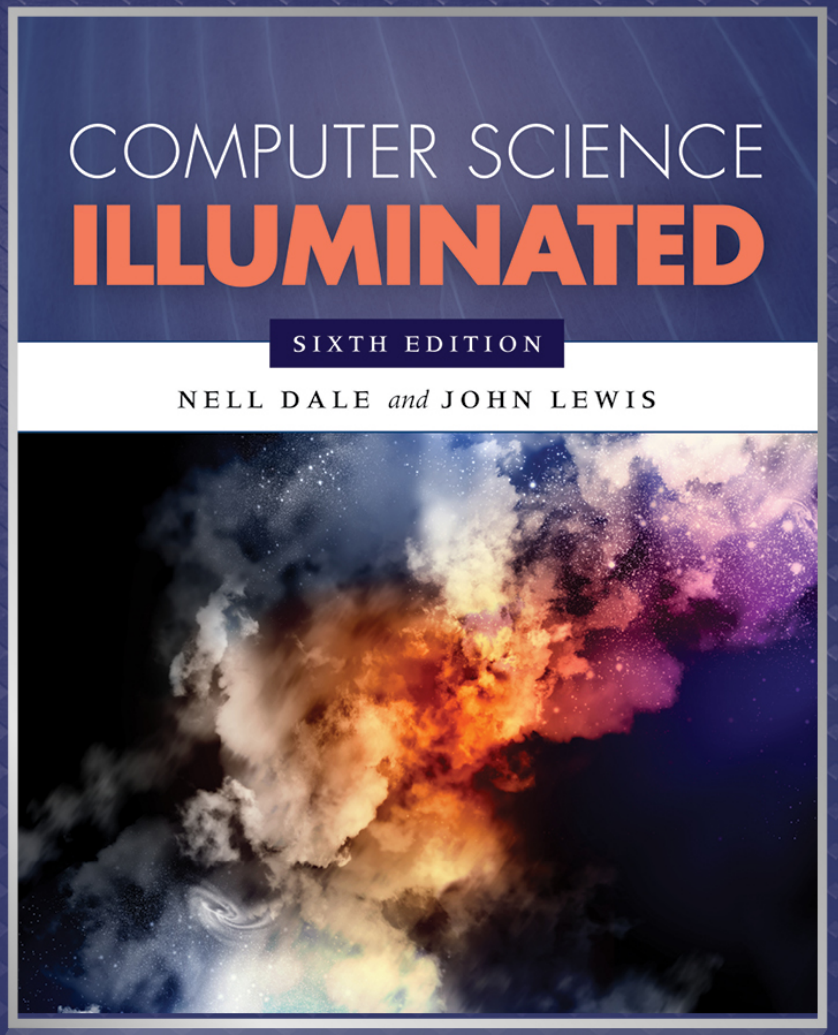
Chapter 2

Binary Values and Number Systems



Chapter Goals

- Distinguish among categories of numbers
- Describe positional notation
- Convert numbers in other bases to base 10
- Convert base-10 numbers to numbers in other bases
- Describe the relationship between bases 2, 8, and 16
- Explain the importance to computing of bases that are powers of 2

Numbers

Natural Numbers

Zero and any number obtained by repeatedly adding one to it.

Examples: 100, 0, 45645, 32

Negative Numbers

A value less than 0, with a – sign

Examples: -24, -1, -45645, -32

Numbers

Integers

A natural number, a negative number

Examples: 249, 0, - 45645, - 32

Rational Numbers

An integer or the quotient of two integers

Examples: -249, -1, 0, 3/7, -2/5

Positional Notation

How many ones are there in 642?

600 + 40 + 2 ?

Or is it

384 + 32 + 2 ?

Or maybe…

1536 + 64 + 2 ?

Positional Notation

Aha!

