Combinatorics HW w6-1

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1. Please prove the following equation of fibonacci sequence F_i:

$$F_1 + F_3 + F_5 + \cdots + F_{2n-1} = F_{2n}$$

This is obvious if we expand the RHS to give the LHS. Reversing the process will collapse LHS into RHS.

But proof by induction looks more fun, so let's do that.

$H_0 : F_1 + F_3 + F_5 + \ldots + F_{2n-1} = F_{2n}$

$H_1 : F_1 + F_3 + F_5 + \ldots + F_{2n-1} \neq F_{2n}$

Initialization, we can see H_0 is true for n = 1:

$$F_1 = F_2 = 1$$

Assume H_0 is true for n-1, we prove H_0 is true for n:

$$F_1 + F_3 + F_5 + \ldots + F_{2(n-1)-1} = F_{2(n-1)}$$

$$F_1 + F_3 + F_5 + \ldots + F_{2n-3} + F_{2n-1} = F_{2n-2} + F_{2n-1}$$

$$F_1 + F_3 + F_5 + \ldots + F_{2n-3} + F_{2n-1} = F_{2n}$$

2. Please provide the corresponding characteristic equations for the following recurrence relation:

$$a_n = 2a_{n-1} + 4a_{n-2} - 5a_{n-3}$$

$C(x) := x^4 - 2x^3 - 4x + 5 = 0$

3. Solve the recurrence relation $h_n=2h_{n-1}+8h_{n-2}$,n≥2, $h_1=1$, $h_2=10$

$C(x) := x^2 - 2x - 8 = 0$

$C(x) := (x - 4)(x + 2) = 0$

The characteristic roots are 4 and -2.

Suppose $h_n = c_1(4)^n + c_2(-2)^n$

Solving initial conditions:

$1 = 4c_1 - 2c_2$ & $10 = 16c_1 + 4c_2$

$c_1 = 0.5$ & $c_2 = 0.5$

$h_n = 0.5(4)^n + 0.5(-2)^n$