

Lectures in Astroparticle Phenomenology

III. Putting it all together with global fits

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Slides available from
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Lecture Plan

Tuesday: (here) Particle Cosmology

- Λ CDM
- Power spectra of cosmological perturbations
- Reheating, Big Bang nucleosynthesis, cosmic strings

Yesterday (SIIfA Redfern): Dark Matter

- Theories
- Production
- Direct + indirect detection

Today: Global Fits

- Techniques, status and coming developments



Outline of Lecture 3

- 1 The Problem
- 2 Progress
 - Gamma-rays
 - Neutrinos
 - CMB constraints
- 3 Future Challenges
 - Respectable LHC likelihoods
 - Statistical/numerical issues (**optional**)
 - Parameter space → Theory space



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Searching for new physics

Many reasons to look for physics Beyond the Standard Model (BSM):

- Higgs mass (hierarchy problem + vacuum stability)
- Dark matter exists
- Baryon asymmetry
- Neutrino masses and mixings



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So what do we do about it?

- Make new particles at high- E colliders
- Study rare processes at high- L colliders
- Hunt for dark matter
- Look at cosmological observables (CMB, reionisation, etc)
- Look for kooky neutrino physics



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Combining searches I

Question

How do we know which models are in and which are out?



Combining searches I

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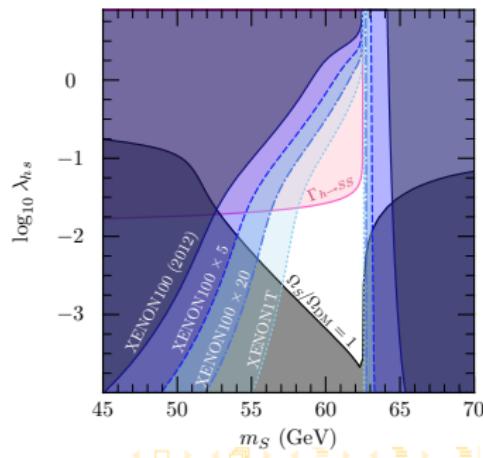
How do we know which models are in and which are out?

Answer

Combine the results from different searches

- Simplest method: take different exclusions, overplot them, conclude things are “allowed” or “excluded”
- Simplest BSM example: the scalar singlet model

(Cline, Kainulainen, PS & Weniger, *PRD*, 1306.4710)



Combining searches II

That's all well and good if there are only 2 parameters and few searches...

Question

What if there are many different **constraints**?



Combining searches II

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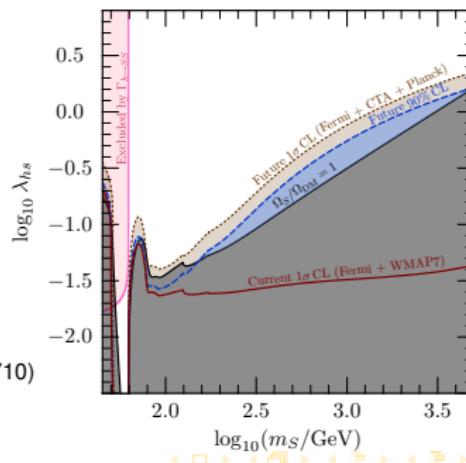
Question

What if there are many different **constraints**?

Answer

Combine constraints in a statistically valid way
→ composite likelihood

(Cline, Kainulainen, PS & Weniger, *PRD*, 1306.4710)



Combining searches III

That's all well and good if there are only 2 parameters and few searches...

Question

What if there are many **parameters**?



Combining searches III

That's all well and good if there are only 2 parameters and few searches...

Question

What if there are many **parameters**?

Answer

Need to

- scan the parameter space (smart numerics)
- interpret the combined results (Bayesian / frequentist)
- project down to parameter planes of interest (marginalise / profile)

→ **global fits**



Bayesian & Frequentist terminology [Statistical aside I]

Likelihood: probability of obtaining observed data D if model parameters Θ are correct

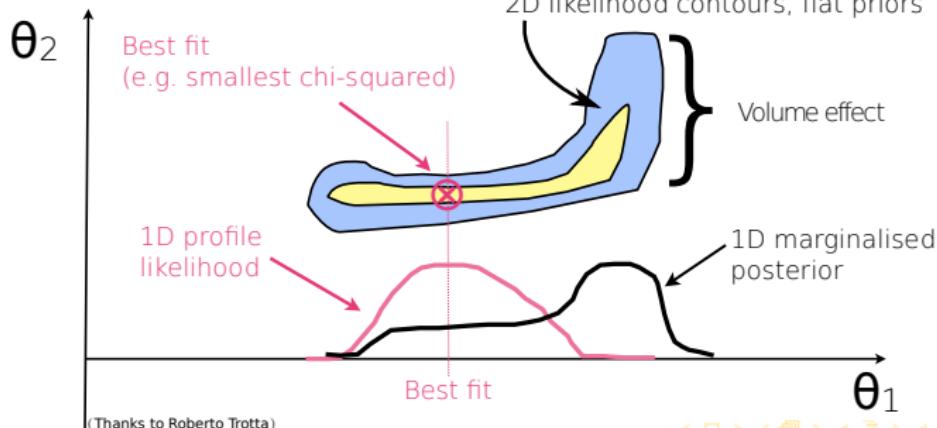
$$\mathcal{L}(D|\Theta) \quad (1)$$

Profiling: maximising the likelihood over a parameter you are not interested in

Posterior probability: probability of parameters Θ being correct given observed data D

$$P(\Theta|D) = \frac{\mathcal{L}(D|\Theta)P(\Theta)}{\mathcal{Z}(D)} \quad (2)$$

Marginalising: integrating the posterior over a parameter you are not interested in



BSM Model Scanning

Goals:

- ① Given a particular theory, determine which parameter combinations fit all experiments, and how well
- ② Given multiple theories, determine which fit the data better, and quantify how much better



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 \implies parameter estimation
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Why simple IN/OUT analyses are not enough...

