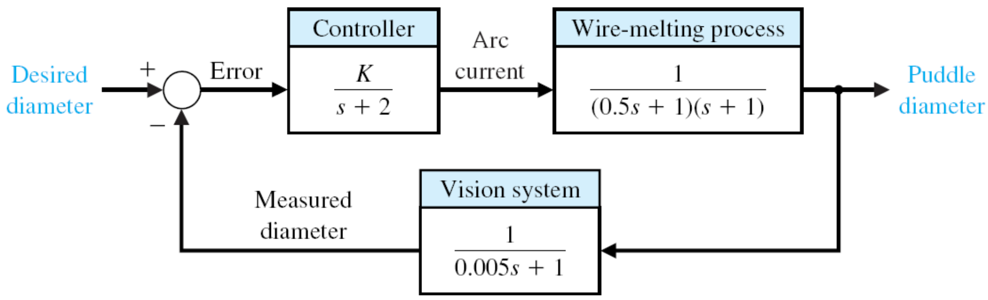
Homework

Chapter 3: The Time-Domain Analysis of Linear Systems

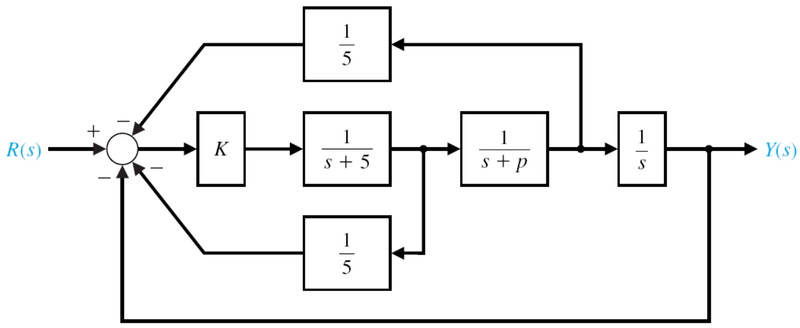
P3.1 Arc welding is one of the most important areas of application for industrial robots. In most manufacturing welding situations, uncertainties in dimensions of the part, geometry of the joint, and the welding process itself require the use of sensors for maintaining weld quality. Several systems use a vision system to measure the geometry of the puddle of melted metal, as show in the following figure. This system uses a constant rate of feeding the wire to be melted.

(a) Calculate the maximum value for K for the system that will result in a stable system.

(b) For half of the maximum value of K found in part (a), estimate the overshoot of the system when it is subjected to a step input. (Hint: *You can use Matlab to determine the poles*)



P3.2 The control of the spark ignition of an automotive engine requires constant performance over a wide range of parameters. The control system is shown in the following figure, with a controller gain K to be selected. Select a gain K that will result in a stable system for both $p=2$ and $p=0$.

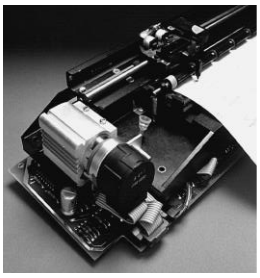


P3.3 A low-inertia plotter and its block diagram are shown in the figure below.

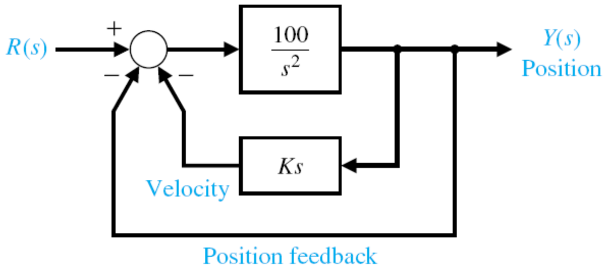
(a) Calculate the steady-state error for a ramp input.

(b) Select a value of K that will result in zero overshoot to a step input, but can provide the most rapid response that is attainable. (Hint: damping ratio 1.0)

(c) Plot the poles and zeros of this system and discuss the dominance of the complex poles. What overshoot for a step input do you expect?



