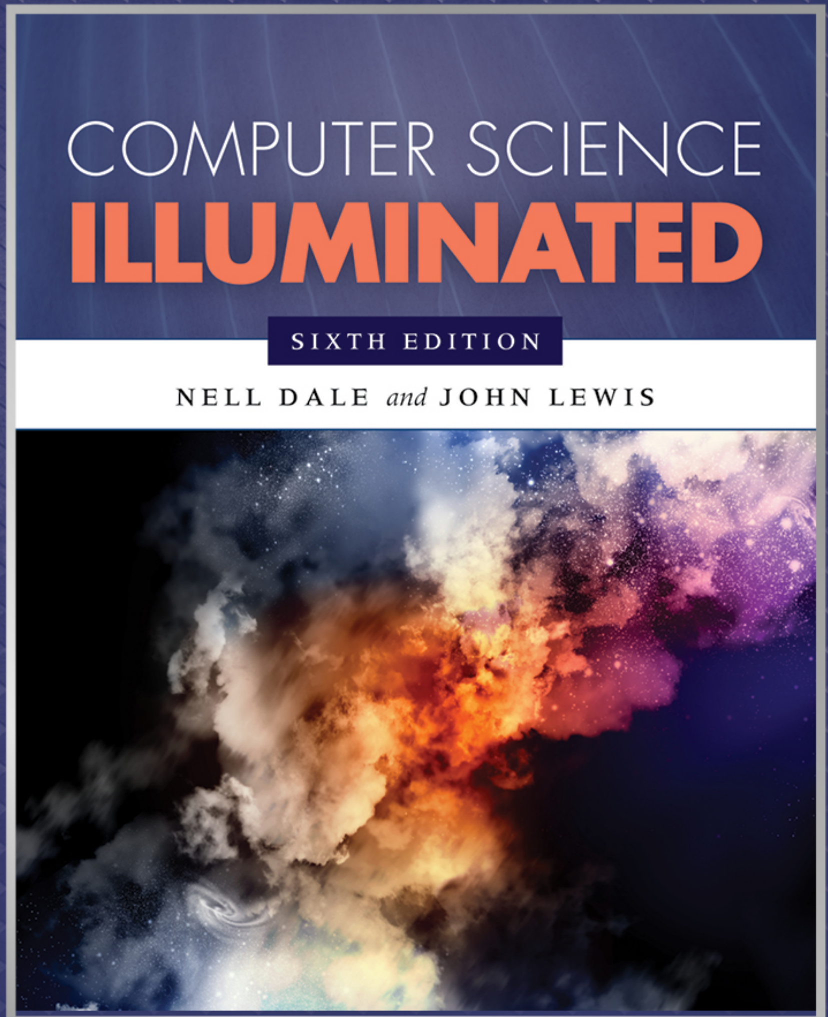
Chapter 7

Problem Solving and Algorithms



Chapter Goals

- Describe the computer problem-solving process and relate it to *Polya's How to Solve It* list
- Distinguish between a simple type and a composite type
- Describe two composite data-structuring mechanisms
- Recognize a recursive problem and write a recursive algorithm to solve it
- Distinguish between an unsorted array and a sorted array
- Distinguish between a selection sort and an insertion sort

Chapter Goals

- Describe the Quicksort algorithm
- Apply the selection sort, the bubble sort, insertion sort, and Quicksort to an array of items by hand
- Apply the binary search algorithm
- Demonstrate an understanding of the algorithms in this chapter by hand-simulating them with a sequence of items

Problem Solving

Problem solving

The act of finding a solution to a perplexing, distressing, vexing, or unsettled question

*How do **you** define problem solving?*

Problem Solving

How to Solve It: A New Aspect of Mathematical Method by George Polya

"How to solve it list" written within the context of mathematical problems

But list is quite general

We can use it to solve computer related problems!

Problem Solving

How do you solve problems?

Understand the problem

Devise a plan

Carry out the plan

Look back

Strategies

Ask questions!

- *What do I know about the problem?*
- *What is the information that I have to process in order the find the solution?*
- *What does the solution look like?*
- *What sort of special cases exist?*
- *How will I recognize that I have found the solution?*

Strategies

Ask questions! Never reinvent the wheel!

Similar problems come up again and again in different guises

A good programmer recognizes a task or subtask that has been solved before and plugs in the solution

Can you think of two similar problems?

Strategies

Divide and Conquer!

Break up a large problem into smaller units and solve each smaller problem

- Applies the concept of abstraction
- The divide-and-conquer approach can be applied over and over again until each subtask is manageable

Computer Problem-Solving

Analysis and Specification Phase
- Analyze
- Specification

Algorithm Development Phase
- Develop algorithm
- Test algorithm

Implementation Phase
- Code algorithm
- Test algorithm

Maintenance Phase
- Use
- Maintain

Can you name a recurring theme?

Phase Interactions

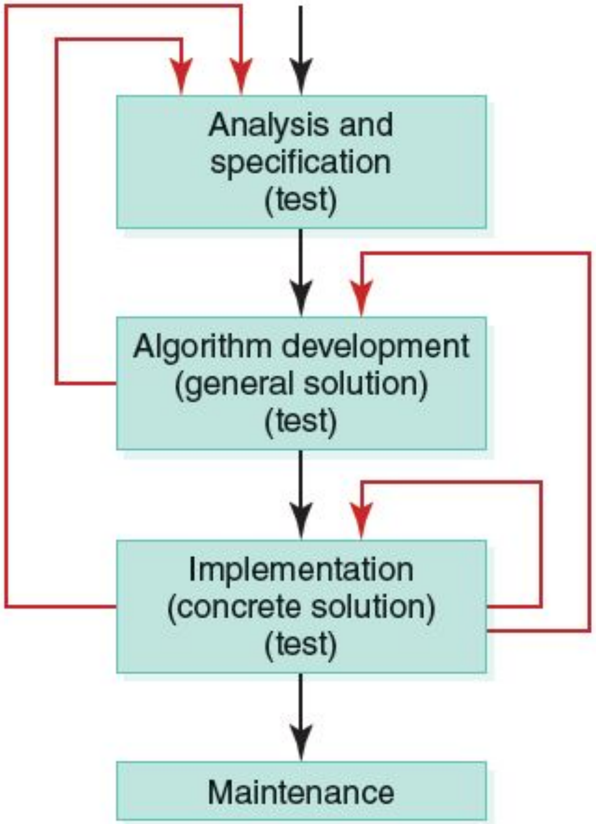


FIGURE 7.3 The interactions among the four problem-solving phases

Should we add another arrow?

(What happens if the problem is revised?)

Algorithms

Algorithm

A set of unambiguous instructions for solving a problem or subproblem in a finite amount of time using a finite amount of *data*

Abstract Step

An algorithmic step containing unspecified details

Concrete Step

An algorithm step in which all details are specified

Developing an Algorithm

Two methodologies used to develop computer solutions to a problem

- Top-down design focuses on the **tasks** to be done
- Object-oriented design focuses on the **data** involved in the solution (We will discuss this design in Ch. 9)

Summary of Methodology

Analyze the Problem

Understand the problem!!