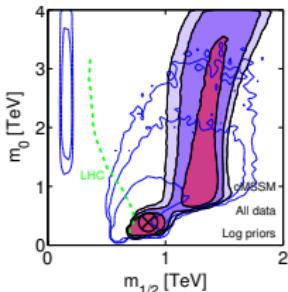
- Only partial goodness of fit, no measure of convergence, no idea how to generalise to regions or whole space.
- Frequency/density of models in IN/OUT scans is **not** proportional to probability \implies means essentially **nothing**.



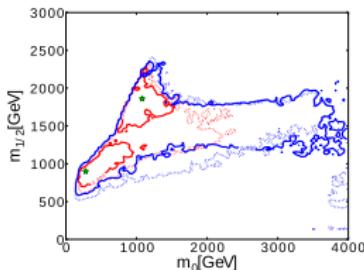
Know your (SUSY) parameter scans

Global fits:

Quantitative?
per-point: always
overall: always



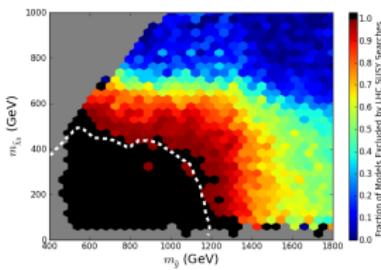
Strege et al *JCAP*, 1212.2636



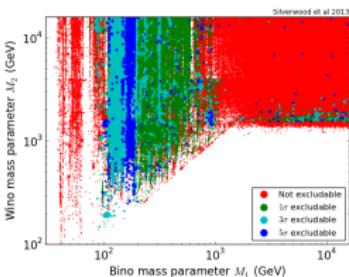
MasterCode, *EPJC*, 1207.7315

Not global fits:

Quantitative?
per-point: sometimes
overall: never



Cahill-Rowley et al, 1307.8444



Silverwood, PS, et al, *JCAP*, 1210.0844

Putting it all together

Issue 1: Combining fits to different experiments

Relatively easy – composite likelihood ($\mathcal{L}_1 \times \mathcal{L}_2 \equiv \chi_1^2 + \chi_2^2$ for simplest \mathcal{L})

- dark matter relic density from WMAP
- precision electroweak tests at LEP
- LEP limits on sparticle masses
- B -factory data (rare decays, $b \rightarrow s\gamma$)
- muon anomalous magnetic moment
- LHC searches, direct detection (only roughly implemented for now)



Putting it all together: global fits

Issue 2: Including the effects of uncertainties in input data

Easy – treat them as *nuisance parameters* and profile/marginalise

Issue 3: Finding the points with the best likelihoods

Tough – MCMCs, nested sampling, genetic algorithms, etc

Issue 4: Comparing theories

Depends – Bayesian model comparison, p values
(TS distribution? \longrightarrow coverage???)



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Two different approaches to including astro data in BSM scans

- ➊ Just use the published limits on $\langle \sigma v \rangle$ (or $\sigma_{\text{SI,SD}}$)
 - Fast – can cover large parameter spaces
 - Not so accurate – experimental limits are invariably based on theoretical assumptions, e.g. $b\bar{b}$ spectrum
 - Full likelihood function almost never available
- ➋ Use the data points directly in BSM scans
 - Slow – requires full treatment of instrument profile for each point
 - Accurate – can test each point self-consistently
 - Allows marginalisation over theoretical assumptions
 - Allows construction of full multi-dimensional likelihood function



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- ➌ (indirect only: use just flux upper limits)



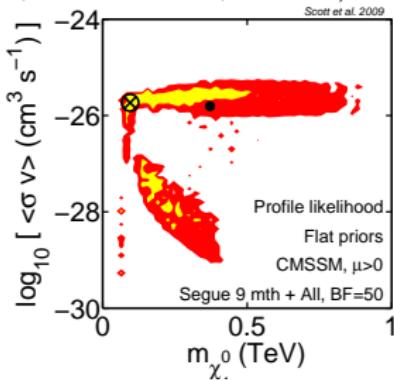
Gamma-rays

Gamma-ray annihilation searches in CMSSM global fits:

Fermi-LAT

Satellite pair conversion telescope
Dwarf galaxy Segue 1

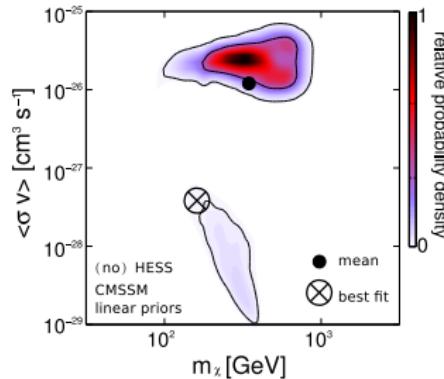
(PS, Conrad et al *JCAP*, 0909.3300)



