Tsinghua University C&A Final Exam –Fall 2010 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 1.5 hours]

Name:

Student ID:

1. Count the number of permutations of *eight* letters: "A,B,C,D,E,F,G and H" that A, C, E and G are not in its natural positions. （4 points）

2. How many inequivalent ways are there to color the *edges* of a cube with the 3 different colors? （6 points）

3. Randomly pick $n+1$ numbers from integers between 1 to $2n$, please prove that there are at least two integers such that one of them is divisible by the other. (5 points)

4. A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled. (7 points)

n grids

Square brick

Rectangle brick
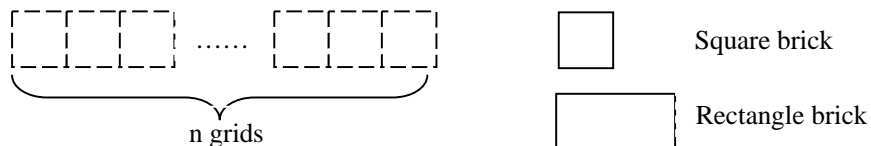
5. How many ways to put 8 *identical* balls into 4 *different* boxes that no empty box is allowed. (3 points)