p.122 5.2-3
p.122 5.2-5
p.143 5-2 (a-e)
p.204 8.4-4

Programming assignment 1: http://www.tsinsen.com p.143 5-2 (a)

```
def RANDOM-SEARCH(array A, x):
    n = length of array A
    countList = [0]*n # Initialise empty list of counters to keep track how many times each index has been searched
    count = 0 # Initialise total counter
    while count < n: # Loop till count = n, ie. each index has been searched
        i = random number [0, n-1]
        if A[i] == x: # Found x
            return i
        elif A[i] != x and countList[i] == 0: # x not at this index i, and index has not been searched before
            countList[i]  += 1
            count += 1
        else: # x not at this index i, and index has been searched before
            continue
    return 'x not found in A'
```

p.143 5-2 (b-e)

Refer to attached handwritten solution.

p.204 8.4-4

We implement bucket sort. This partitions the circle into concentric buckets of radius $r_i = \sqrt{\frac{i}{n}}$ This runs in $\Theta(n)$.

```
def BUCKET-SORT(array A):
    n = length of array A
    B=[0]*n # Initialize empty bucket array
    for i from 0 to n-1:
        B[i] = [] # Make a list for each bucket position
    for i from 1 to n:
        B[j].append(A[i]), where (j-1)/n <= A[i].x** + A[i].y** < j/n # Sort elements in A into n buckets where each bucket has distance sqrt(i/n)
    for i from 0 to n-1:
        sort B[i]
    return B[0]+B[1]+...+B[n-1]
```

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## Q ~~Let~~ 5.2-3

Let $X_i = I\{\text{die rolls } i\}$ $\forall i$ from 1 to 6

$$E(X) = \sum_{i=1}^{6} p(X_i)\cdot i = \frac{1}{6}(1+6)\frac{6}{2} = 3.5$$

$$E(nX) = n\,E(X) = 3.5n$$

## Q 5.2-5

Let $X_{i,j}$ denote the ~~events~~ inversion event,

ie $X_{i,j} = I\{\, i<j \cap A[i] > A[j]\}$ $\forall i \in [1, n-1]$ & $j \in [i+1, n]$

$$E\Big(\sum_{i<j} X_{ij}\Big) = \sum_{i<j} E(X_{ij}) = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n} p(A[i] > A[j])$$

$$= \sum_{i=1}^{n-1} \frac{1}{2} \cancel{j} [n-(j-1)]$$

— for a randomly picked element, $\frac{1}{2}$ of a random array is expected to be smaller than this element

$$= \frac{1}{2}\sum_{i=1}^{n-1}(n-i)$$

$$= \frac{1}{2} \cancel{1} (n-1+1)\frac{n-1}{2}$$

$$= \frac{n(n-1)}{4}$$

Q 5-2 (b)

Let $p_i$ denote the probability of X occuring at index $i$.

Also assume uniform distribution $p_i = p = \frac{1}{n} \ \forall i \in [0, n-1]$

Let $y$ denote the total number of searches.

$$E(y) = \sum_{y=1}^{\infty} y \, P(y) = \sum_{y=1}^{\infty} y \cdot (1-p)^{y-1} p$$

$$= p \sum_{y=1}^{\infty} y \cdot (1-p)^{y-1} \quad \text{———} \quad ①$$

Let $s = \sum_{y=1}^{\infty} y (1-p)^{y-1}$

$$= 1(1-p)^0 + 2(1-p)^1 + 3(1-p)^2 + \dots$$

$$(1-p)\, s = 1(1-p)^1 + 2(1-p)^2 + 3(1-p)^3 + \dots$$

$$(1-(1-p))\, s = (1-p)^0 + (1-p)^1 + (1-p)^2 + \dots$$

$$p\, s = \frac{1}{1-(1-p)}$$

$$s = \frac{1}{p^2} \quad \text{———} \quad ②$$

sub ② into ①

$$E(y) = p \frac{1}{p^2}$$

$$= \frac{1}{p}$$

$$= n \ //$$

Q5-2 (c)

Here $p_i = p = \frac{k}{n}$

$$E(y) = \frac{1}{p} = \frac{n}{k}$$

Q5-2 (d)

$$E(y) = \sum_{y=n}^{\infty} y\, p(y)$$

$$\cancel{= \sum_{y=n}^{\infty}}$$

$p(y)$ denotes the event that the first $y-1$ searches touch $n-1$ indices & the last search looks at $n^{th}$ index

$$p(y) = \left(\frac{n-1}{n} \cdot \frac{n-2}{n} \cdots \frac{1}{n}\right)\left(\frac{n-1}{n}\right)^{y-1-(n-1)} \cdot \frac{1}{n}$$

$$= \frac{(n-1)!}{n^{n-1}} \left(\frac{n-1}{n}\right)^{y-n} \frac{1}{n}$$

$$= \frac{(n-1)!\,(n-1)^{y-n}}{n^{(n+y-n)}}$$

$$= \frac{(n-1)!\,(n-1)^{y-n}}{n^y}$$

$$E(y) = \sum_{y=n}^{\infty} \frac{(n-1)!\,(n-1)^{y-n}}{n^y} \cdot y$$

Stuck here

~~Let~~ Have to use textbook method.

Q5-2 (d)

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Let $n_i$ denote the number of searches to take us from having searched $i-1$ indices to having searched $i$ indices.

$$E(n_i) = \sum_{i=1}^{\infty} i \left(\frac{i-1}{n}\right)^{i-1} \left(\frac{n-i+1}{n}\right)$$

$$= \frac{n-i+1}{i-1} \sum_{i=1}^{\infty} i \left(\frac{i-1}{n}\right)^{i}$$

$$= \frac{n-i+1}{\cancel{i-1}} \frac{\frac{\cancel{i-1}}{n}}{\left(1 - \frac{i-1}{n}\right)^2}$$

$$= \frac{n}{n-i+1}$$

$$E(y) = E\left(\sum_{i=1}^{n} n_i\right)$$

$$= \sum_{i=1}^{n} E(n_i)$$

$$= \sum_{i=1}^{n} \frac{n}{n-i+1}$$

$$= n \sum_{i=1}^{n} \frac{1}{i}$$

No nice close-form solution for $n^{th}$ harmonic series

Approximate harmonic series by integral:

$$\sum_{i=1}^{n} \frac{1}{i} \geqslant \int_1^{n+1} \frac{1}{x}\,dx$$
$$\geqslant [\ln x]_1^{n+1}$$
$$\geqslant \ln(n+1)$$

$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \sum_{i=2}^{n} \frac{1}{i} \leqslant 1 + \int_1^{n} \frac{1}{x}\,dx$$
$$\sum_{i=1}^{n} \frac{1}{i} \leqslant 1 + \ln(n)$$

$$n\ln(n+1) \leqslant E(y) \leqslant n + n\ln(n)$$

## Q5-2(e)

Average-case time:

$$E(y) = \sum_{y=1}^{n} y\, P(y)$$
$$= \frac{1}{n} \sum_{y=1}^{n} y$$
$$= \frac{n+1}{2}$$

. Assume each search cost constant $c$

then $T(n) = \frac{n+1}{2} c$

Worst-case time:

$$T(n) = nc$$