642 is 600 + 40 + 2 in **BASE 10**

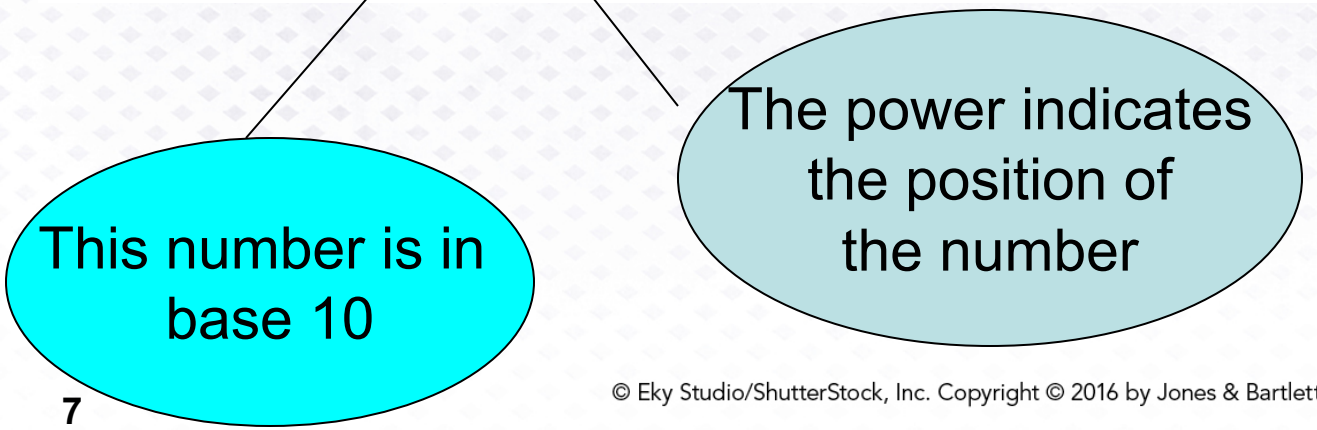
The **base** of a number determines the number of different digit symbols (numerals) and the values of digit positions

