intro

Montag, 25. September 2023 13:06

loop_invariants_intro

$4 = 0 + 2 \cdot 2$

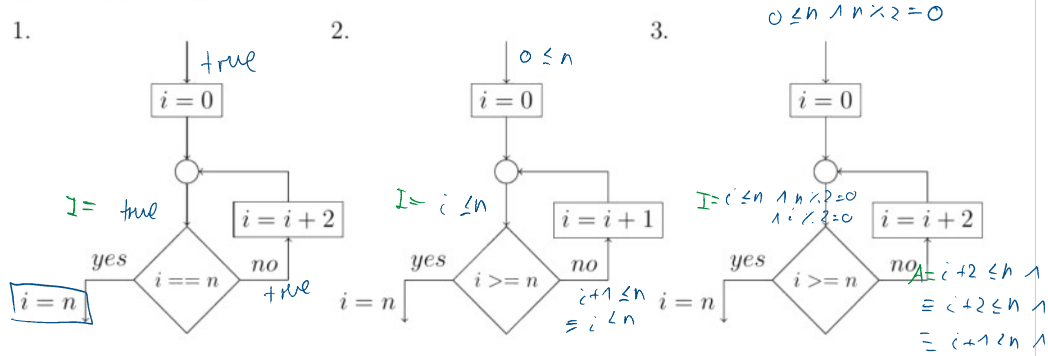
$n \bmod 2 = i$

$n = i + 2 \cdot k$

$n - i = k \cdot 2$

$n \equiv i \bmod 2$

Inspect the following loops and discuss the preconditions that have to hold, such that the assertion $i = n$ is satisfied. In particular, discuss the results for positive and negative inputs, respectively.



$A = i+2 \le n \wedge n\%2=0 \wedge i+2\%2=0$
$\equiv i+2 \le n \wedge n\%2=0 \wedge i\%2=0$
$\equiv i+1 < n \wedge n\%2=0 \wedge i\%2=0$

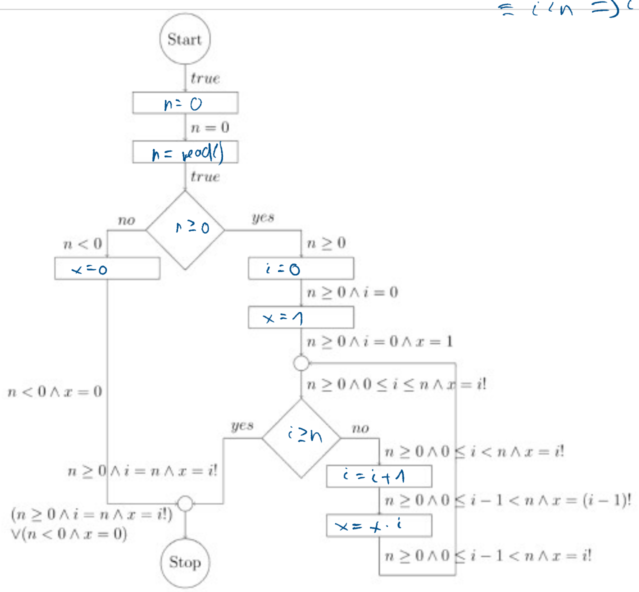
$SP[i==n](true) \equiv i=n$
$\Rightarrow i=n$ ✓

True: $SP[i \ge n](i \le n) \equiv i = n$

False: $SP[i<n](i \le n) \equiv i<n \Rightarrow i<n$

True: $i \le n \wedge i \ge n \wedge n\%2=0 \wedge i\%2=0$
$\equiv i=n \wedge n\%2=0 \Rightarrow i=n$ ✓✓

False: $SP[i<n](I) \equiv i<n \wedge n\%2=0 \wedge i\%2=0$
$\equiv i+1<n \wedge n\%2=0 \wedge i\%2=0$
$\Rightarrow A$ $\Rightarrow$ ✓✓



