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6. $E[g(x)] \geq g(E[x])$ $\Rightarrow$ for convex G, & finite values of LHS & RHS

Jensen's Inequality.

Let's try to take $g(x) = \ln\sqrt{X}$

$g'(X) = \frac{1}{\sqrt{X}}\left(\frac{1}{2}X^{-1/2}\right) = \frac{1}{2X}$

$g''(X) = \frac{-1}{2X^2}$ which is -ive when X is +ive

So if we take $g(x) = -\ln\sqrt{X}$, g'' will be +ive everywhere

$\therefore E[-\ln\sqrt{X}] \geq -\ln\sqrt{E(X)}$

$\hookrightarrow -\ln\sqrt{10}$ which is finite

Also LHS is finite as $\ln(\sqrt{X}) \leq X$ as:

For $X > 1$, $\ln(\sqrt{X}) \leq \sqrt{X} \leq X$

For $0 < X \leq 1$, $\ln(\sqrt{X}) \leq 0 \leq X$

$\therefore -E[\ln\sqrt{X}] \geq -\ln\sqrt{E[X]}$

$\therefore$ $\boxed{E[\ln\sqrt{X}] \leq \ln\sqrt{10}}$