# LakeAnalyzer

Ver. 3.3 User Manual

Authors: Jordan S. Read, Kohji Muraoka

March, 2011

Global Lake Ecological Observatory Network (<a href="http://www.gleon.org/">http://www.gleon.org/</a>)

# **Table of Contents**

TABLE OF CONTENTS	III
1. INTRODUCTION	1
2. INPUT FILE FORMATS	2
Bathymetry (hypsographic curve) file	3
Water Temperature	3
Wind data	4
Salinity	5
Configuration file	6
3. PROGRAM OPERATION	7
Sequence of LakeAnalyzer operation	7
Online application	10
Configuration parameter descriptions	11
4 APPENDIX	13

## 1. Introduction

Lake Analyzer is a numerical code coupled with supporting visualization tools for determining indices of mixing and stratification that are critical to the biogeochemical cycles of lakes and reservoirs. Stability indices, including Lake Number, Wedderburn Number, Schmidt Stability, and thermocline depth are calculated according to established literature definitions and returned to the user in a time series format. The program was created for the analysis of high-frequency data collected from instrumented lake buoys, in support of the emerging field of aquatic sensor network science. Available outputs for the Lake Analyzer program are: water temperature (error-checked and/or down-sampled), wind speed (errorchecked and/or down-sampled), metalimnion extent (top and bottom), thermocline depth, friction velocity, Lake Number, Wedderburn Number, Schmidt Stability, mode-1 vertical seiche period, and Brunt-Väisälä buoyancy frequency. Secondary outputs for several of these indices delineate the parent thermocline depth (seasonal thermocline) from the shallower secondary or diurnal thermocline. Lake Analyzer provides a program suite and best practices for the comparison of mixing and stratification indices in lakes across gradients of climate, hydro-physiography, and time, and enables a more detailed understanding of the resulting biogeochemical transformations at different spatial and temporal scales.

## 2. Input file formats

Full performance of the Lake Analyzer program requires various input files, including a bathymetry file (extension: .bth), water temperature file (.wtr), wind data (.wnd), configuration file (.lke), surface water level file (\*.lvl) and salinity file (.sal). Additionally, users can control plotting defaults by including a plot file (\*.plt). Names must be shared among all the files, and the required file format is tab delimited text file, whereas the bathymetry file is an exception that requires a comma delimited text file format (a tab-delimited file in LA versions 3.4 and higher). A list of the input files required for individual outputs can be found in Table 2.1.

Table 2.1. List of the input files required for the corresponding outputs

Outputs	Bathymetry (*.bth)	Water Temperature (*.wtr)	Wind Speed (*.wnd)	Water Level (*.lvl)	Salinity (*.sal)
Thermocline Depths	Not Required	Required	Not Required	Optional	Optional
Metalimnion Depths	Not Required	Required	Not Required	Optional	Optional
Schmidt Stability	Required	Required	Not Required	Optional	Optional
uStar	Required	Required	Required	Optional	Optional
Lake Number	Required	Required	Required	Optional	Optional
Wedderburn Number	Required	Required	Required	Optional	Optional
Buoyancy Frequency	Not Required	Required	Not Required	Optional	Optional
Mode 1 Seiche Periods	Required	Required	Not Required	Optional	Optional

All the input files should be located in an identical folder with user defined name (i.e. lake name or year). If the salinity file is present in the correct directory, salinity will be used to calculate the density of the water, which affects most indices calculated in LakeAnalyzer.