Develop a plan of attack

List the Main Tasks (becomes Main Module)

Restate problem as a list of tasks (modules)

Give each task a name

Write the Remaining Modules

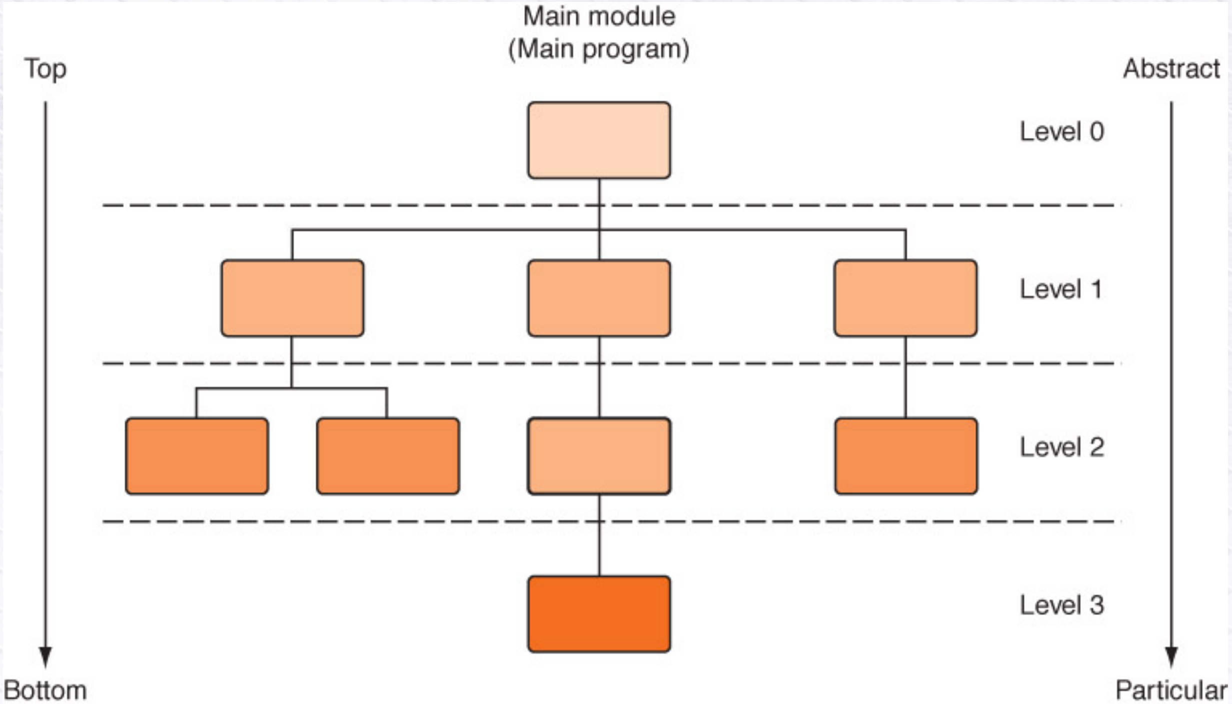
Restate each abstract module as a list of tasks

Give each task a name

Re-sequence and Revise as Necessary

Process ends when all steps (modules) are concrete

Top-Down Design



Process continues for as many levels as it takes to make every step concrete

Name of (sub)problem at one level becomes a module at next lower level

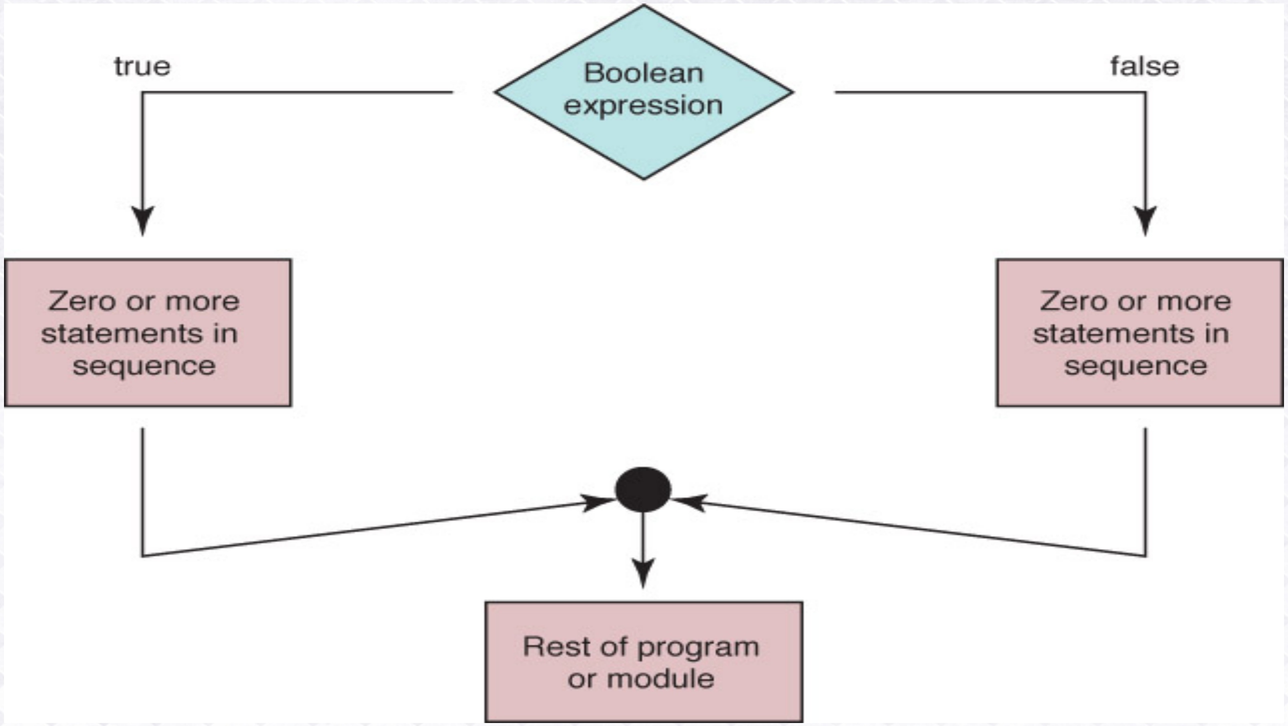
Control Structures

Control structure

An instruction that determines the order in which other instructions in a program are executed

Can you name the ones we defined in the functionality of pseudocode?

Selection Statements



Flow of control of if statement

Algorithm with Selection

Problem: Write the appropriate dress for a given temperature.

Write "Enter temperature"
Read temperature
Determine Dress

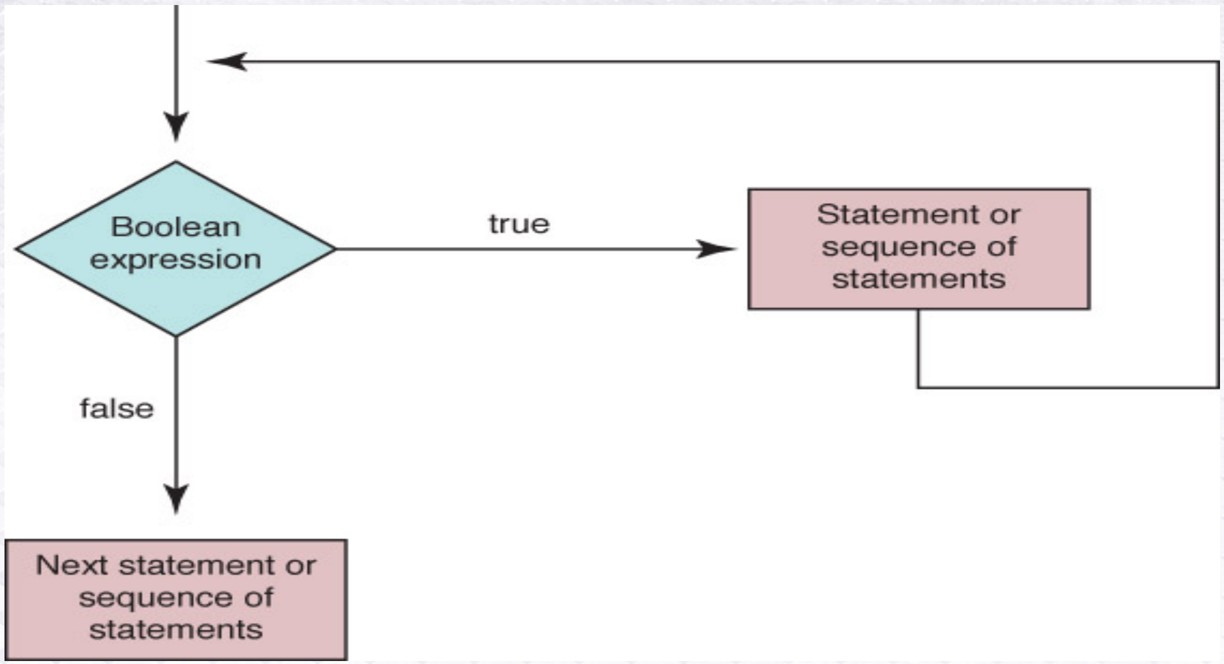
Which statements are concrete?
Which statements are abstract?

