

Analog Proportional Direct Current Servo Motor Controller

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Abstract

The operation, theory, and general usage of operational amplifiers and transistors in relation to analog motor control is discussed. An analog, proportional direct current servo motor controller was designed to gain an understanding of controller theory and architecture. The design was then constructed and tested to further enhance learning and a second improved version was made. Included is the necessary background information, an explanation of critical features of the servo motor controller, design documentation for both iterations, and the test results and issues common to accurate servo control.

Index Terms-Analog Controller, Servomotor, DC circuit, Control Theory, Op-Amps, Proportional Control

I Introduction

This project began by making a prototype controller to precisely control and restrict a direct current (DC) motor's movements as well as makes it reverse direction without physically modifying the circuit. This was all done without the use of a computerized controller or any special hidden things. Ordinarily a DC gear motor turns in one direction continuously when supplied with an appropriate amount of electrical current. The direction a gear motor normally rotates depends on whether polarity (positive or negative value) the electricity flowing through the motor. This is precisely how a commercially available DC servomotor behaves. At the basic level the control circuit transforms an ordinary DC gear motor into a servomotor.

The prototype was created over the summer of 2015 as a practical lesson given with the guidance of semi-retired Woods Hole

Oceanographic Institution (WHOI) engineer Albert Bradley. Not only was the servo controller built from scratch, but the servomotor was also built from scratch. This lesson was in operational amplifiers, control theory, servomotors and use of transistors. Near the end, focus shifted to the skill of soldering was added to make a more permanent and resilient design. It was by no means a trivial accomplishment, many transistors were shorted out or destroyed when initially building and testing the prototype. Eventually the design's flaws were discovered and compensated for in some fashion that collectively gave rise to the final design. This lesson also served as a warning of not exceeding maximum ratings of electrical components. It also taught the importance of double checking the design before connecting power to it.

Over the course of this project two circuits were used. Each has a slightly different

meaning. The first circuit is the previously mentioned prototype design that served as the source test circuit. The second circuit is the improved circuit, which resulted from research and testing described in the rest of this paper.

To fully understand the circuit some basic equations and properties of some components is required. Also needed is an understanding of the components used in the circuit along with how they are represented is vital. The standard symbols for representing components within a circuit used in the figures in the paper are listed in a table of circuit diagram symbols. A black line between two symbols represents a connection between the two. A dot placed on a line where another meets it is an intersection. A break or an intersection with no dot across two lines is where two wires cross, but there is no physical connection. A critical piece to understanding any circuit is

knowing Ohms Laws and the basic units used. Ohms Laws describe mathematically the relationships between voltage (V), current (I) and resistance (R). Refer to equations (1), (2) and (3) at the end of this section [7] for the specific equations. The base unit of voltage is the Volt (V). The base unit of Current is the Ampere (A), but the milliAmpere (mA) is commonly used. The base unit of resistance is the Ohm (Ω). Though other common formats are the kilo-ohm ($k\Omega$) and mega-ohm ($M\Omega$) [9].

One of the most common and basic components is called a Resistor. Resistors are used to reduce current and/or voltage flowing through the rest of the circuit by converting part of the energy into heat [4]. While every component has a tiny amount of resistance (Copper for example has a resistivity of $1.72 \cdot 10^{-6} \Omega \cdot \text{cm}$ [23] which does not amount to anything except in specific circumstances. Typically only

resistors are considered to have a resistance in a diagram [8] [10]. Fixed resistors in a circuit diagram are given the resistance nominal value of resistance. Variable resistors, such as potentiometers, can have the resistance changed from 0Ω to some given maximum value. As such, a potentiometer is not typically given a resistance for clarity.

Another seemingly simple component is a power source; this can be a battery or a voltage power supply. With the term battery there is a difference between what is referred to commonly defined as a battery and what is technically defined as a battery. Batteries technically are comprised of two or more battery cells and are considered a DC voltage source [5]. The common AA alkaline battery is a single cell and is therefore technically not a battery. A 9V alkaline battery is a battery because it contains 6 cells. This distinction, while

seemingly academic is vital to accurately showing a battery as the symbol for a battery consists of the number of cells that make up the battery. When using batteries the nominal voltage is also provided to allow determining of the specific battery chemistry. Each type of battery chemistry not only provides a different performance over its discharge, but also a different cell voltage. This means the number of cells combined with the voltage gives the specific battery type used.

A second power source is a lab power supply, but it provides a constant voltage supply from a wall outlet. The benefit of batteries is they are portable and do not need an electrical plug. The next component used in the circuit is the Transistor, though explaining them in an easily understandable way requires understanding of a different component called a diode.

Diodes are components used to control the flow of current [16]. Diode forces current to flow in a specific direction, like a one-way street in a city. Also like the sign for a one-way street a diode's symbol has an arrow that points in the direction the current will flow. The back of the arrow is called the anode [16]; which is more positive than the cathode[16] at the opposite end. Thus the anode is the positive lead and the cathode is a negative lead. There is a proper way of putting a diode in a circuit for a specific purpose. The most common is to regulate current flow, where cathode is oriented so it is closest to ground as the circuit permits and the anode is connected closest to the applied voltage, this is forward biasing. Inserting a diode backwards makes it reversed biased, meaning the applied voltage is opposite the easiest current flow [15]. A transistor is merely two diodes configured in one of two specific ways.

Transistors have three terminals, referred to as the collector, base and emitter [15], which are typically, abbreviated C, B and E respectively and usually in that order. Current entering any of the terminals is defined by the abbreviation of the terminal name in subscript. Current between two terminals has both terminal abbreviations in subscript. There are several kinds of transistors, but the ones used in the prototype are normal transistors and power transistors, which come in two forms. There are NPN and PNP transistors. These refer to a particular diode-like behavior. NPN transistors, or negative-positive-negative, are like two anodes of a diode connected in the base and the cathodes to the collector and emitter respectively [15, Fig. 2.2]. PNP transistors, or positive-negative-positive are like two cathodes of a diode touching the base and the anodes at the collector and emitter respectively. For the operation of NPN transistors the collector must be more

positive than the emitter. For PNP, the collector must be more negative. All transistors also have a current gain and a voltage drop, the voltage drop, which follows the same format as current mentioned previously. Typically this drop is about 0.7V [11]. The transistor's gain is the ratio of the current from the collector to the emitter in relation to the current from the base to the emitter and uses the term h_{FE} .