#### Bathymetry (hypsographic curve) file

A bathymetry file is a comma delimited (after ver. 3.5, tab delimited) text file with extension of [.bth]. The file starts from one line header and followed by the hypsographic data at each depth (Example 2.1). Depths must start from zero (i.e. surface) with a unit of meters, and hypsographic curve data with area as square meters is followed by comma delimiter. If the hypsographic curve is not concluded with zero at the bottom, LakeAnalyzer program automatically assigns zero to the bottom depth which was defined during the configuration process (see section 3). LakeAnalyzer linearly interpolates the given hypsographic curve. Change to the hypsographic curve due to surface elevation change is not supported by the current version of the LakeAnalyzer.

```
Bathymetry Depths (m), Bathymetry Areas (m<sup>2</sup>)
0, 583054
1, 549139.5
2, 519084.94
...
19, 0
```

Example 2.1 an example bathymetry file used for Sparkling Lake

#### **Water Temperature**

The water temperature file is a tab delimitated text file with a file extension of [.wtr]. The file should contain one header which starts from DateTime, followed by individual thermister depths in meters with format of [temp5] (see Example 2.2). LakeAnalyzer uses header information to acquire thermister depth. Temperature data should be inserted from the following line. The data starts from the date/time inputs, which should be formatted as [yyyymm-dd HH:MM].

```
DateTime temp0
                temp0.5
                         temp1
                                temp1.5
                                        temp2
                                               temp3
                                                      temp4
2009-05-02 10:00
                6.555 6.552 6.445
                                   6.435 6.335 6.405
                                                      6.365
                                   6.495
2009-05-02 10:30
                6.555 6.505 6.425
                                         6.450 6.485
                                                      6.405
2009-05-02 11:00
                6.555 6.540
                            6.455
                                   6.495
                                         6.450 6.485
                                                      6.405
```

Example 2.2 An example temperature file used for Sparkling Lake

#### Wind data

The wind speed file is a tab delimitated text file with extension of [.wnd]. Wind speed data are used for uStar, Lake Number, and Wedderburn Number calculations. Time scale and resolution of the wind speed must match the water temperature inputs. The file starts from one line header [dateTime windSpeed]. From the second line, date/time information with the format of [yyyy-mm-dd HH:MM], and wind speed data in m/s should be described.

```
dateTime windSpeed
2009-05-02 10:00 5.080
2009-05-02 10:30 4.433
2009-05-02 11:00 4.700
...
```

Example 2.3 An example wind file used for Sparkling Lake

#### **Water Level**

The Water Level file is a tab delimited text file with the file extension of [.lvl]. Water level input is optional for all the outputs. It is useful for estuaries and lake with significant level changes which affect hypsographic curve of the water body. If the program locates the water level file in the correct directory with correct file name, the effect of water level fluctuation to the bathymetry area are calculated when calculating stabilities. The water level file contains one header [DateTime level(positive Z down)]. From the second line, date/time information with the format of [yyyy-mm-dd HH:MM], and water level from the highest elevation area measurement available (original depth is the surface level stated in the \*.bth file) should be described. Level depths must be equal or greater than 0.

Example 2.4 An example water level file used for Sparkling Lake

### **Salinity**

The salinity file is a tab delimitated text file with the file extension of [.sal]. Salinity input is optional for all the outputs. If the program locates the salinity file in the correct directory, the effect of salinity on the density is calculated during the process. Salinity time can be independent to the other input files. The salinity file contains one header line starting from DateTime, and followed by depths of measurements in format of [salinity2.0]. The second line is the beginning of the actual data inputs, starting from date/time in format [yyyy-mm-dd HH:MM]. After tab separation, salinity should be indicated Practical Salinity Scale (PSS) units.

DateTime	salinity2.0	0 sal	inity8.0	salinity12.0	salinity18.0
2009-02-24 00	0:00 2	.3 5.2	4.8	5	
2009-04-29 00	0:00 2	.15 2.3	6.8	7	
2009-08-19 00	0:00 2	.13 2.4	7.5	7.743	

Example 2.5 An example salinity file used for Sparkling Lake

#### Plot defaults

The plot file (\*.plt) is a tab delimited file with no header lines, it simply contains a list of parameter names for the plot defaults, and their user-specified value. There are only selected supported modifications that can be made to the plots, and these are listed below in Table 2.2.

Table 2.2: Optional parameters and values for \*.plt file (optional plot modification file).