(a)



(b)

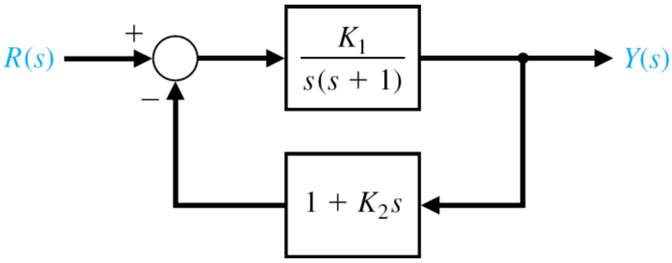
P3.4 A second-order control system has the plant transfer function

$$G(s) = \frac{K}{s\left(s+\sqrt{2K}\right)}.$$

(a) Determine the percent overshoot and settling time (using a 2% settling criterion) due to a unit step input.

(b) For what range of K is the settling time less than 1 second?

P3.5 The model for a position control system using a DC motor is shown below. The goal is to select K_1 and K_2 so that the peak time is 0.2 second and the overshoot (P.O.) for a step input is negligible ($1\% < \text{P.O.} < 4\%$).



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(a)

$$\frac{C(s)}{R(s)} = \frac{\frac{k}{s+2}\,\frac{1}{(0.5s+1)(s+1)}}{1+\frac{k}{s+2}\,\frac{1}{(0.5s+1)(s+1)}\,\frac{1}{0.005s+1}}$$

$$= \frac{k(0.005s+1)}{(s+2)(0.5s+1)(s+1)(0.005s+1)+K}$$

$$= \frac{k(0.005s+1)}{0.0025s^4+0.5125s^3+2.52s^2+4.01s+2+K}$$

Apply Routh's Stability Criterion

s^4	0.0025	2.52	$2+K$
s^3	0.5125	4.01	0
s^2	$\frac{1.2915-0.010025}{0.5125}$	$\frac{1.025+0.5125K}{0.5125}$	
	1.281475	$1.025+0.5125K$	
	2.5	$2+K$	
s^1	$2.5\times4.01-0.5125(2+K)$	0	
	$9-0.5125K$	0	
s^0	$(9-0.5125K)(2+K)$		

$$(9-0.5125K)(2+K) > 0$$

$$2+K > 0$$

$$K > -2$$

$$9-0.5125K > 0$$

$$K < 17.56$$

(b) $K = \cancel{8.75}\ 8.78$

$$E(s) = 1 - \frac{C(s)}{R(s)} = \frac{1}{1+G_1 G_2 H} R(s) \qquad R(s) = \frac{A}{s}$$

$$= \frac{1}{1+\frac{K}{s+2}\frac{1}{(0.5s+1)(s+1)}\frac{1}{(0.005s+1)}} \cdot \frac{A}{s}$$

$$= \frac{(s+2)(0.5s+1)(s+1)(0.005s+1)}{(s+2)(0.5s+1)(s+1)(0.005s+1)+K} \cdot \frac{A}{s}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} \frac{(s+2)(0.5s+1)(s+1)(0.005s+1)A}{(s+2)(0.5s+1)(s+1)(0.005s+1)+K}$$

$$= \lim_{s\to 0} \frac{2(\frac{1}{2}s+1)(0.5s+1)(s+1)(0.005s+1)A}{0.0025s^4+0.5125s^3+2.52s^2+4.01s+10.78}$$

poles/ roots of polynomial $= -200,\ -4.3238,\ -0.338 \pm 2.207j$

$$= \lim_{s\to 0} \frac{(0.0025s^4+0.5125s^3+2.52s^2+4.01s+2)A}{0.0025s^4+0.5125s^3+2.52s^2+4.01s+10.78}$$

计算超调量

$$= \frac{2A}{10.78}$$

$= 0.186$ // if take $A=1$ 超调 61.2%

0–5

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(b) $K = 8.78$

$$\frac{C(s)}{R(s)} = \frac{8.78\,(0.005s+1)}{(s+2)(0.5s+1)(s+1)(0.005s+1)+8.78}$$

$$= \cancel{2.0}\ 17.56\,\frac{s+200}{(s+2)(s+2)(s+1)(s+200)+3512}$$

$$= 17.56\,\frac{s+200}{s^4+205s^3+1008s^2+1604s+4312}$$

$$\cancel{= 17.56\quad s+200}$$

Roots of characteristic polynomial $= -200,\ -4.324,\ -0.338 \pm 2.207j$

$$= 17.56\,\frac{\cancel{s+200}}{\cancel{(s+200)}\cancel{(s+4.3)}(s+0.338+2.207j)(s+0.338-2.207j)}$$

$$= 17.56\,\frac{\cancel{200}}{\cancel{200}\times 4.324(s+0.338+2.207j)(s+0.338-2.207j)}$$ DC gain

$$= 4.061\,\frac{1}{(s+0.338+2.207j)(s+0.338-2.207j)}$$

$$= 0.812\,\frac{5}{s^2+0.676s+5}$$

$\omega_n^2 = 5$

$2\xi\omega_n = 0.676$

$\xi = 0.151$

overshoot $\sigma = e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi} \times 100\%$

