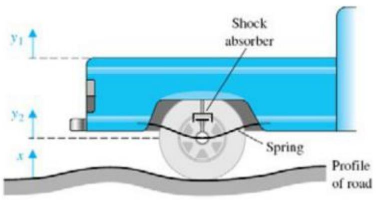
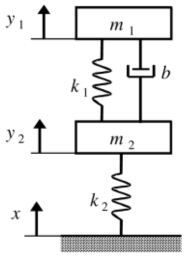
Homework

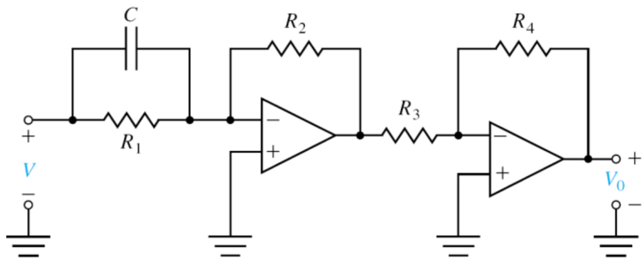
Chapter 2: Mathematical Models of Systems

P2.1 The suspension system for wheels of an old-fashioned pickup truck is illustrated in Fig. P2.1. The mass of the vehicle is m_1 and the mass of the wheel is m_2. The suspension spring has a spring constant k_1, and the tire has a spring constant k_2. The damping constant of the shock absorber is b. Obtain the differential equations describing the system, and determine the transfer function $Y_1(s)/X(s)$ which represents the vehicle response to bumps in the road.

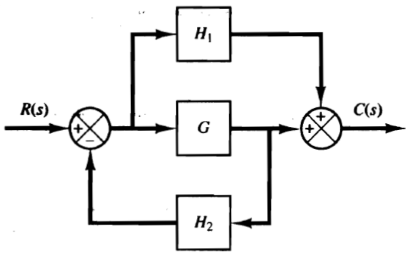




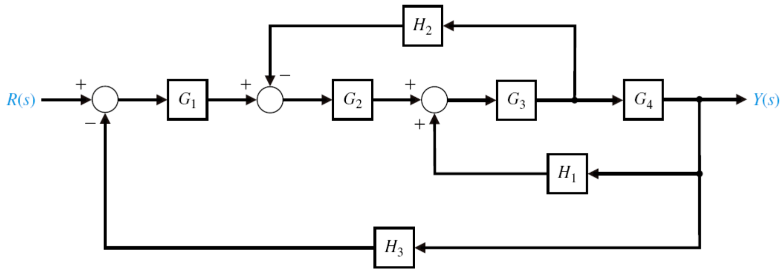
P2.2 Determine the transfer function $V_o(s)/V(s)$ for the op-amp circuit shown below. Let $R_1 = 167\ \text{k}\Omega$, $R_2 = 240\ \text{k}\Omega$, $R_3 = 1\ \text{k}\Omega$, $R_4 = 100\ \text{k}\Omega$, and $C = 1\ \mu\text{F}$. Assume an ideal op-amp.



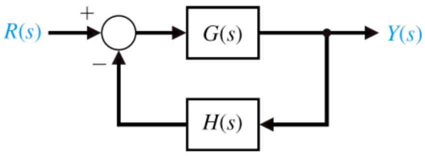
P2.3 Obtain the transfer function $C(s)/R(s)$ of the following block diagram.



P2.4 Simplify the following block diagram by at least two different methods from the one given in Example 2.6.1 of the lecture note.



P2.5 Consider the feedback control system in the following figure,



where $G(s) = \dfrac{s+1}{s+2}$ and

$$H(s) = \frac{1}{s+1}$$

(a) Using MATLAB, determine the closed-loop transfer function.

(b) Obtain the pole-zero map using the "**pzmap**" function. Where are the closed-loop system poles and zeros?

(c) Are there any pole-zero cancellations? If so, use the "**mineral**" function to cancel common poles and zeros in the closed-loop transfer function.

(d) Why is it important to cancel common poles and zeros in the transfer function?