Parameter	Description of parameter	Supported values
figUnits	Units of measure for figure size	inches, centimeters, pixels
figWidth	Width of figure (relative to figUnits)	number
figHeight	Height of figure	number
leftMargin	Space between left edge of figure and y-axis	number < (figWidth-
	(relative to figUnits)	rightMargin)
rightMargin	Space between right edge of figure and right	number < (figWidth-
	axis	leftMargin)
topMargin	Space between the top edge of the figure	number < (figHeight-
	and the top of the plot axis	botMargin)
botMargin	Space between the bottom edge of the figure	number < (figHeight-

	and the bottom of the plot x-axis	topMargin)
figType	Image format that will be saved	png, bmp, eps, jpeg, tiff, pdf
figRes	Resolution of the figure in dots per inch.	50, 100, 200, 300, 400, 500
fontName	Font name for plot text	Arial, Times New Roman,
		Helvetica
fontSize	Font sive for plot text	8, 9, 10, 11, 12, 14
heatMapMin	Value that represents the minimum heatmap color	-5, 0, 5, 10, etc
heatMapMax	Value that represents the maximum heatmap color	15, 20, 25, 30, 35
	•0101	

The plt file lines can be arranged in any order (there is no order for parameters), and can have any number of the supported parameters found in Table 2.2. An example plt can be seen below in example 2.6.

figRes 200	
fontName	Times New Roman
heatMapMin	0
heatMapMax	25
fontSize	11
figWidth	6.65

Example 2.6 An example plot modification file

## **Configuration file**

The configuration file manages operation of LakeAnalyzer with an extension of [.lke]. Configuration file is automatically created by LakeAnalyzer program through the configuration window (see section 3). The user can manually modify the file using abbreviations shown in Table 2.3.

Table 2.3 Abbreviations used in the lake analyzer configuration file

Abbreviation	Full description
N2	Buoyancy frequency
SN2	Parent buoyancy frequency
Ln	Lake number
SLn	Parent lake number

metaB	Metalimnion bottom depth
SmetaB	Parent metalimnion bottom depth
metaT	Metalimnion top depth
SmetaT	Parent metalimnion top depth
T1	Mode one vertical seiche period
ST1	Parent mode one vertical seiche period
St	Schmidt stability
thermD	Thermocline depth
SthermD	Parent thermocline depth
uSt	u star (turblent velocity scale from wind)
SuSt	Parent u star (turblent velocity scale from wind)
wTemp	Water temperature
$\mathbf{W}$	Wedderburn number
SW	Parent Wedderburn number
wndSpd	Wind speed

Configuration file for Sparkling				
SN2, SLn, S	SN2, SLn, SmetaB, SmetaT, ST1, St, SthermD, SW #outputs			
86400	#output resolution (s)			
19	#total depth (m)			
10	#height from surface for wind measurement (m)			
86400	#wind averaging (s)			
86400	#thermal layer averaging (s)			
21600	#outlier window (s)			
40	#max water temp (°C) inf if none			
-12	#min water temp (°C) -inf if none			
98	#max wind speed (m/s) inf if none			
0	#min wind speed (m/s) -inf if none			
0.1	#meta min slope (drho/dz per m)			
0.5	#mixed temp differential (°C)			
N	#plot figure (Y/N)			
Y	#write results to file (Y/N)			

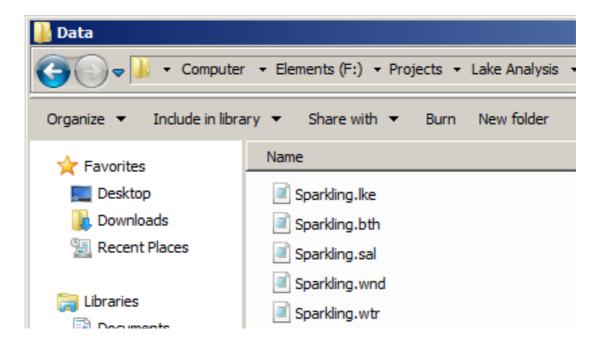
Example 2.7 An example of configuration file used for Sparkling Lake (not all output options are shown).