$= 61.8\%$

$$TF_1: \frac{\frac{K}{s+5}}{1+\frac{K}{5}\frac{1}{s+5}} = \frac{5K}{5(s+5)+K}$$

$$TF_2: \frac{\frac{5k}{5(s+5)+K}\frac{1}{s+p}}{1+\frac{K}{5(s+5)+k}\frac{1}{s+p}} = \frac{5k}{(5(s+5)+k)(s+p)+K}$$

$$= \frac{5k}{(5s+25+K)(s+p)+K}$$

$$TF_3: \frac{\frac{5k}{(5s+25+k)(s+p)+k}\frac{1}{s}}{1+\frac{5k}{(5s+25+k)(s+p)+k}\frac{1}{s}} = \frac{5K}{(5s+25+k)(s+p)s+ks+5k}$$

Characteristic polynomial: $(5s+25+K)(s+p)s + Ks+5k=0$

substitute $p=2$

$$(5s+25+k)(s+2)s + \cancel{2k}\, Ks+5k=0$$

$$5s^3+10s^2+25s^2+50s+Ks^2+2ks+ks+5k=0$$

$$5s^3+(35+K)s^2+(50+3k)s+5k=0$$

$$s^3+\left(7+\frac{k}{5}\right)s^2+\left(10+\frac{3}{5}k\right)s+K=0$$

Apply Routh's stability criteria

$$\begin{array}{lll}
s^3 & 5 & 50+3K \\
s^2 & 35+K & 5K \\
s^1 & \dfrac{(35+K)(50+3K)-25K}{35+K} & \\
s^0 & 5K &
\end{array}$$

$$\frac{(35+K)(50+3K)-25K}{35+K} > 0$$

$$\frac{1750+155K+3K^2-25K}{35+K} > 0$$

$$\frac{3K^2+130K+1750}{35+K} > 0$$

$$35+K > 0$$

$$K > -35$$

$$5K > 0$$

$$K > 0$$

substitute $p=0$

characteristic polynomial: $(5s+25+k)s^2+ks+5k=0$

$5s^3+25s^2+ks^2+ks+5k=0$

Routh's stability criteria:

s^3 5 k

s^2 $25+k$ $5k$

s^1 $\frac{k^2+25k-25k}{25+k}$ 0

$\frac{k^2}{k+25}$ 0

s^0 $5k$

$25+k>0$

$k>-25$

$5k>0$

$k>0$

$p=2$，$p=0$ 都需要 $k>0$

$$\frac{\frac{100}{s^2}}{1+\frac{100}{s^2}ks} = \frac{100}{s^2+100ks}$$

$$\frac{Y(s)}{R(s)} = \frac{\frac{100}{s^2+100ks}}{1+\frac{100}{s^2+100ks}} = \frac{100}{s^2+100ks+100}$$

$$w_n^2 = 100$$

$$2\xi w_n = 100k$$

$$\xi = \frac{100k}{2\times 10}$$

$$= 5k$$

Q 3.2?

(a) $R(s) = \frac{A}{s^2}$

$$Y(s) = \frac{100A}{(s^2+100ks+100)s^2}$$

9.9 +0.1 =10.

$$e_{ss} = \lim_{s\to 0} s\left(R(s)-Y(s)\right)$$

$$= \lim_{s\to 0} s\left(1-\frac{100}{s^2+100ks+100}\right)\frac{A}{s^2}$$

$$= \lim_{s\to 0} \frac{s^2+100ks}{s^2+100ks+100}\cdot\frac{A}{s}$$ +0.1

$$= \lim_{s\to 0} \frac{(s+100k)A}{s^2+100ks+100}$$

$$= \frac{100kA}{100}$$

$$= kA$$ //

~~不需计算稳态误差~~

(b) $\zeta = 5k$

$5k = 1$

$k = 0.2$ //

(c) $$\frac{Y(s)}{R(s)} = \frac{100}{s^2 + 100ks + 100}$$

Characteristic polynomial $0 = s^2 + 100ks + 100$

$$s_1, s_2 = \frac{-100k \pm \sqrt{(100k)^2 - 4 \times 100}}{2}$$

$$s_1, s_2 = -50k \pm 10\sqrt{25k^2 - 1}$$

没有 zeroes. 有 2 个 poles s_1 & s_2.

