A2 Q12

29 September 2020 23:45

Question 12

You are going from College (Point A) to some distant eastern part of Hyderabad (Point B) which is 25 kms away. While your friend starts from (Point B) towards (Point A) with the goal of meeting you. Both of you travel at 50 km/h towards each other. Both your starting time is truly random and uniformly distributed from 1 pm to 2 pm and both your starting time is independent of each other. Let the random variable X denote the distance between college and the point where both of you meet. Find F_X i.e. $P(X \leq x)$.

Ans Let the starting time of both be S_1 and S_2.

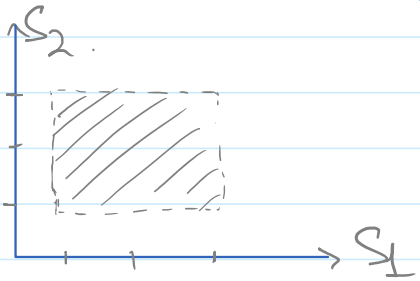
Now if $S_1 > S_2 + 0.5$, you will meet at point A itself since friend can travel 25 kms in 30 mins.

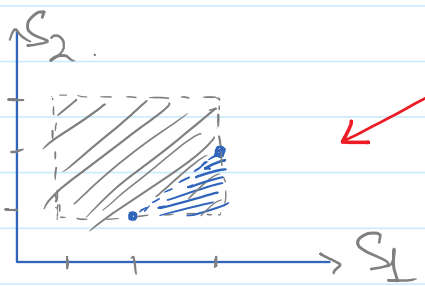
Similarly for all cases when $S_2 > S_1 + 0.5$, you will meet at point B.

The sample space for this would be

$\Omega = [0,1] \times [0,1]$

Since both can start any time in this interval.



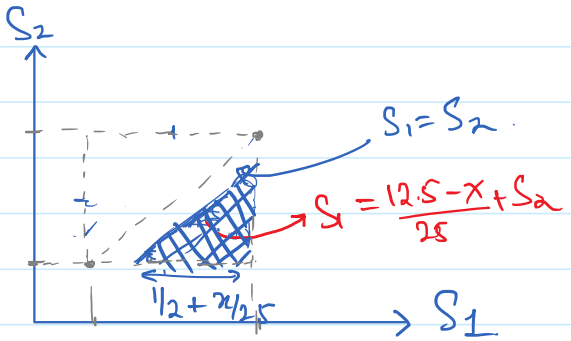


Example

Case: $S_2 < S_1 - 1/2$.

$X = 0, \quad F_X(0)$

$$P(X \le 0) = \frac{Ar(Blue)}{Ar(grey)} = \boxed{\frac{1}{8}}$$



$$S_2 \in [S_1 - 1/2, S_1]$$

$$x = \frac{25 - v(S_1 - S_2)}{2}$$

$x \in [0, 12.5)$

$$(S_1 - S_2) = \frac{1}{2} - \frac{x}{25}$$

$$\Rightarrow \quad S_2 = S_1 - \frac{1}{2} + \frac{x}{25}$$

$$P(X \le x) = \frac{Ar(Blue)}{Ar(Grey)} = \left(\frac{12.5 + X}{25}\right)^2 \frac{1}{2} = \boxed{\left(\frac{1}{2} + \frac{x}{25}\right)^2 \cdot \frac{1}{2}}$$

Similarly

when $S_2 \in [S_1, S_1 + 1/2]$

$x \in [12.5, 25]$

$$x = \vec{v}(S_2 - S_1) + \frac{25 - \vec{v}(S_2 - S_1)}{2}$$

$$x = \frac{25 + \vec{v}(S_2 - S_1)}{2}$$

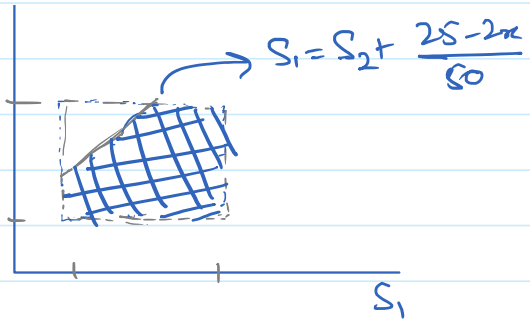
$$S_1 = \frac{25 - 2x}{50} + S_2$$

$$\Rightarrow \quad S_1 = \frac{1}{2} - \frac{x}{25} + S_2$$

S_2

$\rightarrow S_1 = S_2 + \frac{25 - 2x}{50}$

2 25



$$P(X \le x) = 1 - \left(\frac{3}{2} - \frac{x}{25}\right)^2 \cdot \frac{1}{2}$$

$\forall x \in [12.5, 25)$

Finally,

$$F_x(x) = P(\{X \le x\}) = \begin{cases} 0 & x \in (-\infty, 0) \\ \left(\frac{1}{2} + \frac{x}{25}\right)^2 \cdot \frac{1}{2} & x \in [0, 12.5) \\ 1 - \left(\frac{3}{2} - \frac{x}{25}\right)^2 \cdot \frac{1}{2} & x \in (12.5, 25) \\ 1 & x \in [25, \infty) \end{cases}$$