Positional Notation

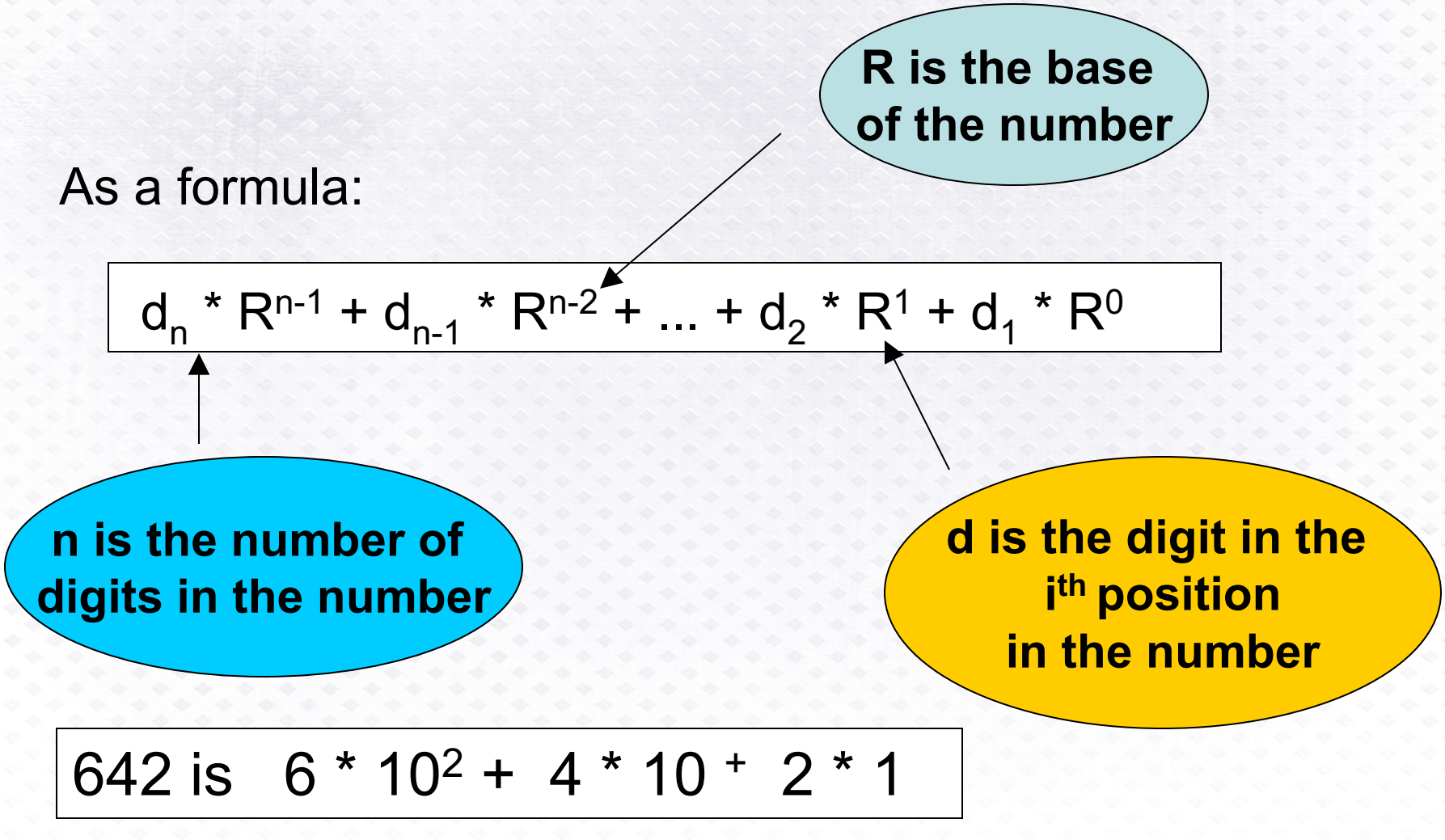
Continuing with our example…

642 in base 10 *positional notation* is:

$$6 \times 10^2 = 6 \times 100 = 600$$
$$+ 4 \times 10^1 = 4 \times 10 = 40$$
$$+ 2 \times 10^0 = 2 \times 1 = 2 = 642 \text{ in base } 10$$



Positional Notation



642 is $6 * 10^2 + 4 * 10 + 2 * 1$

Positional Notation

What if 642 has the base of 13?

$$+ 6 \times 13^2 = 6 \times 169 = 1014$$
$$+ 4 \times 13^1 = 4 \times 13 = 52$$
$$+ 2 \times 13^0 = 2 \times 1 = 2$$
$$= 1068 \text{ in base } 10$$

642 in base 13 is equivalent to 1068 in base 10

Binary

Decimal is base 10 and has 10 digit symbols:

0,1,2,3,4,5,6,7,8,9

Binary is base 2 and has 2 digit symbols:

0,1

For a number to exist in a given base, it can only contain the digits in that base, which range from 0 up to (but not including) the base.

What bases can these numbers be in? 122, 198, 178, G1A4

Bases Higher than 10

How are digits in bases higher than 10 represented?

With distinct symbols for 10 and above.

Base 16 has 16 digits:

0,1,2,3,4,5,6,7,8,9,A,B,C,D,E, and F

Converting Octal to Decimal

What is the decimal equivalent of the octal number 642?

$$
\begin{aligned}
6 \times 8^2 &= 6 \times 64 &&= 384 \\
+\ 4 \times 8^1 &= 4 \times 8 &&= 32 \\
+\ 2 \times 8^0 &= 2 \times 1 &&= 2 \\
&&&= 418 \text{ in base } 10
\end{aligned}
$$

Converting Hexadecimal to Decimal

What is the decimal equivalent of the hexadecimal number DEF?

$$
\begin{aligned}
D \times 16^2 &= 13 \times 256 = 3328 \\
+\ E \times 16^1 &= 14 \times 16 = 224 \\
+\ F \times 16^0 &= 15 \times 1 = 15 \\
&= 3567 \text{ in base } 10
\end{aligned}
$$

Remember, the digit symbols in base 16 are 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Converting Binary to Decimal

