Date ___/___/___

The random variable N is equal to the number of successive interarrival intervals that are smaller than τ. Interarrival intervals are independent and each one is smaller than τ with probability $1 - e^{-\lambda\tau}$.

$\therefore$ $P(N=0) = e^{-\lambda\tau}$

$P(N=1) = e^{-\lambda\tau}(1-e^{-\lambda\tau})$

$P(N=k) = e^{-\lambda\tau}(1-e^{-\lambda\tau})^{k}$

So N has a distribution similar to a geometric one, with parameter $p = e^{-\lambda\tau}$

except that it shifted one place to the left, so that it starts out at 0. Hence,

$$E[N] = \frac{1}{p} - 1 = e^{\lambda\tau} - 1$$