Formulation of the nonlinear equations of motion

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ClearAll["Global`*"]
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(* Shape functions *)
$$\psi[i\_,\ z\_] := \text{Sin}\left[\frac{i\,\pi}{L}\,z\right]$$

(* Normal force over the length (z) and time (t) *)
$$T[z\_,\ t\_] := T_{top} - \gamma\,L + \gamma\,z + \frac{EA}{L}\,A_{top}\,\text{Cos}[\Omega\,t]$$

(* Temporal-spatial separation *)
$$u[z\_,\ t\_] := \psi[1,\ z] \times A_1[t] + \psi[2,\ z] \times A_2[t] + \psi[3,\ z] \times A_3[t]$$

(* Auxiliary parameters*)

(*a commom divisor*) $\text{div} := \mu\,\omega_1{}^2 + \mu_a\,\omega_1{}^2$
(*to collect*)
$$\text{coef2Collect} := \Big\{ A_1[t]^2\,A_2[t],\ A_1[t]^2\,A_3[t],\ A_2[t]^2\,A_3[t],$$
$$A_1[t]\,A_2[t]^2,\ A_1[t]\,A_3[t]^2,\ A_2[t]\,A_3[t]^2,\ A_2[t]\,A_3[t],$$
$$A_1[t]\,A_2[t],\ A_1[t]\,A_3[t],\ A_1[t]^2,\ A_2[t]^2,\ A_3[t]^2,$$
$$A_1[t]^3,\ A_2[t]^3,\ A_3[t]^3,\ A_1[t],\ A_2[t],\ A_3[t],\ A_1{}'[t],$$
$$A_2{}'[t],\ A_3{}'[t] \Big\};$$

(*term used in the Galerkin's method integration*)
Galerkin2Int :=
$$\Bigg(D[u[z,\ t],\ \{t,\ 2\}] + \frac{c\,\omega_1}{\text{div}}\,D[u[z,\ t],\ \{t,\ 1\}] +$$
$$\frac{EI}{\text{div}}\,D[u[z,\ t],\ \{z,\ 4\}] -$$
$$\frac{1}{\text{div}}\,(D[T[z,\ t] \times D[u[z,\ t],\ z],\ z]) -$$
$$\frac{d^2}{\text{div}}\,\frac{EA}{2\,L}\,D[u[z,\ t],\ \{z,\ 2\}] \times$$
$$\text{Integrate}\Big[(D[u[z,\ t],\ \{z,\ 1\}])^2,\ \{z,\ 0,\ L\}\Big] \Bigg)$$

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(* Evaluate the equations of motion *)
eq1 = Collect[Integrate[Galerkin2Int ψ[1, z] 2/L, {z, 0, L}],
    coef2Collect, FullSimplify];

eq2 = Collect[Integrate[Galerkin2Int ψ[2, z] 2/L, {z, 0, L}],
    coef2Collect, FullSimplify];

eq3 = Collect[Integrate[Galerkin2Int ψ[3, z] 2/L, {z, 0, L}],
    coef2Collect, FullSimplify];

(* Show equations *)

(* Formatting *)
styleTitle =
  Style[#, FontSize → 20, FontWeight → Bold,
    Background → LightBlue, FontFamily → "Times New Roman"] &;
styleText =
  Style[#, FontSize → 16, FontWeight → Bold,
    FontFamily → "Times New Roman"] &;
styleEqs =
  Style[#, FontSize → 14, FontFamily → "Cambria Math"] &;

(* Print eqs*)
Print[styleTitle@"Equations of motion: \n\n",
 styleText@" - First equation of motion: \n",
 styleEqs@ TraditionalForm[eq1], "\n\n",
 styleText@" - Second equation of motion: \n",
 styleEqs@TraditionalForm[eq2], "\n\n",
 styleText@" - Third equation of motion: \n",
 styleEqs@TraditionalForm[eq3], "\n"]
```