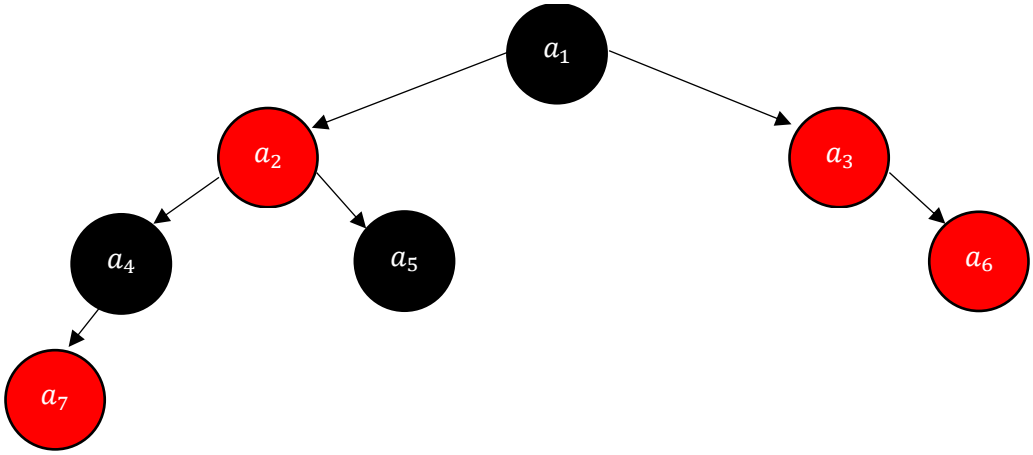
Red – Black Tree

RB Tree $= \{a_1, a_2, a_3, \ldots, a_n\}$

As in the other trees, the conditions of the order must be met, in addition to this, each node must have, these colors are red and black, finally, it must meet certain conditions given, it will be seen in the invariant



$$a_7 < a_4 < a_2 < a_2 < a_1 < a_3 < a_6$$

Note: The red and black trees have a black height, which will be denoted by $hb(x)$.

$$inv = \{root = color.black\}$$
$$inv = \ \forall\, node\ x \rightarrow \{node\ x \mid x = color.red \mid color.black\}$$
$$inv = \forall\, node.sheet\ (null)\ x\{node.sheet\ x \mid x = color.black\}$$
$$inv = \forall\, node.color.red\ x\{node\ x \mid x.sons = color.black\ \}$$
$$inv = \ \forall\, node\ x\ \{node\ x \mid x = hb(x)\}$$

Primitive Operations

RB Tree	…	RB Tree
Add	RB Tree x Key x Value	RB Tree
Delete	RB Tree x Key Value	RB Tree
Search	RB Tree x Key x Value	Node
Left Rotate	RB Tree x Node	RB Tree
Right Rotate	RB Tree x Node	RB Tree
Flip Colors	RB Tree x Node	RB Tree
Is Red	RB Tree x Node	RB Tree

RB Tree( ) : Constructor
Create the RB Tree
$pre = \{true\}$ $pos = \{RB\ Tree\ initialize\}$

Add(Key k, Value v) : Modifier
Add a new element in the RB Tree, this element will be added with red color and after will be rebalance according to the previous conditions
$pre = \{true, element\}$ $pos = \{root\ != null\ and\ new\ element\}$

Delete(Key k, Value v) : Modifier
Search a node with Key k and Value v, after this node is deleted
$pre = \{node\ x \in RBTree\}$ $pos = \{new\ order\ of\ the\ nodes\ and\ different\ colors\}$

Search(Key k, Value v) : Analyzer
Search a node with Key k and Value v
$pre\ \{node\ x \in RBTree\}$ $pos = \{node\ with\ the\ same\ conditions, Key = k, and\ Value = v\}$

Left Rotate(Node x) : Modifier
Rotate to the left to accommodate the colors.
$pre = \{true\}$ $pos = \{new\ order\ of\ the\ nodes\ and\ set\ colors\ in\ the\ nodes\}$

Right Rotate(Node x) : Modifier
Rotate to the right to accommodate the colors.
$pre = \{true\}$ $pos = \{new\ order\ of\ the\ nodes\ and\ set\ colors\ in\ the\ nodes\}$

Flip Colors(Node x) : Modifier
Change the colors of the nodes according to the invariant
$pre = \{true\}$ $pos = \{changed\ colors\ of\ the\ nodes\}$

Is Red(Node x) : Analyzer
Evaluate if the node is color red
$pre = \{node\ x\ != null\}$ $pos = \{true\ or\ false\ according\ its\ color\}$