This is typically around 100 [11] and is usually given in the documentation for the specific transistor. Transistors also have a special set of equations governing their properties refer to (4), (5), (6) at the end of the section [16][11]. Equation 4 may be derived from the Ebers-Moll equation [24].

Another key component that needs to be understood is the DC motor. In essence the motor can be thought of as two magnetic fields interacting with one another. Current going through a coil of copper wire induces

one of the magnetic fields. The other is supplied by permanent magnets. However the sheer amount of copper wire used gives a measurable resistance of a few ohms due to the resistivity of copper [23]. Which while small, must be accounted for.

While a DC motor was used for all designs, the design as a whole transformed the DC motor into a DC servomotor. There are two primary differences between a normal DC motor and a servomotor. The main difference between a normal DC motor and a servomotor is the required use of a controller in a servomotor. A DC servomotor's controller is a special type of circuit that controls when the servomotor moves and by how much. The controller requires an input as well as electrical, which gives rise to typically three wires coming from a servo. A DC motor only has two wires that supply power and does not need an additional circuit to operate correctly.

$$V = I \cdot R \quad (1) \quad I_C = h_{FE} \cdot I_B \quad (4)$$

$$I = \frac{V}{R} \quad (2) \quad h_{FE} = \frac{I_C}{I_B} \quad (5)$$

$$R = \frac{V}{I} \quad (3) \quad V_B = V_E + V_{BE} \quad (6)$$

II Prototype Circuit Operation

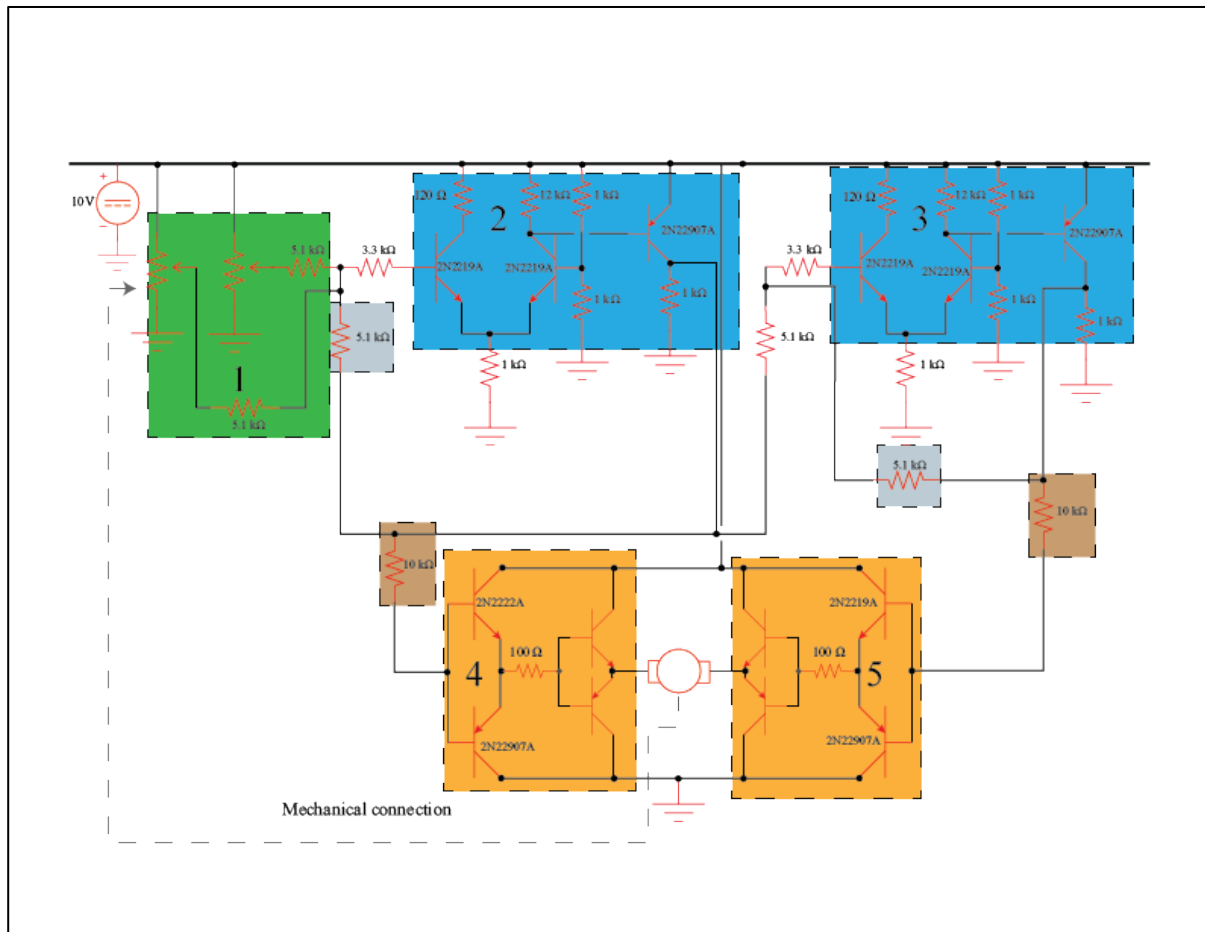


Fig. 1: Testing Circuit Schematic

At the most basic level the circuit takes an input from a potentiometer and that input turns a motor that has a feedback potentiometer mechanically connected to the output shaft. The feedback potentiometer is used to position the motor shaft to the

position of the input potentiometer. Since the prototype was the basis on which the other circuits were based on, understanding the prototype is key to understanding the other circuits. For the following section refer to Figure 1.