P2.1 DEs:

$$m_2 \ddot{y}_2 = \cancel{k_2(x-y_2)}\ k_1(y_1-y_2) + b(\dot{y}_1-\dot{y}_2) - k_2(y_2-x)$$

$$m_1 \ddot{y}_1 = -k_1(y_1-y_2) - b(\dot{y}_1-\dot{y}_2).$$

~~Simply~~ Simplifying:

$$m_2\ddot{y}_2 + k_1 y_2 + b\dot{y}_2 + k_2 y_2 = k_1 y_1 + b\dot{y}_1 + k_2 X$$

$$m_1\ddot{y}_1 + k_1 y_1 + b\dot{y}_1 = k_1 y_2 + b\dot{y}_2$$

Take $\mathcal{L}$ transform:

$$(m_2 s^2 + bs + k_1 + k_2)\,Y_2(s) = (bs+k_1)\,Y_1(s) + k_2 X(s)$$

$$(m_1 s^2 + bs + k_1)\,Y_1(s) = (bs+k_1)\,Y_2(s)$$

$$Y_2(s) = \frac{m_1 s^2 + bs + k_1}{bs+k_1}\,Y_1(s)$$

$$= \left(\frac{m_1 s^2}{bs+k_1} + 1\right) Y_1(s)$$

$$(m_2 s^2 + bs + k_1 + k_2)\left(\frac{m_1 s^2}{bs+k_1} + 1\right) Y_1(s) = (bs+k_1)\,Y_1(s) + k_2 X(s)$$

$$\frac{Y_1(s)}{X(s)} = \frac{k_2\,(bs+k_1)}{m_1 m_2 s^4 + (m_1+m_2)bs^3 + (k_1+k_2)m_1 s^2 + k_1 m_2 s^2 + k_2 bs + k_1 k_2}$$

（科目：P2.2） 清华大学数学作业纸

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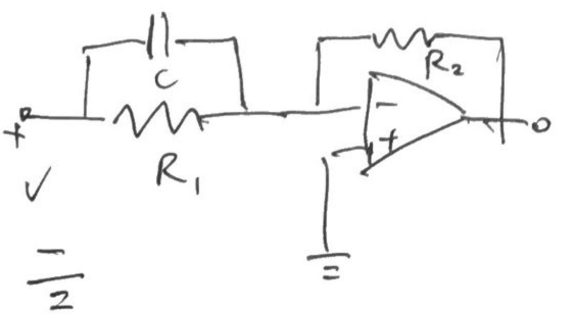
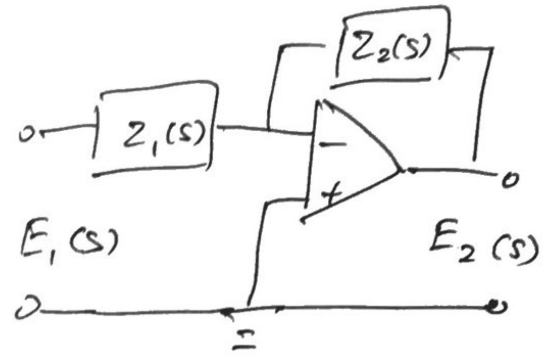
First consider without the inverter



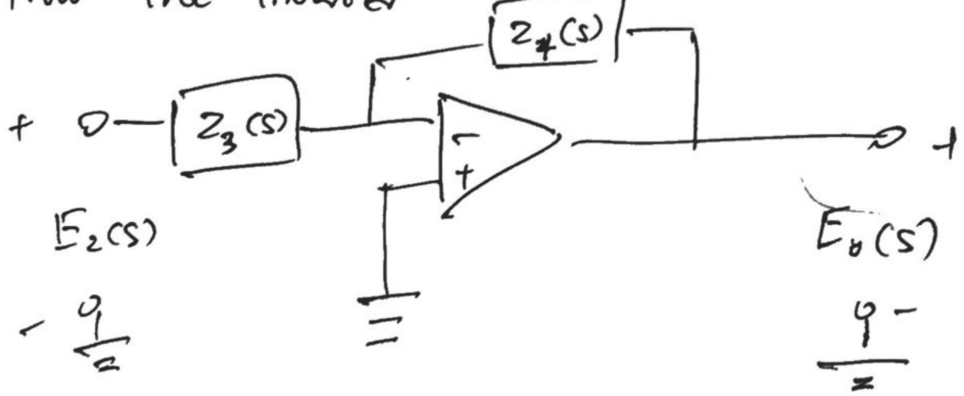
$\Leftrightarrow$



$$Z_1(s) = \frac{R_1 \frac{1}{Cs}}{R_1 + \frac{1}{Cs}} = \frac{R_1}{R_1Cs+1} \qquad Z_2(s) = R_2$$

