Derivatives - Hull

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1. Concepts

Maturity - The end of the life of a contract.
Arbitrage - The simultaneous purchase and sale of the same asset in different markets in order to profit from tiny differences in the asset's listed price.
Risk-neutral - This assumption means investors do not increase the expected return they require from an investment to compensate for increased risk.
Market to Market (MtM) - Price of an asset/product at the present moment.
Short - Short position = Seller position
Long - Long position = Buyer position
Risk factor - Source of uncertainty.

- IR: Interest Rate
  - Yield Curves: Rate curves given by countries
  - Reference rate curves: EURIBOR, LIBOR, SOFR
  - CDS curves
- FX: Foreign Exchange. ex. EUR/USD
- Equity: ex. BNP equity.
- Commodities: ex. Oil price.

Spot Price - The price for immediate delivery.
OTC Market - Over-the-counter. A market where traders deal directly with each other or through an interdealer broker. The traders are usually financial institutions, corporations, and fund managers.
Exchange-traded markets - A derivatives exchange is a market where individuals and companies trade standardized contracts that have been defined by the exchange. Contrary to OTC

2. Notation

S - Stock price, more generally underlying asset price.
Δ - Variation
t - Time
T - Maturity. Time when contract is finished.
ϕ - Normal distribution
σ - Volatility
r - Interest rate
c,f - Price of option
K - Strike price. Price to buy/sell the underlying asset.

3. Assets

Most known:

3.1 Commodities

Gold, Oil, Corn, etc.

3.2 Real State

Houses, buildings, lands, etc.

3.3 Currencies

EUR, USD, GBP, etc.

3.4 Stocks

BNP, Apple, etc.

3.5 Bonds

Types of bonds: Zero coupon bond, Coupon bond, Convertible bond, Callable bond

- Zero coupon bond: Only has one face value payment at maturity.
- Coupon bond: It has multiple coupon payments and one face value payment. Coupon rate $= (k*n)/N, k$: number of coupons per year.
- Convertible bond: Only convert to stock when market price of the bond < value of the stock.
- Callable bond: Have to return the bonds for the company at a fixed price when the company decides to call it back.

Pricing:
- Zero coupon bond:

$$P = \frac{N}{(1+r_T)^T}$$

- Coupon bond:
⇒ Method 1:
Continuously compounded and compounded once per annum:

$$P = Ne^{-r_T T} + \sum_{i=i_0}^{T} Ce^{-ir_i}, P = \frac{N}{(1+r_T)^T} + \sum_{i=i_0}^{T} \frac{C}{(1+r_i)^i}$$

⇒ Method 2 (Yield):
Continuously compounded and compounded once per annum:

$$P = Ne^{-yT} + \sum_{i=i_0}^{T} Ce^{-iy}, P = \frac{N}{(1+y)^T} + \sum_{i=i_0}^{T} \frac{C}{(1+y)^i}$$

P : Bond price
N : Principal / face value / value
C : Coupon
i_0 : Time for receiving first coupon with years as unit. Ex: 1/2 year, 1 year, etc.
i : Time. The difference between two consecutives i is i_0
r_i : Rate interest at time i
T : Maturity
y : Yield

4. Credit Risk

Probability of default by time t

$$Q(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau} = 1 - e^{-\overline{\lambda}(t)t}, \qquad \overline{\lambda}(T) = \frac{s(T)}{1-R}$$

λ : Hazard rate
$s(T)$: Bond yield spread for a T-year bond.
R : Recovery rate. Percentage of the bond value that is recovered in case of default.

$$Q(t) = 1 - e^{-\frac{s(T)}{1-R}t}$$
$$V(t) = e^{-\frac{s(T)}{1-R}t}$$

$V(t)$: Probability of non-default by time t.

CVA and DVA

CVA: Credit Value Adjustment.
DVA: Debt Value Adjustment.

$$CVA = \sum_{i=1}^{N} q_i v_i \qquad DVA = \sum_{i=1}^{N} q_i^* v_i^*$$

q_i : Risk-neutral probability of default of counterparty during the ith interval.
v_i : Present value of the expected loss to the bank if the counterparty defaults during the ith interval.
q_i^* : Risk-neutral probability of default of the bank during the ith interval.
v_i^* : Present value of the expected loss to the counterparty if the bank defaults during the ith interval.
$q_i = V(t_{i-1}) - V(t_i)$
$v_i = f_{nd}(1-R)$

5. Derivatives

A derivative involves two parties agreeing to a future transaction. Its value depends on (or derives from) the values of other underlying variables.