Fill in the gaps, so that everything is Locally Consistant

factorial

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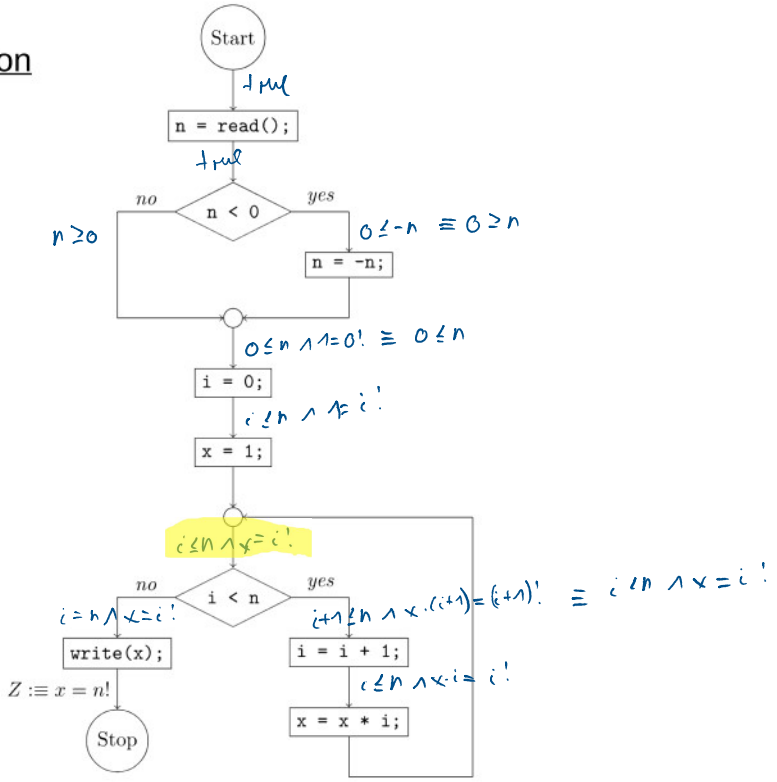
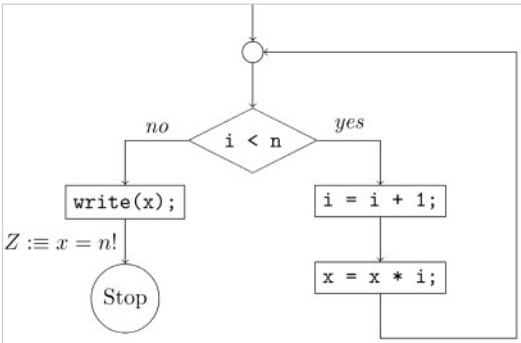
factorial