$$\frac{E_2(s)}{E_1(s)} = -\frac{Z_2(s)}{Z_1(s)} = -\frac{R_2}{R_1}(R_1Cs+1)$$

Now the inverter

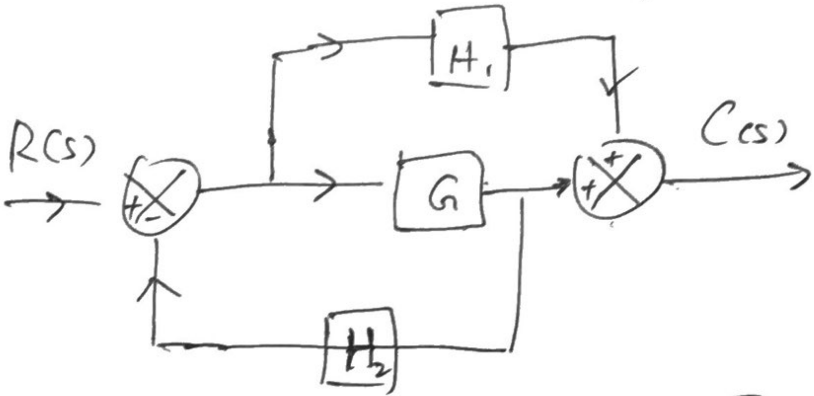


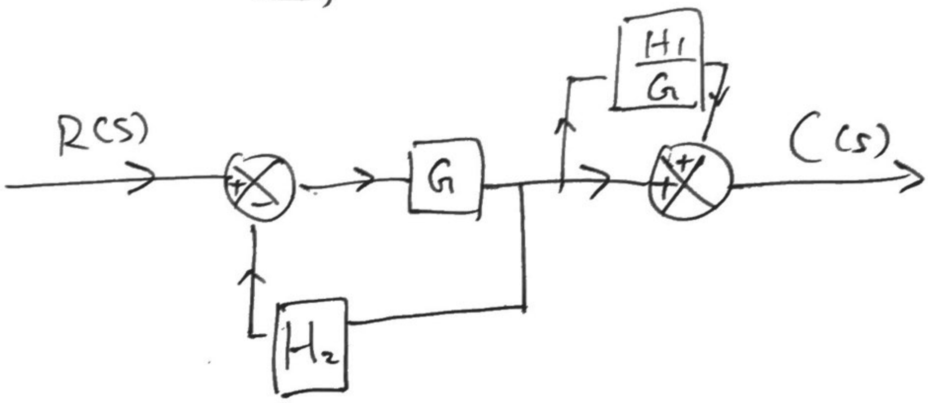
$$\frac{E_o(s)}{E_2(s)} = -\frac{Z_4(s)}{Z_3(s)} = -\frac{R_4}{R_3}$$

代值计算

Together

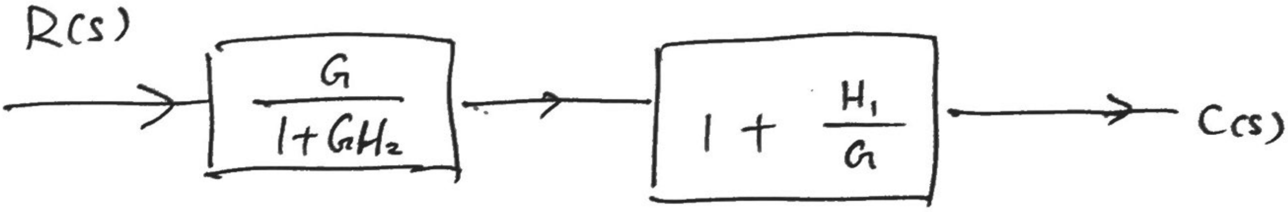
$$\frac{E_o(s)}{E_1(s)} = \frac{E_o(s)}{E_2(s)}\frac{E_2(s)}{E_1(s)} = \frac{R_4 R_2}{R_3 R_1}(R_1Cs+1)$$

