1. p.378 15.2-1
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4. Programming assignment 2 (50% weight of ADw6&7)：p.405 15-4 Printing neatly

1. p.378 15.2-1

Using the notation from CLRS/class, we have the following recursive relation:

$$m[i,j] = \begin{cases} 0 & for\ i = j \\ \min_{i \le k < j}\{m[i,k] + m[k+1,j] + p_{i-1}p_k p_j\} & for\ i < j \end{cases}$$

Let this matrix multiplication be denoted by $A_1A_2A_3A_4A_5A_6$. This corresponds to dimensions <5, 10, 3, 12, 5, 50, 6>, where $p_0 = 5$.

$$m[1,2] = p_0p_1p_2 = 150$$
$$m[2,3] = p_1p_2p_3 = 360$$
$$m[3,4] = p_2p_3p_4 = 180$$
$$m[4,5] = p_3p_4p_5 = 3000$$
$$m[5,6] = p_4p_5p_6 = 1500$$

$$m[1,3] = \min\begin{cases} m[1,1] + m[2,3] + p_0p_1p_3 \\ m[1,2] + m[3,3] + p_0p_2p_3 \end{cases} = \min\begin{cases} 0 + 360 + 5*10*12 \\ 150 + 0 + 5*3*12 \end{cases} = 330$$

$$m[2,4] = \min\begin{cases} m[2,2] + m[3,4] + p_1p_2p_4 \\ m[2,3] + m[4,4] + p_1p_3p_4 \end{cases} = \min\begin{cases} 0 + 180 + 10*3*5 \\ 360 + 0 + 10*12*5 \end{cases} = 330$$

$$m[3,5] = \min\begin{cases} m[3,3] + m[4,5] + p_2p_3p_5 \\ m[3,4] + m[5,5] + p_2p_4p_5 \end{cases} = \min\begin{cases} 0 + 3000 + 3*12*50 \\ 180 + 0 + 3*5*50 \end{cases} = 930$$

$$m[4,6] = \min\begin{cases} m[4,4] + m[5,6] + p_3p_4p_6 \\ m[4,5] + m[6,6] + p_3p_5p_6 \end{cases} = \min\begin{cases} 0 + 1500 + 12*5*6 \\ 3000 + \text{whatever, too big} \end{cases} = 1860$$

$$m[1,4] = \min\begin{cases} m[1,1] + m[2,4] + p_0p_1p_4 \\ m[1,2] + m[3,4] + p_0p_2p_4 \\ m[1,3] + m[4,4] + p_0p_3p_4 \end{cases} = \min\begin{cases} 0 + 330 + 5*10*5 \\ 150 + 180 + 5*3*5 \\ 330 + 0 + 5*12*5 \end{cases} = 405$$

$$m[2,5] = \min\begin{cases} m[2,2] + m[3,5] + p_1p_2p_5 \\ m[2,3] + m[4,5] + p_1p_3p_5 \\ m[2,4] + m[5,5] + p_1p_4p_5 \end{cases} = \min\begin{cases} 0 + 930 + 10*3*50 \\ 360 + 3000 + \text{whatever} \\ 330 + 0 + 10*5*50 \end{cases} = 2430$$

$$m[3,6] = \min\begin{cases} m[3,3] + m[4,6] + p_2p_3p_6 \\ m[3,4] + m[5,6] + p_2p_4p_6 \\ m[3,5] + m[6,6] + p_2p_5p_6 \end{cases} = \min\begin{cases} 0 + 1860 + 3*12*6 \\ 180 + 1500 + 3*5*6 \\ 930 + 0 + 3*50*6 \end{cases} = 1770$$

$$m[1,5] = \min\begin{cases} m[1,1] + m[2,5] + p_0p_1p_5 \\ m[1,2] + m[3,5] + p_0p_2p_5 \\ m[1,3] + m[4,5] + p_0p_3p_5 \\ m[1,4] + m[5,5] + p_0p_4p_5 \end{cases} = \min\begin{cases} 0 + 2430 + 5*10*50 \\ 150 + 930 + 5*3*50 \\ 330 + 3000 + 5*12*50 \\ 405 + 0 + 5*5*50 \end{cases} = 1655$$

$$m[2,6] = \min\begin{cases} m[2,2] + m[3,6] + p_1p_2p_6 \\ m[2,3] + m[4,6] + p_1p_3p_6 \\ m[2,4] + m[5,6] + p_1p_4p_6 \\ m[2,5] + m[6,6] + p_1p_5p_6 \end{cases} = \min\begin{cases} 0 + 1770 + 10*3*6 \\ 360 + 1860 + 10*12*6 \\ 330 + 1500 + 10*5*6 \\ 2430 + \text{whatever} \end{cases} = 1950$$

$$m[1,6] = \min \begin{cases} m[1,1] + m[2,6] + p_0p_1p_6 \\ m[1,2] + m[3,6] + p_0p_2p_6 \\ m[1,3] + m[4,6] + p_0p_3p_6 \\ m[1,4] + m[5,6] + p_0p_4p_6 \\ m[1,5] + m[6,6] + p_0p_5p_6 \end{cases} = \min \begin{cases} 0 + 1950 + 5 * 10 * 6 \\ 150 + 1770 + 5 * 3 * 6 \\ 330 + 1860 + \text{whatever} = 2010 \\ 405 + 1500 + 5 * 5 * 6 \\ 1655 + 0 + 5 * 50 * 6 \end{cases}$$

$(A_1A_2)((A_3A_4)(A_5A_6))$

2. p.389 15.3-3

Assume there is the partition A that splits the sequence of matrices at A_k, such that the scalar multiplications are the maximum, i.e. $A = (A_1 \dots A_k)(A_{k+1} \dots A_n)$. Let $cost(A)$ denote the number of scalar multiplications.

This problem has optimal substructure, which says that $A' = (A_1 \dots A_k)$ must also has the sequence of parenthesis that gives the maximum scalar multiplications.

We prove by contradiction.
Assume there exists $A'' = (A_1 \dots A_k)$ s.t. $cost(A'') > cost(A')$
Then $A^* = A''(A_{k+1} \dots A_n)$ and $cost(A^*) > cost(A)$
The last statement is a contradiction to our initial assumption that $cost(A)$ is the maximum.

3. p.699 25.2-4

To prove that updating in place gives the correct answer, we need to show that the term min $(d_{ij}^{k-1}, d_{ik}^{k-1} + d_{kj}^{k-1})$ is the same as min $(d_{ij}, d_{ik} + d_{kj})$.
$d_{ij} = d_{ij}^{k-1}$ since k is in the outermost loop.
$d_{ik} = d_{ik}^{k-1}$ or $d_{ik} = d_{ik}^{k}$. In the former case, there is no problem. In the latter case, we can note that $d_{ik}^{k} = d_{ik}^{k-1}$ because going through vertex k with all intermediate vertices in set {1…k} is the same as having all intermediate vertices in set {1…k-1}.
Same logic applies to d_{kj}