

Sorting (chapter 7)

1. P144 5-2 (g-i), for (h) only consider  $k=0$  and  $k=1$ .
2. p173 7.1-1
3. p178 7.2-2
4. Programming assignment 1 (50% weight of ADw5)

# P144 5-2 (g)

Since DETERMINISTIC-SEARCH would search through the entire array in the event of not finding  $x$ , both average-case and worst-case would be  $\Theta(n)$ .

# P144 5-2 (h)

Assume search cost dominates permutation cost, i.e.  $g(n) = o(f(n))$  where  $f(n)$  denotes search cost and  $g(n)$  permutation cost.

$k = 0$

Both average-case and worst-case would be  $\Theta(n)$ .

$k = 1$

Assume searching each index incurs constant cost  $C$ .

Average-case:  $T(n) = C(n+1)/2 + g(n) = \Theta(n)$

Worst-case:  $T(n) = Cn + g(n) = \Theta(n)$

# P144 5-2 (i)

DETERMINISTIC-SEARCH is obviously better than RANDOM-SEARCH because  
1. it would not check the same index more than once, and 2. it is guaranteed to terminate.

If the underlying distribution of  $A$  is heavily skewed towards the worst-case for DETERMINISTIC-SEARCH, i.e.  $x$  towards the end of  $A$ , SCRAMBLE-SEARCH might be better, if the cost of scrambling is less than the cost of having to search through the whole array  $A$ . In practice, however, in the time it takes to scramble  $A$ , we might as well have done a DETERMINISTIC-SEARCH.

So DETERMINISTIC-SEARCH is likely to be the best.

# p173 7.1-1

Following Figure 7.1, we use the last element 11 as pivot. We let  $p$  denote the index of the first element, and  $r$  the index of the last element.

i	p	j										r
	13	19	9	5	12	8	7	4	21	2	6	11
i	p	j										r
	13	19	9	5	12	8	7	4	21	2	6	11
i	p	j										r
	13	19	9	5	12	8	7	4	21	2	6	11
	p	i		j								r
	9	19	13	5	12	8	7	4	21	2	6	11
	p	i		j								r
	9	5	13	19	12	8	7	4	21	2	6	11
	p	i		j								r
	9	5	13	19	12	8	7	4	21	2	6	11

p	i				j				r			
9	5	8	19	12	13	7	4	21	2	6	11	
p	i				j				r			
9	5	8	7	12	13	19	4	21	2	6	11	
p	i				j				r			
9	5	8	7	4	13	19	12	21	2	6	11	
p	i				j				r			
9	5	8	7	4	13	19	12	21	2	6	11	
p	i				j				r			
9	5	8	7	4	2	19	12	21	13	6	11	
p	i				j				r			
9	5	8	7	4	2	6	12	21	13	19	11	
p	i				j				r			
9	5	8	7	4	2	6	11	21	13	19	12	

p178 7.2-2

The function PARTITION( $A, p, r$ ) on page 171 is reproduced below for complexity analysis.

```

1. PARTITION (A, p, r):
2.   x = A[r]
3.   i = p-1
4.   for j=p to r-1:
5.     if A[j] ≤ x:
6.       i = i+1
7.       exchange A[i] with A[j]
8.   exchange A[i+1] with A[r]
9.   return i+1

```

Let the constant cost in line 2, 3, 8 & 9 be denoted by  $C_1$ , and the cost in line 6 & 7 be  $C_2$ . Since the if condition in line 5 is always fulfilled, lines 6 & 7 would be executed for  $(r-1)-(p-1) = r-p = n-1$  times. Therefore the cost of PARTITION( $A, p, r$ ) is

$$T_{\text{partition}}(n) = (n-1)C_2 + C_1$$

Every run of PARTITION( $A, p, r$ ) returns  $i+1=r$ , meaning each time QUICKSORT needs to perform PARTITION( $A, p, r$ ) on an array with one fewer element. Therefore,

$$T_{\text{quicksort}}(n) = T_{\text{quicksort}}(n-1) + T_{\text{partition}}(n)$$

$$T_{\text{quicksort}}(N) = \sum_{n=1}^N (n-1)C_2 + C_1$$

$$T_{\text{quicksort}}(N) = (N-1)\frac{N}{2}C_2 + NC_1 = \Theta(N^2)$$