Prove Z using Weakest Precondition





simple_sum

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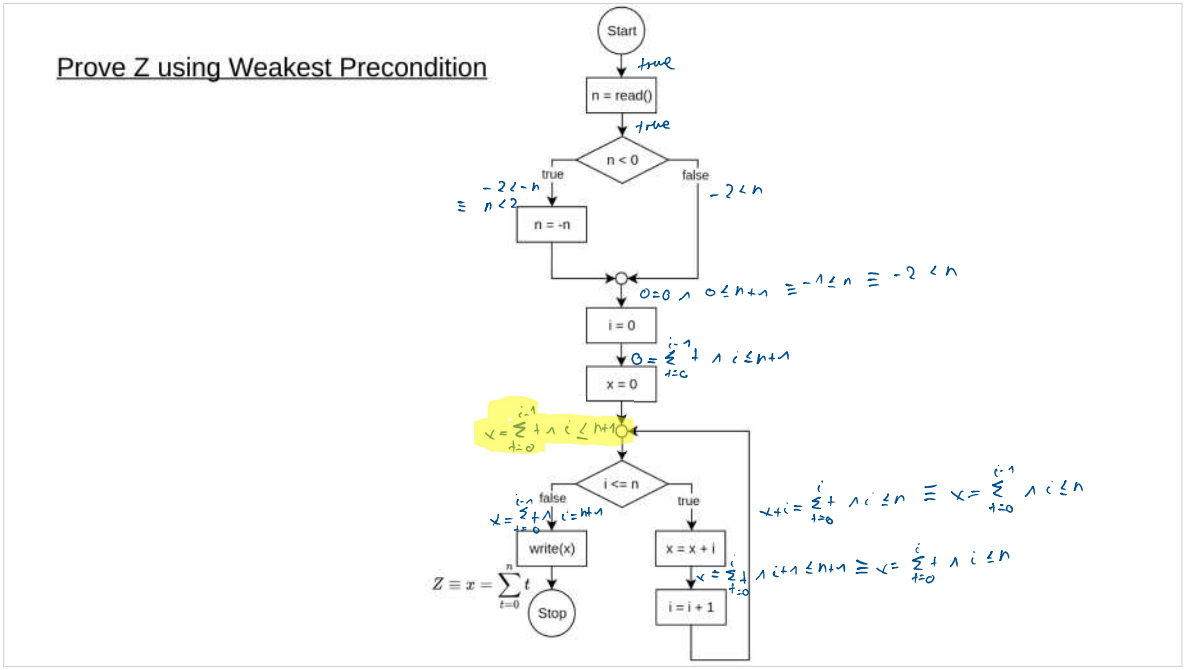
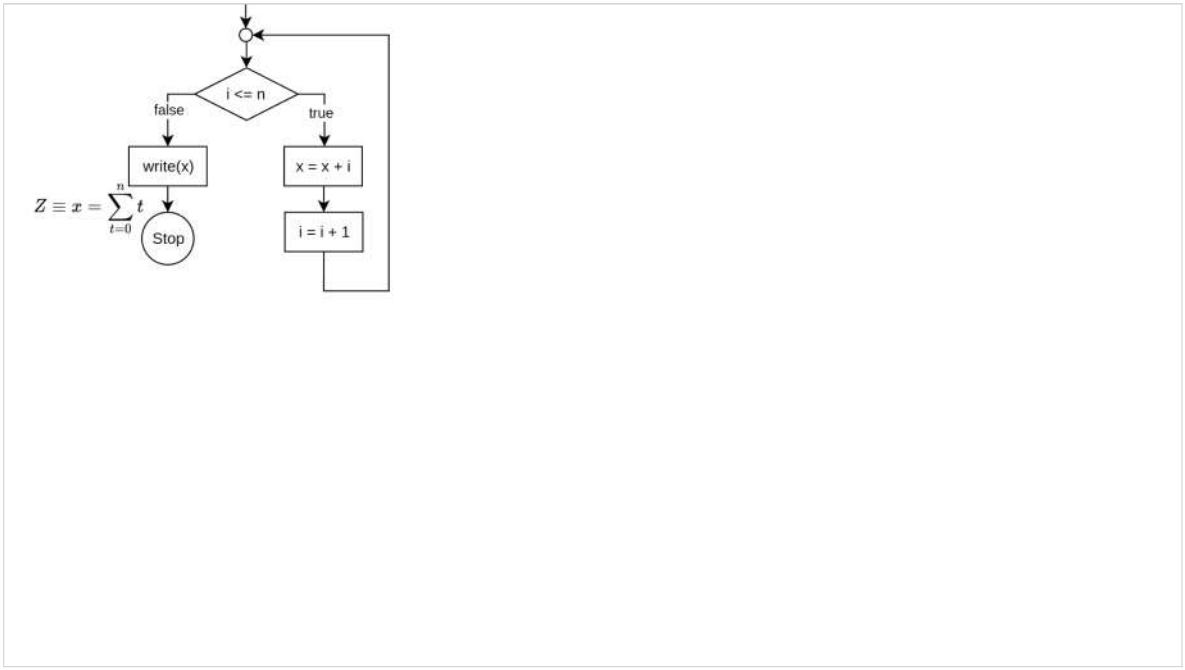
simple_sum