## 3. Program operation

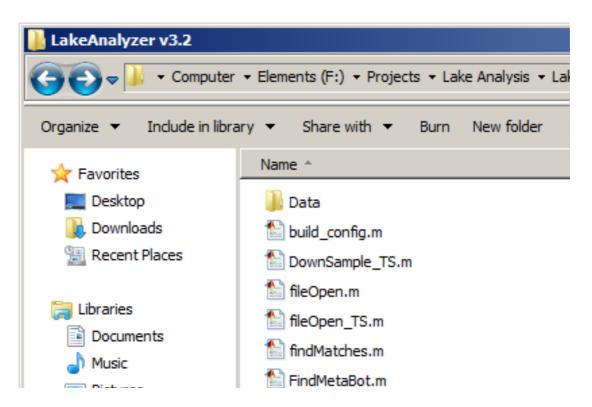
## Sequence of LakeAnalyzer operation

This section describes the processes of LakeAnalyzer operation, step by step.

#### 1) Setup the input file



2) Allocate the folder with inputs under the directory of LakeAnalyzer



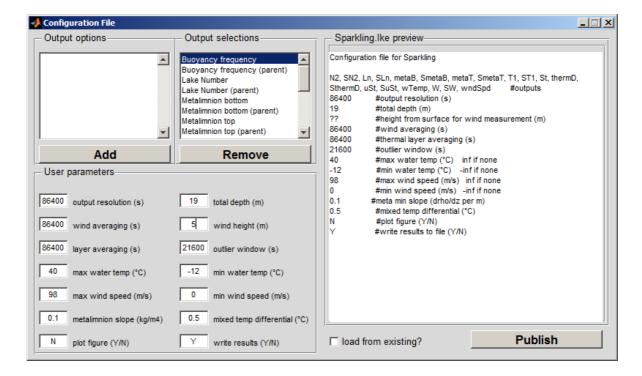
3) Start Matlab program. Set the current directory to the folder where the LakeAnalyzer is allocated

Current Folder: F:\Projects\Lake Analysis\LakeAnalyzer v3.2

4) On the command window, type: >> Run\_LA('LakeName','FolderName'). LakeName is the file name shared in the input files, and the FolderName is the folder name which contains input files. Configuration window will appear.



5) The configuration window automatically creates configuration file [.lke]. Select the outputs and clike [Add]. Set total depth and wind observation height. Temporal averaging should exceed one quarter of the first vertical mode internal seiche period, which may require a trial run with all the averaging values set to 1. Click [Publish] to operate the program.



6) When the analysis has successfully finished, the folder which the input files are allocated should contain new files. If the user selected [Water temperature] as an output, there should be another file with an identical file name with additional [wtr] which contains averaged water temperature output using

selected resolution. The output files have tab delimited format, thus the files can be viewed by Microsoft Excel or text editors.

#### **Online application** (http://lakeanalyzer.gleon.org/)

Lake Analyzer has a web application which operates exactly the same way as the Matlab application. However, the program does not show configuration window, thus the user must prepare configuration file [.lke] prior to the program operation. Please follow the page 5 example and abbreviation list to create the configuration file.

All the input files should have common names. The input files should be zipped into one file. The zip file must share its name with the other input files. Example Sparkling lake file can be found on the website. Onece the input files are prepared, the file name and its location should be chosen by following the instruction on the website.

lake analyzer web  This is the web front end to Lake Analzyer, a tool that allows users to calculate common metrics for lake physical states.
If used, please cite
Read JS, DP Hamilton, ID Jones, K Muraoka, LA Winslow, R Kroiss, CH Wu, E Gaiser. 2011. <u>Derivation of lake mixing</u> and stratification indices from high-resolution lake <u>buoy data.</u> Environmental Modelling and Software. 26: 1325-1336.
Code and Documentation
Additional information can be found in the <u>LakeAnalyzer Manual</u> . Code and more info can be found <u>here</u> .
Web Submit
Please zip up your input files and upload them to run. An input example set is provided to help you create input of your own.
Lake Name: Input files (zipped): Choose File No file chosen Submit
Input file format
The input files need to be in the format as described below.
*Lake Name*.lke: Configuration file for Lake Analyzer run
*Lake Name*.wtr: Time series of water temperature measurements
• *Lake Name*.wnd: Time series of wind speed measurements (only required for certain outputs)
• *Lake Name*.bth: Depth/area measurements of the lake (only required for certain outputs)
*Lake Name*.sal: Time series of salinity measurements (optional)
*Lake Name*.lvl: Time series of water level measurements (optional)
© Jordan Read & Luke Winslow 2012

