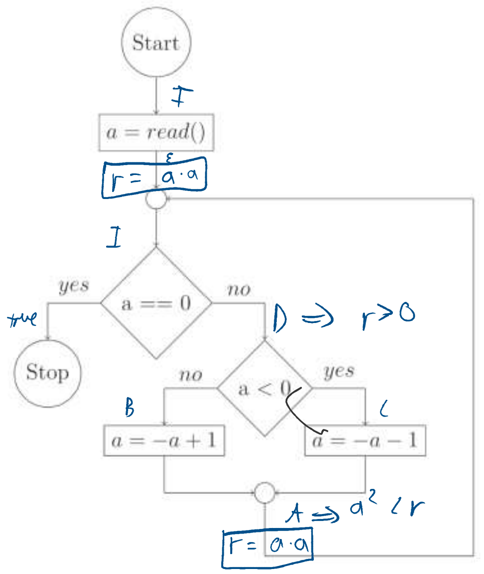
closer_to_zero

Montag, 25. September 2023 17:58

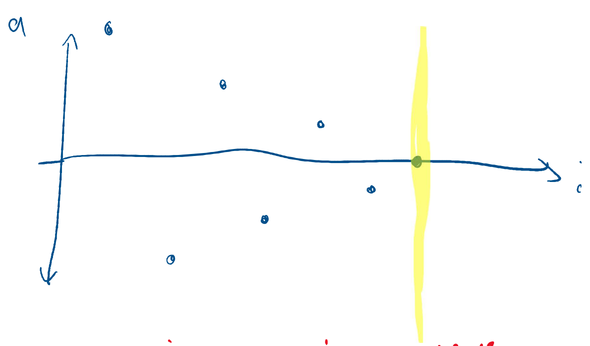
closer_to_zero

Prove termination of the following program



Example Run

a	6	-5	4	-3	2	-1	0



- Find strictly decreasing sequence bound by 0

$r = |a|$ works but no $|\cdot|$ operator in MiniJava

$\Rightarrow r = a \cdot a$ also works

$I \equiv r = a^2$

$A \equiv$ true (doesn't work)

$\Rightarrow$ chose something stronger

e.g. $\underline{a^2 < r}$

$B \equiv (-a+1)^2 < r$

$C \equiv (-a-1)^2 < r$

$D \equiv (a < 0 \wedge (-a-1)^2 < r) \vee (a \geq 0 \wedge (-a+1)^2 < r)$

$\equiv (a < 0 \wedge a^2 + 2a + 1 < r) \vee (a \geq 0 \wedge a^2 - 2a + 1 < r)$

$\equiv (a < 0 \wedge a^2 - 2|a| + 1 < r) \vee (a \geq 0 \wedge a^2 - 2|a| + 1 < r)$

$\equiv (a < 0 \vee a \geq 0) \wedge (|a| - 1)^2 < r$

$\equiv (|a| - 1)^2 < r$

$r > \underbrace{(|a| - 1)^2}_{\text{Squares are always } \geq 0} \geq 0 \Rightarrow r > 0$ special Assertion holds

LC - Check

$r = a^2 \wedge a \neq 0 \overset{?}{\Rightarrow} (|a| - 1)^2 < r$

$\Leftrightarrow (|a|-1)^2 < a^2 \wedge a \neq 0$

$\Leftrightarrow a^2 - 2|a| + 1 < a^2 \wedge a \neq 0$

$\Leftrightarrow 1 < 2|a| \wedge a \neq 0$

$\Leftrightarrow |a| > \frac{1}{2} \wedge a \neq 0$

$\Leftrightarrow$ true $\checkmark$

$\xi \equiv$ true

$\bar{f} =$ true

$\Rightarrow$ Valid Proof

step_into_the_k-loop

Montag, 25. September 2023 22:29

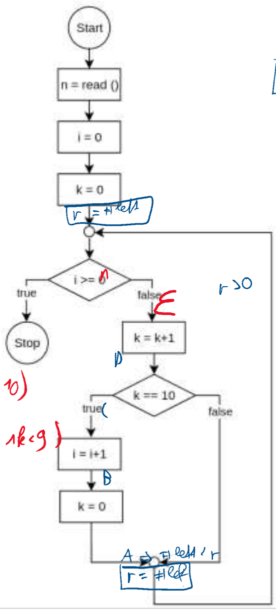
step_into_the_k-loop

Prove termination of the following program

$I \equiv r > n \cdot 10 - (i \cdot 10 + k) \wedge k < 10$

$A \equiv k < 10$ (to weak for special Assertion)

