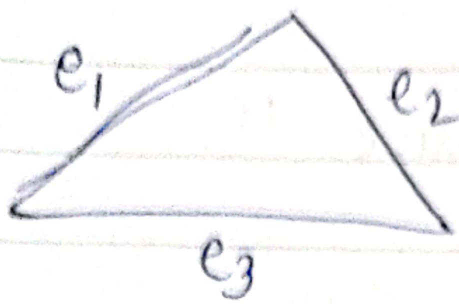
Q.14 a. Counter example



$$P(X_{e_1}=1 \cap X_{e_2}=1 \cap X_{e_3}=1)$$

$$= \frac{1+1+1}{3\times3\times3} = 1/9$$

$$P(X_{e_1}=1) \times P(X_{e_2}=1) \times P(X_{e_3}=1)$$

$$= \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} = 1/27$$

$$P(A \wedge B \wedge C) \neq P(A)\,P(B)\,P(C)$$

Hence, Proved.

[1.5 Points]

b. $Y = |E \setminus E(a)|$

$$= |E - \{E(a)\}|$$

$$E[Y] = \underbrace{E[|E|]}_{\text{constant}} - E[E(a)]$$

(Using Linearity of Expectation)

$$= |E| - E[E(a)]$$

$$= |E| - |E| \times E[X_e]$$

(because $E(a)$ is the set of all ~~edjes~~ edges that are monochromatic, hence its expectation will be the ~~su~~ ~~expectation~~ ~~of~~ sum of expectation of all edges to be monochromatic)

$$= |E| [1 - E[X_e]]$$

$$E[X_e] = P(X_e = 1) \times 1 + P(X_e = 0) \times 0$$

$$= \frac{1}{3} \times 1 + 0$$

$$= 1/3$$

$$E[Y] = |E| [1 - 1/3]$$

$$\boxed{= \frac{2|E|}{3}}$$

[3 Points]

6 c. Any valid explanation that states that atleast one assignment should have greater than $\frac{2|E|}{3}$ edges non monochromatic so that the average comes out to be $\frac{2|E|}{3}$.

Average cannot be x if all elements are less than x.

(1.5 Points)