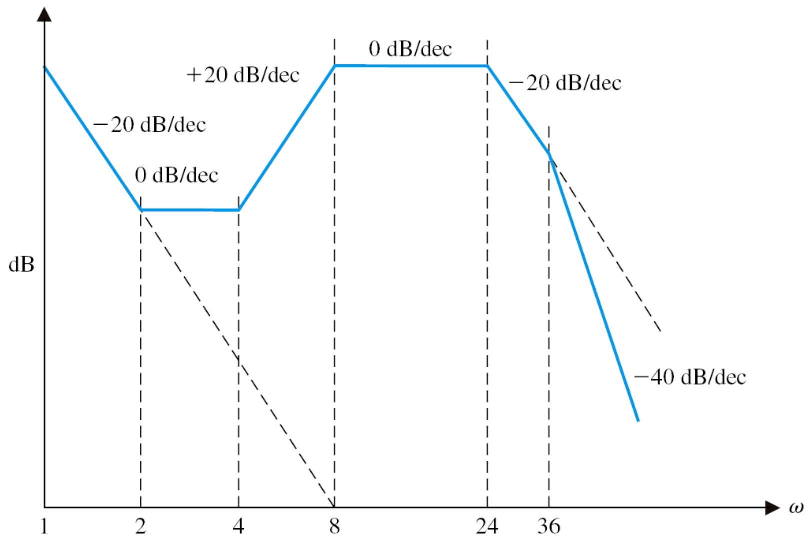
Homework

Chapter 4: Frequency Response Methods

P4.1 The magnitude plot of a transfer function

$$G(s)=\frac{K(1+0.5s)(1+as)}{s(1+s/8)(1+bs)(1+s/36)}$$

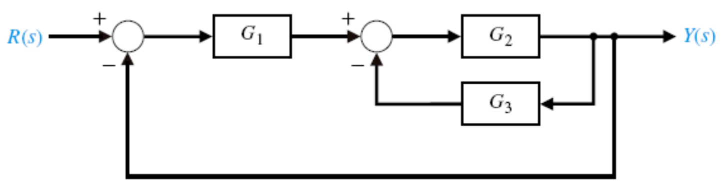
is shown in the following figure. Determine K, a, and b from the plot.



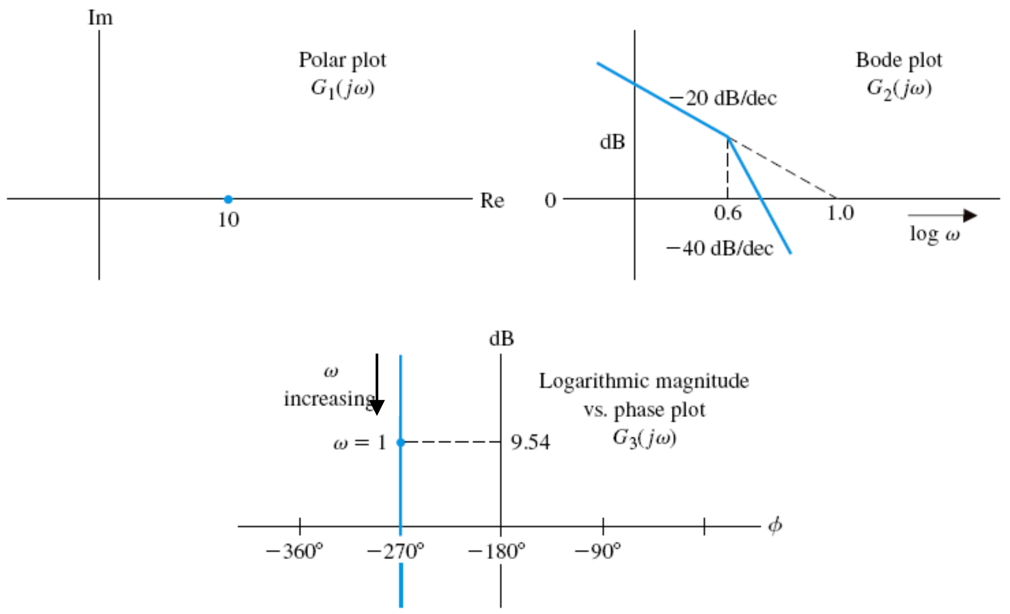
P4.2 The block diagram of a feedback control system is shown in Figure (a). The transfer functions of the blocks are represented by the frequency response curves shown in Figure (b).

(1)Determine the transfer function of the feedback control system

(2)When G_3 is disconnected from the system, determine the damping ratio ζ of the system.