The input and feedback potentiometers shown in the green functional block labeled with the 1. The potentiometer with the grey dashed arrow is the feedback potentiometer. The mechanical connection arrow was unnecessary, but provides greater clarity as to which potentiometer is the input and which provides the output. The dashed grey line was used to avoid confusion with an electrical connection. These potentiometers output signals are sent into the functional portion labeled 2 inside a blue square. The contents of this functional part are a group of transistors and resistors. These form an operational amplifier (Op-Amp) this is Op-Amp 1. This particular Op-Amp consists of a comparator formed by the 2N2219A NPN transistors [12]. A comparator is a device which compares two inputs such as voltages and switches its output to select the larger value [25]. These provide the two inputs for the Op-Amp and as the name suggests

compares the two values and outputs the difference. The potentiometers specifically go into the negative input of Op-Amp 1, making it an inverted Op-Amp. The positive input is subject to a voltage divider formed by the two resistors in series. This divider provides an input of half the input source voltage (10V), which would be 5V. The output of the comparator is actually above second transistor. This signal is then fed through a PNP transistor, which provides an output signal. This signal is sent to three places. One is sent back to the negative input across a feedback resistor, represented by the grey box and it is also sent to the second Op-amp, which also inverts the from the first Op-Amp. The third goes to the box labeled 4.

The second Op-Amp, labeled 3, is identical to the first except that it takes in the output of the first Op-Amp and the output goes to the box labeled 5.

The orange box labeled part 4 is comprised of two pairs of NPN and PNP transistors that are connected at the base and emitters forming what is called a push-pull emitter follower [14]. The push-pull emitter follower also known as a class B amplifier. The resistor between the two push-pull emitter-followers is in the circuit because push-pull emitter-followers are prone to thermal runaway [13]. This particular setup adds to the dead-band of the servomotor due to crossover distortion [28]. The outside pair of push-pull emitters are normal transistors, the inside pair are power transistors. Power transistors behave like normal transistors except with a much lower gain and a higher maximum current rating. The two pairs can be thought of as stages. With the prototype circuit both sets are required to kick up the circuit current to the DC gear motor's required operating current of 2.5A. 2.5A would short out any of the

transistors on hand with a 600mA maximum current rating, hence the power transistors are used to get the current up to the level required and the resistors marked by the brown box are safety resistors to protect the rest of the circuit from harm, this was calculated using values a little higher than the values likely to be generated by the circuit to provide extra protection in case something unexpected happened. The push-pull amplifiers amplify the signal received so the motor will turn. The block labeled 4 amplifies the first op-amp's output. The block labeled 5 boosts the second op-amp's output. The motor uses the difference between the amplified signals to turn. Attached to one end of the output shaft is the potentiometer. The other end has an omni-wheel attached; it served as an easy method to determine if an output had occurred during testing.

For the most part the resistor values used are

unimportant with the exception of two inserted as safety resistors and the ones preventing thermal runaway between the two push-pull stages. The safety resistor values were calculated using the transistor gains used in the power amplifiers, the operating current of the motor and a power supply of 10V for easier calculations.

Through these calculations the initial safety resistor needed a current across it of 1mA. The second resistor would get a current of 100mA due to transistor gain that would increase to 2.5A by the power transistor gain. The initial resistors would need to be 10k Ω each and the second would be around 100 Ω each.

III Comparison with other Servo Motor Control Setups

There are many commercially available servomotors available in hobby shops. Most are designed for use in remote control model planes and use digital proportional control instead of analog. Digital control allows for the servo to be controlled in a separate circuit and the signal is digitally sent to the servomotor's control circuit, typically embedded within the servomotor casing, it is smaller and thus lighter than an analog version. All servo motor control systems have limitations in resolution, dead-band and gain [29]. The impact of these limitations much depends on the specific type of controller used. Nearly all servo controllers also contain a feedback input to check the output for error and adjust accordingly. These are referred to as closed-loop control systems. Systems without a

feedback input are referred to as open-loop control; these are highly inaccurate and limited in use. Commercially available servomotors are split into categories: Consumer and Industrial. Consumer grade servomotors are typically DC and are restricted usually to a fraction of a turn or some finite number of turns. These are cheap to make, small and cannot handle large forces. Industrial grade servomotors are entirely different. These are designed to operate robotic arms on an assembly line. Otherwise known as automated storage and retrieval. Regardless of the application industrial grade motors are AC, but their method for positioning control is applicable to DC servomotors. A vertical rack type or moving a box to a storage area, high precision is not extremely useful, but speed is more important. The feedback input in this case is provided by a digital encoder connected to the shaft that uses a micro-

controller and some programming to figure out the distance and deceleration distance [3]. A common problem in AC motors is providing the servomotor with the correct frequency and voltage. AC, or alternating current is the type of current that comes from an electrical socket. A typical AC voltage in the US is 120 volts inverting about every 8.4 milliseconds [9]. This means every 8.4 milliseconds AC current flips from positive to negative. AC power also comes in 240 volts and 480 volts [3]. However some AC servos might not operate on the frequency of AC voltage provided [3]. To compensate for this problem the AC power coming from the wall can be converted into DC current and then fed through an oscillator that mimics the inversion of AC current but be set to the operating frequency of the servomotor [3].

