QUESTION ONE

$$CDF(Y=y) = P(Y \le y)$$
$$= P(aX+b \le y)$$
$$= P\left(X \le \frac{(y-b)}{a}\right)$$
$$= CDF\left(X=\frac{(y-b)}{a}\right)$$

This assumes a > 0

Therefore, $PDF(Y=y) = \frac{1}{a} PDF\left(X = \frac{y-b}{a}\right)$

$$\Rightarrow f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right)$$ for $a>0$.

Similarly, for $a<0$, $f_Y(y) = \frac{-1}{a} f_X\left(\frac{y-b}{a}\right)$

Overall, $\boxed{f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)}$ for non-zero a

(b) $f_Y(y) = \frac{1}{|a|} \lambda e^{-\lambda(y-b)/a}$ — 1

If $f_Y(y)$ is exponential,

$$\Rightarrow f_Y(y) = \begin{cases} \mu e^{-\mu x} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$ — 2

for some μ.

Comparing 1 and 2,

$\underline{\underline{b=0}}$ and $\underline{\underline{a>0}}$ $\left(\text{So}, \mu = \frac{\lambda}{a}\right)$

(c) $f_Y(y) = \frac{1}{|a|} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\left(\frac{y-b}{a} - \mu\right)^2 / 2\sigma^2\right)$

$= \frac{1}{\sqrt{2\pi}\,(\sigma|a|)} \exp\left(-\frac{(y-b-a\mu)^2}{2(\sigma|a|)^2}\right)$

This is a Normal R.V. with
mean $= b + a\mu$
variance $= \sigma^2 a^2$
hence no special condition necessary.

NOTE:
Some of you have integrated the f_Y(Y) and set the integral value to 1. It is a necessary condition, but it is NOT sufficient enough to guarantee that Y is an exponential or normal R.V.

Some students have written Y>=b as condition for exponential, however, we cannot impose any condition on the R.V Y, by definition, it will be from -infinity to +infinity.