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Note: contact me if any doubts w/ question or soln

Question 6

First look for λ

500 pages − 500 errors avg
1 page − 1 error avg
$\Rightarrow \lambda = 1$ per day

$$P(k \text{ misprints in a page}) = \frac{\lambda^k e^{-\lambda}}{k!} = \frac{e^{-1}}{k!} = \frac{1}{e\,k!}$$

$$\begin{aligned}P(\text{atleast } 3 \text{ misprints}) &= 1 - P(<3 \text{ misprints}) \\ &= 1 - P(k=2) - P(k=1) - P(k=0) \\ &= 1 - \frac{1}{2e} - \frac{1}{e} - \frac{1}{e} = \frac{2e-5}{2e}\end{aligned}$$

(Ans)

Question 17

(a) Let X_i be a R.V. which is 1 when k balls are assigned to the color i.

$$P(X_i = 1) = \left({}^{n}C_{k}\right)\left(\frac{1}{255}\right)^{k}\left(\frac{254}{255}\right)^{n-k}$$

Now, Let X = number of colors assigned to exactly x balls.

$$X = \sum_{i=1}^{255} X_i$$

$$E[X] = E\left[\sum X_i\right] = \sum_{i=1}^{255} E[X_i] \quad \text{(Linearity of expectation)}$$

$$= 255 \cdot E[X_1]$$

$$= 255 \cdot \left(1 \cdot {}^{n}C_{k}\left(\frac{1}{255}\right)^{k}\left(\frac{254}{255}\right)^{n-k} + 0\right)$$

(b) $Y_i = 1$ if ith color assigned to more than 1 ball.

Y = no. of colors assigned to >1 ball.

$$= \sum_{i=1}^{255} Y_i$$

$$E[Y] = 255 \cdot E[Y_i]$$

$$= (255)\left[1 - n\frac{(254)^{n-1}}{(255)^{n}} - \frac{(254)^{n}}{(255)^{n}}\right]$$

$P(Y_i = 1)$

$= 1 - P$(ith color assigned to 1 ball) $- P$(ith color assigned to 0 balls)

$$= \left[1 - n \cdot \left(\frac{1}{255}\right)\left(\frac{254}{255}\right)^{n-1} - \left(\frac{254}{255}\right)^{n}\right]$$

(c) From (b),

$$E[Y] = 255 \cdot E[Y_i]$$

$$= (255)\left[1 - n\frac{(254)^{n-1}}{(255)^n} - \frac{(254)^n}{(255)^n}\right] \geq 1 \qquad \text{Let } c = \frac{254}{255}$$

$$\Rightarrow \quad 255 - n\,c^{n-1} - 255\,c^n \geq 1$$

Just use Wolfram Alpha / Python at this point to get the ans. :)

$n = \underline{\underline{23}}$.