HESS

Air Čerenkov telescope
Milky Way+Carina+Sculptor+Sag dwarf

(Ripken, Conrad & PS *JCAP*, 1012.3939)



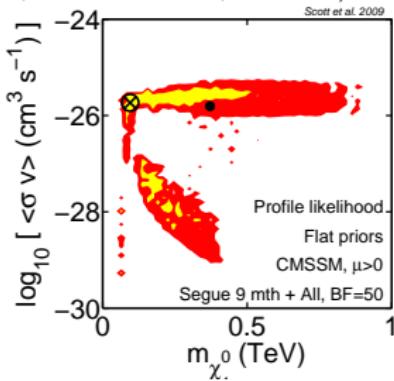
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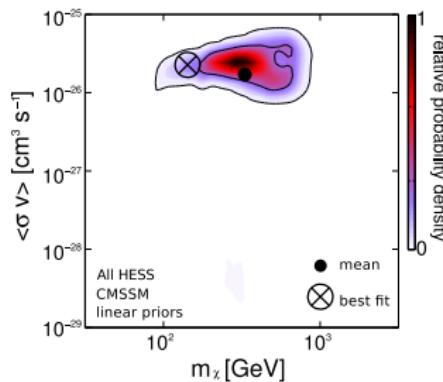
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Advanced IceCube Likelihood for Model Testing

Simplest way to do anything is to first make it a counting problem...

Compare observed number of events n and predicted number θ for each model, taking into account error σ_ϵ on acceptance:

$$\mathcal{L}_{\text{num}}(n|\theta_{\text{BG}} + \theta_{\text{sig}}) = \frac{1}{\sqrt{2\pi}\sigma_\epsilon} \int_0^\infty \frac{(\theta_{\text{BG}} + \epsilon\theta_{\text{sig}})^n e^{-(\theta_{\text{BG}} + \epsilon\theta_{\text{sig}})}}{n!} \frac{1}{\epsilon} \exp \left[-\frac{1}{2} \left(\frac{\ln \epsilon}{\sigma_\epsilon} \right)^2 \right] d\epsilon. \quad (3)$$

Nuisance parameter ϵ takes into account systematic errors on effective area, etc. $\sigma_\epsilon \sim 20\%$ for IceCube.



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Then: upgrade to full unbinned likelihood with number (\mathcal{L}_{num}), spectral ($\mathcal{L}_{\text{spec}}$) and angular (\mathcal{L}_{ang}) bits:

$$\mathcal{L} = \mathcal{L}_{\text{num}}(n|\theta_{\text{signal+BG}}) \prod_{i=1}^n \mathcal{L}_{\text{spec},i} \mathcal{L}_{\text{ang},i} \quad (4)$$

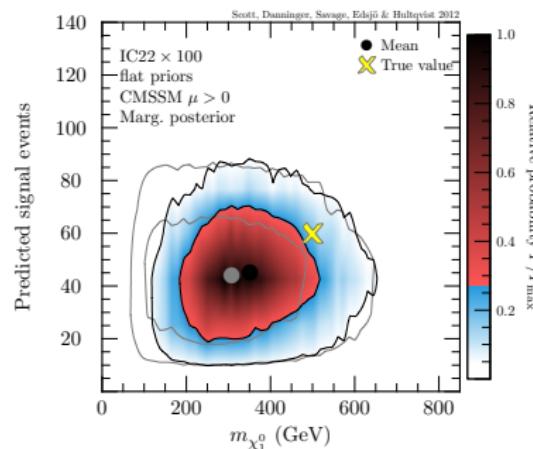
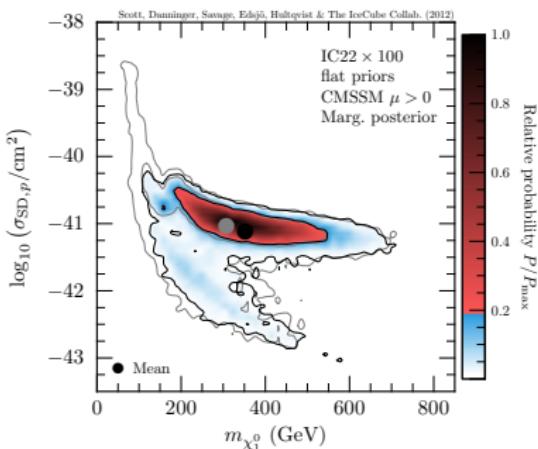
All available in DarkSUSY v5.0.6 and later: www.darksusy.org



CMSSM model reconstruction with IceCube event data

Benchmark recovery with 22-string IceCube WIMP-search neutrino events + full likelihood:

Mock signal: 60 events, $m_\chi = 500 \text{ GeV}$, 100% $\chi\chi \rightarrow W^+W^-$