Figure 1. Online Lake Analyzer application (http://lakeanalyzer.gleon.org/)

#### Configuration parameter descriptions

#### **Output resolution**

Output resolution specifies the time-step (s) of the calculations made for Lake Analyzer. If the temporal resolution of the input data is coarser than the entry for this input, calculations will be made according to input data resolution.

#### Wind averaging

Wind averaging (s) is the backwards-looking smoothing window used for the calculation of uSt and SuSt. This calculation allows for the relevant wind duration to influence the calculation of wind-derived parameters.

#### Layer averaging

Thermal averaging (s) (.lke {6}) is the smoothing window used for metaT, metaB, thermD, SmetaT, SmetaB, and SthermD. Temporal smoothing for thermal layers is intended to minimize the effects of internal waves on these parameters.

#### **Outlier window**

Outlier window (s) (.lke  $\{7\}$ ) is the window size (seconds) for outlier removal, where measurements outside of the bounds ( $\mu \pm 2.5 \cdot \sigma$ ) based on the standard deviation and the mean inside the outlier window are removed. Outlier removal is performed on .wtr and .wnd files prior to down-sampling (if applicable).

#### Total depth

Total depth (m) (.lke {3}) must be greater or equal to than the maximum depth given in the .bth file. If the total depth is not included in the .bth file, it is assumed that the area at total depth is 0 (m2) and the depth area curve is linearly interpolated from this depth to the values in the .bth file.

#### Wind height

Height of wind measurement (m) (.lke {4}) is used for the wind speed correction factor in Eqn 11.

#### Max/min water temp

Maximum and minimum allowed water temperatures (°C), where all values of .wtr file not fitting this criteria are removed before outlier checking.

#### Max/min wind speed

Maximum and minimum allowed wind speeds (m s-1), where all values of .wnd file not fitting this criteria are removed before outlier checking.

#### **Metalimnion slope**

Minimum slope for the range of the metalimnion (kg m-3 per meter), which is used to calculated values of metaT, metaB, SmetaT, and SmetaB according to Eqn 2.

#### Mixed temp differential

Minimum surface to bottom thermistor temperature differential (°C) before the case of 'mixed' is applied. When 'mixed' is true, all thermal layer calculations are no longer applicable, and values are given as the depth of the bottom thermistor.

## 4. Appendix

#### A.1 Thermocline depth

Water density ( $^{\rho}$ ) is calculated according to the contributions of temperature and solutes (if applicable) (eqn 8). For k number of measurements referenced from the surface, for i = 1 to i = k - 1.

$$\frac{\partial \rho}{\partial z_{i\Delta}} = \frac{\rho_{i+1} - \rho_i}{z_{i+1} - z_i},\tag{1}$$

which applies to the depth characterized by  $z_{i\Delta} = (z_{i+1} + z_i)/2$ , where  $z_{i\Delta}$  represents a midpoint depth between measurements i and i+1. If the maximum  $\partial \rho/\partial z_{i\Delta}$  is found when  $i = \zeta$  for discrete measurements (Figure 1c), the true depth of the maximum change in density  $(z_T)$  likely occurs within the bounds defined by the two depths at which the discrete measurements were taken  $(z_{\zeta} < z_T < z_{\zeta+1})$ . An improvement on the initial guess of  $z_T \approx z_{\zeta\Delta}$  can be made by weighting the magnitudes of the difference between the maximum calculated density change and the adjacent calculations (Figure 1c; 1d);