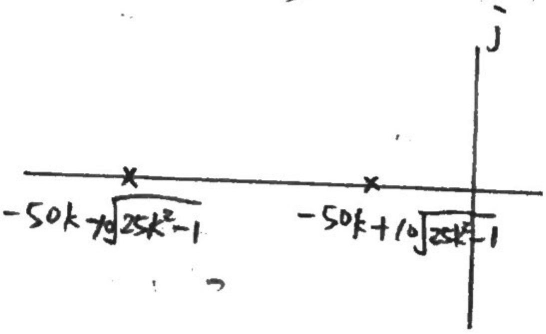
Case 1: $25k^2 - 1 \geq 0$

$\therefore k^2 \geq \frac{1}{25}$

$\therefore k \geq \frac{1}{5}$ ~~or $k \leq -\frac{1}{5}$~~

poles 全为实部

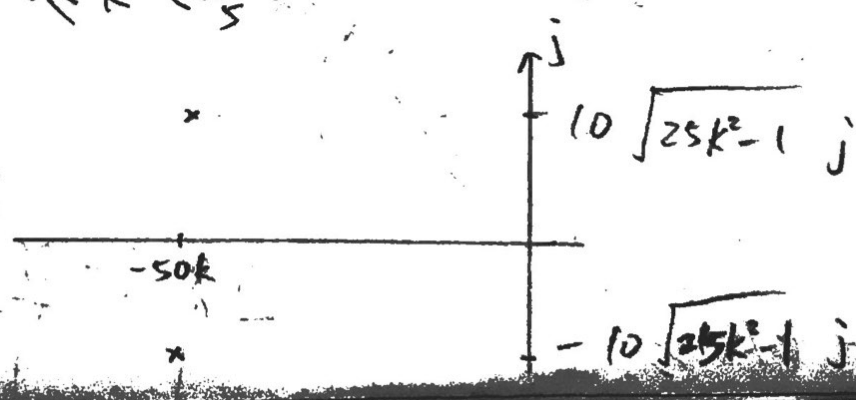
假设 $k \geq 0$



Case 2: $0 \leq k < \frac{1}{5}$

设 $k \geq 0$

poles 为复数极点对



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(c) overshoot $\sigma = e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi} \times 100\%$

for $0<k<0.2$ $= e^{-\frac{5k}{\sqrt{1-25k^2}}\pi} \times 100\%$

for $k \geq 0.2$ $\sigma = 0$ No overshoot for critically damped & overdamped systems

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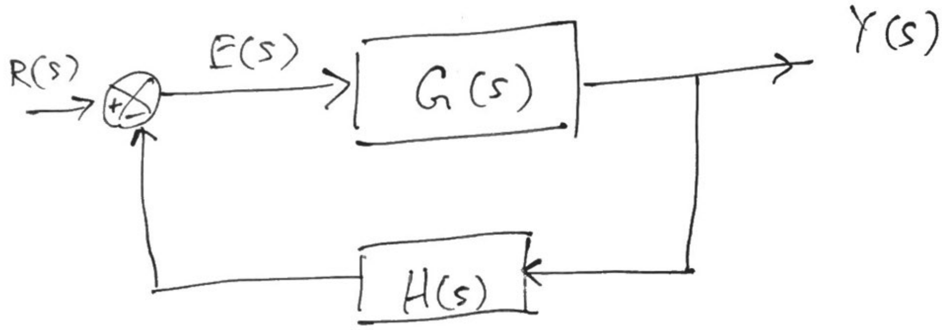
$H(s) = 1$

