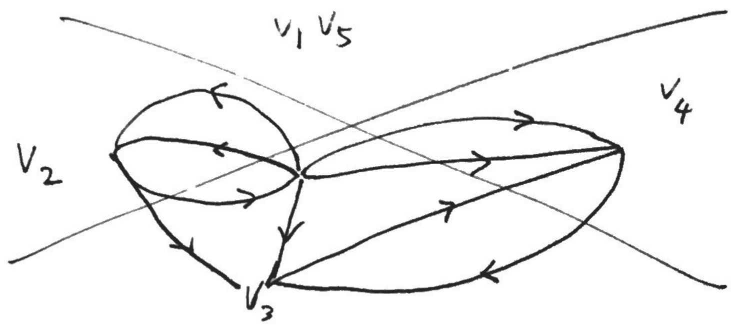
Q4 (a)

$$B_5B_5^T = \begin{pmatrix} 3 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -2 \\ -1 & 0 & -2 & 4 \end{pmatrix}$$

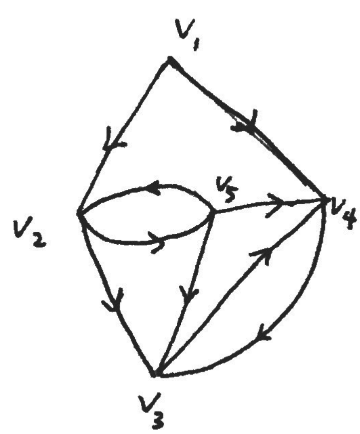
$$\det\left(B_5 B_5^T\right) = 101$$

(b)



~~$B_5 B_5^T =$~~

去掉 $(v_1 \ v_5)$

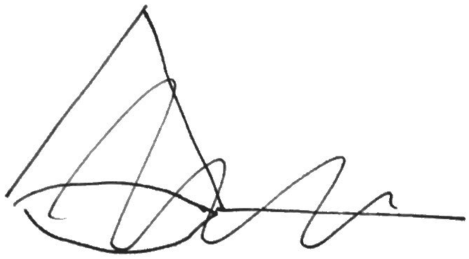


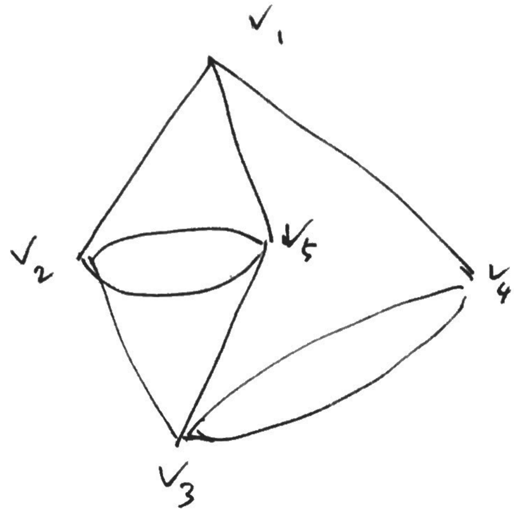
$$B_5 B_5^T = \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -2 \\ -1 & 0 & -2 & 4 \end{pmatrix}$$

$$\det\left(B_5 B_5\right)^T = 57$$

$$101 - 57 = 44$$

Q4 (c) 去掉 $(v_4\ v_5)$





$$B_5 B_5^T = \begin{pmatrix} 3 & -1 & 0 & -1 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -2 \\ -1 & 0 & -2 & 3 \end{pmatrix}$$

$$\det\left(B_5\ B_5^T\right) = 60$$

Q5 (a)

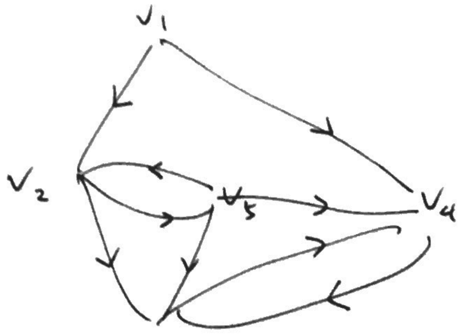
$$B_1 = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & -1 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\vec{B}_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\vec{B}_1\ B_1^T = \begin{pmatrix} 2 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 2 \end{pmatrix}$$

$$\det\left(\vec{B}_1\ B_1^T\right) = 24$$

Q5 (b)



去掉 $(v_1\ v_5)$

$$B_1 = \begin{pmatrix} -1 & 0 & 1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

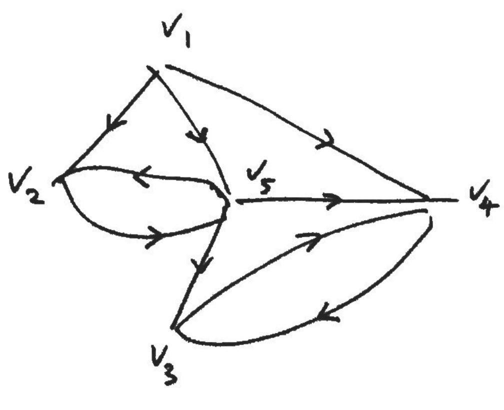
$$\vec{B}_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\vec{B}_1 B_1^T = \begin{pmatrix} 2 & 0 & 0 & -1 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

$\det(\vec{B}_1 B_1^T) = 8$

~~24 = 8 = 16~~

Q5 (c) 去掉 (v_2, v_3)



$$B_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Q5(c)

$$B_1 = \begin{pmatrix} -1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$\vec{B}_1 = \begin{pmatrix} -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\vec{B}_1 B_1^T = \begin{pmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & 0 & 2 \end{pmatrix}$$

$$\det(\vec{B}_1 B_1^T) = 15$$

$$24 - 15 = 9$$

Q9 题 5(c) 中 $\det(\vec{B}_1 \vec{B}_1^T) = 24 \neq \det(\vec{B}_1 B_1^T)$

因此 $\det(\vec{B}_k B_k^T)$ 不是从 v_1 为根的根树数目.

Q13 $v \in V(G)$, $G - v$ 仍为连通图 $\Rightarrow d(v_i) > 1$ for $v \in V(G)$

G的基本割集矩阵每行偶数个1元素

$\Rightarrow$ 每个点 v 都是此矩阵的一行

$\Rightarrow d(v_i) = 2m$

$\therefore$ 因此 G 中有欧拉回路

Q11 余树边 $\{e_1\ e_2\ e_3\ e_4\}$

先求对树 $\{e_5\ e_6\ e_7\ e_8\}$ 的基本回路矩阵

$\cancel{C'' = PC}$

$$C' = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C'' = PC' = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 1 \end{pmatrix}$$

$$C_f = \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 1 \end{pmatrix} \\ \begin{matrix} e_1 & e_3 & e_2 & e_4 & e_5 & e_6 & e_7 & e_8 \end{matrix} \end{array}$$

$C_f = \begin{pmatrix} I & C_{f12} \end{pmatrix}$ $\qquad S_f = \begin{pmatrix} S_{f11} & I \end{pmatrix}$

$$S_{f11} = -C_{f12}^T = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$$

$$S_f = \begin{array}{c} \begin{pmatrix} 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \\ \begin{matrix} e_1 & e_3 & e_2 & e_4 & e_5 & e_6 & e_7 & e_8 \end{matrix} \end{array}$$