Analog servomotor controllers have the highest resolution and a continuous output with a resolution that is theoretically infinite

[1][2]. Though in reality the resolution is finite and is limited by component tolerances and variations from an ideal circuit. Analog servomotor controllers can be connected to a digital input, but with a loss of resolution and increased complexity. Most analog types use a potentiometer as it is the most accurate and one of the simplest to construct, some use an optical feedback, which is more complicated and less accurate [2]. Analog servomotors are much simpler than digital alternatives. Regardless of whether the servomotor is analog or digitally controlled they have a dead-band. Dead-band is merely a range over which a change in the input does not give an output [19]. Once the input is outside the dead-band range the motor turns. Dead-band can be estimated by using a least-squares indirect parameter adaptation algorithm. [24]

Dead-band can be caused by a few things. One is mechanical backlash in the case of a geared motor [20], another is static friction

[21] and the third is the motor's inertia. Anti-backlash gears are often used in gear trains to reduce power loss. Friction between sliding surfaces at the gear tooth is a major source of power loss in gear trains [27]. The laws of physics govern the motor's inertia and static friction. Servos typically have

gears to decrease the rotation of the motor's shaft down to a more controllable level. Gears cannot be changed without taking apart the servo. Any thoughtful comparison of servomotors has to be made on resolution, accuracy and dead-band.

IV Circuit testing

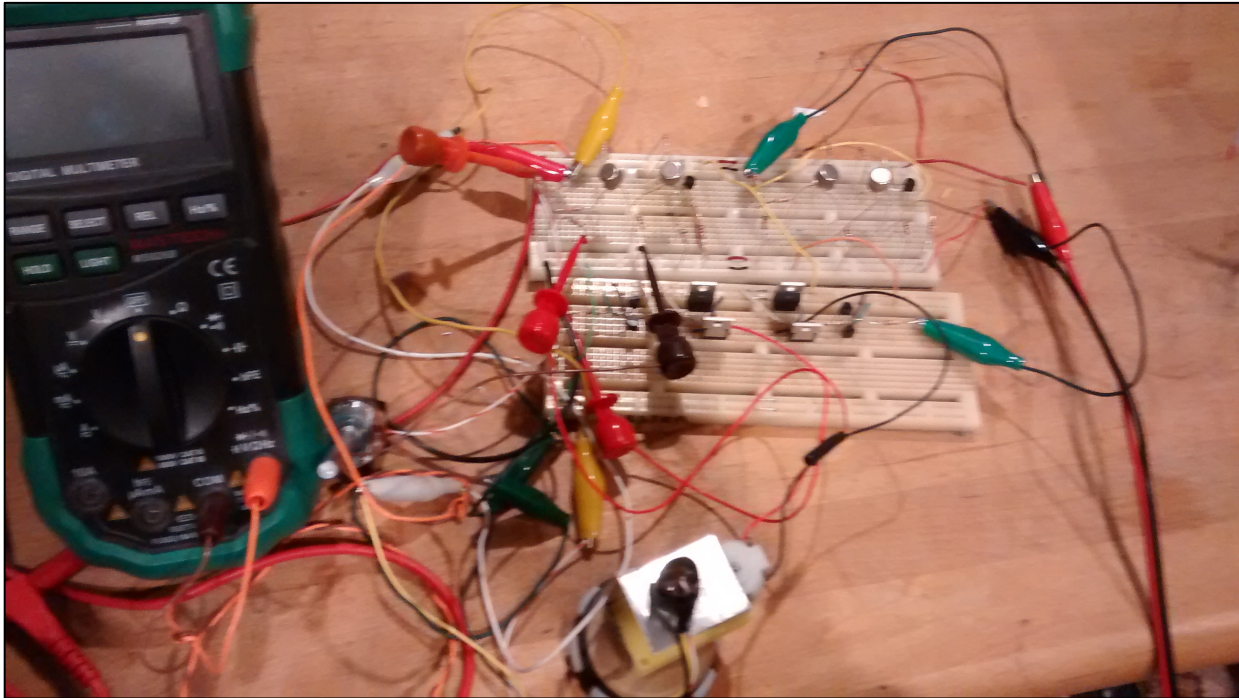


Figure 2: Testing Setup

Circuit testing was done using an adjustable lab power supply set to 10V output from an electrical outlet to provide a consistent voltage for accurate testing and simpler calculations. A total of two tests were conducted with two trials for each test. The trials were resolution and dead-band. Analyzing the dead-band and resolution test results can determine accuracy. Below is the

general setup of the testing, minor changes to this setup for each test are detailed as the tests are presented along with the graphs of the data. The data values for each graph are listed in Appendix B. Both tests were conducted using the same setup as depicted in Fig. 2.

The first test concerned the input-to-output

resolution of the servomotor. Data points were gathered at two different intervals using a Digital Multi-Meter that was switched between the two potentiometers every time the input potentiometer changed

by the selected interval. The first interval was at every 0.5V the input potentiometer changed the voltage across the feedback potentiometer was measured. The second was at every 0.4V