5.1 Forward contracts

- A contract that obligates the holder to buy or sell an asset for a predetermined delivery price at a predetermined future time. The contract has a virtual zero cost at time zero.
- **Forward's delivery price when created**

$$F_0 = S_0 e^{rT}$$

F_0 : Forward delivery price
S_0 : Price of underlying asset
T : Time to maturity
r : Risk-free rate
⇒**Assumptions**:
- The gain is the free-risk rate
- Not coupon neither yield from underlying asset.

- **MtM of a Forward** (Payoff)

$$f = (F_0 - K)e^{-rT}$$

f : MtM of Forward
F_0 : Forward price at present time (if it was created at present time). Formula is above.
K : Delivery price for a contract that was negotiated some time ago. (F_0 when forward was created.)
T : Time to maturity
r : Risk-free rate

5.2 Future contracts

- A contract that obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. The contract is settled daily.

Difference Forward and Future

- Future are traded in exchange markets whereas Forwards are OTC.

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5.3 Options

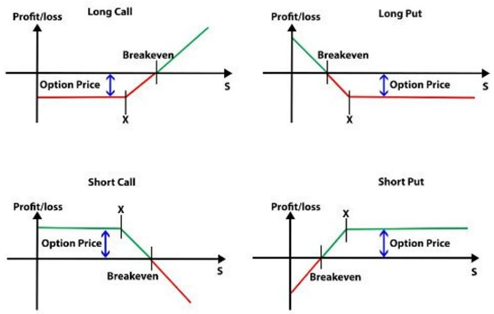
Call: Right to buy, Put: Right to sell

Spot options

Assumptions:

- 1. There are no transaction costs.
- 2. All trading profits (net of trading losses) are subject to the same tax rate.
- 3. Borrowing and lending are possible at the risk-free interest rate.

Analyse Delta:



European call option:
* Stochastic because S_T is stochastic

$$c = e^{-rT}\hat{E}[max(S_T - K, 0)]$$

Applying Black & Scholles:

$$c = SN(d_1) - Ke^{-rT}N(d_2)$$

European put option

$$p = e^{-rT}\hat{E}[max(K - S_T, 0)]$$

$$p = Ke^{-rT}N(-d_2) - SN(-d_1)$$

Where

$$d_1 = \frac{ln(S/K) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

N : Normal quantile function

$\hat{E}$: Expectation

American call option
American put option

Future options

5.4 Swaps: Interest rate swap

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5.5 Swaps: Currency swap

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5.6 Swaps: CDS(Credit default swap)

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5.7 Swaps: Quanto

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6. Black and Scholles

Assumptions

- Stock price assumes that percentage changes in very short period of time are normally distributed.

$$\frac{\Delta S}{S} \backsim \phi(\mu\Delta t, \Delta t)$$

Equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

7. Risk

7.1 Greeks or Risk sensitivities

Greek	Symbol	Measures	Definition
Delta	$\Delta = \frac{\partial c}{\partial S}$	Underlying variable (S) exposure	Change in option price due to spot
Gamma	$\Gamma = \frac{\partial^2 c}{\partial S^2}$	Underlying variable (S) convexity	Curvature of option price with respect to spot
Theta	$\Theta = \frac{\partial c}{\partial t}$	Time decay	Change in option price due to time passing
Vega	$\upsilon = \frac{\partial c}{\partial \sigma}$	Volatility exposure	Change in option price due to volatility
Rho	$\rho = \frac{\partial c}{\partial r}$	Interest rate exposure	Change in option price due to interest rates
Volga	$\frac{\partial^2 c}{\partial \sigma^2}$	Volatility convexity	Curvature of option price with respect to spot
Vanna	$\frac{\partial c}{\partial S \partial t}$		Change in Delta due to Volatility

7.2 VaR & ES

7.3 PnL