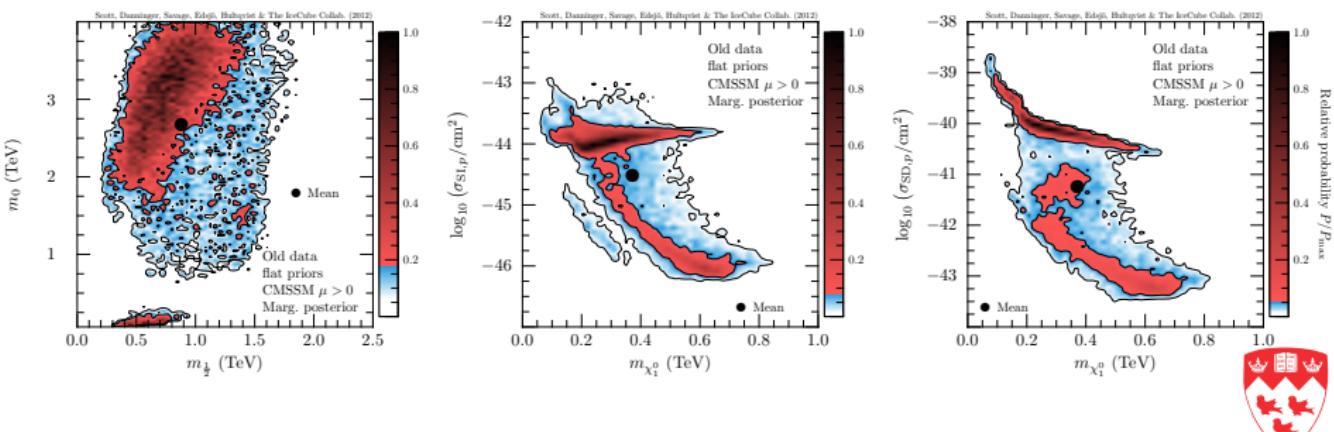


(PS, Savage, Edsjö & The IceCube Collaboration, *JCAP*, 1207.0810)



Example of Combined Direct + Indirect + LHC constraints

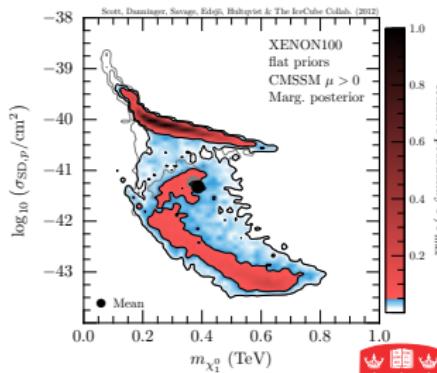
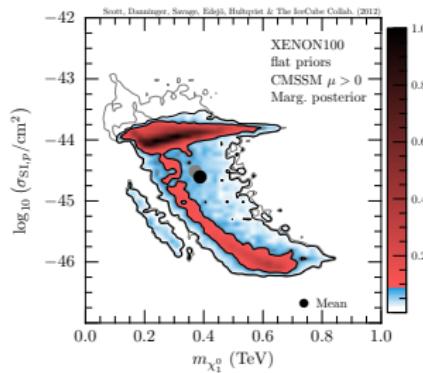
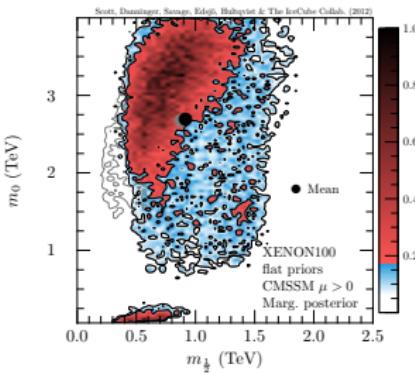
Base Observables



Example of Combined Direct + Indirect + LHC constraints

Base Observables + XENON-100 (2011)

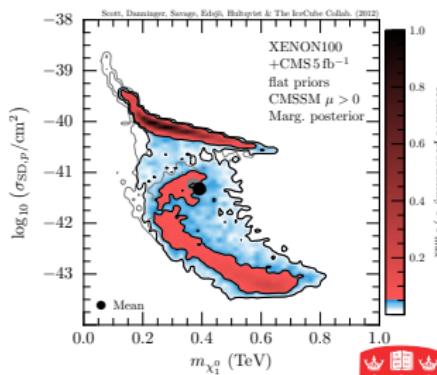
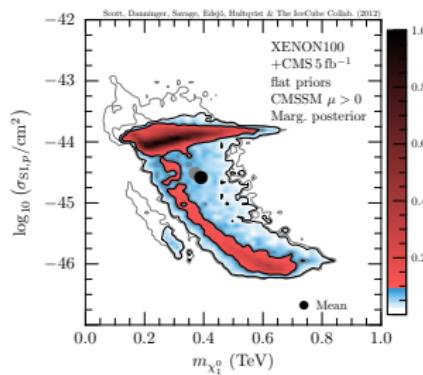
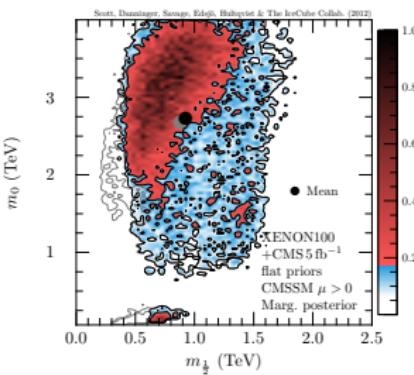
Grey contours correspond to Base Observables *only*



