded in exchange markets whereas Forwards are OTC.

### Options

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### Swaps: Interest rate swap

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### Swaps: Currency swap

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### Swaps: CDS(Credit default swap)

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### Swaps: Quanto

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Frank Facundo

## Black and Scholles

### Assumptions

– Stock price assumes that percentage changes in very short period of time are normally distributed.

$$\frac{\Delta S}{S} \backsim \phi(\mu \Delta t, \Delta t)$$

### Equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

## Risk

### Greeks or Risk sensitivities

| Greek | Symbol | Measures | Definition |
|---|---|---|---|
| Delta | $\Delta = \frac{\partial c}{\partial S}$ | Underlying variable (S) exposure | Change in option price due to spot |
| Gamma | $\Gamma = \frac{\partial^2 c}{\partial S^2}$ | Underlying variable (S) convexity | Curvature of option price with respect to spot |
| Theta | $\Theta = \frac{\partial c}{\partial t}$ | Time decay | Change in option price due to time passing |
| Vega | $\upsilon = \frac{\partial c}{\partial \sigma}$ | Volatility exposure | Change in option price due to volatility |
| Rho | $\rho = \frac{\partial c}{\partial r}$ | Interest rate exposure | Change in option price due to interest rates |
| Volga | $\frac{\partial^2 c}{\partial \sigma^2}$ | Volatility convexity | Curvature of option price with respect to spot |
| Vanna | $\frac{\partial c}{\partial S \partial t}$ | | Change in Delta due to Volatility |

## VaR & ES

## PnL