Algorithm with Selection

Determine Dress

IF (temperature > 90)
Write "Texas weather: wear shorts"
ELSE IF (temperature > 70)
Write "Ideal weather: short sleeves are fine"
ELSE IF (temperature > 50)
Write "A little chilly: wear a light jacket"
ELSE IF (temperature > 32)
Write "Philadelphia weather: wear a heavy coat"
ELSE
Write "Stay inside"

Looping Statements



Flow of control of while statement

Looping Statements

A count-controlled loop

```
Set sum to 0
Set count to 1
While (count <= limit)
    Read number
    Set sum to sum + number
    Increment count
Write "Sum is " + sum
```

Why is it called a count-controlled loop?

Looping Statements

An event-controlled loop

```
Set sum to 0
Set allPositive to true
WHILE (allPositive)
    Read number
    IF (number > 0)
        Set sum to sum + number
    ELSE
        Set allPositive to false
Write "Sum is " + sum
```

Why is it called an event-controlled loop?
What is the event?

Looping Statements

Calculate Square Root

Read in square
Calculate the square root
Write out square and the square root

Are there any abstract steps?

Looping Statements

Calculate Square Root

```
Set epsilon to 1
WHILE  (epsilon > 0.001)
    Calculate new guess
    Set epsilon to abs(square - guess * guess)
```

Are there any abstract steps?

Looping Statements

Calculate New Guess

Set newGuess to
(guess + (square/guess)) / 2.0

Are there any abstract steps?

Looping Statements

Read in square
Set guess to square/4
Set epsilon to 1
WHILE (epsilon > 0.001)
    Calculate new guess
    *Set epsilon to abs(square - guess * guess)*
Write out square and the guess

Composite Data Types

Records

A named heterogeneous collection of items in which individual items are accessed by name. For example, we could bundle name, age and hourly wage items into a record named *Employee*

Arrays

A named homogeneous collection of items in which an individual item is accessed by its position (index) within the collection

Composite Data Types

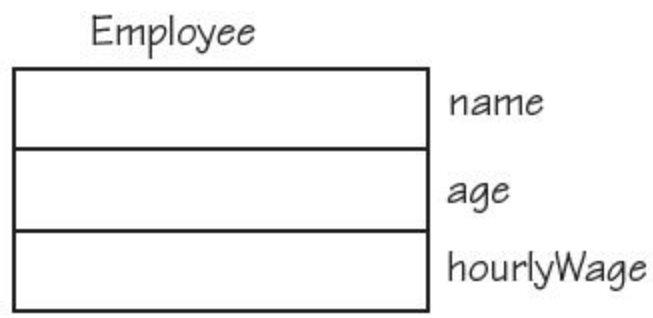


FIGURE 7.6 Record Employee

Following algorithm, stores values into the fields of record:

Employee employee *// Declare and Employee variable*
Set employee.name to "Frank Jones"
Set employee.age to 32
Set employee.hourlyWage to 27.50

Composite Data Types

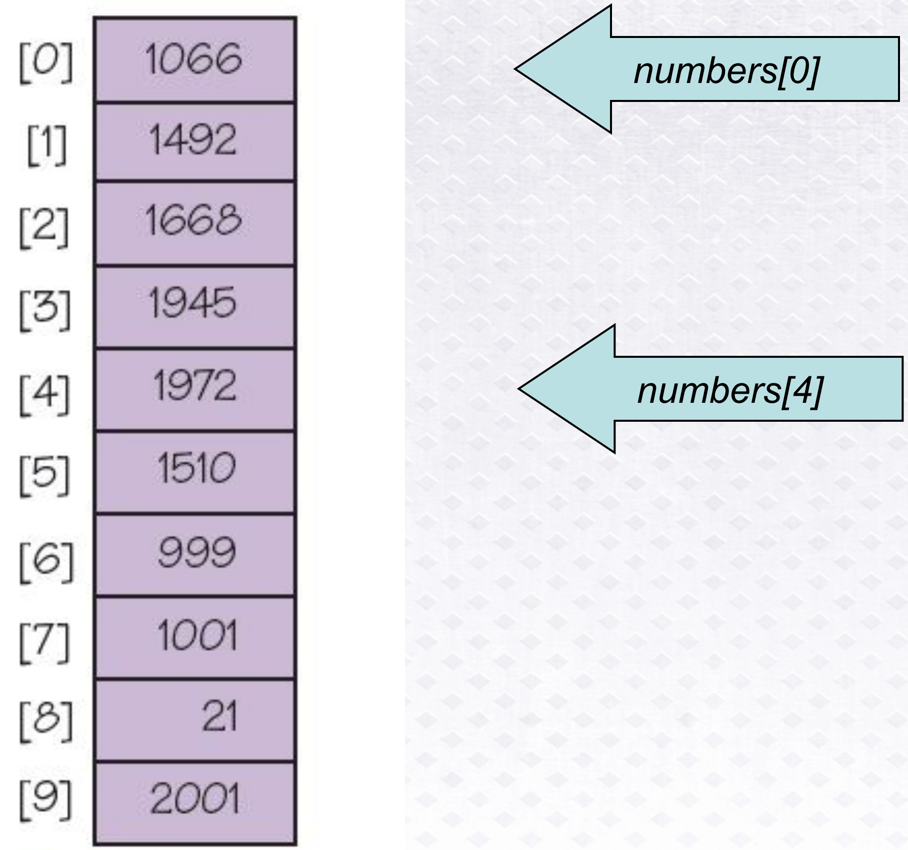


FIGURE 7.5 An array of ten numbers

Arrays

As data is being read into an array, a counter is updated so that we always know how many data items were stored

If the array is called *list*, we are working with

list[0] to list[length-1] *or*

list[0]..list[length-1]

An Unsorted Array

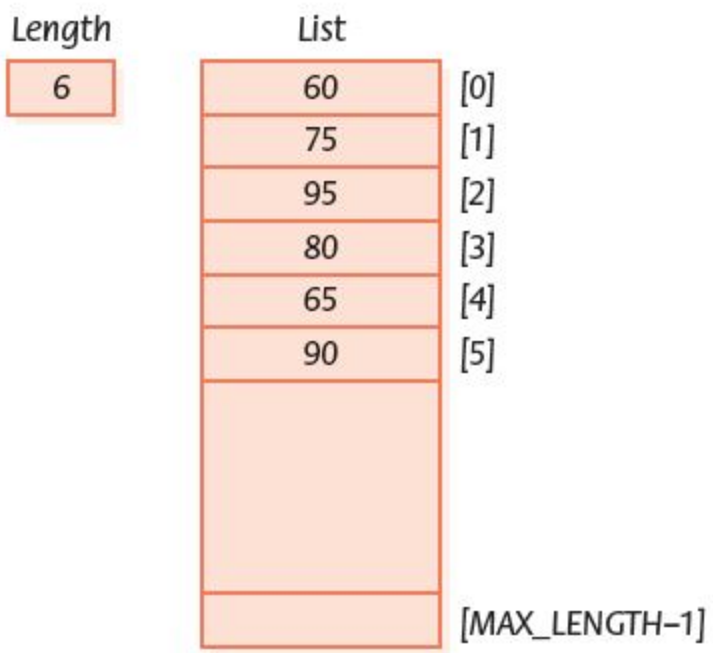


FIGURE 7.7 An unsorted array

data[0]...data[length-1] is of interest

Composite Data Types

Fill array numbers with *limit* values

```
integer data[20]
Write "How many values?"
Read length
Set index to 0
WHILE (index < length)
    Read data[index]
    Set index to index + 1
```

Sequential Search of an Unsorted Array

A sequential search examines each item in turn and compares it to the one we are searching.