Example of Combined Direct + Indirect + LHC constraints

Base Observables + XENON-100 + CMS 5 fb⁻¹

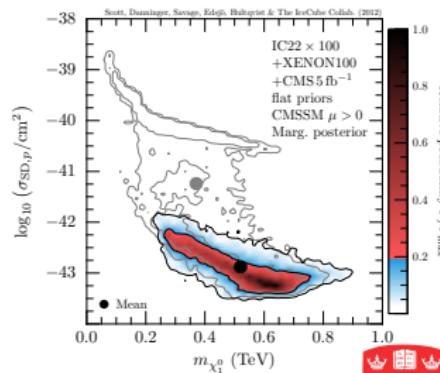
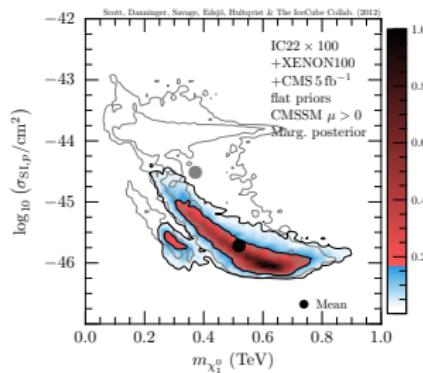
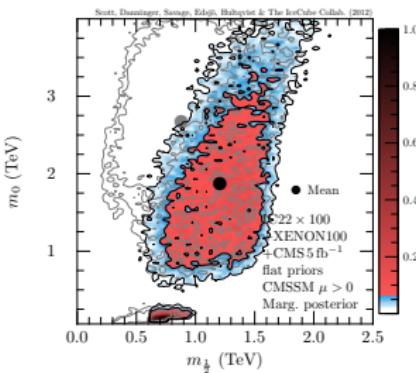
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Example of Combined Direct + Indirect + LHC constraints

Base Observables + XENON-100 + CMS 5 fb^{-1} + projected IC86-DeepCore

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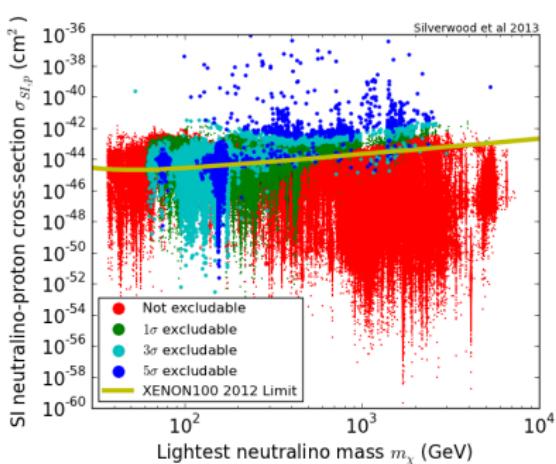
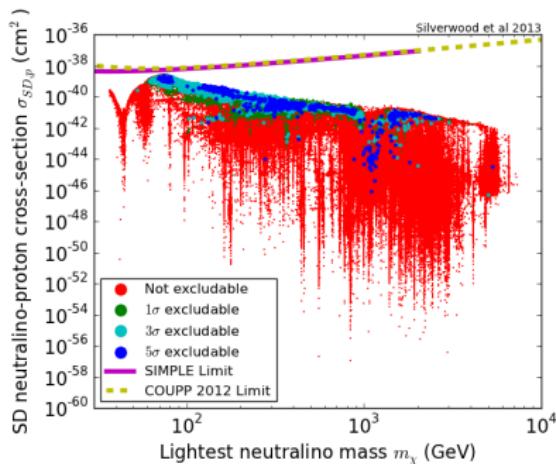


CMSSM, IceCube-22 with $100\times$ boosted effective area
(kinda like IceCube-DeepCore)



Prospects for detection in the MSSM-25

86-string IceCube vs Direct Detection (points pass $\Omega_\chi h^2$, $b \rightarrow s\gamma$, LEP)



(Silverwood, PS, et al, JCAP, 1210.0844)

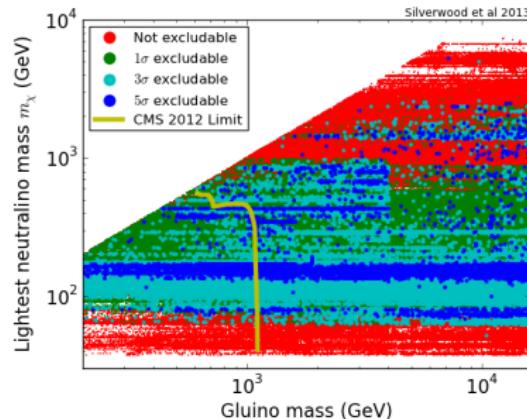
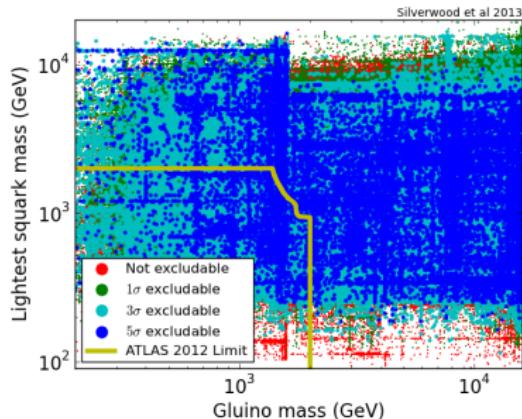
Many models that IceCube-86 can see are not accessible to direct detection...