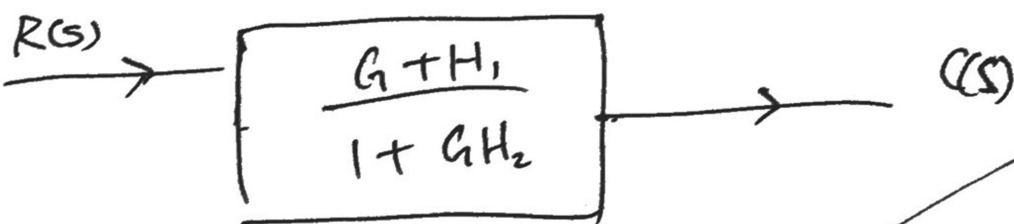




$$\frac{G}{1+GH_2}$$

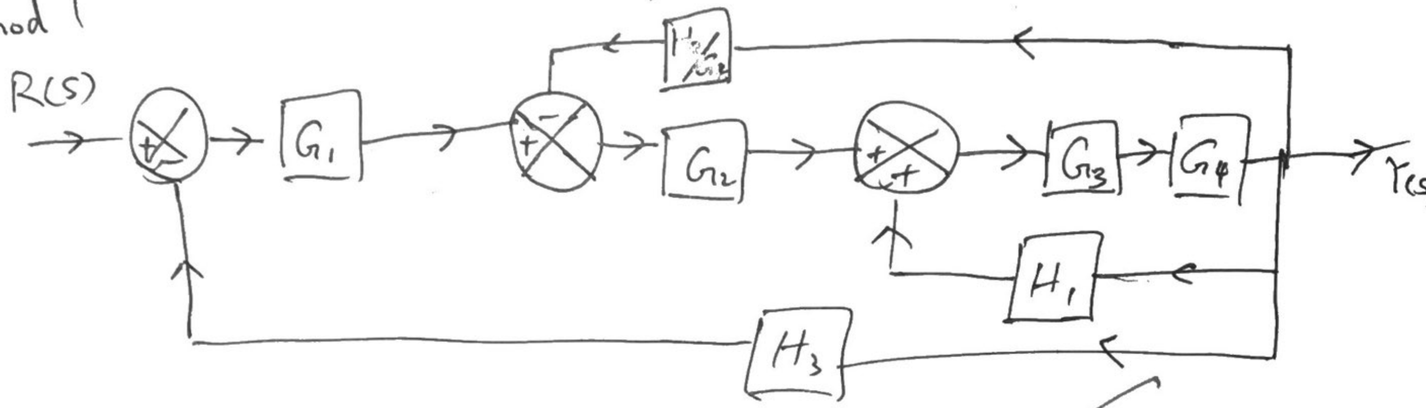
$$\cancel{\frac{1}{1-\frac{H_1}{G}}} \cancel{=} \cancel{\frac{G}{G-H_1}}$$