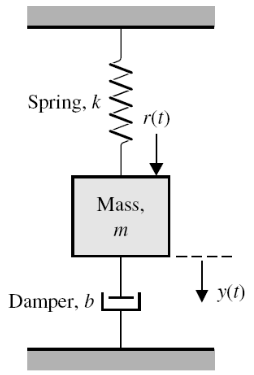


(a)

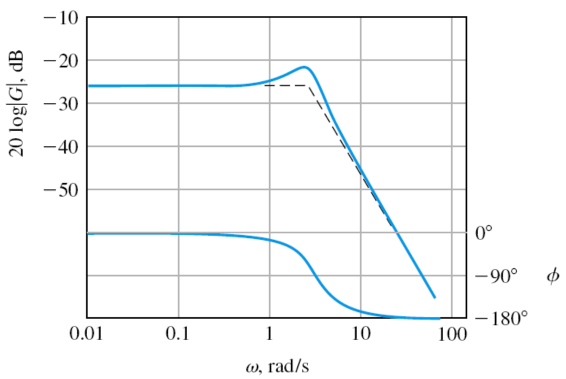


(b)

P4.3 A spring-mass-damper system is shown in Figure (a). The Bode diagram obtained by experimental means using a sinusoidal forcing function is shown in Figure (b). Determine the numerical values of $m, b,$ and k.



(a)



(b)

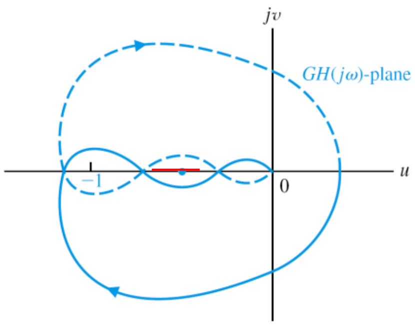
P4.4 Consider a unity feedback system with

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

(a) For $K = 4$, show that the gain margin is 3.5 dB.

(b) If we wish to achieve a gain margin equal to 16 dB, determine the value of the gain K.

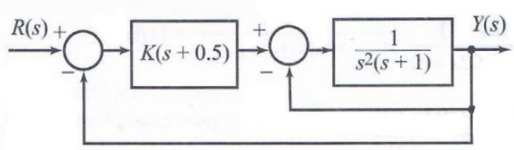
P4.5 The polar plot of a conditionally stable system is shown below for a specific gain K.



(a) Determine whether the system is stable, and find the number of roots (if any) in the right-hand s-plane. The system has no poles of $GH(s)$ in the right half-plane.

(b) Determine whether the system is stable if the -1 point lies at the colored dot on axis.

P4.6 Consider the control system shown below. Using the Nyquist criterion, determine the range of $K > 0$ for stability of the system.



$$G(s) = \frac{k(1+0.5s)(1+as)}{s(1+0.125s)(1+bs)(1+\frac{1}{36}s)}$$

$$G(j\omega) = \frac{k(1+0.5j\omega)(1+aj\omega)}{j\omega(1+0.125j\omega)(1+bj\omega)(1+\frac{1}{36}j\omega)}$$

corner frequencies in numerator:

$\omega_1 = 2$ $\qquad$ $\omega_2 = \frac{1}{a}$

corner frequencies in denominator:

$\omega_3 = 8$ $\qquad$ $\omega_4 = \frac{1}{b}$ $\qquad$ $\omega_5 = 36$

$\cancel{\frac{1}{a} > 8}$ $\qquad$ $\frac{1}{a} = 4$ $\qquad$ $\frac{1}{b} = 24$

$\cancel{a = \frac{1}{8}}$ $\qquad$ $a = \frac{1}{4}$ $\qquad$ $b = \frac{1}{24}$

$$20\log\left|\frac{K}{j\omega}\right| = 20(\log K - \log\omega)$$

$$20(\log K - \log 8) = 0$$

$$K = 8$$

Only consider $g(j\omega) = \frac{K}{j\omega}$ for the $\omega < 2$ part of diagram