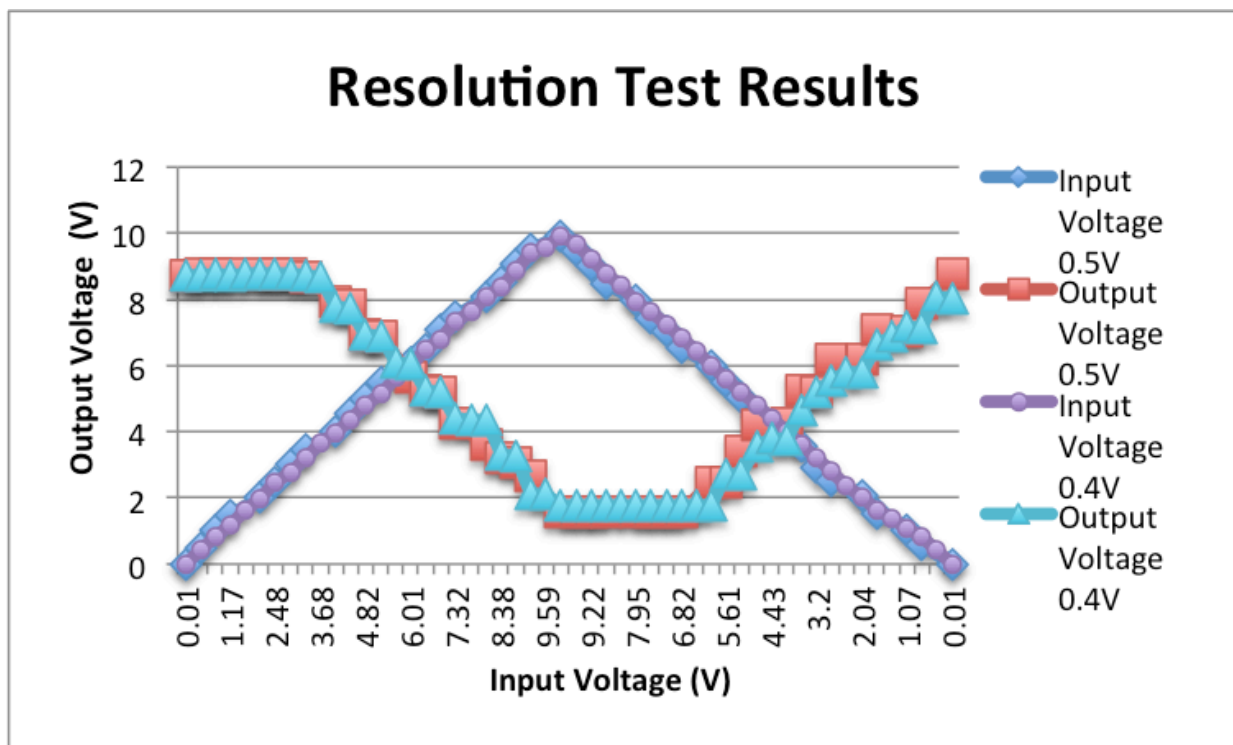


Fig. 3: Resolution testing Graph

The second test concerned dead-band. Dead-band is the range over which when the signal oscillates the motor does not respond to the new input. This is best tested in the middle range of the input signal and the

output signal using three points at which the motor rotates. For the first test those points are based off of the results from the resolution test. The central point of the graph occurs where the input voltage and

output voltage graphs intersect. This occurs at two points based on the resolution test. The first (reading from left to right) at 6V input voltage and 5.7V output voltage; and the second 3.8V input voltage and 4.4V output voltage. The key is to select the most accurate, regularly changing grouping as possible. This is best represented in the 6V

input grouping. The process is rather simplistic, go to the central point, reverse direction until the motor turns, continue past the point until the motor turns, then reverse direction until the motor turns. This cycle was done twice with measurements taken every 0.4V

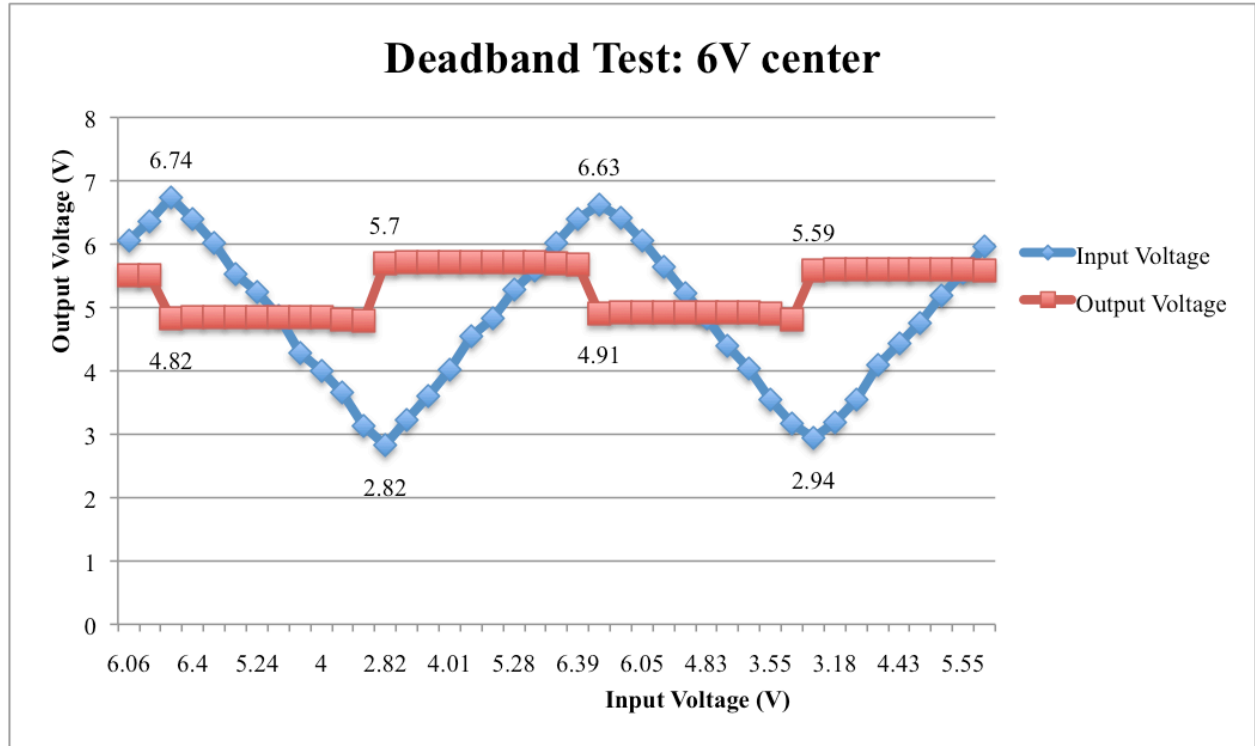


Fig. 4: Deadband test graph

V Testing Result Analysis

In order to achieve highly accurate servomotor control, it is essential to secure accurate positional feedback for the controller [26]. The resolution test was problematic due to issues with the circuit, the data was all over the place and was not consistent or stable in the reading. The problems were eventually solved and testing resumed. Both tests resulted in a graph that was in agreement with the expectations based on background research and initial testing of the prototype. The input voltage was a linear line, and the output voltage was

a stair-stepping progression as expected.