If it matches, we have found the item. If not, we look at the next item in the array.

We stop either when we have found the item or when we have looked at all the items and not found a match

Thus, a loop with two ending conditions

Sequential Search Algorithm

Set Position to 0

Set found to FALSE

WHILE (position < length AND NOT found)

    IF (numbers [position] equals searchitem)

        Set Found to TRUE

    ELSE

        Set position to position + 1

Booleans

Boolean Operators

A Boolean variable is a location in memory that can contain either *true* or *false*

Boolean operator AND returns *TRUE* if both operands are true and *FALSE* otherwise

Boolean operator OR returns *TRUE* if either operand is true and *FALSE* otherwise

Boolean operator NOT returns *TRUE* if its operand is false and *FALSE* if its operand is true

Sorted Arrays

The values stored in an array have unique keys of a type for which the relational operators are defined

Sorting rearranges the elements into either ascending or descending order within the array

A sorted array is one in which the elements are in order

Sequential Search in a Sorted Array

If items in an array are sorted, we can stop looking when we pass the place where the item would be it were present in the array

Is this better?

A Sorted Array

FIGURE 7.8 A sorted array

A sorted array of integers

A Sorted Array

Read in array of values

Write "Enter value for which to search"

Read searchItem

Set found to TRUE if searchItem is there

IF (found)

    Write "Item is found"

ELSE

    Write "Item is not found"

A Sorted Array

Set found to TRUE if searchItem is there

```
Set index to 0
Set found to FALSE
WHILE (index < length AND NOT found)
        IF (data[index] equals searchItem)
                Set found to TRUE
        ELSE IF (data[index] > searchItem)
                Set index to length
        ELSE
                Set index to index + 1
```

Binary Search

Sequential search

Search begins at the beginning of the list and continues until the item is found or the entire list has been searched

Binary search (list must be sorted)

Search begins at the middle and finds the item or eliminates half of the unexamined items; process is repeated on the half where the item might be

Say that again…

Binary Search

```
Set first to 0
Set last to length-1
Set found to FALSE
WHILE (first <= last AND NOT found)
    Set middle to (first + last)/ 2
    IF (item equals data[middle]))
        Set found to TRUE
    ELSE
        IF (item < data[middle])
            Set last to middle – 1
    ELSE
        Set first to middle + 1
RETURN found
```

Binary Search

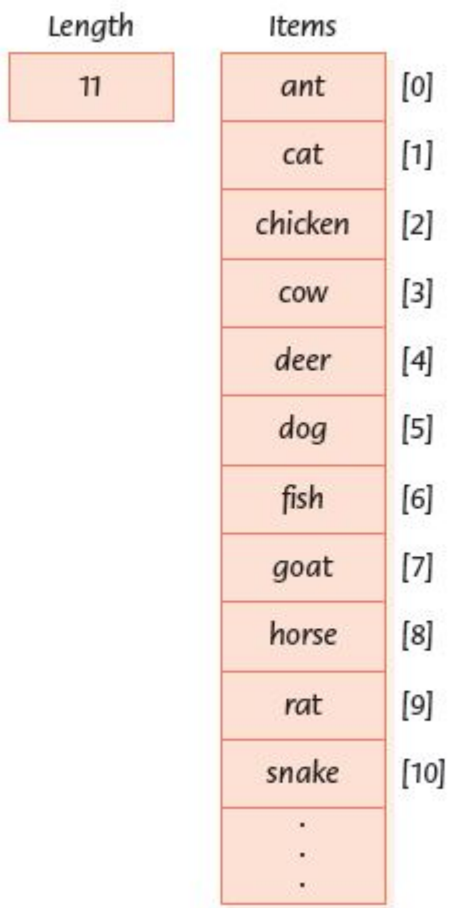


FIGURE 7.9 Binary search example

Searching for *cat*

First	Last	Middle	Comparison	
0	10	5	*cat < dog*	
0	4	2	*cat < chicken*	
0	1	0	*cat > ant*	
1	1	1	*cat = cat*	**Return: true**

Searching for *fish*

First	Last	Middle	Comparison	
0	10	5	*fish > dog*	
6	10	8	*fish < horse*	
6	7	6	*fish = fish*	**Return: true**

Searching for *zebra*

First	Last	Middle	Comparison	
0	10	5	*zebra > dog*	
6	10	8	*zebra > horse*	
9	10	9	*zebra > rat*	
10	10	10	*zebra > snake*	
11	10		*first > last*	**Return: false**

FIGURE 7.10 Trace of the binary search

Binary Search

TABLE 7.1 Average Number of Comparisons

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Length	Sequential Search	Binary Search
10	5.5	2.9
100	50.5	5.8
1000	500.5	9.0
10000	5000.5	12.0

Is a binary search always better?

Sorting

Sorting

Arranging items in a collection so that there is an ordering on one (or more) of the fields in the items

Sort Key

The field (or fields) on which the ordering is based

Sorting algorithms

Algorithms that order the items in the collection based on the sort key

Why is sorting important?

Selection Sort

Given a list of names, put them in alphabetical order

- Find the name that comes first in the alphabet, and write it on a second sheet of paper
- Cross out the name off the original list
- Continue this cycle until all the names on the original list have been crossed out and written onto the second list, at which point the second list contains the same items but in sorted order

Selection Sort

A slight adjustment to this manual approach does away with the need to duplicate space

- As you cross a name off the original list, a free space opens up
- Instead of writing the value found on a second list, exchange it with the value currently in the position where the crossed-off item should go

Selection Sort

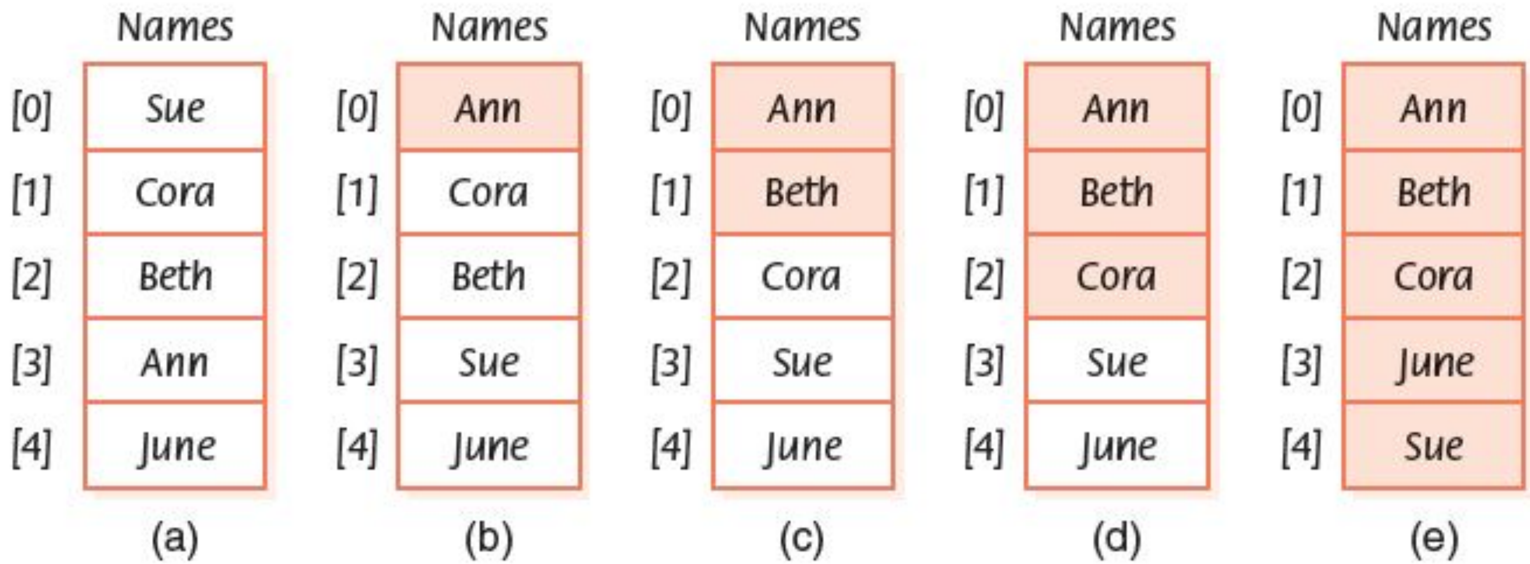


FIGURE 7.11 Examples of selection sort (sorted elements are shaded)