P4.2 (1) Consider inner loop Transfer function $G'(s)$

$$G'(s) = \frac{G_2}{1+G_2G_3}$$

Consider outer loop Transfer Function $G(s)$

$$G(s) = \frac{G_1 G'}{1+G_1G'}$$

$$= \frac{\dfrac{G_1G_2}{1+G_2G_3}}{1+\dfrac{G_1G_2}{1+G_2G_3}}$$

$$= \frac{G_1G_2}{1+G_2G_3+G_1G_2}$$

From Polar plot $G_1(s) = 10$

From Bode plot:

$$G_2(s) = \frac{K}{s(Ts+1)} = \frac{K}{s}\ \frac{1}{Ts+1}$$

consider $\frac{K}{s}$ part

$$20\log\left|\frac{K}{s}\right|_{s=j\omega} = 20\log K - 20\log|j\omega|$$

$$20\log K - 20\log|j\omega|_{\omega=1} = 20\log K = 0$$

$$K = 1$$

Consider $\frac{1}{Ts+1}$ part

corner frequency $\omega = 0.6 = \frac{1}{T}$

$$T = \frac{5}{3}$$

$$G_2(s) = \frac{1}{s\left(\frac{5}{3}s+1\right)}$$

From log-magnitude vs phase plot

$$G_3 = \frac{k'}{(j\omega)^3}$$

$$20 \log \left|\frac{k'}{(j\omega)^3}\right| = 20 \log k' - 20 \log \left|(j\omega)^3\right|$$

$$20 \log k' - 20 \log \left|(j\omega)^3\right|_{\omega=1} = 20 \log k' = 9.54$$

$$k' = 3$$

$$G_3(s) = \frac{3}{s^3}$$

$$G_1(s) = 10 \qquad G_2(s) = \frac{1}{s\left(\frac{5}{3}s+1\right)} \qquad G_3(s) = \frac{3}{s^3}$$

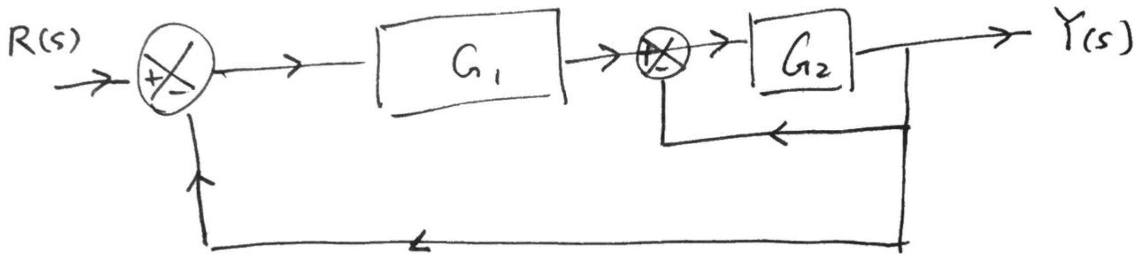
$$G(s) = \frac{G_1 G_2}{1 + G_1 G_2 + G_2 G_3}$$

$$= \frac{10/s\left(\frac{5}{3}s+1\right)}{1 + 10/s\left(\frac{5}{3}s+1\right) + 3/s^4\left(\frac{5}{3}s+1\right)}$$

$$= \frac{10s^3}{s^4\left(\frac{5}{3}s+1\right) + 10s^3 + 3}$$

P4.2 (2) Not sure if the entire loop is removed.

If question means



~~characteristic equation~~

$$G(s) = \frac{G_1 G_2}{1 + G_2 + G_1 G_2} = \frac{10}{\frac{5}{3}s^2 + s + 11} = \frac{6}{s^2 + 0.6s + 6.6}$$

$\omega_n^2 = 6.6$

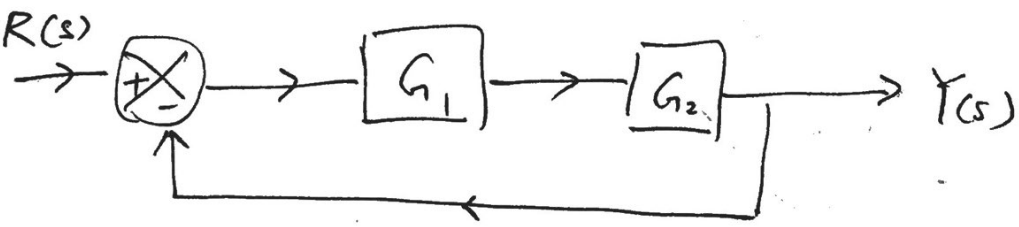
$\omega_n = \sqrt{6.6}$

$2\xi\omega_n = 0.6$

$\xi = \frac{0.6}{2\sqrt{6.6}}$

$= 0.117$ //

If question means



$$G(s) = \frac{G_1 G_2}{1 + G_1 G_2} = \frac{10}{\frac{5}{3}s^2 + s + 10} = \frac{6}{s^2 + 0.6s + 6}$$

