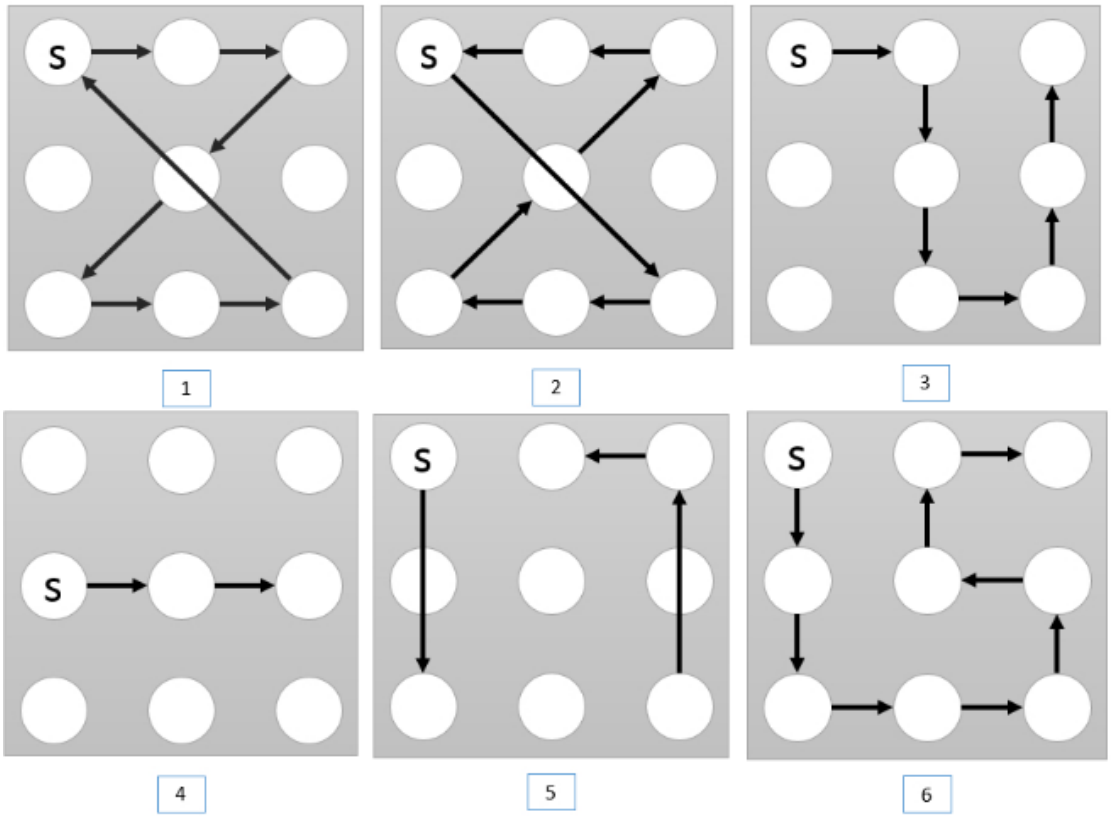
Combinatorics HW 1.1

Student ID: 2018280351 Name: Zhang Naifu Score:

1. A T-shirt will be printed with a magic square of size 3. How many different prints are possible?

   Refer to attached.

2. According to the video, which pattern(s) below match the description of a logical passcode for Android phones? (You can pick more than one)



   1, 3, and 6

3. A large tournament has 569 entrants in total. If it is a single elimination tournament, how many matches have to be played out before the champion can be decided? (Please calculate the precise value)

   Each match eliminates 1 player. 569-1=568 matches.

4. The figure below shows a partial 4X4 matrix，is there some way of filling up the rest of the omitted entries to produce a magic square of size 4？

$$\begin{bmatrix} 2\ 3 \\ 4 \end{bmatrix}$$

Refer to attached.

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Let a 3×3 magic square be represented by following matrix:

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$$

Property 1 : $m_{22} = 5$

$$(m_{11} + m_{22} + m_{33}) + (m_{31} + m_{22} + m_{13}) + (m_{21} + m_{22} + m_{23}) + (m_{12} + m_{22} + m_{32})$$

~~$= 4M = 4 \times 15 = 60$~~

$$= 4M = 60$$

$$= m_{11} + m_{12} + \dots + m_{33} + 3m_{22} \quad \text{——} \quad ①$$

$$m_{11} + m_{12} + \dots + m_{33} = 45 \quad \text{——} \quad ②$$

$$① - ② = 3m_{22}$$

$$3m_{22} = 15$$

$$m_{22} = 5$$ //

Q1

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Property 2: Corner cells contain even no;
Non-corner cells contain odd no.