Selection Sort

Selection Sort

Set firstUnsorted to 0

WHILE (not sorted yet)

Find smallest unsorted item

Swap firstUnsorted item with the smallest

Set firstUnsorted to firstUnsorted + 1

Not sorted yet

current < length – 1

Selection Sort

```
Find smallest unsorted item
Set indexOfSmallest to firstUnsorted
Set index to firstUnsorted + 1
WHILE (index <= length – 1)
        IF (data[index] < data[indexOfSmallest])
                Set indexOfSmallest to index
        Set index to index + 1
Set index to indexOfSmallest
```

Selection Sort

Swap firstUnsorted with smallest

Set tempItem to data[firstUnsorted]

Set data[firstUnsorted] to data[indexOfSmallest]

Set data[indexOfSmallest] to tempItem

Bubble Sort

Bubble Sort uses the same strategy:

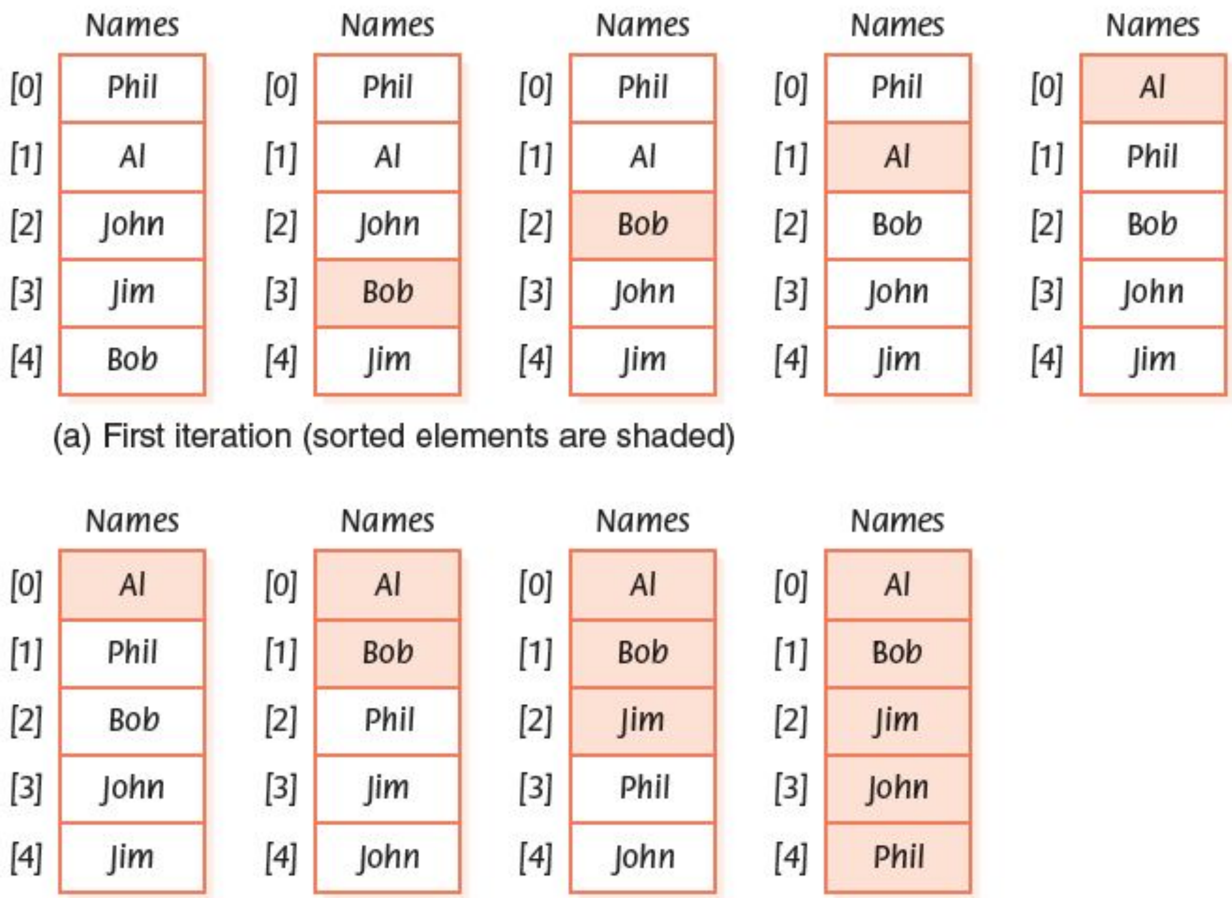
Find the next item

Put it into its proper place

But uses a different scheme for finding the next item

Starting with the last list element, compare successive pairs of elements, swapping whenever the bottom element of the pair is smaller than the one above it

Bubble Sort



(a) First iteration (sorted elements are shaded)

(b) Remaining iterations (sorted elements are shaded)

FIGURE 7.12 Examples of a bubble sort

Bubble Sort

Bubble sort is very slow!

Can you see a way to make it faster?

Under what circumstances is bubble sort fast?

Bubble Sort

Bubble Sort

Set firstUnsorted to 0

Set index to firstUnsorted + 1

Set swap to TRUE

WHILE (index < length AND swap)

Set swap to FALSE

"Bubble up" the smallest item in unsorted part

Set firstUnsorted to firstUnsorted + 1

Bubble Sort

Bubble up

```
Set index to length – 1
WHILE (index > firstUnsorted + 1)
        IF (data[index] < data[index – 1])
                Swap data[index] and data[index – 1]
                Set swap to TRUE
                Set index to index - 1
```

Insertion Sort

If you have only one item in the array, it is already sorted.

If you have two items, you can compare and swap them if necessary, sorting the first two with respect to themselves.