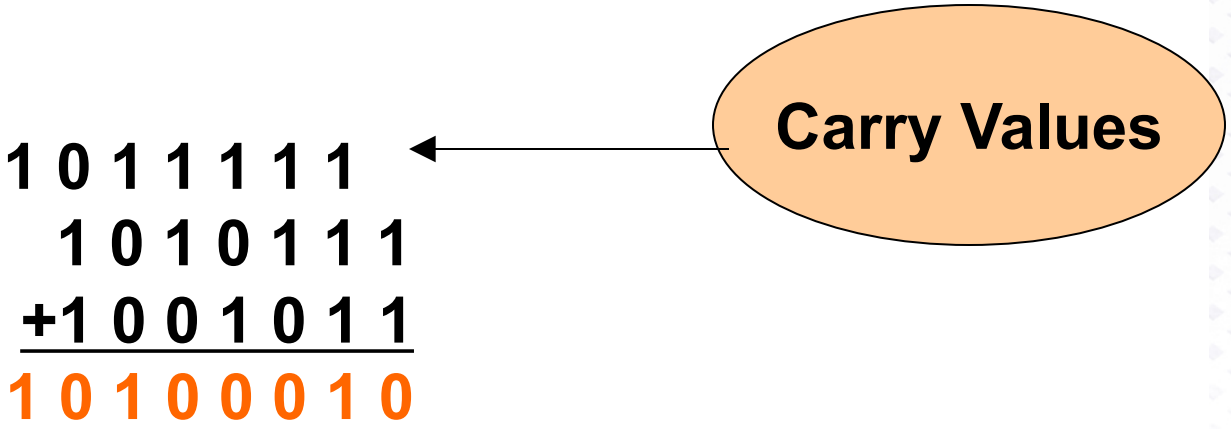
What is the decimal equivalent of the binary number 1101110?

$$
\begin{aligned}
&1 \times 2^6 = 1 \times 64 = 64 \\
+\ &1 \times 2^5 = 1 \times 32 = 32 \\
+\ &0 \times 2^4 = 0 \times 16 = 0 \\
+\ &1 \times 2^3 = 1 \times 8 = 8 \\
+\ &1 \times 2^2 = 1 \times 4 = 4 \\
+\ &1 \times 2^1 = 1 \times 2 = 2 \\
+\ &0 \times 2^0 = 0 \times 1 = 0 \\
&= 110 \text{ in base } 10
\end{aligned}
$$

Arithmetic in Binary

Remember that there are only 2 digit symbols in binary, 0 and 1

1 + 1 is 0 with a carry



Subtracting Binary Numbers

Remember borrowing? **Apply that concept here:**

```
  0 1 2
    0 2
  1 0 1 0 1 1 1
- 1 1 1 0 1 1
-------------
  0 0 1 1 1 0 0
```

Counting in Power-of-2 Bases

BINARY	OCTAL	DECIMAL
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	10	8
1001	11	9
1010	12	10

