Part (A)

→ Replacing bulb 3 times means that he used 4 bulbs - 2 of type A and 2 of type B.

→ $E[\text{total lifetime}] = E[8 \times (\text{life A}) + 8 \times (\text{life B})]$

$= 2\,E[\text{life A}] + 2\,E[\text{life B}]$ (linearity of expectation)

$= 0.5 + 1 = 1.5$ years

Part (b)

⊛ $X_n = p E_{n,A} + (1-p) E_{n,B}$

⊛ $E_{n,A} = l_A + p E_{n-1,A} + (1-p) E_{n-1,B}$

⊛ $E_{n,B} = l_B + \underbrace{p E_{n-1,A} + (1-p) E_{n-1,B}}_{X_{n-1}}$

① $E_{0,A} = l_A$, $E_{0,B} = l_B$, $X_0 = p l_A + (1-p) l_B$

② $E_{1,A} = l_A + X_0$, $E_{1,B} = l_B + X_0$

$X_1 = p l_A + p X_0 + (1-p) l_B + (1-p) X_0$

$\Rightarrow X_1 = 2X_0$

③ $E_{2,A} = l_A + 2X_0$, $E_{2,B} = l_B + 2X_0$,

$X_2 = 3X_0$

⋮

(n) $E_{n,A} = l_A + nX_0$, $E_{n,B} = l_B + nX_0$

here $X_0 = p l_A + (1-p) l_B$.

NOTES

- E(n, A) is the expected total illumination time given you start with bulb A and would do exactly `n` replacements.
- E(n, B) is the expected total illumination time given you start with bulb B and would do exactly `n` replacements.
- l_A is the expected lifetime of a single bulb of type A (=0.25 years)
- l_B is the expected lifetime of a single bulb of type B (=0.5 years)
- This question has nothing to do with the bulb lifetime being a Poisson variable, that's just to trick you.