Take the third item and put it into its place relative to the first two

Now the first three items are sorted with respect to one another

Insertion Sort

The item being added to the sorted portion can be bubbled up as in the bubble sort

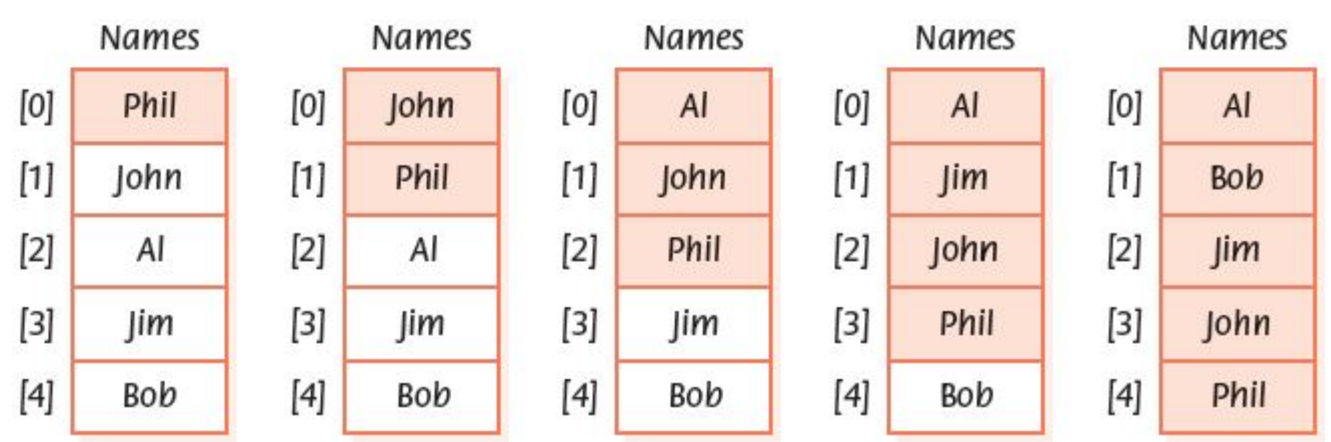


FIGURE 7.13 Insertion sort

Insertion Sort

InsertionSort

```
Set current to 1
WHILE (current < length)
Set index to current
Set placeFound to FALSE
WHILE (index > 0 AND NOT placeFound)
        IF (data[index] < data[index – 1])
                Swap data[index] and data[index – 1]
                Set index to index – 1
        ELSE
                Set placeFound to TRUE
Set current to current + 1
```

Subprogram Statements

We can give a section of code a name and use that name as a statement in another part of the program

When the name is encountered, the processing in the other part of the program halts while the named code is executed

Remember?

Subprogram Statements

What if the subprogram needs data from the calling unit?

Parameters

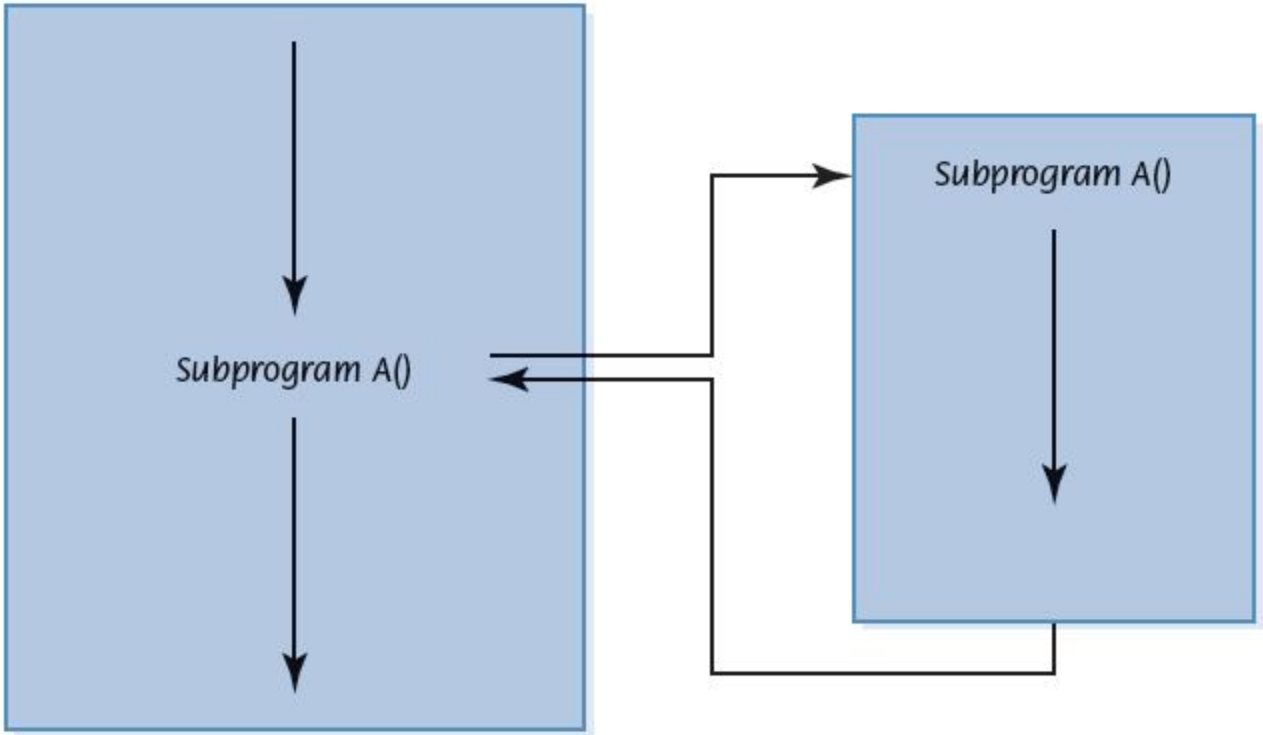
Identifiers listed in parentheses beside the subprogram declaration; sometimes called **formal parameters**

Arguments

Identifiers listed in parentheses on the subprogram call; sometimes called **actual parameters**

Subprogram Statements

(a) Subprogram A does its task and calling unit continues with next statement



Subprogram Statements

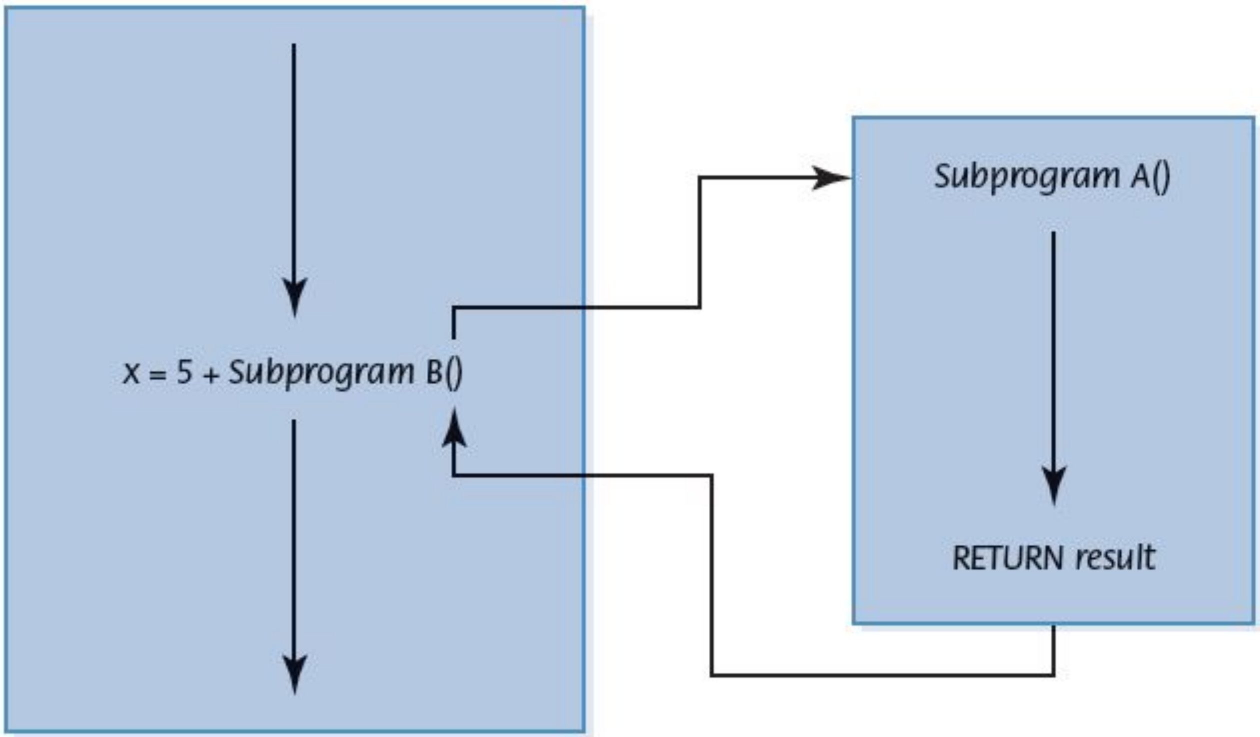


FIGURE 7.14 Subprogram flow of control

Recursion

Recursion

The ability of a subprogram to call itself

Base case

The case to which we have an answer

General case

The case that expresses the solution in terms of a call to itself with a smaller version of the problem

