Combinatorics 2018 HW 4.1

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1. Find out the number of lattice paths from (0,0) to (n,n+2), which are above but do not cross y=x line? List the formula with n.

   Every invalid path has to touch the line y=x-1 at some point (x,x-1). From this point, the invalid path has to move (n-x,n+2-(x-1)) in order to reach (n,n+2). Applying André's reflection method, every invalid path ends at (n+3,n-1) after reflection.

   No. of valid paths = C(2n+2,n) – C(2n+2,n-1)

2. Find out the number of lattice paths from (0,0) to (n,n+2), which are above but do not touch y=x line? List the formula with n.

   From (0,0), we can get to either (1,0) or (0,1). Every path from (1,0) to (n,n+2) is invalid. So we need to find no. of valid paths from (0,1) to (n,n+2). But this is Question1, with (n,n+1) as the end point.

   No. of valid paths = C(2n+1,n) – C(2n+1,n-1)

3. If we want to use positive integers from 1 until 7 to form a ring in order. Since 1 and 7 are adjacent to each other in the ring. Due to their neighboring position, 1 and 7 are also considered as neighbor numbers. Then if we want to pick 3 non-neighboring numbers from this ring of 7 numbers, how many different solutions are there?

   One way to think about this is how to pick m non-neighboring numbers out of n in a circle, where m<=rounddown(n/2).
   After m (Picked, Unpicked) sequences are chosen, there are (n-2m) Unpicked left. Then we sort the (n-2m) Unpicked into m (Picked, Unpicked) pairs. Applying the bar method, there are C(n-m-1,m-1) ways to do this. The first Picked element can be placed anywhere from 1 to n along the circle, and each combination is repeated m times, so multiply by n/m.
   No. of solutions = **n/m x C(n-m-1,m-1) = 7/3 x C(3,2) = 7**

   Another way: Picking 3 non-neighboring numbers amounts to picking 2 adjacent numbers, so **7.**