$$Z_{T} \approx Z_{\zeta+1} \left( \frac{\Delta \rho_{+1}}{\Delta \rho_{-1} + \Delta \rho_{+1}} \right) + Z_{\zeta} \left( \frac{\Delta \rho_{-1}}{\Delta \rho_{-1} + \Delta \rho_{+1}} \right)$$

$$(2)$$

where  $(z_{\zeta\Delta+1} - z_{\zeta\Delta})/(\partial \rho/\partial z_{\zeta\Delta} - \partial \rho/\partial z_{\zeta\Delta+1})$  has been simplified to  $\Delta \rho_{+1}$  and  $(z_{\zeta\Delta} - z_{\zeta\Delta-1})/(\partial \rho/\partial z_{\zeta\Delta} - \partial \rho/\partial z_{\zeta\Delta-1})$  to  $\Delta \rho_{-1}$ .

#### A.2 Mixed layer depth

From  $i = \zeta$  to i = 1, find i where  $\partial \rho / \partial z_{i\Delta} \leq \delta_{\min}$ , interpolate between i and i+1 to yield the approximate depth of the base of the mixed layer, ze (also referred to as the top of the metalimnion)

$$z_{e} = z_{i\Delta} + \left(\delta_{\min} - \frac{\partial \rho}{\partial z_{i\Delta}}\right) \frac{z_{i\Delta} - z_{i\Delta+1}}{\frac{\partial \rho}{\partial z_{i\Delta}} - \frac{\partial \rho}{\partial z_{i\Delta+1}}}$$
(3)

Likewise, for the base of the metalimnion, zh (the theoretical division between the metalimnion and the hypolimnion), from  $i=\zeta$  to i=k-1, find i where  $\partial \rho/\partial z_{i\Delta} \leq \delta_{\min}$ , interpolate between i-1 and i:

$$z_{h} = z_{i\Delta-1} + \left(\delta_{\min} - \frac{\partial \rho}{\partial z_{i\Delta-1}}\right) \frac{z_{i\Delta} - z_{i\Delta-1}}{\frac{\partial \rho}{\partial z_{i\Delta}} - \frac{\partial \rho}{\partial z_{i\Delta-1}}}$$

$$(4)$$

The searching algorithm used to perform these calculations (Eqns 3 and 4) requires knowledge of the pycnocline index ( $^{\xi}$  from section A.1), and searches upward towards the water surface from  $^{\xi}$  to find the depth of the top of the metalimnion, and downward towards the lake bottom from  $^{\xi}$  to find the depth of the base of the metalimnion.

#### A.3 Schmidt Stability

$$S_T = \frac{g}{A_s} \int_0^{z_D} (z - z_v) \rho_z A_z \partial z \tag{5}$$

where g is the acceleration due to gravity,  $A_s$  is the surface area of the lake,  $A_z$  is the area of the lake at depth z,  $z_D$  is the maximum depth of the lake, and  $z_V$  is the depth to the centre of volume of the lake, written as  $z_V = \int_0^{z_D} z A_z \partial z / \int_0^{z_D} A_z \partial z$ 

#### A.4 Wedderburn Number

$$W = \frac{g'z_e^2}{u_*^2 L_s} \tag{6}$$

where  $g' = g \cdot \Delta \rho / \rho_h$  is the reduced gravity due to the change in density  $(\Delta \rho)$  between the hypolimnion  $(\rho_h)$  and epilimnion  $(\rho_e)$ ,  $z_e$  is the depth to the base of the mixed layer (Eqn 3),  $L_s$  is the lake fetch length and  $u_s$  is the water friction velocity due to wind stress (Eqn 9).

#### A.5 Lake Number

$$L_{N} = \frac{S_{T}(z_{e} + z_{h})}{2\rho_{h}u_{*}^{2}A_{s}^{1/2}z_{v}}$$
(7)

where  $z_e$  and  $z_h$  are the depths to the top and bottom of the metalimnion, respectively (Eqns 3 and 4).