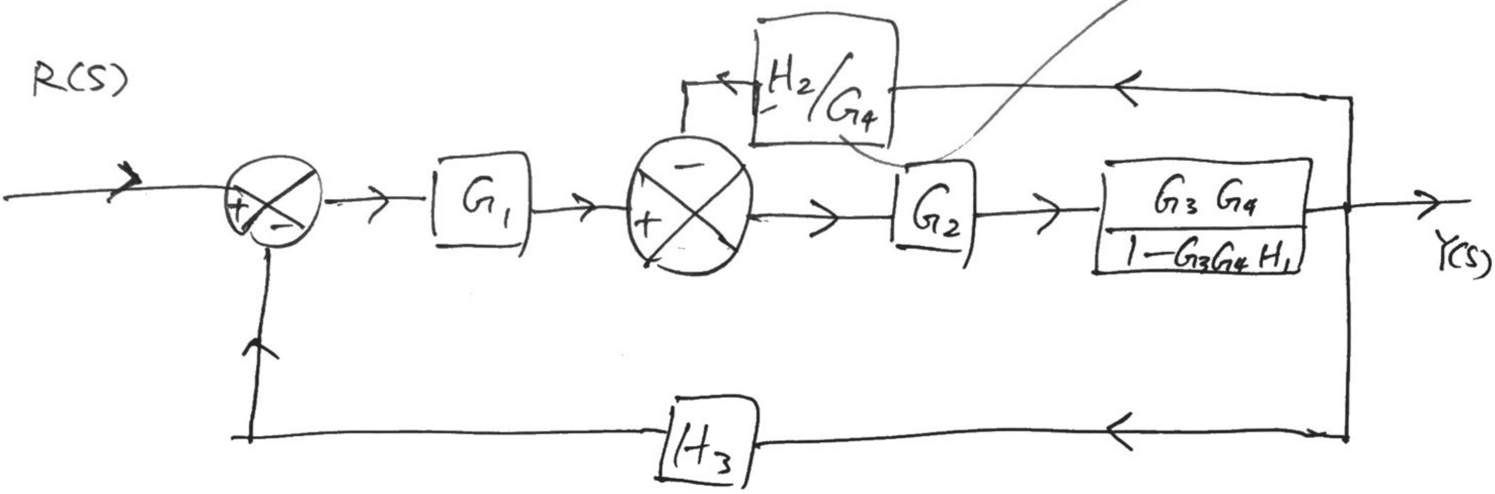


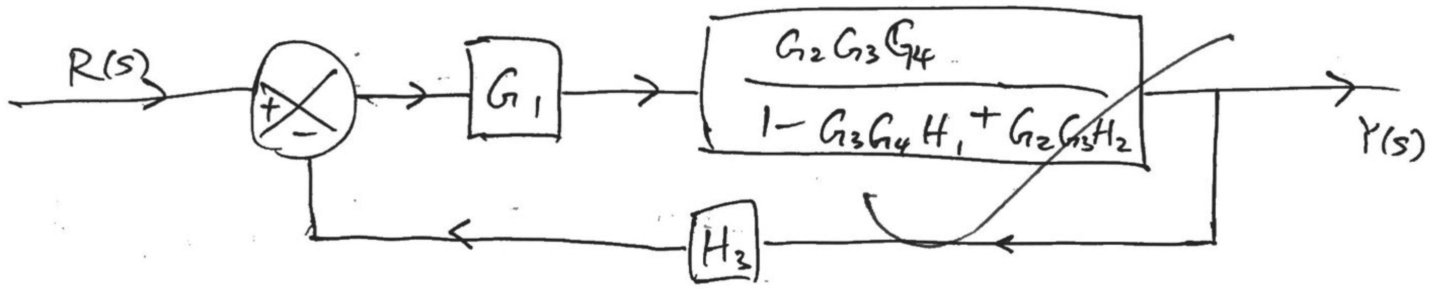


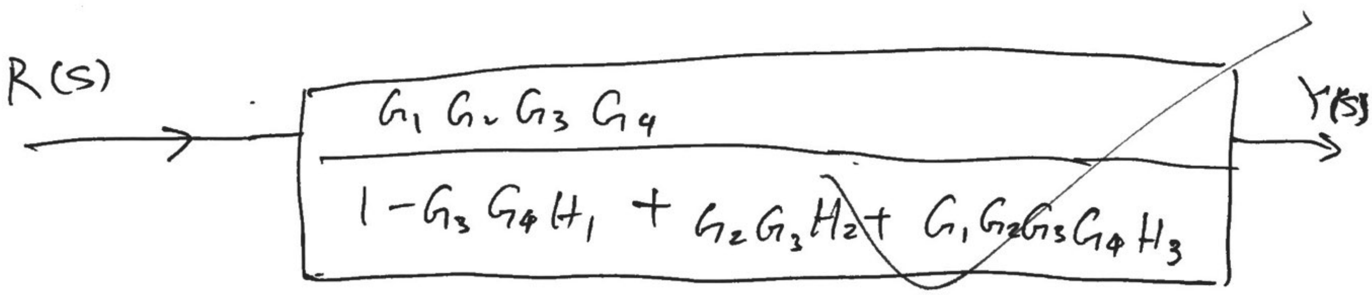
$$\frac{C(s)}{R(s)} = \cancel{G\&G} \quad \frac{G+H_1}{1+GH_2}$$

