Note that by linearity of expectation, we have

$$\mathbf{EY} = \mathbf{AEX} + \mathbf{b}.$$

By definition, we have

$$\begin{aligned}
\mathbf{C_Y} &= \mathbf{E}[(\mathbf{Y} - \mathbf{EY})(\mathbf{Y} - \mathbf{EY})^{\mathbf{T}}] \\
&= \mathbf{E}[(\mathbf{AX} + \mathbf{b} - \mathbf{AEX} - \mathbf{b})(\mathbf{AX} + \mathbf{b} - \mathbf{AEX} - \mathbf{b})^{\mathbf{T}}] \\
&= E[\mathbf{A}(\mathbf{X} - \mathbf{EX})(\mathbf{X} - \mathbf{EX})^{\mathbf{T}}\mathbf{A}^{\mathbf{T}}] \\
&= \mathbf{AE}[(\mathbf{X} - \mathbf{EX})(\mathbf{X} - \mathbf{EX})^{\mathbf{T}}]\mathbf{A}^{\mathbf{T}} && \text{(by linearity of expectation)} \\
&= \mathbf{AC_X A^T}.
\end{aligned}$$