Combinatorics HW 5-2

Student ID: 2018280351 Name: Zhang Naifu Score:

1. Integer composition: Integer 5 is partitioned into orderly partitions which are made up by numbers 1,2,3,4. Such as (1+1+3, or 1+3+1 or 2+3, 4+1,....) How many different ways are there?

   $\sum_{i=1}^{5} C(5-1, i-1) = 2^{5-1} = 16$
   Since the question states 5 itself is not a valid partition, there are 16-1=**15** ways.

2. Integer partition: How many ways to partition n into several numbers that the order between numbers is ignored. Please write the corresponding generating function.

   $$G(x) = (1 + x + x^2 + \dots)(1 + x^2 + x^4 + \dots) \dots (1 + x^m + x^{2m} + \dots) \dots = \prod_{i=1}^{\infty} \frac{1}{1-x^i}$$

   so $p(n) = [x^n]\left(\prod_{i=1}^{\infty} \frac{1}{1-x^i}\right)$

3. Provide proof that the partition number of the summation of the partitioning of integer *n* into odd numbers, is equaled to the partition number of *n* being partitioned into the self-conjugated Ferrers Diagram. (1st row exchanged with 1st column, 2nd row exchanged with 2nd column, …, as image is rotated by following the dotted line as axis; is still Ferrers diagram. 2 Ferrers diagrams are known as a pair of conjugated Ferrers diagram. If both the conjugated Ferrers Diagram and it original diagram are the same, the diagram is called self-conjugated.)
   We consider a partitioning into odd numbers, $n = 1a_1+3a_3+\cdots+ka_k+\cdots+ ma_m$ where m=n-1 is the largest odd number in the partitioning. Every odd number k, where $1 \leqslant k \leqslant m$, can be represented by k=(k-1)/2+1+(k-1)/2. This corresponds to a corner of the self-conjugated Ferrers diagram, where length of row = length of column = 1+(k-1)/2. E.g., k=5= 2+1+2 corresponds to the following Ferrers diagram corner.
   • • •
   •
   •
   For m such corners to form a Ferrers diagram together, each successive odd number must be strictly greater than the previous one, so the odd numbers must be distinct.
   For every partitioning into distinct odd numbers, we can make a self-conjugated Ferrers diagram using the method above, and vice versa. The mapping between self-conjugated Ferrers diagram and partitioning into odd numbers is bijective. QED.