设



$$传递函数 = \frac{G(s)}{1+G(s)\,H(s)} = \frac{\dfrac{K}{s(s+\sqrt{2K})}}{1+\dfrac{K}{s(s+\sqrt{2K})}}$$

$$\frac{Y(s)}{R(s)} = \frac{K}{s^2+\sqrt{2K}\,s+K}$$

$\omega_n^2 = K$ $\qquad 2\xi\omega_n = \sqrt{2K}$

$$\xi = \frac{\sqrt{2K}}{2\sqrt{K}} = \frac{1}{\sqrt{2}}$$

(a)

$$\sigma = e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi} \times 100\%$$

$$= e^{-\frac{\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}}\pi} \times 100\%$$

$$= 4.32\%$$

$T_s \approx \dfrac{4}{\xi\omega_n}$ for 2% settling criterion

$\approx 4\sqrt{\dfrac{2}{K}}$

(b) $T_s \approx 4\sqrt{\frac{2}{k}} < 1$

$$\sqrt{\frac{2}{k}} < \frac{1}{4}$$

$$\sqrt{k} > 4\sqrt{2}$$

$$k > 32$$ //

$T_p = 0.2 \quad 1\% < P.O. < 4\%$

$$\frac{Y(s)}{R(s)} = \frac{\frac{k_1}{s(s+1)}}{1+\frac{k_1}{s(s+1)}(1+k_2 s)}$$

$$= \frac{k_1}{s(s+1)+k_1(1+k_2 s)}$$

$$= \frac{k_1}{s^2+s+k_1+k_1k_2 s}$$

$$= \frac{k_1}{s^2+(1+k_1k_2)s+k_1}$$

$\omega_n^2 = k_1 \qquad 2\xi\omega_n = 1+k_1k_2$

$$\xi = \frac{1+k_1k_2}{2\sqrt{k_1}}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\sqrt{1-\xi^2}\,\omega_n}$$

$$= \frac{\pi}{\sqrt{1-\frac{(1+k_1k_2)^2}{4k_1}}\sqrt{k_1}}$$

$$0.2 = \frac{2\pi}{\sqrt{4k_1-2k_1k_2-k_1^2k_2^2-1}}$$

$$4k_1-2k_1k_2-k_1^2k_2^2 = 987.96$$

$$k_2^2k_1^2+(2k_2-4)k_1+987.96=0$$

$$k_1 = \frac{(4-2k_2)\pm\sqrt{(2k_2-4)^2-4k_2^2\times 987.96}}{2k_2^2}$$

$$P.O. = e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi} \times 100\%$$

$$0.04 > e^{-\frac{\xi}{\sqrt{1-\xi^2}}\pi}$$

$$\ln 0.04 > -\frac{\xi}{\sqrt{1-\xi^2}}\pi$$

$$\ln 0.04 \sqrt{1-\xi^2} > -\xi\pi$$

$$1-\xi^2 \cancel{\sqrt{1-\xi^2}} < \left(\frac{-\xi\pi}{\ln 0.04}\right)^2$$

$$\xi^2 > \frac{1}{\left(\frac{\pi}{\ln 0.04}\right)^2 + 1}$$

$$\xi > \sqrt{\frac{1}{\left(\frac{\pi}{\ln 0.04}\right)^2 + 1}}$$

$$\cancel{0.716 < \xi < 1}$$

$$1 > \frac{1+K_1K_2}{2\sqrt{K_2}} > 0.716$$

$$1 > \xi > 0.716$$

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$$T_p = \frac{\pi}{\omega_d}$$

$$0.2 = \frac{\pi}{\sqrt{1-\xi^2}\ \omega_n} \qquad \xi > 0.716$$

~~take $\xi = 0.716$~~

$$\omega_n > \frac{\pi}{\sqrt{1-0.716^2}\ 0.2}$$

$$> 22.501$$

$$k_1 = \omega_n^2 > 506.296$$

$$\xi = \frac{1 + k_1 k_2}{2\sqrt{k_1}}$$

$$2\xi\sqrt{k_1} = 1 + k_1 k_2$$

$$k_2 = \frac{2\xi\sqrt{k_1} - 1}{k_1}$$

$$\cancel{= 0.0597}$$

$$= \frac{2\xi}{\sqrt{k_1}} - \frac{1}{k_1}$$

$$k_2 < 0.0617$$

$$\begin{cases} k_1 = 685.4 \\ k_2 = \cancel{0.0597} \end{cases}$$

works