Doubts

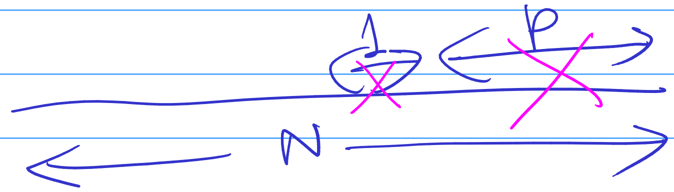
N doors, N−1 goats, 1 car

host reveals 'p' wrong doors

P(win if you switch)

$$= P\left(\text{choose a goat initially}\right) \times P\left(\frac{\text{choose car}}{\text{given prev was goat}}\right)$$

→ $$P(\text{win on switch}) = \left(\frac{N-1}{N}\right) \times \left(\frac{1}{N-p-1}\right)$$



→ N is very large, $p = N-2$, almost always win on switch

$$P(\text{win if not switch}) = \frac{1}{N}$$

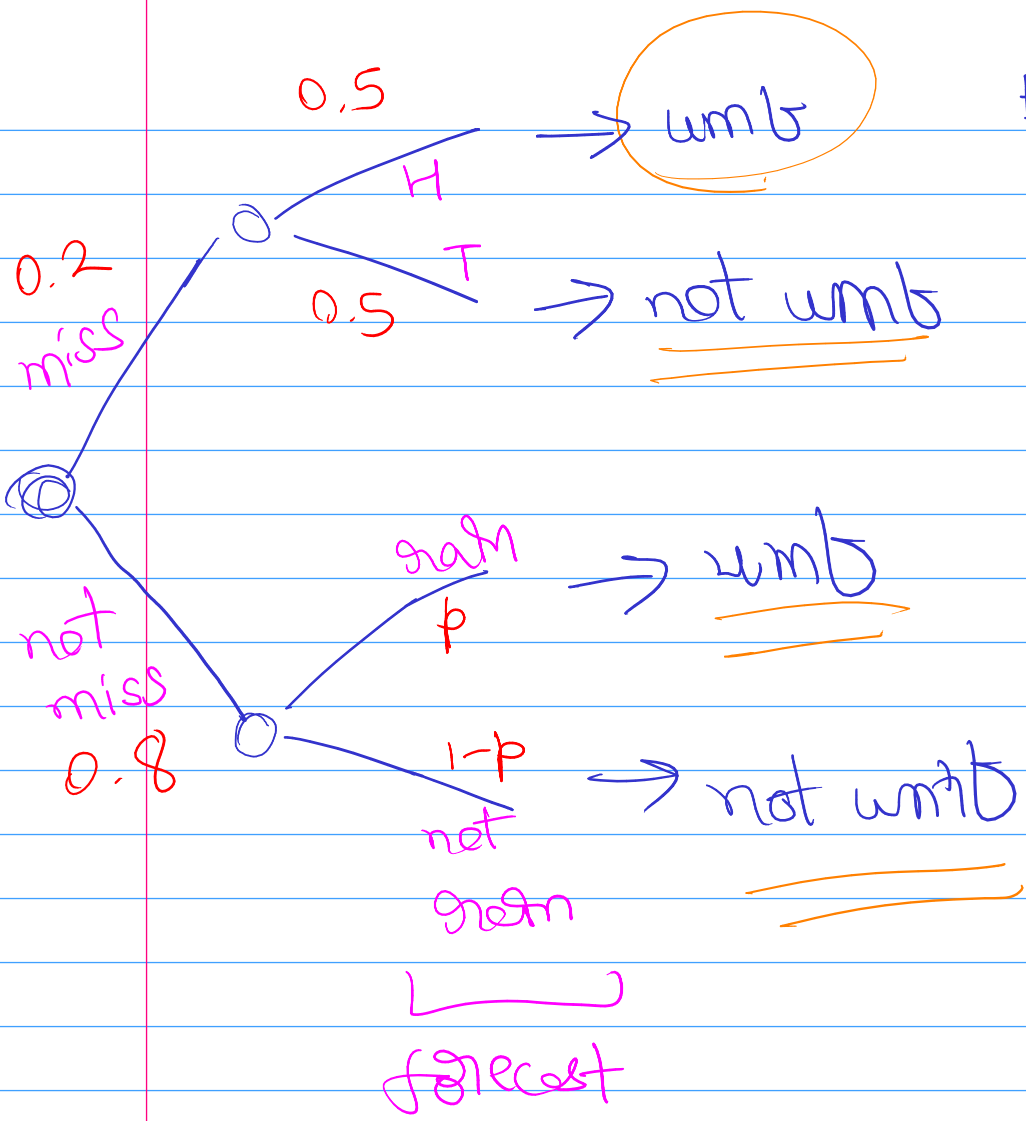
$\Rightarrow$ Always better to switch (left as exercise)

