HW w7-1

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1. Using 5 numbers 1，2，3，4, 5 to fill in 1x n grids, each grid is filled with one digit. If there are odd number of grids that have 1 written on them, and an even number of grids with 2, please write the corresponding exponential generating function and figure out how many arrangements there for 1*6 grids?____

$$G(x)=\left(\frac{x^1}{1!}+\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots\right)\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)\left(1+\frac{x^1}{1!}+\frac{x^2}{2!}+\cdots\right)^3$$

$$G(x)=\frac{1}{4}(e^x-e^{-x})(e^x+e^{-x})e^{3x}=\frac{1}{4}(e^{5x}-e^x)=\frac{1}{4}\sum_{n=0}^{\infty}(5^n-1)\frac{x^n}{n!}$$

$$a_n=\frac{1}{4}(5^n-1)$$

$$a_6=\frac{1}{4}(5^6-1)=3906$$

2. There are six people in a library queuing up, three of them want to return the book “Interviewing Skills”, and 3 of them want to borrow the same book. If at the beginning, all the books of “Interviewing Skills” are out of stock in the library, how many ways can these people line up?____

If we don’t consider the elements (people) in the subsets (the 2 groups of return/borrow) as distinct, this is the same as the stack problem. A person returning the book represents a push, a person borrowing the book represents a pop.

The solution is therefore given by the Catalan number sequence, {c_1, c_2 c_3…}, where

$$c_n=\binom{2n}{n}-\binom{2n}{n+1}$$

$$c_3=5$$

However, since the elements of the subsets are distinct, solution is c_3*3!*3! = 180