Recursion

For example, the factorial of a number is defined as the number times the product of all the numbers between itself and 0:

$$N! = N * (N - 1)!$$

Base case

Factorial(0) = 1 (0! is 1)

General Case

Factorial(N) = N * Factorial(N-1)

Recursion

```
Write "Enter n"
Read n
Set result to Factorial(n)
Write result + " is the factorial of " + n

Factorial(n)
IF (n equals 0)
     RETURN 1
ELSE
     RETURN n * Factorial(n-1)
```

Recursion

```
BinarySearch (first, last)
IF (first > last)
        RETURN FALSE
ELSE
        Set middle to (first + last)/ 2
        IF (item equals data[middle])
                RETURN TRUE
        ELSE
                IF (item < data[middle])
                        BinarySearch (first, middle – 1)
                ELSE
                        BinarySearch (middle + 1, last
```

Quicksort

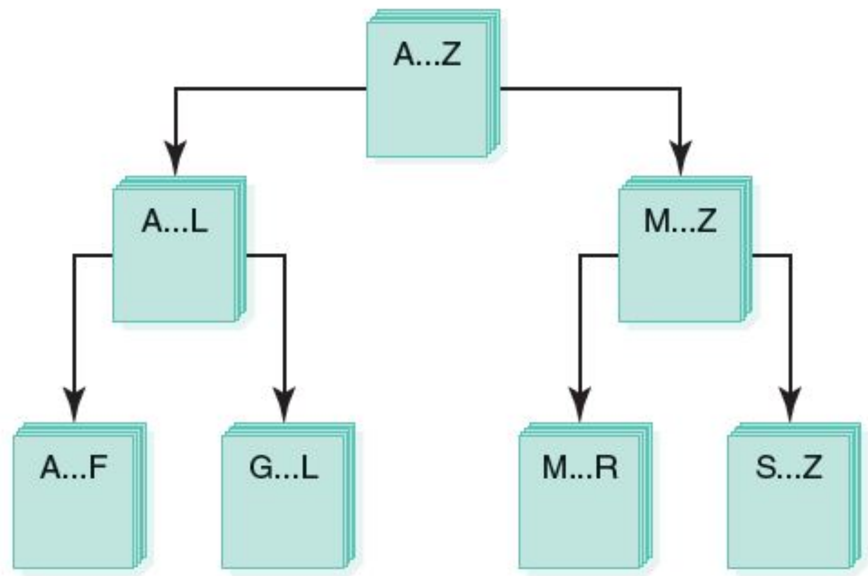


FIGURE 7.15 Ordering a list using the Quicksort algorithm

It is easier to sort a smaller number of items: Sort A…F, G…L, M…R, and S…Z and A…Z is sorted

Ordering a list using the Quicksort algorithm

Quicksort algorithm

With each attempt to sort the stack of data elements, the stack is divided at a splitting value, *splitVal*, and the same approach is used to sort each of the smaller stacks (a smaller case)

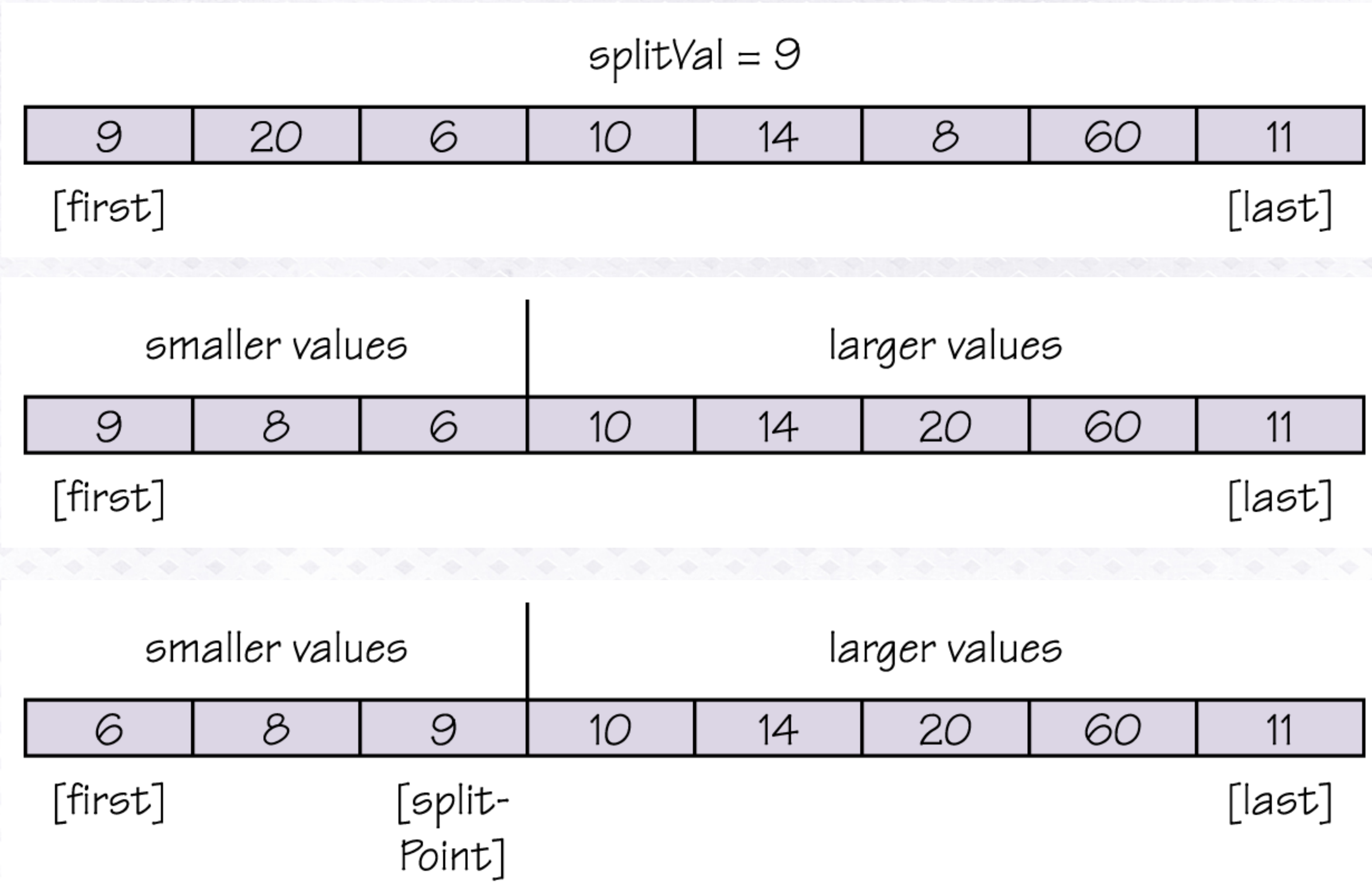
Process continues until the small stacks do not need to be divided further (the base case)

The variables *first* and *last* in Quicksort algorithm reflect the part of the array *data* that is currently being processed

Quicksort

```
Quicksort(first, last)
IF (first < last)              // There is more than one item
    Select splitVal
    Split (splitVal)           // Array between first and
                               //   splitPoint–1 <= splitVal
                               //  data[splitPoint] = splitVal
                               // Array between splitPoint + 1
                               //   and last > splitVal
    Quicksort (first, splitPoint - 1)
    Quicksort (splitPoint + 1, last)
```

