

Derivatives - Hull

Last Updated May 16, 2023

1. Concepts

Maturity - The end of the life of a contract.

Arbitrage - The simultaneous purchase and sale of the same asset in different markets in order to profit from tiny differences in the asset's listed price.

Market to Market (MtM) - Price of an asset/product at the present moment.

Short - Short position = Seller position

Long - Long position = Buyer position

Risk factor - Source of uncertainty.

- IR: Interest Rate
 - Yield Curves: Rate curves given by countries
 - Reference rate curves: EURIBOR, LIBOR, SOFR
 - CDS curves
- FX: Foreign Exchange. ex. EUR/USD
- Equity: ex. BNP equity.
- Commodities: ex. Oil price.

Spot Price - The price for immediate delivery.

OTC Market - Over-the-counter. A market where traders deal directly with each other or through an interdealer broker. The traders are usually financial institutions, corporations, and fund managers.

Exchange-traded markets - A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Contrary to OTC

2. Notation

S - Stock price, more generally underlying asset price.

Δ - Variation

t - Time

ϕ - Normal distribution

σ - Volatility

r - Interest rate

c, f - Price of option

3. Assets

Most known:

- Commodities

- Real State

- Currencies

- Stocks

- Bonds

Types of bonds: Zero coupon bond, Coupon bond, Convertible bond, Callable bond

- Zero coupon bond: Only has one face value payment at maturity.
- Coupon bond: It has multiple coupon payments and one face value payment. Coupon rate = $(k * n)/N$, k : number of coupons per year.

- Convertible bond: Only convert to stock when market price of the bond < value of the stock.
- Callable bond: Have to return the bonds for the company at a fixed price when the company decides to call it back.

Pricing:

- Zero coupon bond:

$$P = \frac{N}{(1 + r_T)^T}$$

- Coupon bond:

⇒ Method 1:

Continuously compounded and compounded once per annum:

$$P = Ne^{-r_T T} + \sum_{i=i_0}^T Ce^{-ir_i}, P = \frac{N}{(1 + r_T)^T} + \sum_{i=i_0}^T \frac{C}{(1 + r_i)^i}$$

⇒ Method 2 (Yield):

Continuously compounded and compounded once per annum:

$$P = Ne^{-yT} + \sum_{i=i_0}^T Ce^{-iy}, P = \frac{N}{(1 + y)^T} + \sum_{i=i_0}^T \frac{C}{(1 + y)^i}$$

P : Bond price

N : Principal / face value / value

C : Coupon

i_0 : Time for receiving first coupon with years as unit. Ex: 1/2 year, 1 year, etc.

i : Time. The difference between two consecutives i is i_0

r_i : Rate interest at time i

T : Maturity

y : Yield

4. Credit Risk

Probability of default by time t

$$Q(t) = 1 - e^{-\int_0^t \lambda(\tau) d\tau} = 1 - e^{-\bar{\lambda}(t)t}, \quad \bar{\lambda}(T) = \frac{s(T)}{1 - R}$$

λ : Hazard rate

$s(T)$: Bond yield spread for a T-year bond.

R : Recovery rate. Percentage of the bond value that is recovered in case of default.

$$Q(t) = 1 - e^{-\frac{s(T)}{1-R}t}$$

$$V(t) = e^{-\frac{s(T)}{1-R}t}$$

$V(t)$: Probability of non-default by time t .

Frank Facundo

CVA and DVA

CVA: Credit Value Adjustment.

DVA: Debt Value Adjustment.

$$CVA = \sum_{i=1}^N q_i v_i \quad DVA = \sum_{i=1}^N q_i^* v_i^*$$

q_i : Risk-neutral probability of default of counterparty during the i th interval.

v_i : Present value of the expected loss to the bank if the counterparty defaults during the i th interval.

q_i^* : Risk-neutral probability of default of the bank during the i th interval.

v_i^* : Present value of the expected loss to the counterparty if the bank defaults during the i th interval.

$q_i = V(t_{i-1}) - V(t_i)$

$v_i = f_{nd}(1 - R)$

5. Derivatives

A derivative involves two parties agreeing to a future transaction. Its value depends on (or derives from) the values of other underlying variables.

Forward contracts

- A contract that obligates the holder to buy or sell an asset for a predetermined delivery price at a predetermined future time. The contract has a virtual zero cost at time zero.

- **Forward's delivery price when created**

$$F_0 = S_0 e^{rT}$$

F_0 : Forward delivery price

S_0 : Price of underlying asset

T : Time to maturity

r : Risk-free rate

⇒ **Assumptions:**

- The gain is the free-risk rate
- Not coupon neither yield from underlying asset.

- **MtM of a Forward** (Payoff)

$$f = (F_0 - K)e^{-rT}$$

f : MtM of Forward

F_0 : Forward price at present time (if it was created at present time). Formula is above.

K : Delivery price for a contract that was negotiated some time ago. (F_0 when forward was created.)

T : Time to maturity

r : Risk-free rate

Future contracts

- A contract that obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. The contract is settled daily.

Difference Forward and Future

- Future are traded in exchange markets whereas Forwards are OTC.

Options

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Swaps: Interest rate swap

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Swaps: Currency swap

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Swaps: CDS(Credit default swap)

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Swaps: Quanto

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6. Black and Scholles

Assumptions

– Stock price assumes that percentage changes in very short period of time are normally distributed.

$$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \Delta t)$$

Equation

$$\frac{\partial f}{\partial t} + rS \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

7. Risk

Greeks or Risk sensitivities

Greek	Symbol	Measures	Definition
Delta	$\Delta = \frac{\partial c}{\partial S}$	Underlying variable (S) exposure	Change in option price due to spot
Gamma	$\Gamma = \frac{\partial^2 c}{\partial S^2}$	Underlying variable (S) convexity	Curvature of option price with respect to spot
Theta	$\Theta = \frac{\partial c}{\partial t}$	Time decay	Change in option price due to time passing
Vega	$v = \frac{\partial c}{\partial \sigma}$	Volatility exposure	Change in option price due to volatility
Rho	$\rho = \frac{\partial c}{\partial r}$	Interest rate exposure	Change in option price due to interest rates
Volga	$\frac{\partial^2 c}{\partial \sigma^2}$	Volatility convexity	Curvature of option price with respect to spot
Vanna	$\frac{\partial c}{\partial S \partial t}$		Change in Delta due to Volatility

VaR & ES

PnL