The Dead-band test resulted in a significant dead-band of 3.75V when reversing direction. Since the voltage input is 10V this dead-band spans 37.5% of the input. While this functions as a prototype this certainly is not practically useable and is a large limitation. The resulting dead-band is likely the result the use of a facsimile of the original prototype circuit. While the operation of the circuit remained the same, the variations in resistor values, input and output errors and biases from the op-amps [22].

VI Improved Circuit Design Compared to the Prototype Design

The prototype was made with parts at hand, some improvised repairs on components that had no spares, parts from a non-functioning robot named Murphy, after Murphy's law, and just a touch of duct tape. As such the values of resistors were not ideal, but were the closest available at the time. The improved circuit has resistor values much closer to the ones desired. The use of smaller transistors along with the removal of the power transistors in the improved circuit design allowed the entire circuit to fit onto a single breadboard as opposed to two breadboards in the original. A 12V, 100mA [18] unloaded gear motor was selected to make the improved circuit. Initially the two 10k Ω safety resistors remained in the improved circuit. However with them in the circuit they prevented the motor from running and were thus removed with no

issues for the circuit as a whole. With the jump from 5V to 12V motors figuring out the minimum voltage needed to turn the motor on as well as the current being used by the motor at that current was critical to ensuring the circuit operated with the available power sources. This required some voltage testing and measuring to find the minimum operating voltage. The result came as the surprising result of 0.7V. The natural expected result would be that with a higher voltage there would need to be a higher minimum turn on voltage.

After which the internal resistance was measured at 4.5 Ω and the current derived is 0.16A or 160mA. To ensure sure that the first test was accurate and to determine if there was a difference when the potentiometer was attached a second of voltage testing was conducted. This time the motor and a second potentiometer were secured in place so that the motor shaft

would turn the potentiometer shaft. This came to the same result as the first test. That permitted the removal of the power transistors and accompanying resistors due

to the current of 160mA being far less than 600mA maximum tolerance of the normal transistors.