power_sum

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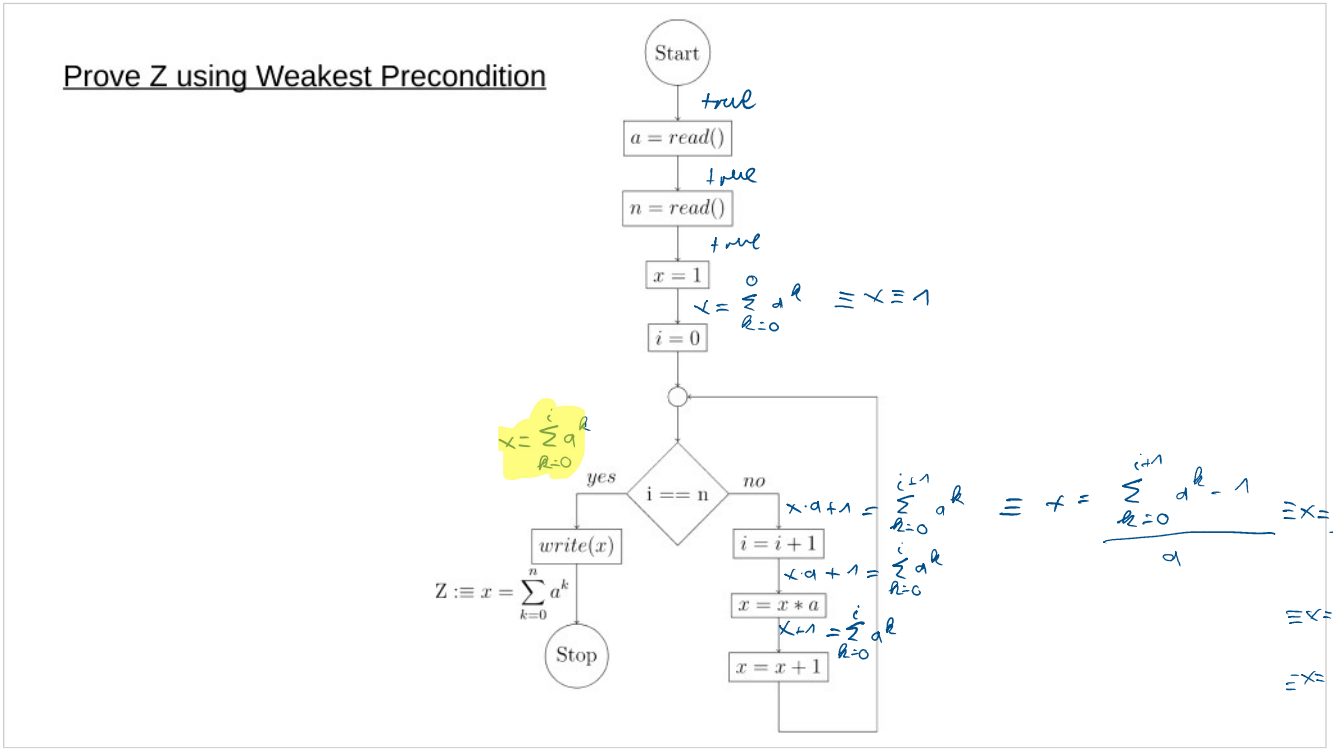
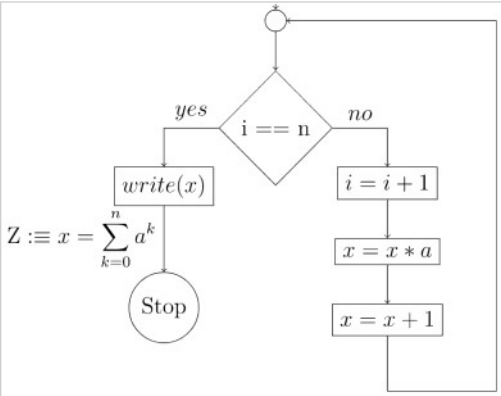
power_sum





double_power

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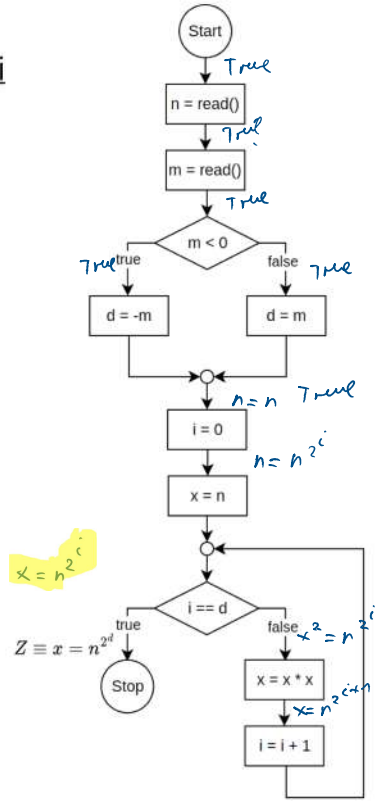
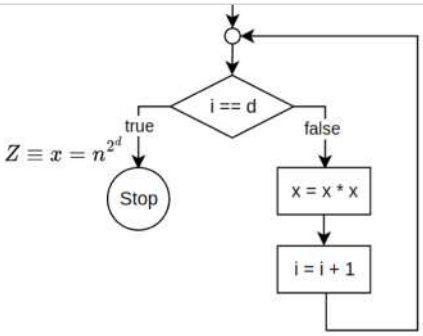
double_power

