HW W6-2:

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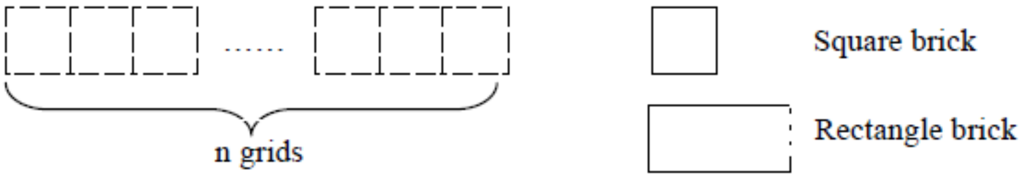
1. A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled.



Let the no. of ways be denoted by f(n).
The recurrence relation is then f(n) = f(n-1) + f(n-2)
The corresponding characteristic function is $x^2-x-1 = 0$
The characteristic roots are $x_1 = \frac{1+\sqrt{5}}{2}$ $x_2 = \frac{1-\sqrt{5}}{2}$
So $f(n) = c_1\left(\frac{1+\sqrt{5}}{2}\right)^n + c_2\left(\frac{1-\sqrt{5}}{2}\right)^n$
Initial conditions f(1) = 1, f(2) = 2
See attached for derivation. Anyhow, $c_1 = \frac{1+\sqrt{5}}{2\sqrt{5}}$ $c_2 = \frac{3-\sqrt{5}}{5-\sqrt{5}}$

$$f(n) = \frac{1+\sqrt{5}}{2\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n + \frac{3-\sqrt{5}}{5-\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

2. How many different ways to color n grids in a line with red, white or blue colors but no two adjacent grids are colored with red?

Let the no. of ways be denoted by f(n).
If we color the n^{th} tile red, then there are 2f(n-2) ways to color the remaining n-1 tiles. Otherwise, there are f(n-1) ways to color the remaining n-1 tiles.

The recurrence relation is then f(n) = 2f(n-1) + 2f(n-2)
The corresponding characteristic function is $x^2-2x-2 = 0$
The characteristic roots are $x_1 = 1+\sqrt{3}$ $x_2 = 1-\sqrt{3}$
So $f(n) = c_1(1+\sqrt{3})^n + c_2(1-\sqrt{3})^n$
Initial conditions f(0) = 1, f(1) = 3
See attached for derivation. Anyhow, $c_1 = \frac{\sqrt{3}+2}{2\sqrt{3}}$ $c_2 = \frac{\sqrt{3}-2}{2\sqrt{3}}$

$$f(n) = \frac{\sqrt{3}+2}{2\sqrt{3}}(1+\sqrt{3})^n + \frac{\sqrt{3}-2}{2\sqrt{3}}(1-\sqrt{3})^n$$

$$f(1) = c_1\left(\frac{1+\sqrt{5}}{2}\right) + c_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

Q1

$$c_1(1+\sqrt{5}) + c_2(1-\sqrt{5}) = 2$$

$$c_1(1+\sqrt{5}) = 2 - c_2(1-\sqrt{5}) \quad \text{——} \quad ①$$

$$f(2) = c_1\left(\frac{1+\sqrt{5}}{2}\right)^2 + c_2\left(\frac{1-\sqrt{5}}{2}\right)^2 = 2$$

$$c_1(1+\sqrt{5})^2 + c_2(1-\sqrt{5})^2 = 8$$

$$\cancel{(2 - c_2(1-\sqrt{5}))(1+\sqrt{5}) + c_2(-4) = 8}$$

$$\cancel{2(1+\sqrt{5}) - c_2(}$$

$$\cancel{c_1(1+\sqrt{5})^2 +}$$

$$(2 - c_2(1-\sqrt{5}))(1+\sqrt{5}) + c_2(1-\sqrt{5})^2 = 8$$

$$2(1+\sqrt{5}) - c_2(-4) + c_2(1-2\sqrt{5}+5) = 8$$

$$c_2(10-2\sqrt{5}) = 6-2\sqrt{5}$$

$$c_2 = \frac{3-\sqrt{5}}{5-\sqrt{5}} \quad //$$

$$c_1(1+\sqrt{5}) = 2 - \frac{3-\sqrt{5}}{5-\sqrt{5}}(1-\sqrt{5})$$

$$c_1(1+\sqrt{5})(5-\sqrt{5}) = 2(5-\sqrt{5}) - (3-\sqrt{5})(1-\sqrt{5})$$

$$c_1(5-4\sqrt{5}-5) = 10-2\sqrt{5}-3+4\sqrt{5}-5$$

$$c_1(4\sqrt{5}) = 2+2\sqrt{5}$$

$$c_1 = \frac{1+\sqrt{5}}{2\sqrt{5}} \quad //$$

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Q2 $$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-4(-2)}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

$$f(0) = c_1 + c_2 = 1$$

$$c_2 = 1 - c_1 \quad \text{—— ①}$$

$$f(1) = c_1(1+\sqrt{3}) + c_2(1-\sqrt{3}) = 3 \quad \text{—— ②}$$

$$c_1(1+\sqrt{3}) + (1-c_1)(1-\sqrt{3}) = 3$$

$$\cancel{c_1 + c_1\sqrt{3} + 1 - \sqrt{3} - c_1 + 3\sqrt{3}c_1 = 3}$$

$$c_1(1+\sqrt{3}) + (1-\sqrt{3}) - c_1(1-\sqrt{3}) = 3$$

$$c_1(2\sqrt{3}) = 3 - 1 + \sqrt{3}$$

$$c_1 = \frac{2+\sqrt{3}}{2\sqrt{3}} = \frac{2\sqrt{3}+3}{6}$$

$$c_1 = \frac{\sqrt{3}+2}{2\sqrt{3}} \quad //$$

$$c_2 = 1 - c_1$$

$$= 1 - \frac{2+\sqrt{3}}{2\sqrt{3}}$$

$$= \frac{\sqrt{3}-2}{2\sqrt{3}} \quad //$$