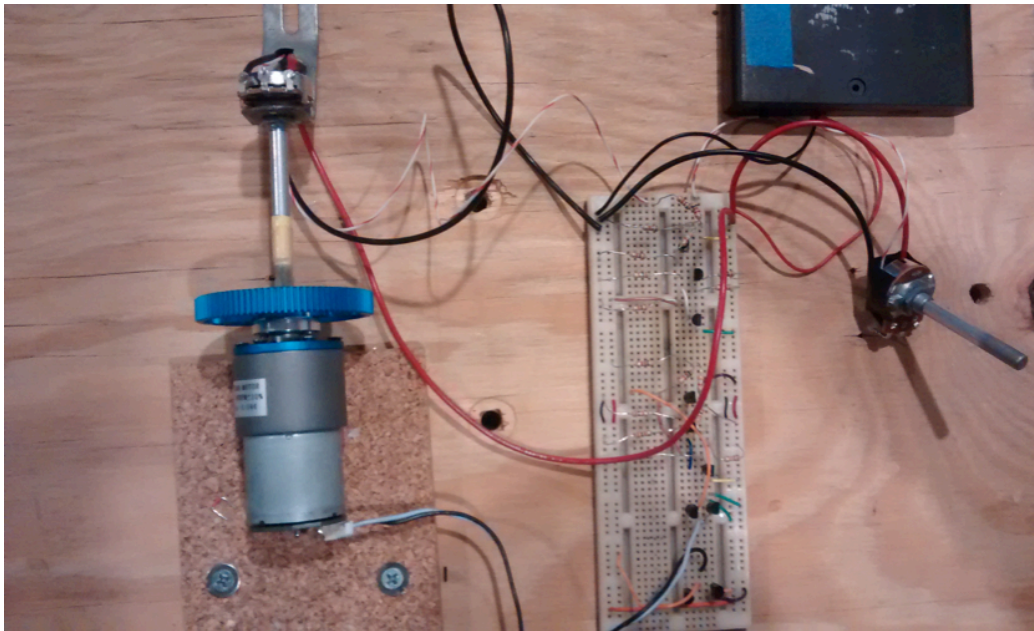


Fig. 5: Improved Circuit Actual Layout Picture

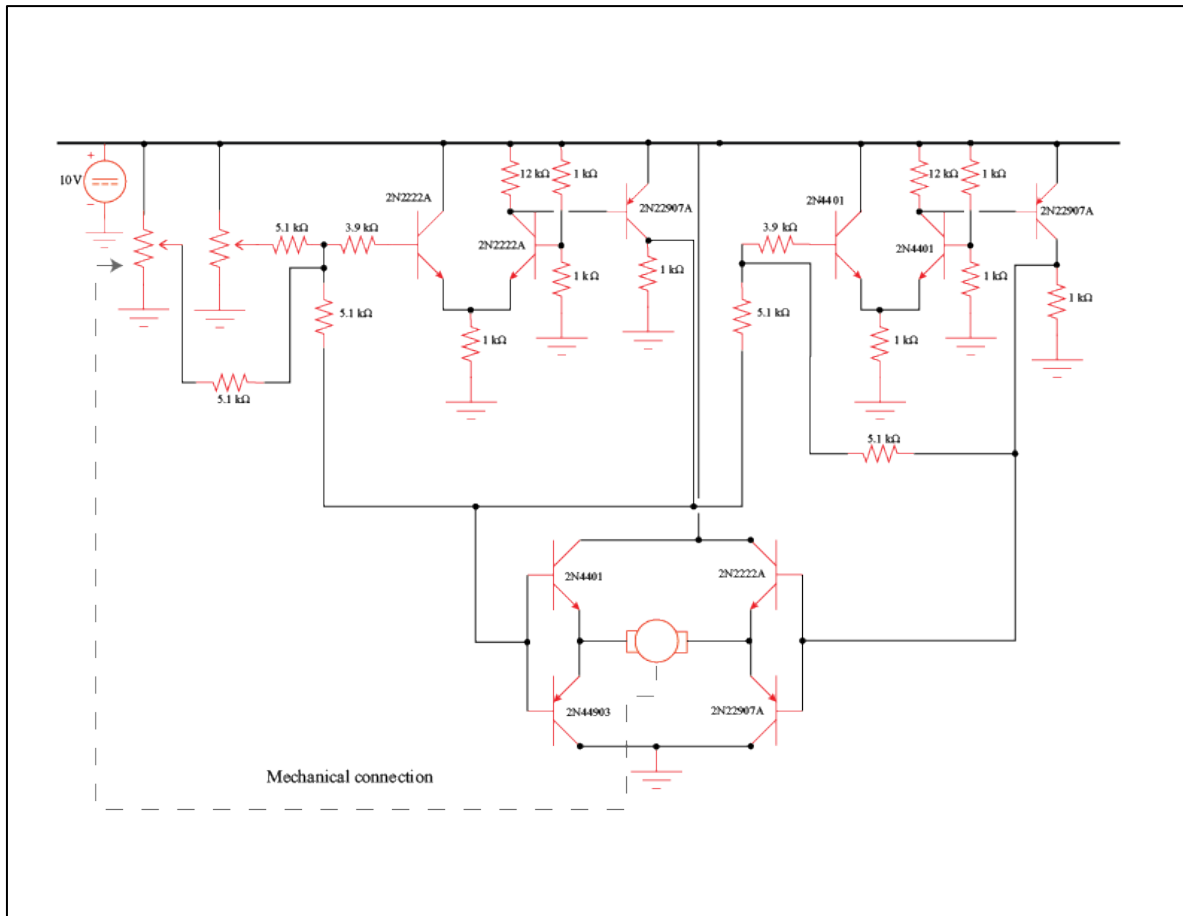


Fig. 6: Improved Circuit Schematic

7 Conclusion

While nearly all servo motor controllers have gone digital, there are still some advantages to an analog design. For one it does not require computer programming and its resolution is theoretically limitless. Digital control has the advantage of using fewer components, smaller boards and an overall lighter design. Digital is limited in the resolution by the number of bits and the feedback input sensor resolutions. For many applications Digital is acceptable, but for other designs it may not be good enough.

The results from testing the prototype circuit gave mixed results. The overall accuracy of the testing circuit was not totally erratic, but it was not a reliable or robust design either. While the operation of the circuit remained the same, the different resistor values and transistor gains likely were the cause of the inaccuracies due to the combined

differences in the current and voltage delivered to the motor resulted in different outcomes. The improved circuit design currently resides on a breadboard, but moving it to post-hole board and soldered together at a later date for a more robust and permanent construction is planned. Lead-free solder will be used for safety considerations. In going further in developing this design there are other controller types that increase accuracy; among them Proportional Derivative Control and Proportional Integral Derivative control. Both of these increase accuracy further than any proportional controller ever could. But that is another subject entirely.

One more pressing matter is the art of creating circuit diagrams. While many consider Circuit Schematics as merely informative, a circuit diagram when drawn haphazardly is completely useless and a nightmare to read. When drawn according to

established conventions can become not just only a method of communicating information, but a work of art. That is something lost on many. A correctly drawn circuit diagram is a thing of beauty. For

example, Fig. 7 was based on of an attempt to make sense out of Fig. 1.9 in Horowitz and Hill [17, Fig. 1.9]. The design, completely by accident, resulted in an artistic layout that is easy to follow.