Prove Z using Weakest Precondi





the_first_strengthening

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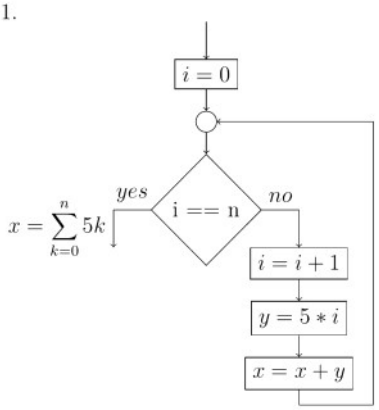
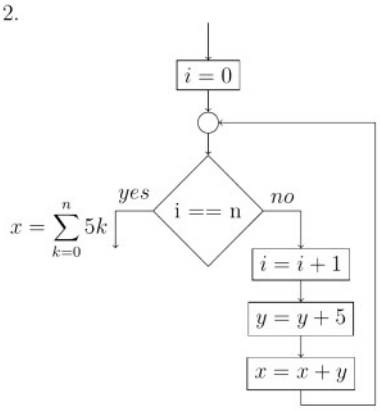
the_first_strengthening

Consider these control flow graph fragments (assume x and y to be 0 initially):

1.

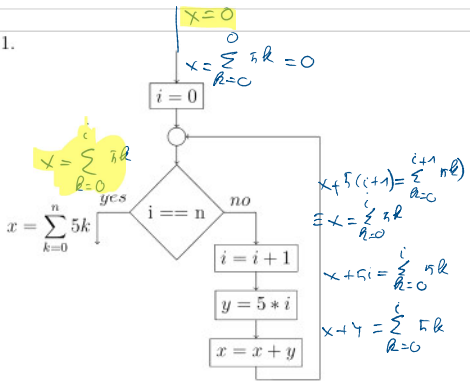


2.

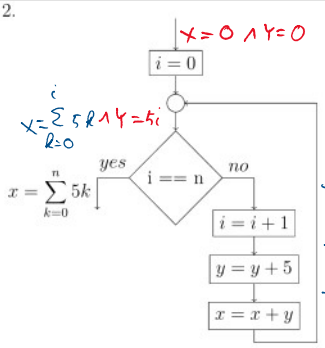


Find suitable loop invariants and prove them locally consistent. Discuss, why these invariants have to be like that.

1.



2.



1. Try the same loop invariant again
2. Why didn't it work? What information are we missing?
3. How can add that information to out proof?

i	0	1	2	3	4
y	0	5	10	15	20
x	0	5	15	30	50

$y = 5i$

$x = \sum_{k=0}^{i} 5k$

Y	0	5	10	15	20	
x	0	5	15	30	50	

$Y = 5i$

$x = \sum_{k=0}^{i} 3k$

the_second_strengthening

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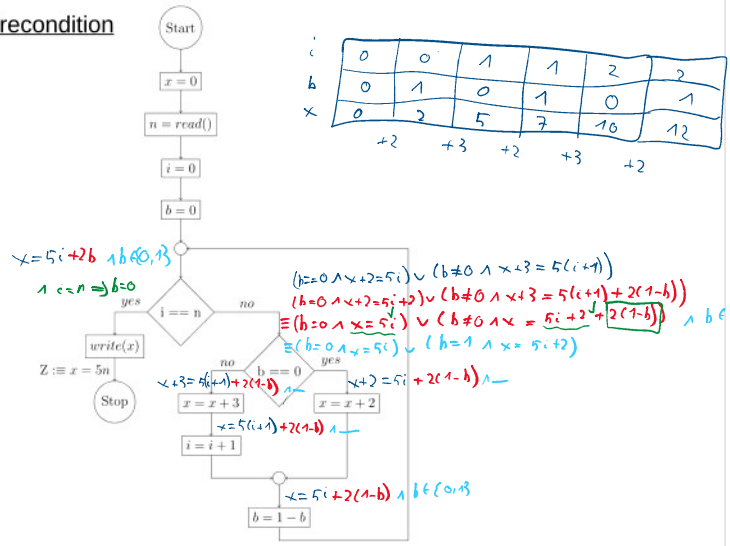
the_second_strengthen...

Prove Z using Weakest Precondition



Prove Z using Weakest Precondition