

Combinatorics 2018 HW 5.1

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1. A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruits that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?

Extreme case: 7 apples, 5 bananas, 8 oranges

We need $7+5+8+1 = 21$ fruits

2. Show that for any given 52 integers there exists two of them whose sum, or else whose difference, is divisible by 100.

We can write each of the 52 integers as $n_i = a_i \cdot 100 + b_i$, so that $n_i \equiv b_i \pmod{100}$.

Hence, when we only consider differences, the problem has 100 holes & 52 pigeons.

If we allow addition, then some pairs of holes become complementary, e.g. if $b_j=1$ and $b_k=99$, for $j \neq k$, then $n_j+n_k \equiv 0 \pmod{100}$. There are 49 such pairs, on top of 0 and 50. So the 100 holes are reduced to 51 holes when addition is considered.

51 holes, 52 pigeons.