Converting Binary to Octal

- Mark groups of *three* (from right)
- Convert each group

10101011 **10 101 011**

2 5 3

10101011 is 253 in base 8

Converting Binary to Hexadecimal

- Mark groups of *four* (from right)
- Convert each group

10101011 **1010 1011**

A B

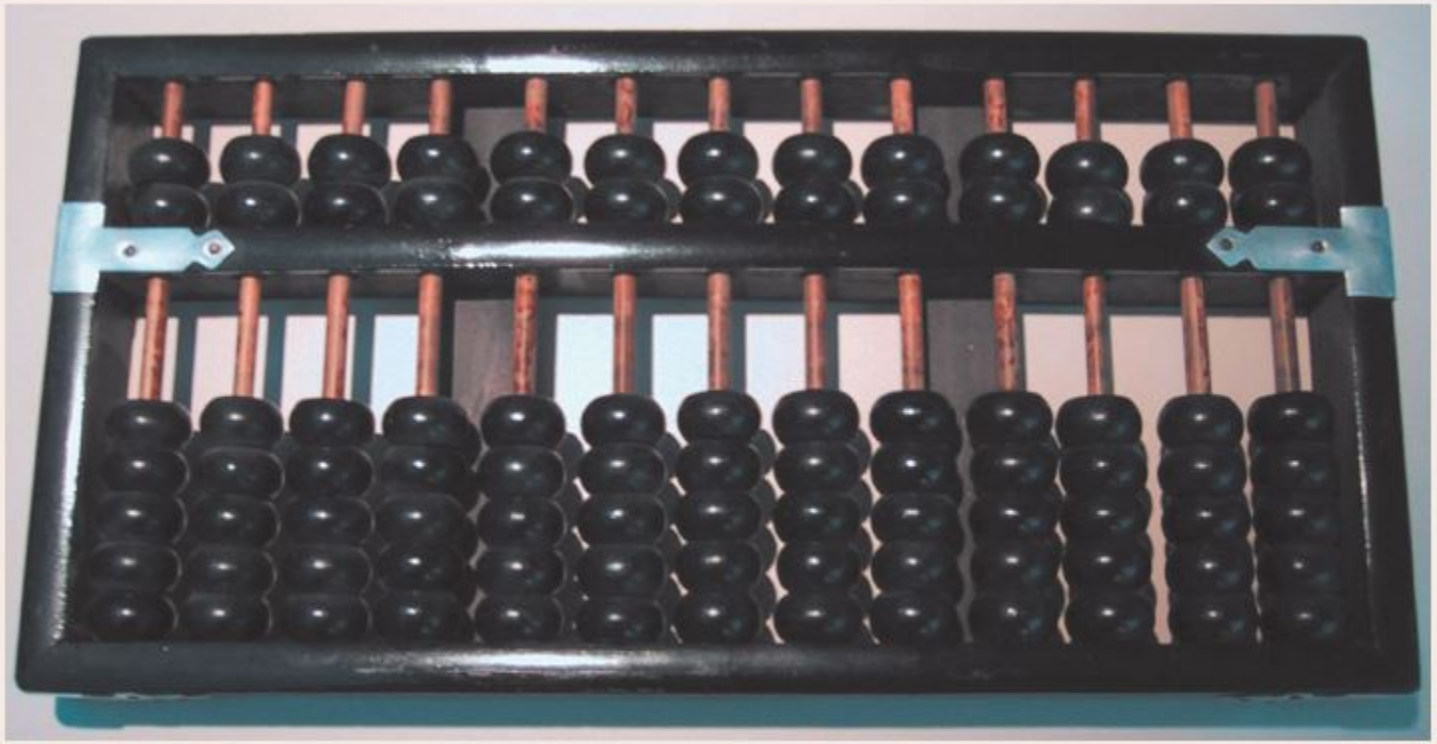
10101011 is AB in base 16

Converting Decimal to Octal

Try some!

http://fclass.vaniercollege.qc.ca/web/mathematics/real/Calculators/BaseConv_calc_1.htm

Abacus



Courtesy of Theresa DiDonato

Converting Decimal to Other Bases

Algorithm for converting number in base 10 to other bases

While (the quotient is not zero)
    Divide the decimal number by the new base
    Make the remainder the next digit to the left in the answer
    Replace the original decimal number with the quotient

Converting Decimal to Octal

What is 1988 (base 10) in base 8?

Try it!

Converting Decimal to Octal

```
   248        31         3         0
8 | 1988   8 | 248    8 | 31    8 | 3
  16         24          24        0
  38         08           7        3
  32          8
   68         0
   64
    4
```

