

Combinatorics HW 1.2

Student ID: 2018280351 Name: Zhang Naifu Score:

1. How many odd numbers between 1000 and 9999 whose digits are distinct with each other?

Consider the ones' place, there are 5 odd numbers to choose from, i.e. {1, 3, 5, 7, 9}. Consider the thousands' place, there are 8 numbers to choose from, i.e. {1, 2, 3, 4, 5, 6, 7, 8, 9} – {odd number at ones' place}. Now the hundreds' place, there are 8 numbers to choose from i.e. {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} – {ones' place & thousands' place}. By the same logic, there are 7 numbers to choose from in the tens' place.

$$5 \times 8 \times 8 \times 7 = 2240$$

2. How many 7-digit numbers are there such that the digits are distinct integers taken from {1, 2, ..., 9} and such that the digits 5 and 6 do not appear consecutively in either order?

Set 1: 7-digit numbers whose digits are distinct integers taken from {1, 2, ..., 9}

$$| \text{Set 1} | = 9! \div 2! = 181440$$

Set 2: {Set 1} \cap {digits 5 and 6 DO appear consecutively in either order}

$$| \text{Set 2} | = 2 \times 6 \times P(7, 5) = 30240$$

$$| \text{Set 1} | - | \text{Set 2} | = 151200$$

3. How many different lattice paths from (-1,1) to (5,4)?

Assume we can only move in 1 direction along x-axis and y-axis, e.g. no loop or go up and down etc. allowed.

$$(5,4) - (-1,1) = (6,3)$$

$$C(9, 6) = 84$$