Prospects for detection in the MSSM-25

86-string IceCube vs LHC (very naively)

SMS limits: 7 TeV, 4.7 fb^{-1} , jets + $E_{T,\text{miss}}$; 0 leptons (ATLAS), razor + M_{T2} (CMS)



(Silverwood, PS, et al, JCAP, 1210.0844)

Many models that IceCube-86 can see are also not accessible at colliders.



Generalised DM CMB likelihood functions

Simple CMB likelihood function, for

- Any combination of annihilation or decay channels
 - Any dark matter mass
 - Any decay lifetime/annihilation cross-section
- just requires interpolating one number in a table.



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f_{eff} for annihilation:

$$\ln \mathcal{L}(\langle\sigma v\rangle | m_\chi, r_i) = -\frac{1}{2} f_{\text{eff}}^2(m_\chi, r_i) \lambda_1 c_1^2 \left(\frac{\langle\sigma v\rangle}{2 \times 10^{-27} \text{cm}^3 \text{s}^{-1}} \right)^2 \left(\frac{\text{GeV}}{m_\chi} \right)^2 \quad (5)$$



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η for decay:

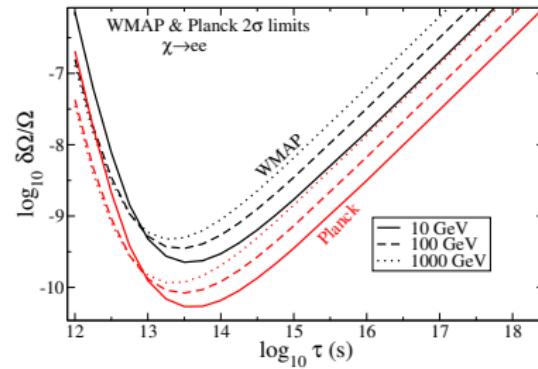
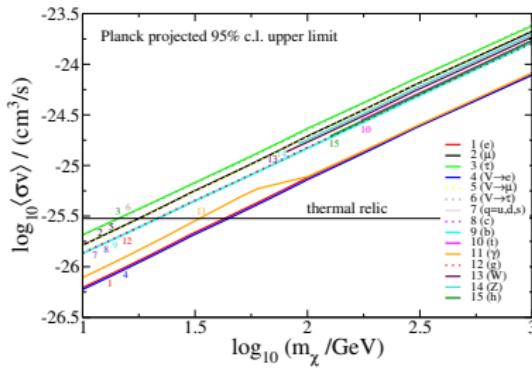
$$\ln \mathcal{L}(\tau | m_\chi, r_i) = -\frac{1}{2} \left(\frac{\delta\Omega}{\Omega_{\text{DM}} \tau} \right)^2 \eta^2(\tau, m_\chi, r_i) \quad (6)$$



Generalised DM CMB likelihood functions

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The LHC likelihood monster

Time per point:

$\mathcal{O}(\text{minute})$ in **best** cases



The LHC likelihood monster

Time per point:

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Time per point for global fits to converge:

$\mathcal{O}(\text{seconds})$ in **worst** cases



The LHC likelihood monster



Time per point:

$\mathcal{O}(\text{minute})$ in **best** cases

Time per point for global fits to converge:

$\mathcal{O}(\text{seconds})$ in **worst** cases

Challenge:

About 2 orders of magnitude too slow to actually include LHC data in global fits properly



Taming the LHC monster

Zeroth Order Response:

“Stuff it, just use the published limits and ignore the dependence on other parameters”



Taming the LHC monster

Zeroth Order Response:

“Stuff it, just use the published limits and ignore the dependence on other parameters”

Obviously naughty – plotted limits assume CMSSM, and fix two of the parameters

- Don't really know dependence on other parameters
- Don't have a likelihood function, just a line
- Can't use this at all for non-CMSSM global fits – e.g. MSSM-25

SuperBayeS



Taming the LHC monster

First Order Response:

“Test if things depend on the other parameters (hope not),
re-simulate published exclusion curve”



Taming the LHC monster

First Order Response:

“Test if things depend on the other parameters (hope not),
re-simulate published exclusion curve”

Not that great, but OK in some cases

- At least have some sort of likelihood this time
- Still a bit screwed if things do depend a lot on other parameters, but
- allows (potentially shaky) extrapolation, also to non-CMSSM models

Fittino, Mastercode



Taming the LHC monster

Second Order Response:

“That’s ridiculous. I’ve never met a calculation I can’t speed up.
There must be some way to have my cake and eat it too”



Taming the LHC monster

Second Order Response:

“That’s ridiculous. I’ve never met a calculation I can’t speed up.
There must be some way to have my cake and eat it too”

Maybe – this is the challenge.

- Interpolated likelihoods (how to choose nodes?)
- Neural network functional approximation (how to train accurately?)
- Some sort of smart reduction based on event topology?
- Something else?