Method 1

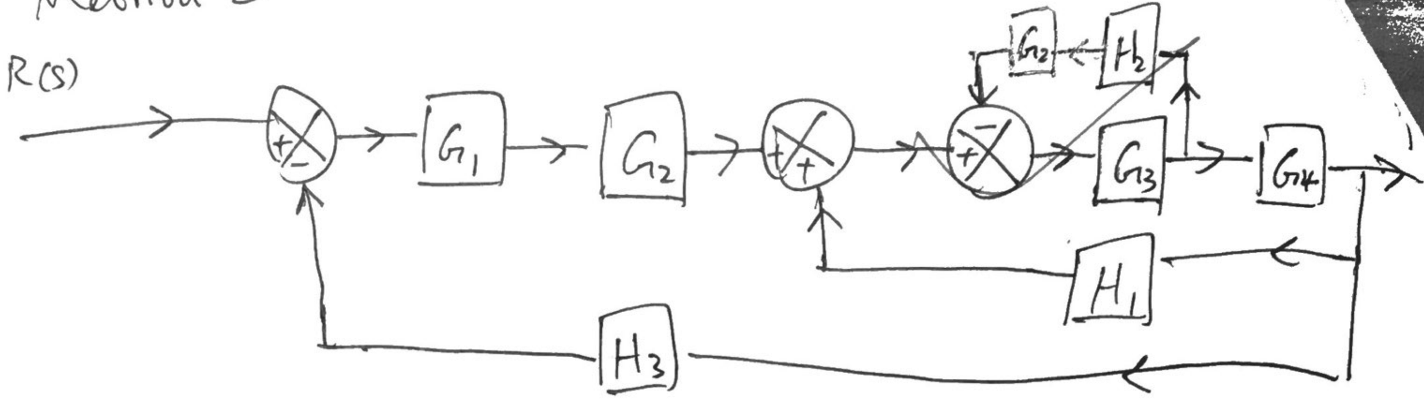

R(s)
G_1
G_2
G_3
G_4
H_2/G_4
H_1
H_3
Y(s)

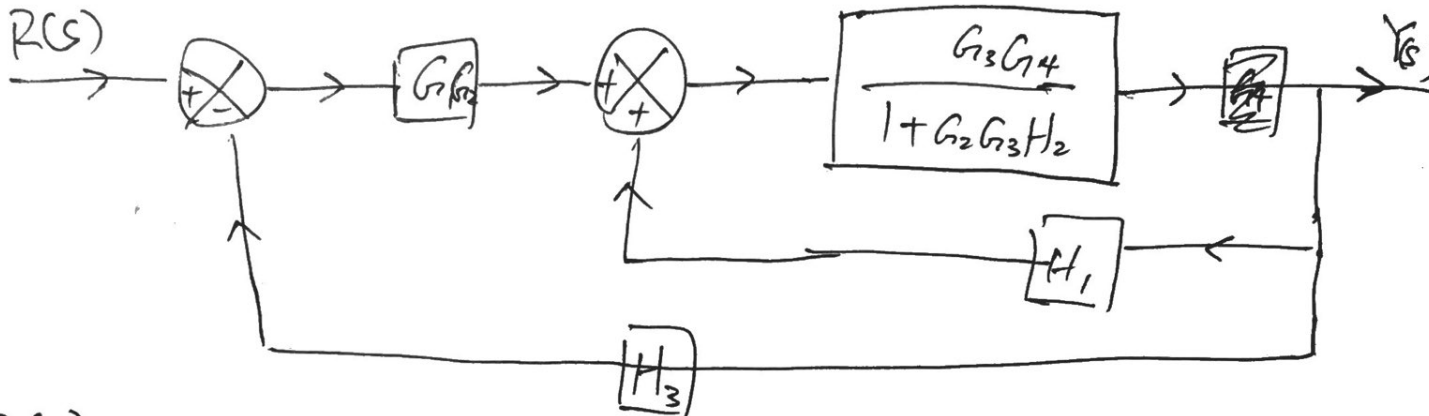

R(s)
G_1
G_2
$\frac{G_3G_4}{1-G_3G_4H_1}$
H_2/G_4
H_3
Y(s)