$\omega_n^2 = 6$

$\omega_n = \sqrt{6}$

$2\xi\omega_n = 0.6$

$\xi = \frac{0.6}{2\sqrt{6}}$

$= 1.225$ //

4.3

$$m\ddot{y} + b\dot{y} + ky = r$$

$$Y(s)\left(ms^2 + bs + k\right) = R(s)$$

$$G(s) = \frac{Y(s)}{R(s)} = \frac{1}{ms^2 + bs + k}$$

$$G(j\omega) = \frac{1}{m(j\omega)^2 + bj\omega + k}$$

$$= \frac{1}{k} \quad \frac{1}{\frac{m}{k}(j\omega)^2 + \frac{b}{k}j\omega + 1}$$

At low frequencies $\omega \ll \omega_n$

$$|G(j\omega)| = -20\log k - 20\log 1$$

$$-25 = -20\log k$$

$$\log k = \frac{5}{4}$$

$$k = 10^{1.25}$$

//

standard form $g(j\omega) = \dfrac{1}{1 + 2\xi j\frac{\omega}{\omega_n} + \left(j\frac{\omega}{\omega_n}\right)^2} = \dfrac{1}{1 + \frac{b}{k}j\omega + \left(\sqrt{\frac{m}{k}}j\omega\right)^2}$

$$\sqrt{\frac{m}{k}} = \frac{1}{\omega_n}$$

$\cancel{\omega_n = \sqrt{\frac{k}{m}}}$ $\omega_n = 3.16 = 10^{\frac{1}{2}}$

$$\sqrt{m} = \frac{\sqrt{k}}{\omega_n}$$

$$m = \frac{k}{\omega_n^2}$$

$$= \frac{10^{1.25}}{(10^{0.5})^2}$$

$$= 10^{0.25}$$

//

No way to tell ξ

Can know $0 < \xi < \frac{1}{\sqrt{2}}$ because ~~由~~ $M_r > 0$

↳ 根据谐振频率来估算 ξ.

$$2\xi \frac{1}{\omega_n} = \frac{b}{k}$$

$$b = 2\xi \frac{k}{\omega_n}$$

$$= 2\xi \frac{10^{1.25}}{10^{0.5}}$$

$$= 2 \cdot 10^{0.75} \xi$$

$$0 < b < 2 \cdot 10^{0.75} \cdot 2^{-0.5}$$

$$0 < b < 2^{0.5} \cdot 10^{0.75}$$

$$0 < b < 7.95$$

P4.4 (a) $G(s) = \dfrac{K}{s(s+1)(s+2)}$

$$20\log|G(s)| = 20\left(\log\frac{K}{2} - \log|s| - \log|s+1| - \log\left|\tfrac{1}{2}s+1\right|\right)$$

$$\phi(G(s)) = -90^\circ - \tan^{-1}(j\omega) - \tan^{-1}\left(\tfrac{1}{2}j\omega\right) = -180^\circ$$

find phase crossover frequency ω

$$\tan^{-1}(j\omega) + \tan^{-1}\left(\tfrac{1}{2}j\omega\right) = 90^\circ$$

$$\tan^{-1}\left(\frac{j\omega + \frac{1}{2}j\omega}{1-\frac{1}{2}\omega^2}\right) = 90^\circ$$

$$1 - \tfrac{1}{2}\omega^2 = 0$$

$$\omega = \sqrt{2}$$

$$-20\log|G(s)| = -20\left(\log 2 - \log|j\sqrt{2}| - \log|j\sqrt{2}+1| - \log\left|j\tfrac{\sqrt{2}}{2}+1\right|\right)$$

$$= -20\left(\log 2 - \log\sqrt{2} - \log\sqrt{3} - \log\sqrt{\tfrac{3}{2}}\right)$$

$$= 3.522$$

P4.4 (b) K does not affect phase ϕ, so phase crossover frequency $\omega = \sqrt{2}$

$$-20\log|G(s)| = -20\left(\log K - \log\sqrt{2} - \log\sqrt{3} - \log\sqrt{6}\right)$$

$$-\frac{4}{5} = \log K - \log\sqrt{2} - \log\sqrt{3} - \log\sqrt{6}$$

$$K = 0.951$$

P4.5

(a) $N = Z - P$ $\quad P = 0$

$N = Z$

N = no. of clockwise encirclement of $(-1, 0j) = 2$

$Z = 2$

System has 2 roots in right-hand s-plane.
Not stable.

