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C.3-1

Let x denote number on a single die, X denote sum, Y denote maximum.

$$E(X) = 2\sum_{1}^{6} x \cdot P(x) = 7$$

There are 36 possible outcomes, so enumeration method to get Y is quick:

$$E(Y) = \frac{1}{36}[(1+2+3+4+5+6)+(2+2+3+4+5+6)+\cdots+(6+6+6+6+6+6)]$$

$$E(Y) = \frac{161}{36}$$

C.3-2

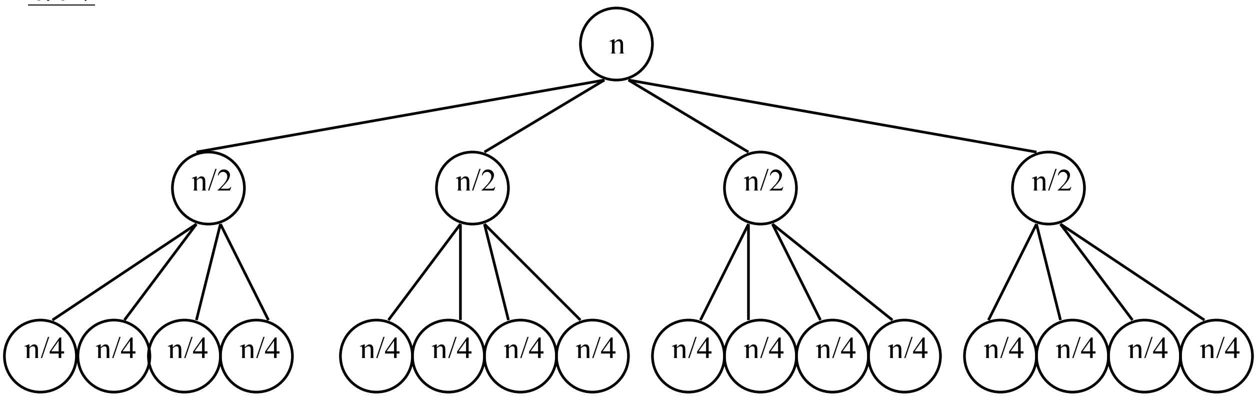
Let the index of the maximum value be denoted by *x*. $P(x = i) = \frac{1}{n}\ \forall\, i = 1 \ldots n$

$$E(x) = \sum_{1}^{n} \frac{x}{n} = \frac{n+1}{2}$$

Let the index of the minimum value be denoted by *y*. By the same logic:

$$E(y) = \frac{n+1}{2}$$

4.4-7



Using master method, a = 4, b = 2, f(n) = cn

$$log_b a = 2$$

$$f(n) = O(n^{log_b a - \epsilon}) \quad where\ \epsilon = 1$$

$$T(n) = \Theta(n^{log_b a}) = \Theta(n^2)$$

（科目：　　）数 学 作 业 纸

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4-1 (b) $T(n) = T(7n/10) + n$

$a = 1$

$b = \cancel{\frac{7}{10}}\ \frac{10}{7}$

$f(n) = n = \Theta(n)$

$\log_b a = 0$

$f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$ where $\epsilon = 1$

Also, $af(n/b) = \frac{7n}{10} \leq c\,f(n) = \frac{8n}{10}$ where $c = \frac{8}{10} < 1$

for large n

$\therefore T(n) = \Theta(n)$ //

4-1 (c) $T(n) = 16T(n/4) + n^2$

$a = 16 \quad b = 4 \quad f(n) = n^2 = \Theta(n^2)$

$\log_b a = 2$

$f(n) = \Theta\left(n^{\log_b a}\right) = \Theta(n^2)$

$\therefore T(n) = \Theta\left(n^{\log_b a} \lg n\right) = \Theta(n^2 \lg n)$ //

4-1 (d) $T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2 = \Theta(n^2)$

$\log_b a = \log_3 7 \approx 1.77$

$f(n) = \Omega\left(n^{\log_b a + \epsilon}\right)$ where $\epsilon \approx 0.23$

Also, $af(n/b) = \frac{7}{9}n^2 \leq c\,f(n) = \frac{8}{9}n^2$ where $c = \frac{8}{9} < 1$

for large n

$\therefore T(n) = \Theta(f(n)) = \Theta(n^2)$ //

4-2 Worst case happens when we have to traverse to the bottom of the binary tree of depth $\lg N$

(a) $T(n) = T(\lceil \frac{n}{2} \rceil) + \Theta(1)$

$a = 1 \quad b = 2 \quad f(n) = \Theta(1)$

$\log_b a = 0$

$f(n) = \Theta(n^{\log_b a}) = \Theta(1)$

$\therefore T(N) = \Theta(N^{\log_b a} \lg N) = \Theta(\lg N)$ //

(b) $T(n) = T(\lceil \frac{n}{2} \rceil) + \Theta(N)$

Since each constant cost is enlarged by a factor of $\Theta(N)$ compared to (a), guess $T(n) = \Theta(N \lg n)$.

We can also tell by recursion tree

$$\begin{array}{c} \Theta(N) \\ | \\ T(\lceil \frac{n}{2} \rceil) \end{array} \Rightarrow \begin{array}{c} \Theta(N) \\ | \\ \Theta(N) \\ \vdots \\ \Theta(N) \end{array}$$

$T(N) = \Theta(N) \cdot \lg N$

$= \Theta(N \lg N)$ //

4-2

(c) $T(n) = T(\lceil \frac{n}{2} \rceil) + \Theta(\lceil \frac{n}{2} \rceil)$

$a=1 \quad b=2 \quad f(n) = \Theta(\frac{n}{2}) \cancel{= \Theta(n)}$

$f(n) = \Theta(n) = \Omega(n^{\log_b a + \epsilon})$ where $\epsilon = 1$

$a f(\frac{n}{b}) = f(\frac{n}{2}) = \Theta(\frac{n}{4}) \leq c f(n) = c\Theta(\frac{n}{2})$

~~where $c = 0.99$~~

where $c_1 \frac{n}{4} \leq c \; c_1 \frac{n}{2}$

$\frac{1}{2} \leq c < 1$

$\therefore T(N) = \Theta(f(N)) = \Theta(\frac{N}{2}) = \Theta(N)$ //