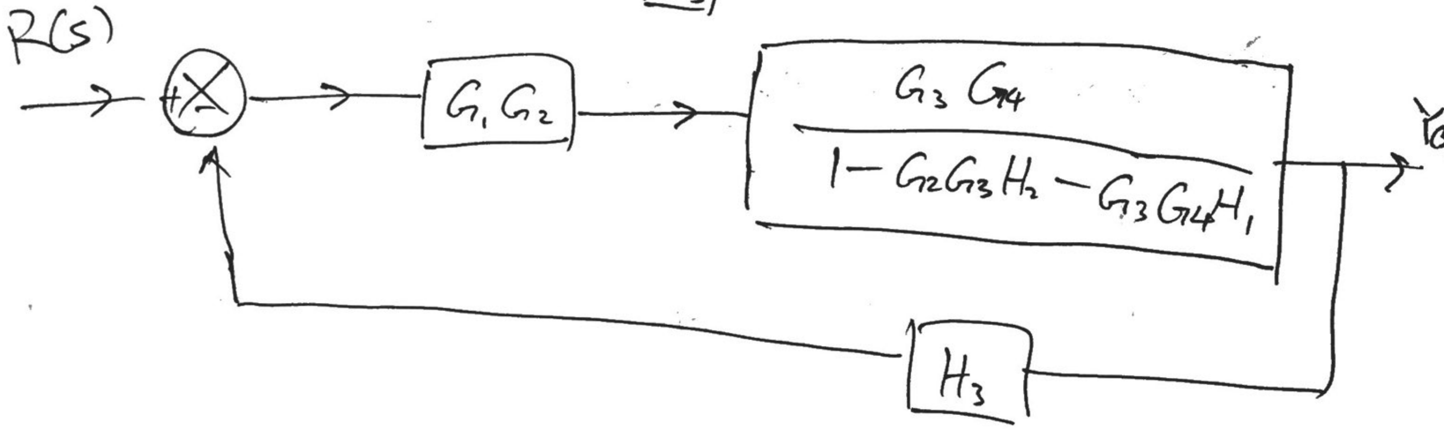

R(s)
G_1
$\frac{G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2}$
H_3
Y(s)

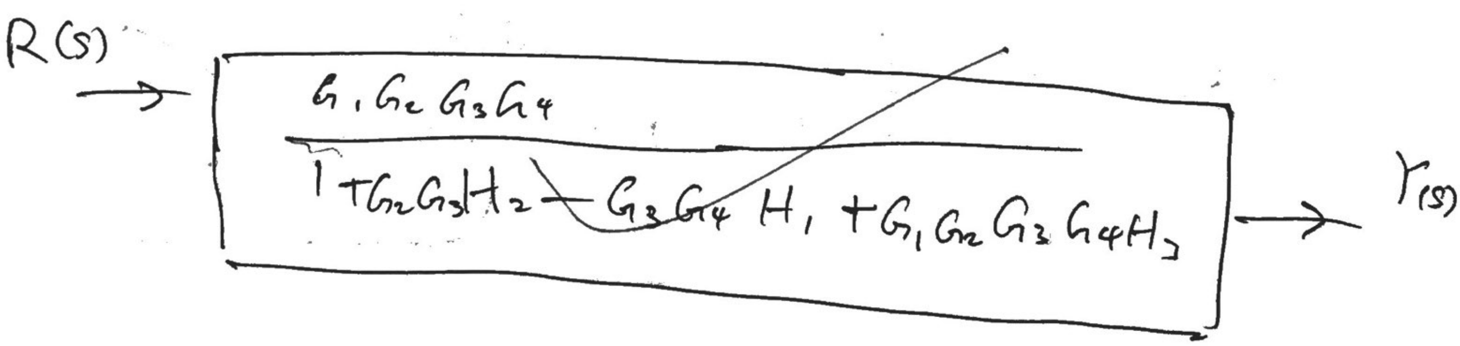

R(s)
$\frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$
Y(s)

Method 2


R(S)
G1
G2
G3
G4
H1
H2
H3
G2


R(S)
G1G2
G3G4 / 1+G2G3H2
H1
H3
Y(S)


R(S)
G1G2
G3G4 / 1−G2G3H2−G3G4H1
H3
Y(S)


R(S)
G1G2G3G4 / 1+G2G3H2−G3G4H1+G1G2G3G4H3
Y(S)

(a) $TF = \dfrac{s^2+2s+1}{s^2+4s+3}$

(b) $p = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ $z = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

(c) TF minreal $= \dfrac{s+1}{s+3}$

(d) In stability analysis, ~~poles in the~~ right-half-plane poles correspond ~~to~~ locations of instability. If we don't cancel common poles & zeroes, the common poles would lead to wrong stability analysis.

It also does not make sense the TF is both 0 & infinite at the same location if we don't cancel.

```
sysg=tf([1,1],[1,2]);
sysh=tf(1,[1,1]);

[num, den] = feedback([1,1],[1,2],[1],[1,1]);
printsys(num,den);

sys = tf(num, den);
p = pole(sys);
z = zero(sys);
[p,z] = pzmap(sys)

sysmin = minreal(sys)
```

```
num/den = 

   s^2 + 2 s + 1
   -------------
   s^2 + 4 s + 3

p =

    -3
    -1


z =

    -1
    -1


sysmin =
 
  s + 1
  -----
  s + 3
 
Continuous-time transfer function.
```