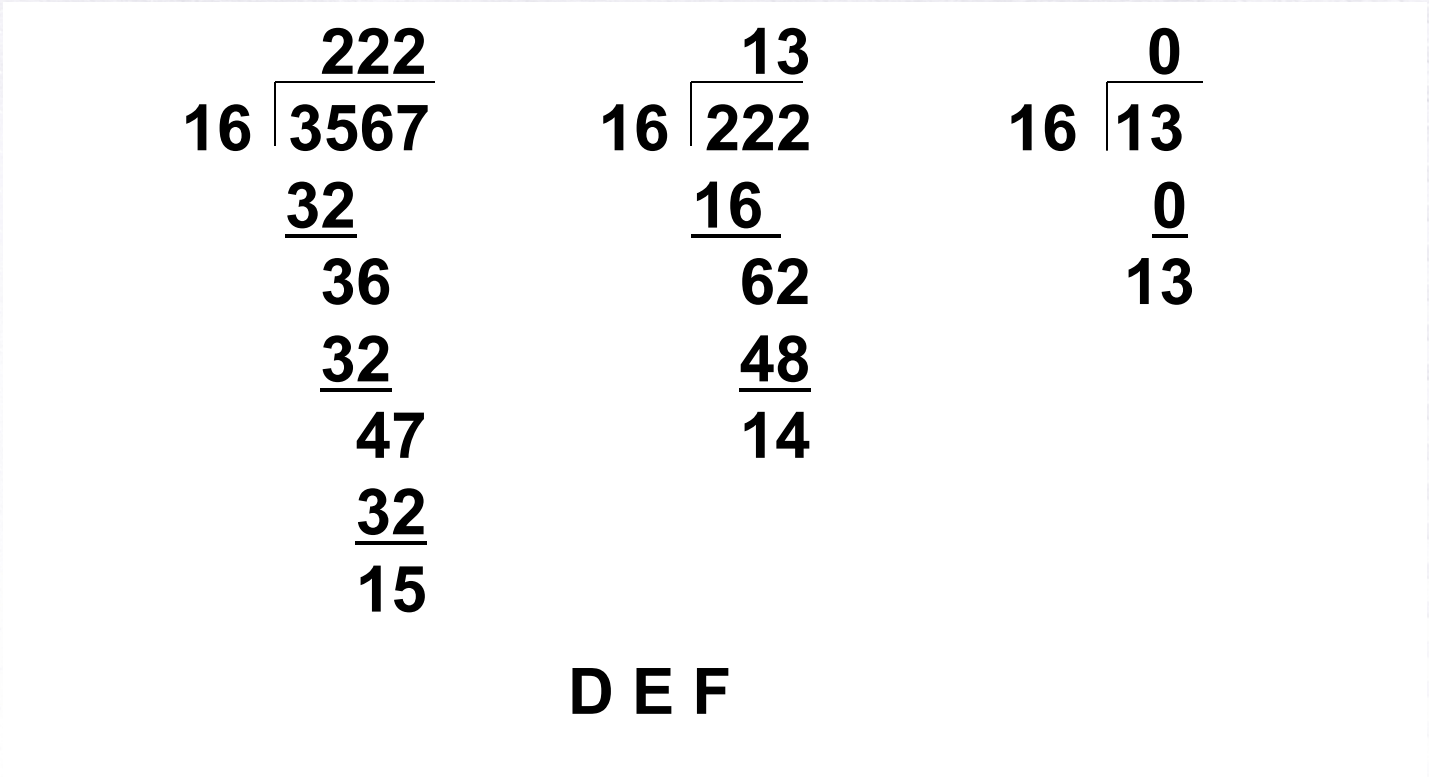
Answer is : **3 7 0 4**

Converting Decimal to Hexadecimal

What is 3567 (base 10) in base 16?

Try it!

Converting Decimal to Hexadecimal



Binary Numbers and Computers

Computers have storage units called binary digits or bits

Low Voltage = 0
High Voltage = 1 all bits have 0 or 1

… or the other way around, but we don't need to worry about that

Binary and Computers

Byte
8 bits

The number of bits in a word determines the word length of the computer, which is usually a multiple of 8

- 32-bit machines
- 64-bit machines etc.

Courtesy of IBM Corporate Archives, © International Business Machines Corporation.

Ethical Issues

The FISA Court

What does the United States Foreign Intelligence Surveillance Court do?

When did most people first hear of the FISA Court?

What checks and balances are there between the FISA Court and other government entities?

What is the stated intent of the FISA Court?

Who am I?

© Cynthia Johnson/Getty Images

Can you tell the person sitting next to you three things about me?

Do you know?

What concept makes positional notation possible?

What three sets can pre-school children identify? What words represent the third set?

How does an abacus work?

How does bi-quinary work?