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Reading: Chapter 2.1, 2.2, 3.1, 3.2
Exercises:
1. P22, 2.1-3
2. P29, 2.2-3
3. P53, 3.1-3
4. Show that $2n2 + 3n + 1 = \Theta(n2)$

*5. reading: Appendix B.5,
*6. Compute the height of a complete 3-ary tree with n nodes.

1. P22, 2.1-3

Pseudo code loosely based on python syntax conventions

```
i = NIL
for index from 1 to n:
// assume indexing of A starts from 1
    if A[index] == v:
        i = index
        break
return i
```

Loop invariant: At the start of each iteration, none of the elements in the checked subarray, i.e. <a_1, a_2 … $a_{index-1}$> is equal to *v*.

Initialization: Trivially true at initialization, since checked subarray is NULL.
Maintenance: If a_{index} is not equal to *v*, a_{index} is appended to the checked subarray, and loop invariant is maintained. Otherwise, exit loop and we are done.
Termination: Loop terminates because either a_{index} == *v* or we have exhausted all *index*. In either case, nothing is appended to the checked subarray in the current iteration, so loop invariant is unchanged. In the latter case, no element of *A* is equal to *v* and *NIL* is returned.

2. P29, 2.2-3

Let *X* ~ number of elements checked. Assume each element could be *v* with equal probability *p*.

Case 1: Assume at most one element of *A* is equal to *v* (so that elements are *NOT* i.i.d.).

$$E(X) = 1 \cdot p + 2p \cdot + \cdots + n \cdot p + n(1 - np) = \frac{n(1+n)p}{2} + n(1 - np)$$

The term $n(1 - np)$ corresponds to the case where *v* is not found in *A*.
In the case of *v* existing in *A* with certainty, we sub in $p = \frac{1}{n}$,

$$E(X) = \frac{1+n}{2} = \Theta(n)$$

Case 2: Any number of elements of *A* could be equal to *v* (so that elements are i.i.d.).

$$E(X) = 1 \cdot p + 2 \cdot p(1-p) + \cdots + np(1-p)^{n-1} + n(1-p)^n$$

The last term corresponds to the case where *v* is not found in *A*. Simplifying,

$$E(X) = \frac{1-(1-p)^n}{p}$$

Since $0 < \frac{(1-p)^n}{p} < \frac{1-p}{p}$, $1 < E(X) < \frac{1}{p}$. So, curiously, $E(X) = \Theta(1)$

In either case, we have to check the entire *A* in the worst case, and the worst case time complexity is trivially $\Theta(n)$.

3. P53, 3.1-3

The statement is equivalent to saying the asymptotic upper bound on running time is at least $C_0 n^2$ for some constant C_0. This seems to suggest the upper bound could be greater – unbounded in fact.

4. Show that $2n^2 + 3n + 1 = \Theta(n^2)$

Let *f(n)* $=2n^2 + 3n + 1$

$$\exists\, C_1 = 1 \;\&\; C_2 = 3 \;\&\; n_0 = 4$$

$$s.t. \quad 0 \leq C_1 n^2 \leq f(n) \leq C_2 n^2 \quad for\ all\ n \geq n_0$$

Hence,

$$f(n) = \Theta(n^2)$$

*6. Compute the height of a complete 3-ary tree with n nodes.

For a k-ary complete tree, with h levels,

$$n = 1 + k + k^2 + \cdots + k^{h-1} = \frac{k^h - 1}{k - 1}$$

Rearranging and taking logs,

$$h = log_k(n(k-1) + 1)$$

Sub in k = 3

$$h = log_3\,(2n + 1)$$