(b) $N = 1 - 1 = 0$ $\quad$ 1 clockwise. 1 anti-clockwise

~~Stab~~ $Z = N = 0$

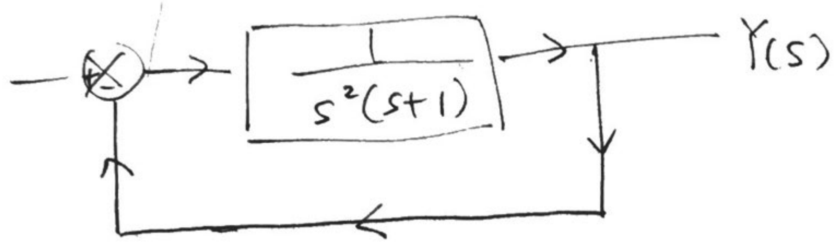
Stable.

P4.6 $N = Z - P$

First, determine the number of poles P.

$k(s+0.5)$ component has 0 poles.

Consider



$G(s)$

$$G(s) = \cancel{\frac{s^3+s^2}{s^3}} \quad \frac{1}{s^3+s^2+1}$$

Characteristic equation $s^3+s^2+1=0$ has 3 roots

$s_1 = -1.466$

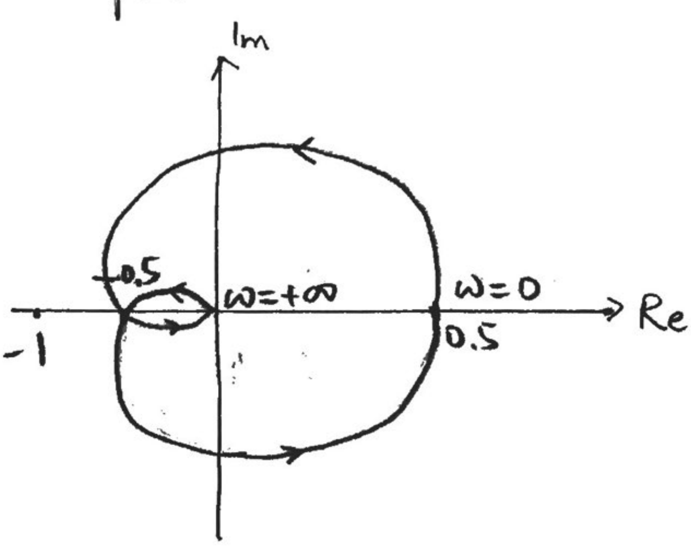
$s_2 = 0.233 + 0.793j$

$s_3 = 0.233 - 0.793j$

2 roots on right-half plane. $P = 2$

Let $G'(s) = \frac{s+0.5}{s^3+s^2+1}$

Nyquist plot



P4.6 Nyquist plot

Type 0 system $\qquad n-m=2$

$$\lim_{\omega\to 0} G'(j\omega) = \frac{0.5}{1} = \frac{1}{2} = \frac{1}{2}\angle 0^\circ$$

$$\lim_{\omega\to +\infty} G'(j\omega) = \frac{j\infty}{(j\infty)^3} = \frac{\cancel{j}\infty}{-\cancel{j}\infty^3} = 0^- \angle -180^\circ$$

$$G'(j\omega) = \frac{j\omega+0.5}{(j\omega)^3+(j\omega)^2+1} = x+jy$$

$$(j\omega+0.5)\frac{1}{1-\omega^2-j\omega^3} = x+jy$$

$$(j\omega+0.5)\left(\frac{1-\omega^2}{(1-\omega^2)^2+\omega^6} + \frac{\omega^3}{(1-\omega^2)^2+\omega^6}j\right) = x+jy$$

$$y = \frac{\omega(1-\omega^2)}{(1-\omega^2)^2+\omega^6} + \frac{0.5\omega^3}{(1-\omega^2)^2+\omega^6} = 0$$

$$\omega - \frac{1}{2}\omega^3 = 0$$

$$\omega_1 = 0 \quad \omega_2 = \sqrt{2} \quad \omega_3 = -\sqrt{2}$$

$$x = \frac{0.5(1-\omega^2)}{(1-\omega^2)^2+\omega^6} - \frac{\omega^4}{(1-\omega^2)^2+\omega^6}$$

$$x\Big|_{\omega=\sqrt{2}} = \frac{0.5(1-2)}{(-1)^2+8} - \frac{4}{1+8}$$

$$= -\frac{4.5}{9}$$

$$= -\frac{1}{2}$$

$$kG'(s) = \frac{k(s+0.5)}{s^3+s^2+1}$$

For $kG'(s)$ to be always stable, Nyquist plot needs to encircle $(-1+j0)$ twice anti-clockwise:

$$-0.5K < -1$$

$$k > 2 \quad //$$

$$N = Z + P = 0$$

$$Z + 2 = 0$$

$$Z = -2$$