NUMERICAL METHODS

THIRD EDITION

Doug Faires and Dick Burden

PREFACE

The teaching of numerical approximation techniques to undergraduates is done in a variety of ways. The traditional Numerical Analysis course emphasizes both the approximation methods and the mathematical analysis that produces them. A Numerical Methods course is more concerned with the choice and application of techniques to solve problems in engineering and the physical sciences than with the derivation of the methods.

The books used in the Numerical Methods courses differ widely in both intent and content. Sometimes a book written for Numerical Analysis is adapted for a Numerical Methods course by deleting the more theoretical topics and derivations. The advantage of this approach is that the leading Numerical Analysis books are mature; they have been through a number of editions, and they have a wealth of proven examples and exercises. They are also written for a full year coverage of the subject, so they have methods that can be used for reference, even when there is not sufficient time for discussing them in the course. The weakness of using a Numerical Analysis book for a Numerical Methods course is that material will need to be omitted, and students might then have difficulty distinguishing what is important from what is tangential.

The second type of book used for a Numerical Methods course is one that is specifically written for a service course. These books follow the established line of service-oriented mathematics books, similar to the technical calculus books written for students in business and the life sciences, and the statistics books designed for students in economics, psychology, and business. However, the engineering and science students for whom the Numerical Methods course is designed have a much stronger mathematical background than students in other disciplines. They are quite capable of mastering the material in a Numerical Analysis course, but they do not have the time for, nor, often, the interest in,

the theoretical aspects of such a course. What they need is a sophisticated introduction to the approximation techniques that are used to solve the problems that arise in science and engineering. They also need to know why the methods work, what type of error to expect, and when a method might lead to difficulties. Finally, they need information, with recommendations, regarding the availability of high quality software for numerical approximation routines. In such a course the mathematical analysis is reduced due to a lack of time, not because of the mathematical abilities of the students.

The emphasis in this Numerical Methods book is on the intelligent application of approximation techniques to the type of problems that commonly occur in engineering and the physical sciences. The book is designed for a one semester course, but contains at least 50% additional material, so instructors have flexibility in topic coverage and students have a reference for future work. The techniques covered are essentially the same as those included in our book designed for the Numerical Analysis course (See [BF], Burden and Faires, Numerical Analysis, Seventh Edition, 2001, Brooks/Cole Publishing.) However, the emphasis in the two books is quite different. In Numerical Analysis, a book with about 800 text pages, each technique is given a mathematical justification before the implementation of the method is discussed. If some portion of the justification is beyond the mathematical level of the book, then it is referenced, but the book is, for the most part, mathematically self-contained. In this Numerical Methods book, each technique is motivated and described from an implementation standpoint. The aim of the motivation is to convince the student that the method is reasonable both mathematically and computationally. A full mathematical justification is included only if it is concise and adds to the understanding of the method.

In the past decade a number of software packages have been developed to produce symbolic mathematical computations. Predominant among them are *DERIVE*, Maple, *Mathematica* and MATLAB. There are versions of the software packages for most common computer systems and student versions are available at reasonable prices. Although there are significant differences among the packages, both in performance and price, they all can perform standard algebra and calculus operations. Having a symbolic computation package available can be very useful in the study of approximation techniques. The results in most

of our examples and exercises have been generated using problems for which exact values can be determined, since this permits the performance of the approximation method to be monitored. Exact solutions can often be obtained quite easily using symbolic computation.

We have chosen Maple as our standard package, and have added examples and exercises whenever we felt that a computer algebra system would be of significant benefit. In addition, we have discussed the approximation methods that Maple employs when it is unable to solve a problem exactly. The Maple approximation methods generally parallel the methods that are described in the text.

Software is included with and is an integral part of this Numerical Methods book, and a program disk is included with the book. For each method discussed in the text the disk contains a program in C, FORTRAN, and Pascal, and a worksheet in Maple, *Mathematica*, and MATLAB. The programs permit students to generate all the results that are included in the examples and to modify the programs to generate solutions to problems of their choice. The intent of the software is to provide students with programs that will solve most of the problems that they are likely to encounter in their studies.

Occasionally, exercises in the text contain problems for which the programs do not give satisfactory solutions. These are included to illustrate the difficulties that can arise in the application of approximation techniques and to show the need for the flexibility provided by the standard general purpose software packages that are available for scientific computation. Information about the standard general purpose software packages is discussed in the text. Included are those in packages distributed by the International Mathematical and Statistical Library (IMSL), those produced by the National Algorithms Group (NAG), the specialized techniques in EISPACK and LINPACK, and the routines in MATLAB.

New for this Edition

This edition includes two new major additions. The Preconditioned Conjugate Gradient method has been added to Chapter 7 to provide a more complete treatment of the

numerical solution to linear systems of equations. It is presented as an iterative approximation technique for solving positive definite linear systems. In this form, it is particularly useful for approximating the solution to large sparse linear systems.

In Chapter 10 we have added a section on Homotopy and Continuation methods. These methods provide a distinctly different technique for approximating the solutions to nonlinear systems of equations, one that has attracted a great deal of recent attention.

We have also added extensive listings of Maple code throughout the book, since reviewers found this feature useful in the second edition. We have updated all the Maple code to Release 8, the current version. Since versions of the symbolic computation software are commonly released between editions of the book, we will post updated versions of the Maple, *Mathematica*, and MATLAB worksheets at the book website:

http://www.as.ysu.edu/∼faires/NumericalMethods3

when material in new versions of any the symbolic computation systems needs to be modified. We will post additional information concerning the book at that site as well, based on requests from those using the book.

Although the major changes in this edition may seem quite small, those familiar with our past editions will find that virtually every page has been modified in some way. All the references have been updated and revised, and new exercises have been added where appropriate. We hope you will find these changes beneficial to the teaching and study of Numerical Methods. These changes have been motivated by the presentation of the material to our students and by comments from users of previous editions of the book.

A *Student Solutions Manual* is available with this edition. It includes solutions to representative exercises, particularly those that extend the theory in the text. We have included the first chapter of the *Student Solutions Manual* in Adobe Reader (PDF) format at the book website so that students can determine if the *Manual* is likely to be sufficiently useful to them to justify purchasing a copy.

The publisher can also provide instructors with a complete *Instructor's Manual* that provides solutions to all the exercises in the book. All the results in this *Instructor's*

Manual were regenerated for this edition using the programs on the disk. To further assist instructors using the book, we plan to use the book website to prepare supplemental material for the text, *Student Solutions Manual*, and *Instructor's Manual* based on user requests. Let us know how we can help you improve your course, we will try to accommodate you.

The following chart shows the chapter dependencies in the book. We have tried to keep the prerequisite material to a minimum to allow greater flexibility.