(chose $A \equiv n \cdot 10 - (i \cdot 10 + k) < r \wedge k < 10$ instead)



$n = 6$

i	0	0	0	...	0	1	1	1	...	1	2
k	0	1	2	...	9	0	1	2	..	0	

$\#left = n \cdot 10 - (i \cdot 10 + k)$

$B \equiv n \cdot 10 - i \cdot 10 < r$

$C \equiv 10(n - (i+1)) < r$

$D \equiv (k = 10 \wedge 10(n - i - 1) < r) \vee (k \neq 10 \wedge n \cdot 10 - (i \cdot 10 + k) < r \wedge k < 10)$

$E \equiv (k+1 = 10 \wedge 10(n-i-1) < r) \vee (k+1 \neq 10 \wedge n \cdot 10 - 10i - k - 1 < r \wedge k < 9)$

$\equiv (k = 9 \wedge 10(n-i) - 10 < r) \vee (10(n-i) - 1 - k < r \wedge k < 9)$

Show $r > 0$ using SP

$SP[i<n](I) \equiv \underbrace{10(n-i)}_{\geq 10} - \underbrace{k}_{<10} < r \wedge k < 10 \wedge i < n$

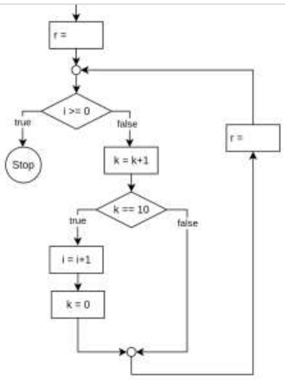
$\Rightarrow 0 < r$ $\Rightarrow$ Both extra Assertion hold $\Rightarrow$ terminates

False Branch

$n \cdot 10 - i \cdot 10 - k < r \wedge k < 10 \wedge i < n$

$\equiv 10(n-i) - k < r \wedge k < 10 \wedge i < n \Rightarrow E \Rightarrow LC$

$10(n-i) - k < r \wedge k < 10 \wedge i < n \Rightarrow E$



step_out_of_the_k-loop

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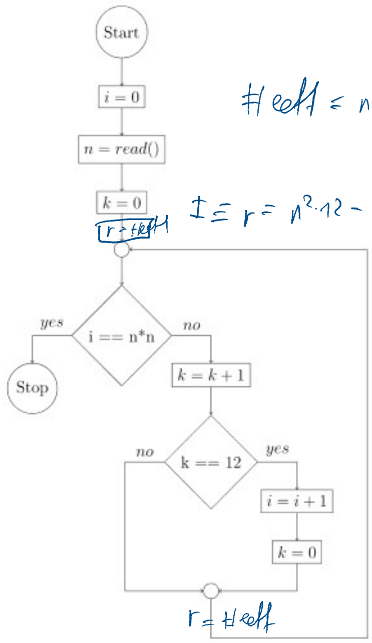
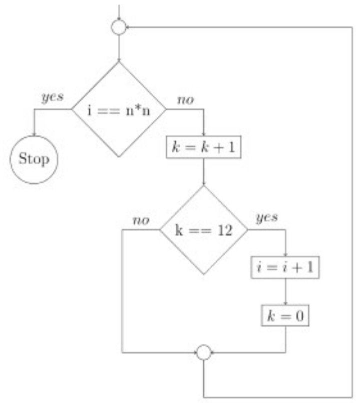
step_out_of_the_k-l...