Question 2

(a)

$$P\left(\frac{\text{forecast} = \text{rain}}{\text{Rain actual}}\right) = \frac{0.7 \times 0.8}{0.7 \times 0.8 + 0.3 \times 0.1}$$

$$= \frac{P(\text{act rain} \mid f = \text{rain})\, P(\text{f rain})}{P(\text{act rain})}$$



Event A = umb

$= 0.2 \times 0.5 + 0.8p$

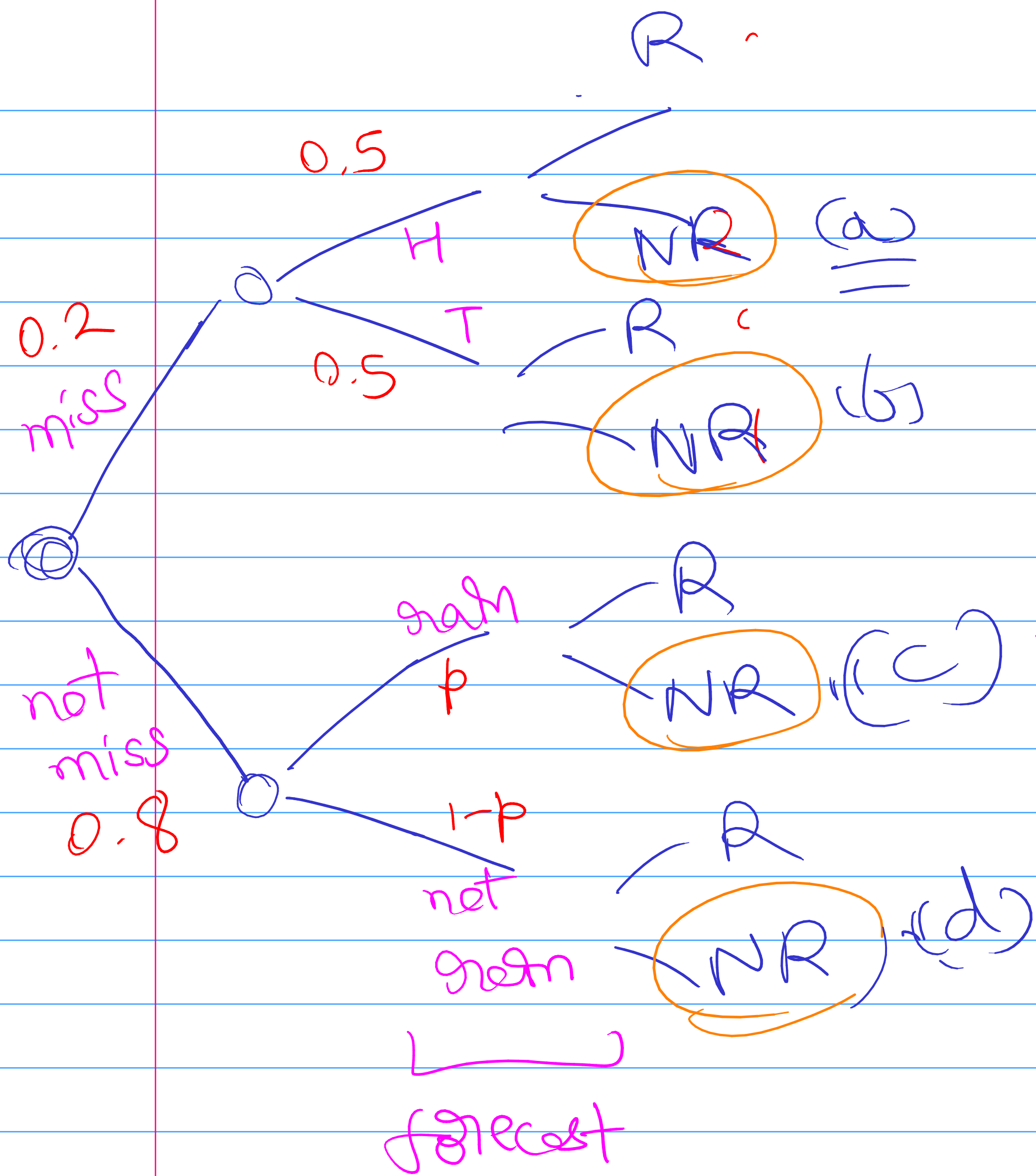
Event B = no rain

$= 1 - p$

$$P(A \cap B) = 0.2 \times 0.5 \neq P(A)P(B)$$

dependent

→ $p = 0$ independent



summer ↓

$= \underset{①}{0.8} \times \underset{②}{0.2} \times \underset{③}{0.2}$

$P\left(\dfrac{\text{saw forecast}}{\text{not raining} \cap \text{carryumb}}\right)$

$P(NR \cap \text{carryumb})$

$= (c) + (a)$

P(not rain on any jump | saw forecast)

= (c)