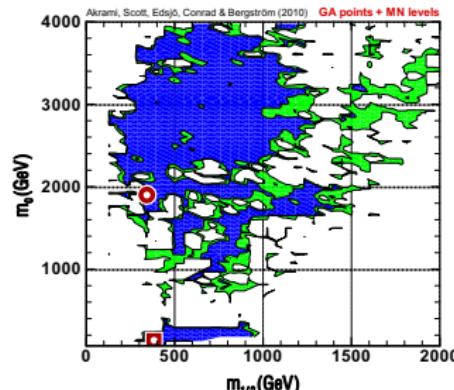
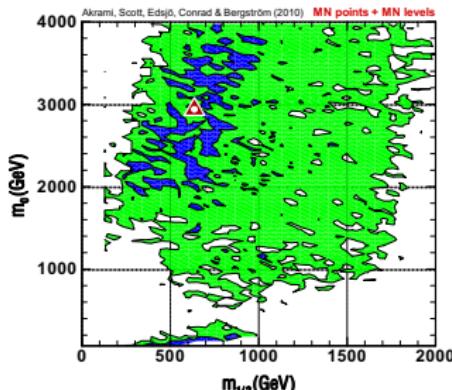
Balázs, Buckley, Farmer, White et al (1106.4613, 1205.1568);
GAMBIT



Scanning algorithms

Convergence remains an issue, especially for profile likelihood
Messy likelihood \implies best-fit point can be (and often is) easily missed
(Akrami, PS et al *JHEP*, 0910.3950, Feroz et al *JHEP*, 1101.3296)

- frequentist CLs are off, as isolikelihood levels are chosen incorrectly
- can impact coverage (overcoverage, or masking of undercoverage due to non- χ^2 TS distribution)
- need to use multiple priors and scanning algorithms (one optimised for profile likelihoods?)



Coverage [Statistical aside II]

Test statistic: a measure on data used to construct statistical tests (e.g. χ^2 , $\ln\mathcal{L}$, etc.)

Coverage: the percentage of the time that a supposed ‘ $x\%$ ’ confidence region actually contains the true value

- Distribution of the test statistic and design of the test it's used in determine coverage.
- p -value calculation *requires* the test statistic distribution to be well known.

We don't **really** know the distribution of our test statistic in BSM global fits, as it is too expensive to Monte Carlo

- coverage is rarely spot-on unless mapping from parameters to data-space is linear
(Akrami, Savage, PS et al *JCAP*, 1011.4297, Bridges et al *JHEP*, 1011.4306, Strege et al *PRD*, 1201.3631)
- p -value assessments of goodness of fit should be viewed with serious scepticism (→MasterCode)



CMSSM, SMS ≠ BSM

(SMS = Simplified Model Spectrum)

Want to do model comparison to actually work out which theory is right...

Challenge:

How do I easily adapt a global fit to different BSM theories?



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(SMS = Simplified Model Spectrum)

Want to do model comparison to actually work out which theory is right...

Challenge:

How do I easily adapt a global fit to different BSM theories?

Somehow, we must recast things quickly to a new theory

- data
- likelihood functions
- scanning code ‘housekeeping’
- even predictions

⇒ a new, very abstract global fitting framework



Hitting the wall

Issues with current global fit codes:

- Strongly wedded to a few theories (e.g. constrained MSSM / mSUGRA)
- Strongly wedded to a few theory calculators
- All datasets and observables basically hardcoded
- Rough or non-existent treatment of most experiments (astroparticle + collider especially)
- Sub-optimal statistical methods / search algorithms
- ⇒ *already hitting the wall on theories, data & computational methods*



GAMBIT: a second-generation global fit code

GAMBIT: Global And Modular BSM Inference Tool

Overriding principles of GAMBIT: flexibility and modularity

- General enough to allow fast definition of new datasets and theoretical models
- Plug and play scanning, physics and likelihood packages
- Extensive model database – not just small modifications to constrained MSSM (NUHM, etc), and not just SUSY!
- Extensive observable/data libraries (likelihood modules)
- Many statistical options – Bayesian/frequentist, likelihood definitions, scanning algorithms
- A smart and *fast* LHC likelihood calculator
- Massively parallel
- Full open-source code release



The GAMBIT Collaboration

22 Members, 13 Institutes

8 Experiments, 3 major theory codes



Fermi-LAT	J. Conrad, J. Edsjö, G. Martinez, P. Scott (leader)
CTA	C. Balázs, T. Bringmann, J. Conrad, M. White (dep. leader)
ATLAS	A. Buckley, P. Jackson, C. Rogan, A. Saavedra, M. White
IceCube	J. Edsjö, C. Savage, P. Scott
LHCb	N. Serra
HESS	J. Conrad
AMS-02	A. Putze
DARWIN	J. Conrad
Theory	C. Balázs, T. Bringmann, J. Cornell, L.-A. Dal, J. Edsjö, B. Farmer, A. Krislock, A. Kvellestad, F.N. Mahmoudi, A. Raklev, C. Savage, P. Scott, C. Weniger, M. White