Quicksort



splitVal = 9
9 20 6 10 14 8 60 11
[first]
[last]
smaller values
larger values
9 8 6 10 14 20 60 11
[first]
[last]
smaller values
larger values
6 8 9 10 14 20 60 11
[first]
[split-
Point]
[last]

Quicksort

Split(splitVal)

Set left to first + 1

Set right to last

WHILE (left <= right)

    Increment left until data[left] > splitVal OR left > right

    Decrement right until data[right] < splitVal

        OR left > right

    IF(left < right)

        Swap data[left] and data[right]

Set splitPoint to right

Swap data[first] and data[splitPoint]

Return splitPoint

Quicksort

(a) Initialization

9	20	6	10	14	8	60	11
[first]	[left]						[right]

(b) Increment *left* until list[*left*] > *splitVal* or *left* > *right*

9	20	6	10	14	8	60	11
[first]	[left]						[right]

(c) Decrement *right* until list[*right* > *splitVal* or *left* > *right*

9	20	6	10	14	8	60	11
[first]	[left]				[right]		

(d) Swap list[*left*] and list[*right*]; move *left* and *right* toward each other

9	8	6	10	14	20	60	11
[first]		[left]		[right]			

(e) Increment *left* until list[*left*] > *splitVal* or *left* > *right*
Decrement *right* until list[*right*] <= *splitVal* or *left* > *right*

9	8	6	10	14	20	60	11
[first]		[right]	[left]				

(f) *Left* > *right* so no swap occurs within the loop
Swap list[*first*] and list[*right*]

6	8	9	10	14	20	60	11
[first]		[right] (splitPoint)					

FIGURE 7.16 Splitting algorithm

Important Threads

Information Hiding

The practice of hiding the details of a module with the goal of controlling access to it

Abstraction

A model of a complex system that includes only the details essential to the viewer

Information Hiding and **Abstraction** are two sides of the same coin

Important Threads

Data abstraction

Separation of the logical view of data from their implementation

Procedural abstraction

Separation of the logical view of actions from their implementation

Control abstraction

Separation of the logical view of a control structure from its implementation

Important Threads

Identifiers

Names given to data and actions, by which

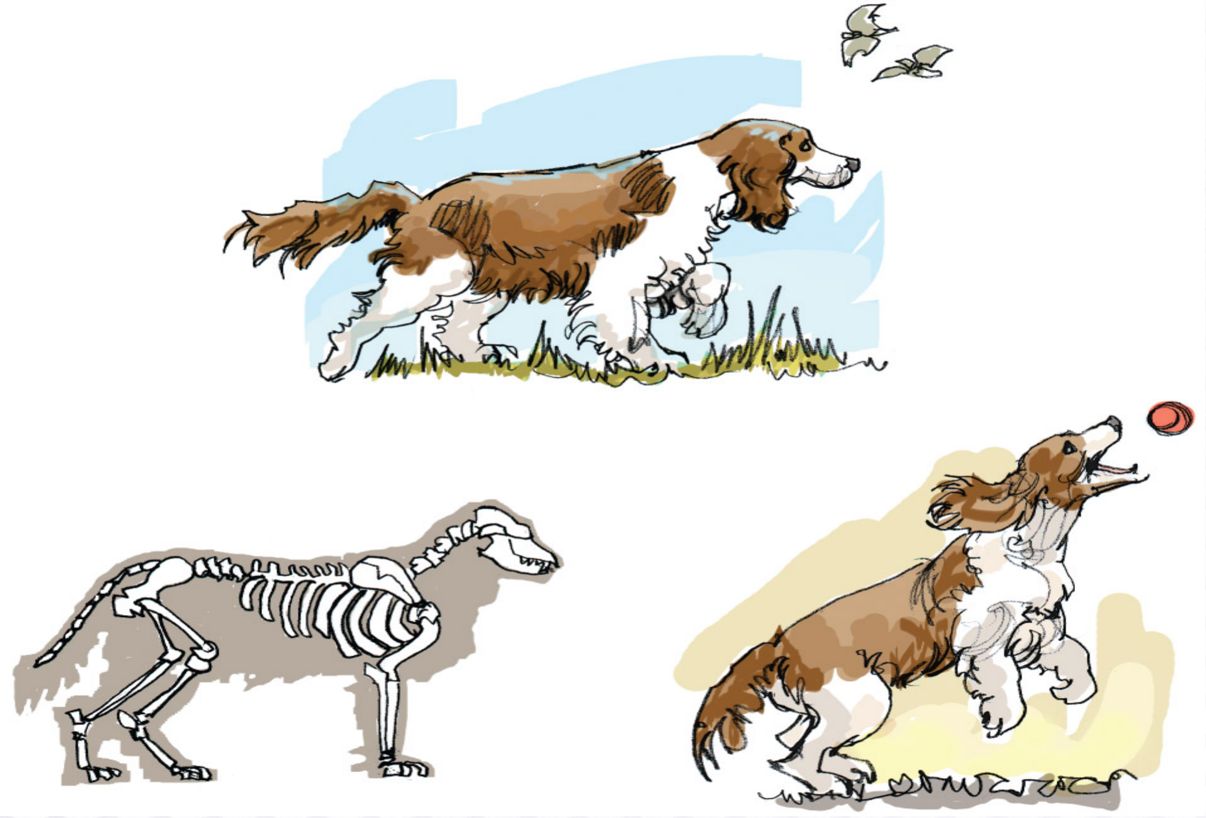
- we access the data and

  Read firstName, Set count to count + 1
- execute the actions

  Split(splitVal)

Giving names to data and actions is a form of abstraction

Important Threads



Abstraction is the most powerful tool people have for managing complexity!

Ethical Issues

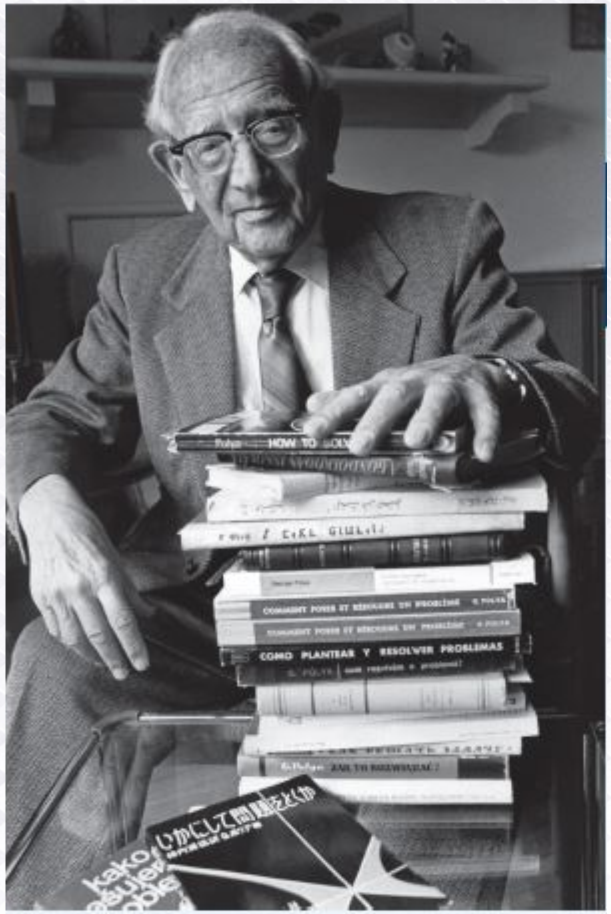
Open-Source Software Development

What are the advantages and disadvantages of open-source software?

What does the success of Linux suggest about the future of open-source software?

Should open-source software be licensed and subject to standard copyright laws?

Who am I?



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I am a mathematician. Why is my picture in a book about computer science?

Do you know?

?

What writing system did the Rosetta stone serve as a key to translating?

What did the National Intellectual Property Rights Coordination Center warn the American people about in 2013?

What is piggybacking? Is it ethical?

What parallels are there between philosophy and object oriented software engineering?