P(saw forecast) = 0.8

Question 5

A = event that n consecutive red balls

B = " " (n+1) " " "

$$P\left(\frac{A}{\text{basket } k}\right) = \frac{k}{N} \times \frac{k}{N} \times \frac{k}{N} \times \ldots$$

$$= \left(\frac{k}{N}\right)^n$$

k red

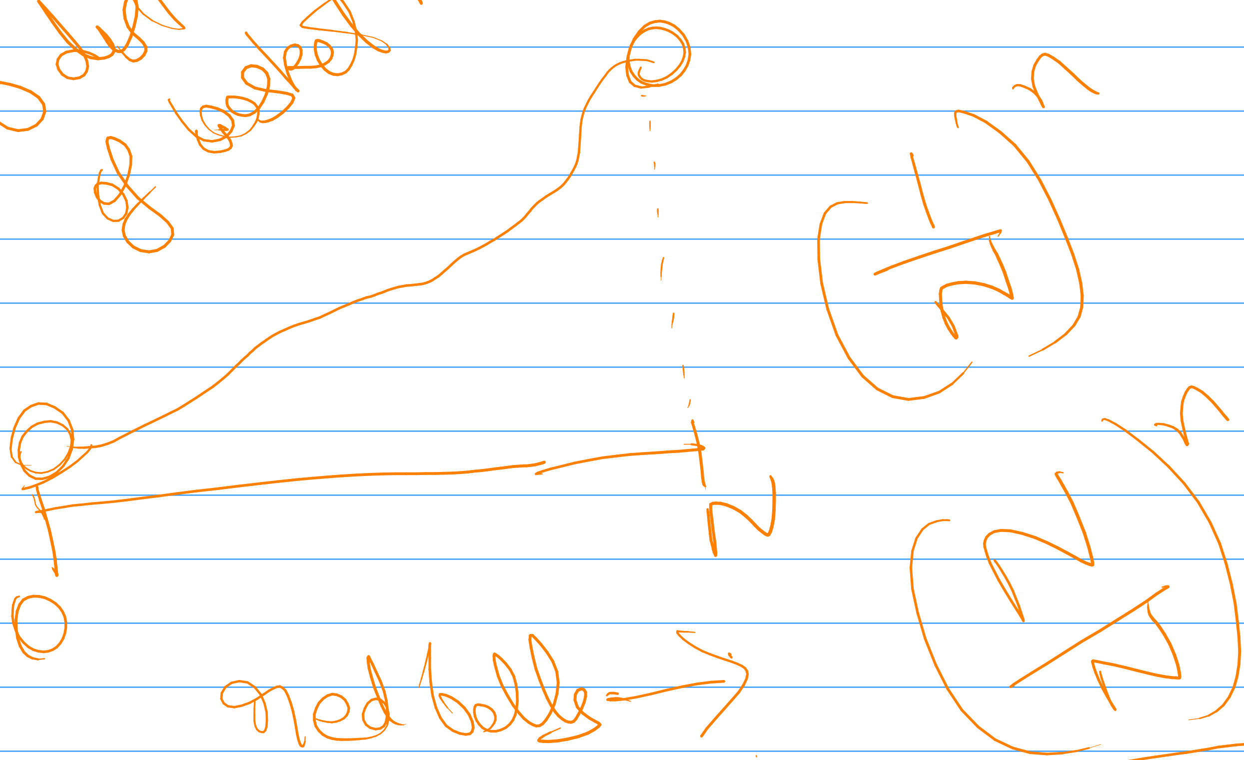
(N-k) blue balls

$$P(A) = \sum_{k=0}^{N} P(A \mid \text{basket } k)\, \underbrace{P(\text{basket } k)}_{= \frac{1}{N+1}}$$

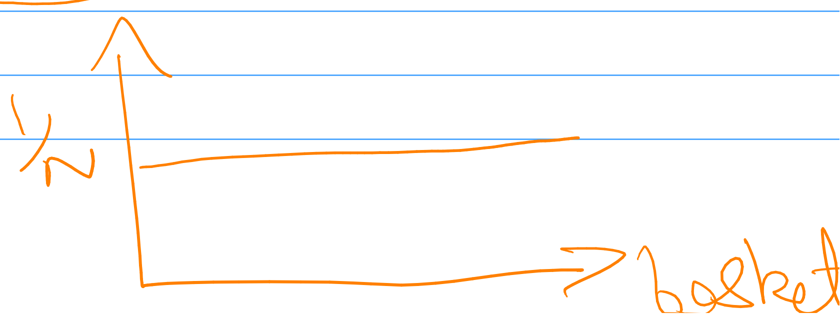
$$= \frac{1^n + 2^n + \dots + N^n}{N^n (N+1)}$$

After event A

prob distribution of basket N



Before event A



$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$P(B \cap A) = \frac{\dfrac{1^{n+1} + 2^{n+1} + \dots + N^{n+1}}{N^{n+1}(N+1)}}{P(A)}$$

$$P\{A\} = \frac{1}{N^{n}(N+1)} \int_0^N x^n \, dx$$

$$P(B|A) \approx \frac{n+1}{n+2} \quad \text{N large}$$