Without loss of generality, consider m_{11}

Case 1: $m_{11} \neq 1$

If $m_{11} = 1 \Rightarrow m_{33} = 9$ & $\{m_{12}, m_{13}\} = \{6, 8\}$

$\Rightarrow m_{21} + m_{31} < 14$ & $m_{13} + m_{23} + m_{33} > 15$

$\therefore$ Contradiction

By same logic, $m_{11} \neq 9$

Case 2: $m_{11} \neq 3$

If $m_{11} = 3 \Rightarrow m_{33} = 7$ & $\{m_{12}, m_{13}\} = \{4, 8\}$

If $m_{13} = 8$, $\Rightarrow m_{13} + m_{23} + m_{33} > 15$

If $m_{13} = 4 \Rightarrow m_{23} = 3$ but 3 is already used

$\therefore$ Contradiction

By same logic $m_{11} \neq 7$

Property 3: The even no.s & odd nos must be paired.
ie. if we set m_{21}, then m_{23} is determined.

Proof is trivial.

△

Q1

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Property 1 & 2 & 3 imply that all ~~3x3~~ variations of 3x3 magic square ~~is~~ are rotation/reflection of 1 unique square.

$$M_1 = \begin{pmatrix} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{pmatrix}$$ is an example.

M_1 can be rotated by 90°, 180°, 270° & reflected along either horizontal, vertical or diagonal axis.

The size of the entire set $\{M_i\}$ where M_i is a 3x3 magic square is

$$|M_i| = 4 \times 2 = 8$$

Q4

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Let the 4×4 magic square be represented by the following matrix:

$$\begin{pmatrix} 2 & 3 & m_{13} & m_{14} \\ 4 & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix}$$

To have row 1 sum up to $M=34$,

$\{m_{13}, m_{14}\} = \{(16, 13), (14, 15)\}$ ignoring order

To have column 1 sum up to $M=34$

$\{m_{31}, m_{41}\} = \{(16, 12), (13, 15)\}$ ignoring order

The only jointly feasible solution between the 2 sets is

$\{(m_{13}, m_{14}), (m_{31}, m_{41})\} = \{(14, 15), (16, 12)\}$

Next, $m_{41}=12$, as assigning it 16 would not work for diagonal $(m_{41}, m_{32}, m_{23}, m_{14})$.

Eg. $m_{41}=16$, $m_{32}=1 \sim 5$ $m_{23}=1$ or 5, $m_{14}=14 \sim 15$

$\min(m_{41}+m_{32}+m_{23}+m_{14}) = 36$ => Invalid

Q4

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So $m_{31} = 16$ & $m_{41} = 12$

Redrawing matrix

$$\begin{pmatrix} 2 & 3 & \{14,15\} & \{14,15\} \\ 4 & m_{22} & m_{23} & m_{24} \\ 16 & m_{32} & m_{33} & m_{34} \\ 12 & m_{42} & m_{43} & m_{44} \end{pmatrix}$$

Case 1 : $m_{13} = 14$ & $m_{14} = 15$

$\Rightarrow \{m_{23}, m_{32}\} = \{1, 6\}$

max ~~12~~ $(3 + m_{22} + m_{32} + m_{42}) = 3 + 6 + 13 + 11 < 34$

$\therefore$ ~~Invad~~ Invalid

Case 2: $m_{13} = 15$ & $m_{14} = 14$

$\rightarrow \{m_{23}, m_{32}\} = \{1, 7\}$

Consider column 2:

max $(3 + m_{22} + m_{32} + m_{42}) = 3 + 7 + 13 + 11 = 34$

The only feasible solution at this step is $m_{32} = 7$ & $\{m_{22}, m_{42}\} = \{13, 11\}$ & $m_{23} = 1$

Consider row 2:

max $(4 + m_{22} + m_{23} + m_{24}) = 4 + 13 + 1 + 10 < 34$

$\therefore$ Invalid

Q4

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We have exhausted all possible ~~solutions~~ cases & none is valid solution