

Tsinghua University C&A Final Exam –Fall 2011 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.

[Total time: 2 hours]

Name:

Student ID:

1. Count the number of distinct permutations  $i_1 i_2 i_3 i_4 i_5$  of  $\{1, 2, 3, 4, 5\}$ , where  $i_1 \neq 1$ ;  $i_2 \neq 2, 3$ ; and  $i_3 \neq 4, 5$  (4 points)
2. How many in-equivalent ways to paint the faces on a **tetrahedron** in 3 different colors? (5 points)
3. A bag contains 10 apples, 12 bananas, 14 oranges, and 16 pears. If a boy picks one piece of fruit each time, how many picks are needed to make sure at least a dozen pieces of fruit of the same kind? (3 points)

4. Solve the following recurrence relations. (6 points)

$$f(n+1) = 1 + \sum_{i=0}^{n-1} f(i), f(0) = 1.$$

5. How many integral solutions of  $x_1 + x_2 + x_3 + x_4 = 14$ , satisfying  $x_1 \geq -2$ ,  $5 \geq x_2 \geq 0$ ,  $x_3 \geq -10$ ,  $x_4 \geq 8$ . (4 points)

6. Transform the following problems into augmented form.

$$\min z = 3x_1 + 6x_2 + 2x_3$$

$$s.t. \quad 3x_1 + 4x_2 - x_3 \geq -2 \quad (3 \text{ points})$$

$$x_1 - 3x_2 + 2x_3 \leq 4$$

$$x_1 \leq 0, x_2 \geq 1.$$