Prove termination of the following program

$\#eff = n^2 \cdot 12 - (12i + k)$

$I \equiv r = n^2 \cdot 12 - 12i - k \quad \land \; k < 12$

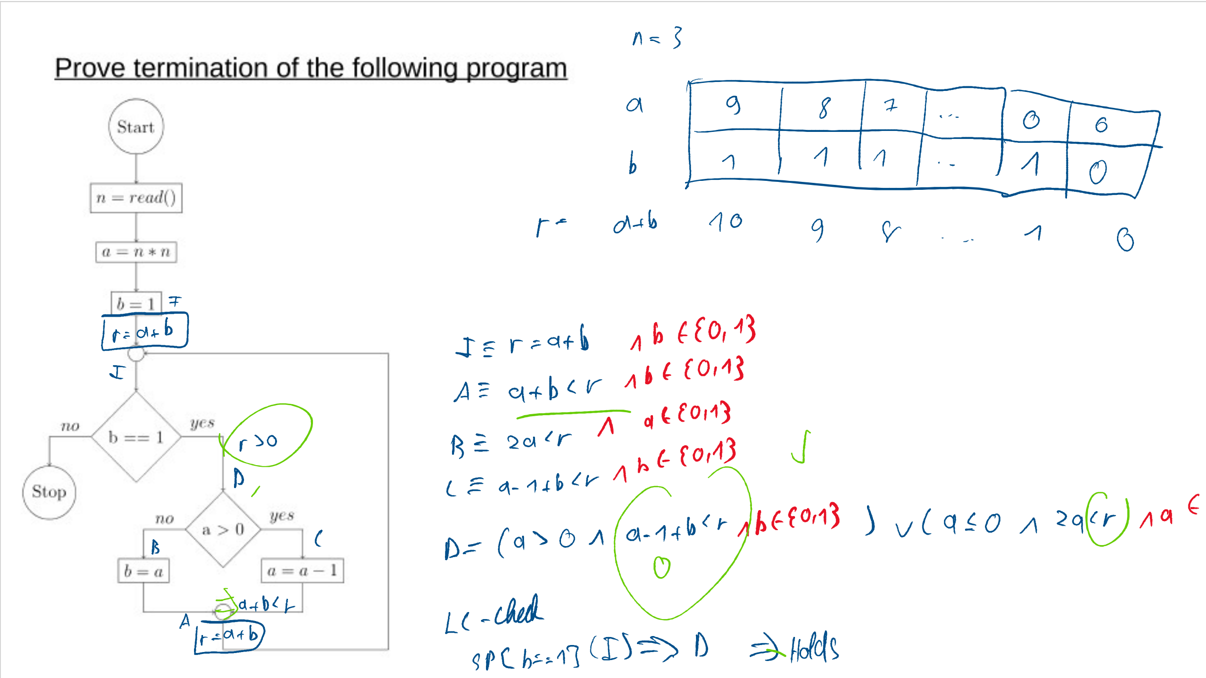




what_about_b

Montag, 25. September 2023 23:28

what_about_b



Check $r > 0$

$D \Rightarrow r > 0$ ✓

$SP[b=1](I) \equiv r = a+1 \land b \in \{0,1\}$

$\Rightarrow D$ ✓

$\Rightarrow LC$

$A \Rightarrow a+b < r$

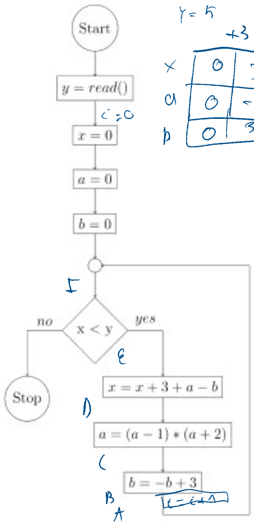
$a+b < r \Rightarrow a+b < r$

what_is_r

Montag, 25. September 2023 23:52

what_is_r

Prove termination of the following program



0 1 2 3 4

y = 5

+3 −2 +3 −2

x	0	3	1	4	2	5
a	0	−2	0	−2	0	
b	0	3	0	3	0	

$$x = \begin{cases} \frac{i}{2} & \text{if } i \text{ is even} \\ \frac{i-1}{2} + 3 & \text{if } i \text{ is odd} \end{cases}$$

x strictly decreasing

x bound by y

$\Rightarrow$ finitely many loops