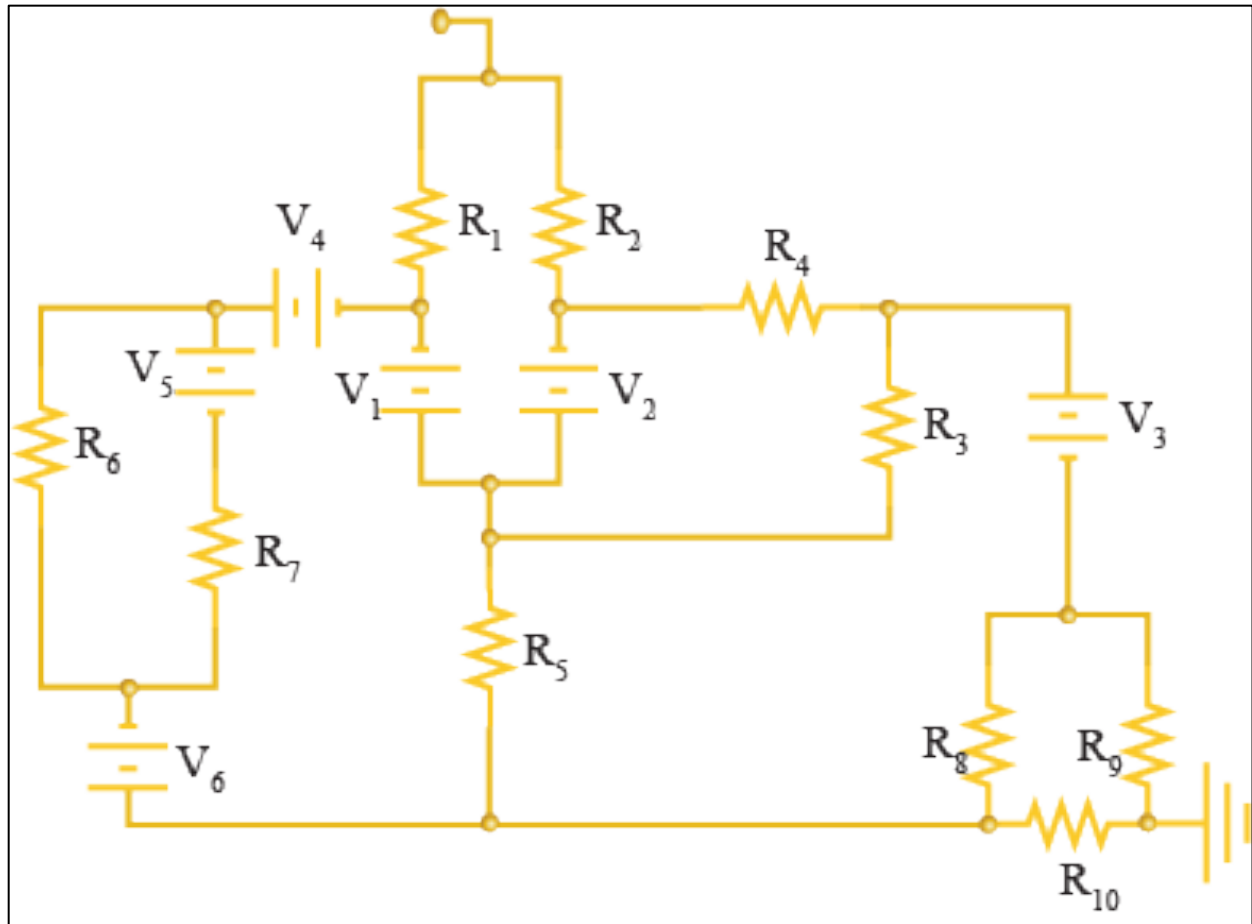


Fig. 7: We All Live in a Yellow Resistor-ine

Acknowledgements

Unless noted otherwise figures and diagrams used in this paper were created by the author. Measurements and photos unless otherwise noted were taken by the author for this project. Circuit diagrams are made in the convention stated and used consistently in *The Art Of Electronics* [10].

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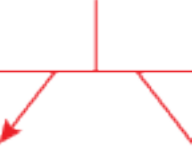
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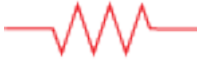





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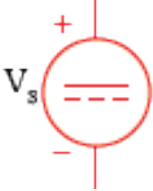

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Appendix A: Circuit Diagram Symbols

Diagram Symbol	Name	Usage
	Ground	Ground means any piece of metal that completes a circuit. Usually in place of a common ground
	Diode	Current regulator.
	NPN Transistor	Can source current, not sink it.

	PNP Transistor	Can sink current, but not source it.
	Resistor	A component, very common for reducing current and/or voltage.
	Potentiometer	A specific type of variable resistor, different types have a slightly different symbols.
	DC Motor	The most common symbol for an electric motor, others are less commonly used.
	Operational Amplifier or Op-Amp	Used to simplify complex or large diagrams involving transistors or certain types of integrated circuits. Has up to two inputs, usually one is used and the other is designated a virtual ground.
	Battery Cell	Voltage source of a single cell, a AA battery is a battery cell
	Battery	Voltage Source of two or more battery cells

	DC Voltage source	The short hand symbol for either a battery or power supply.
		
This is common knowledge in electrical engineering		

Appendix B: Testing Data Tables

Resolution Test 1: 0.5V increments	
Input Voltage	Output Voltage
0	8.73
0.5	8.76
1.03	8.77
1.47	8.77
2	8.77
2.48	8.77
3	8.75
3.47	8.68
4.03	7.91
4.55	7.84
5	6.96
5.5	6.91
6.08	5.64
6.49	5.23
7.09	5.19
7.47	4.27
8.06	3.62
8.52	3.19
9.07	3.06
9.5	2.68
9.91	1.6
9.47	1.6
9.01	1.59
8.49	1.59
8	1.59
7.44	1.59
7.04	1.58
6.55	1.57
5.98	2.48
5.49	2.45
5.06	3.41
4.47	4.19
3.96	4.28
3.55	5.25
2.94	5.2
2.5	6.21
2.07	6.18

1.53	7.07
1.05	7.02
0.57	7.88
0	8.78

Resolution Test 2: 0.4V increments	
Input Voltage	Output Voltage
0.01	8.74
0.43	8.73
0.81	8.75
1.17	8.74
1.64	8.76
1.99	8.77
2.48	8.77
2.78	8.77
3.23	8.71
3.68	8.66
3.97	7.78
4.37	7.73
4.82	6.94
5.16	6.9
5.66	6.09
6.01	6.06
6.49	5.24
6.81	5.21
7.32	4.39
7.64	4.37
8.08	4.34
8.38	3.29
8.85	3.26
9.41	2.14
9.59	2.13
9.94	1.79
9.65	1.79
9.22	1.78
8.79	1.78
8.42	1.78
7.95	1.78
7.64	1.78
7.21	1.78
6.82	1.77

6.42	1.77
5.99	1.77
5.61	2.74
5.21	2.73
4.81	3.55
4.43	3.81
4	3.78
3.63	4.63
3.2	5.21
2.83	5.53
2.38	5.81
2.04	5.78
1.64	6.66
1.39	6.88
1.07	7.18
0.81	7.15
0.44	8.1
0.01	8.05

Dead-band Test 1: 6V center	
Voltage Input	Voltage Output
6.06	5.51
6.35	5.5
6.74	4.82
6.4	4.85
6.02	4.84
5.53	4.84
5.24	4.84
4.87	4.84
4.28	4.84
4	4.84
3.65	4.81
3.13	4.79
2.82	5.7
3.23	5.72
3.6	5.72
4.01	5.72
4.55	5.72
4.83	5.72
5.28	5.72

5.59	5.72
6.02	5.69
6.39	5.67
6.63	4.91
6.42	4.93
6.05	4.93
5.65	4.92
5.23	4.92
4.83	4.92
4.39	4.92
4.03	4.92
3.55	4.9
3.17	4.81
2.94	5.59
3.18	5.61
3.54	5.61
4.09	5.61
4.43	5.61
4.75	5.61
5.19	5.61
5.55	5.61
5.97	5.59