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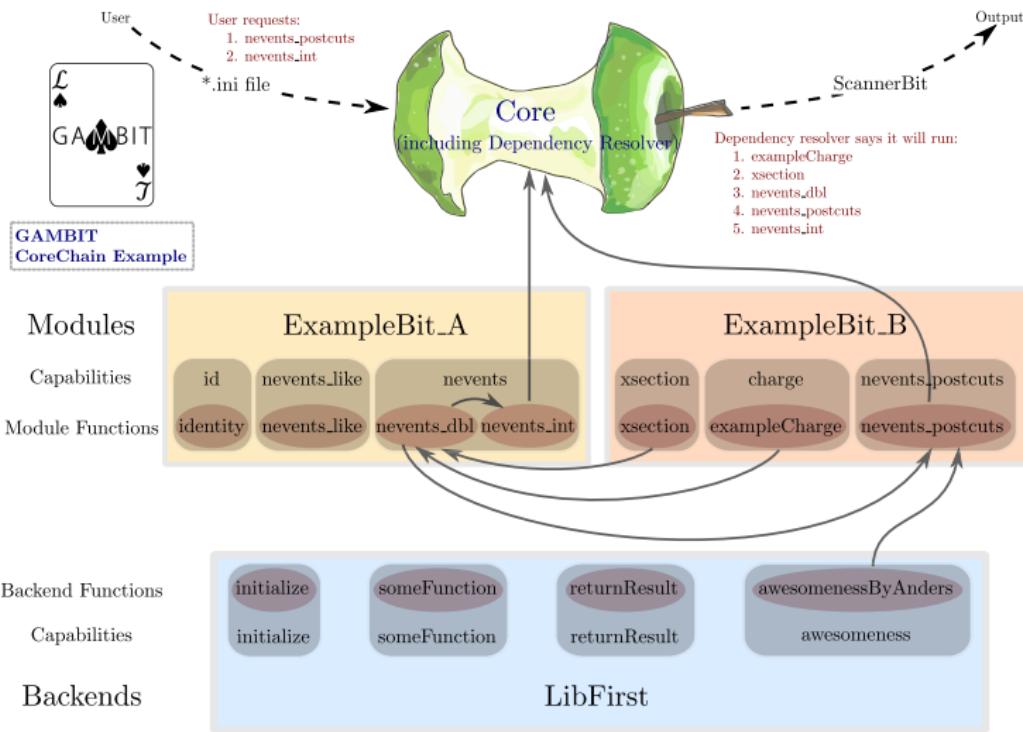
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GAMBIT: sneak peek



Closing remarks

- Robust analysis of dark matter and BSM physics requires multi-messenger global fits
- GAMBIT is coming:
 - Lots of interesting particle, astronomical, cosmological and astroparticle observables to include in global fits
 - Serious theoretical, experimental, statistical and computational detail to work through
 - Many opportunities for good students

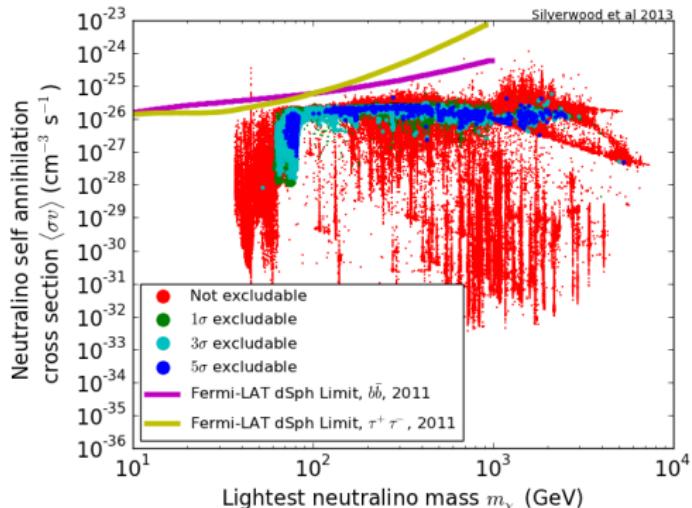


Outline of Lecture 3



Prospects for detection in the MSSM-25

86-string IceCube vs Gamma Rays



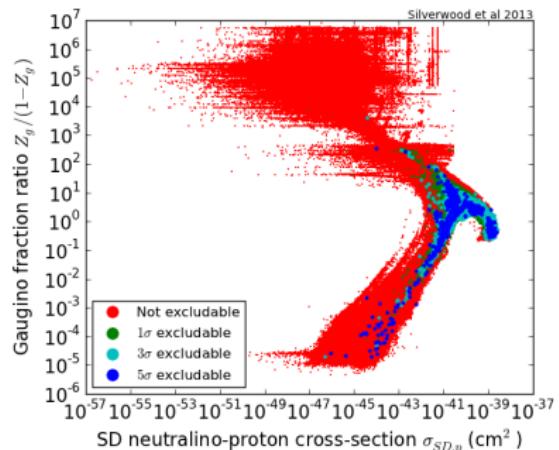
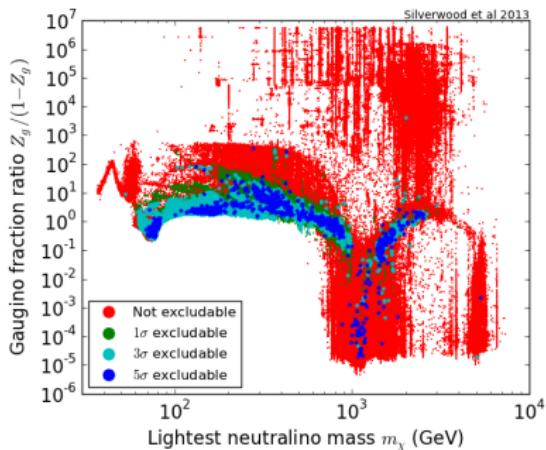
(Silverwood, PS, et al, JCAP, 1210.0844)

Many models that IceCube-86 can see are not accessible by other indirect probes...



Prospects for detection in the MSSM-25

Gaugino fractions



(Silverwood, PS, et al, JCAP, 1210.0844)

Mainly mixed models, a few Higgsinos