## A.6 Density of water ( $\rho_i$ )

Density of water from a given temperature (in °C), assuming negligible effects of any solutes on density, can be calculated as in Martin and McCutchen (1999) to be:

$$\rho_i = \left[1 - \frac{T_i + 288.9414}{508929.2 \cdot (T_i + 68.12963)} (T_i - 3.9863)^2\right] \cdot 1000$$
(8a)

If solutes are non-negligible and a salinity file (.sal) is provided, density will be calculated according to the combined effects of salinity ( $S_i$ ) and water temperature based on the methods of Millero and Poisson (1981):

$$\rho_{i} = \rho_{*} + A \cdot S_{i} + B \cdot S_{i}^{\frac{3}{2}} + C \cdot S_{i}$$
where 
$$\rho_{*} = 999.842594 + 6.793952 \times 10^{-2} \cdot T_{i} - 9.09529 \times 10^{-3} \cdot T_{i}^{2} + 1.001685 \times 10^{-4} \cdot T_{i}^{3} - 1.120083 \times 10^{-6} \cdot T_{i}^{4} + 6.536336 \times 10^{-9} \cdot T_{i}^{5} ,$$

$$A = 8.24493 \times 10^{-1} - 4.0899 \times 10^{-3} \cdot T_{i} + 7.6438 \times 10^{-5} \cdot T_{i}^{2} - 8.2467 \times 10^{-7} \cdot T_{i}^{3} + 5.3875 \times 10^{-9} \cdot T_{i}^{4} ,$$

$$B = -5.72466 \times 10^{-3} + 1.0227 \times 10^{-4} \cdot T_{i} = 1.6546 \times 10^{-6} \cdot T_{i}^{2} , \text{ and } C = 4.8314 \times 10^{-4} .$$

#### A.7 Seasonal thermocline (SthermD)

Lake Analyzer defines the dominant thermocline (thermD) as an estimate of the depth of the greatest density change with respect to depth ( $\partial \rho/\partial z$ ). If a secondary local maximum is found in  $\partial \rho/\partial z$  at a greater depth than thermD, the location of SthermD is calculated using Eqn 1. This calculation is not performed (and SthermD output will be the same as thermD) when either the secondary local maximum is less than 20% of the absolute maximum gradient, or no secondary maximum exists. We found the 20% threshold to work best for a variety of lakes, but users can modify this parameter in the script 'FindThermoDepth.m' by changing the value of 'dRhoPerc', which represents this ratio.

A.8 u-star (
$$u_*$$
)
$$u_* = \sqrt{\frac{\tau_w}{\rho_e}}$$
(9)

With  $\rho_e$  as the average density (kg m-3) of the epilimnion, and  $\tau_w$  is the wind shear (N m-2) on the water surface, given by  $\tau_w = C_D \rho_{air} U^2$ .  $\rho_{air}$  is the density of air (kg m-3), and U is wind speed (m s-1) measured at 10 m above the water surface. Wind speed measurements at any height ( $U_z$ ) other than 10 m (as specified by input #4 in the .lke file) are corrected according to Amorocho and DeVries (1980):

$$U = U_z \left[ 1 - \frac{C_D^{0.5}}{\kappa} \ln \left( \frac{10}{z} \right) \right]^{-1} \tag{10}$$

Where  $\kappa$  is von Karman's constant (taken to be 0.4), z is the height above the water surface for the measurement of  $U_z$ . Values for  $C_D$  are given by Hicks (1972) as

$$C_D = 1 \times 10^{-3}$$
 for U < 5 (m s-1)  
 $C_D = 1.5 \times 10^{-3}$  for U > 5 (m s-1)

A.9 Buoyancy frequency ( $N^2$ )

$$N^2 = \frac{g}{\rho} \frac{\partial \rho}{\partial z} \tag{11}$$

Where  $N^2$  (s-2) represents the local stability of the water column, based on the density gradient  $\partial \rho/\partial z$ .

A.10 Mode 1 vertical seiche period ( $^{T_1}$ )

$$T_{1} = \frac{2z_{D}L_{T}}{g'z_{T}(z_{D} - z_{T})}$$
(12)

From Monismith (1986) where  $L_T$  is the basin length at the depth of the thermocline ( $^{Z_T}$ ).