Combinatorics 2017 HW 1009

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1. how many integer numbers from 1 to 10000 are not squares of integers or cubes of integers?

A={squares}; |A|=100
B={cubes}; |B|=21
$A\cap B$={5^{th} power}; $|A\cap B|=6$

$|N| - |A\cup B| = |N| - |A| - |B| + |A\cap B| = 9885$

2. How many permutations of 1, 2, 3, ……, 9 have at least one odd number in its natural position?

Let A_i be the set such that ith odd numbers is in its natural position.

$|A_1 \cup A_2 \cup \ldots \cup A_m| = \Sigma|A_1| + \ldots + (-1)^{k+1}\Sigma|A_1\cap \ldots \cap A_k| + \ldots + (-1)^{m+1}|A_1\cap \ldots \cap A_m|$

$|A_1 \cup A_2 \cup \ldots \cup A_5| = C(5,1)*8! - C(5,2)*7! + C(5,3)*6! - C(5,4)*5! + C(5,5)*4! = 157824$

3. $x_1 + x_2 + x_3 + x_4 = 20, where\ 1 \le x_1 \le 6, 0 \le x_2 \le 7, 4 \le x_3 \le 8, 2 \le x_4 \le 6$
please calculate the number of integral solutions.

Given $a_i \le x_i \le b_i\ \forall i$
Let $y_i = x_i - a_i$
Rewriting the conditions: $\Sigma y_i = 20 - \Sigma a_i = 13$, where $0 \le y_i \le b_i - a_i\ \forall i$
Also, let A_i denote the set of non-negative integral solutions, where $y_i \ge b_i - a_i + 1$

First consider all solutions where $0 \le y_i\ \forall i$
No. of non-negative integral solutions = C(13+4-1, 4-1) = C(16, 3)

A_1: $y_1 \ge 6$ and $0 \le y_i\ \forall i\neq 1$, $\Sigma y_i - 6 = 7$
$|A_1|$ = C(7+4-1, 4-1) = C(10,3)
A_2: $y_2 \ge 8$ and $0 \le y_i\ \forall i\neq 2$, $\Sigma y_i - 8 = 5$
$|A_2|$ = C(5+4-1, 4-1) = C(8,3)
A_3: $y_3 \ge 5$ and $0 \le y_i\ \forall i\neq 3$, $\Sigma y_i - 5 = 8$
$|A_3|$ = C(8+4-1, 4-1) = C(11,3)
A_4: $y_4 \ge 5$ and $0 \le y_i\ \forall i\neq 4$, $\Sigma y_i - 5 = 8$
$|A_4|$ = C(8+4-1, 4-1) = C(11,3)

$|A_1\cap A_2|$ = C(13-6-8+4-1, 4-1) = 0
$|A_1\cap A_3|$ = C(13-6-5+4-1, 4-1) = C(5,3)
$|A_1\cap A_4|$ = C(13-6-5+4-1, 4-1) = C(5,3)
$|A_2\cap A_3|$ = C(13-8-5+4-1, 4-1) = C(3,3)

$|A_2 \cap A_4| = C(13-8-5+4-1, 4-1) = C(3,3)$
$|A_3 \cap A_4| = C(13-5-5+4-1, 4-1) = C(6,3)$

$|A_1 \cap A_2 \cap A_3| = 0$
$|A_1 \cap A_2 \cap A_4| = 0$
$|A_1 \cap A_3 \cap A_4| = C(13-6-5-5+4-1, 4-1) = 0$
$|A_2 \cap A_3 \cap A_4| = C(13-8-5-5+4-1, 4-1) = 0$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0$

Finally = C(16, 3) - C(10,3) - C(8,3) - C(11,3) - C(11,3) + C(5,3) + C(5,3) + C(3,3) + C(3,3) + C(6,3) = 96

4. For the permutation P=P1 P2 P3 P4 of {1,2,3,4}, how many feasible permutations are there if we constrain that P1≠2，P2≠2、3， P3≠3、4， P4≠4? (4 points)

A_1: P1=2
A_2: P2=2 or P2=3
A_3: P3=3 or P3=4
A_4: P4=4

No. of permutations without constraints = 4!

$|A_1| = 3!$
$|A_2| = 2*3!$
$|A_3| = 2*3!$
$|A_4| = 3!$

$|A_1 \cap A_2| = 2!$
$|A_1 \cap A_3| = 2*2!$
$|A_1 \cap A_4| = 2!$
$|A_2 \cap A_3| = 3*2!$
$|A_2 \cap A_4| = 2*2!$
$|A_3 \cap A_4| = 2!$

$|A_1 \cap A_2 \cap A_3| = 1$
$|A_1 \cap A_2 \cap A_4| = 1$
$|A_1 \cap A_3 \cap A_4| = 1$
$|A_2 \cap A_3 \cap A_4| = 1$

$|A_1 \cap A_2 \cap A_3 \cap A_4| = 0$

Total valid = 4