Tutorial 6

Question 2: Poisson Distribution

A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

(a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.

(b) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.

(c) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.

(d) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge

>2 5th

Poisson Distribution -

λ = avg. no of times event occurs in the time period.

$$F(k;\lambda) = Pr(X=k)_\lambda = \frac{\lambda^k e^{-\lambda}}{k!}$$

X = no. of times of occurrence of Event.

Binomial : Geometric = Poisson : Exponential

(a) P(no train on day 1,2,3 | train arrives on day 0)

$\boxed{\lambda=3}$

$= P(\text{no train in 3 days})$ $\quad \lambda' = 3\lambda = 9$

$$= \frac{\lambda^k e^{-\lambda}}{k!} = \frac{9^0 e^{-9}}{0!} = e^{-9}$$

(b) P(next train takes more than 3 days | first train on day zero)

$= P(\text{no train arrives in 3 days})$

$= e^{-9}$

$P(X \cap Y) = P(X)\,P(Y)$ if X, Y ind.

(c) P(no trains in first 2 days $\cap$ 4 trains on 4th day)

$= P(\qquad\qquad) \cdot P(\qquad\qquad)$

$$= \frac{6^0 e^{-6}}{0!} \cdot \frac{3^4 e^{-3}}{4!}$$

(d) P(more than 2 days for 5th train to arrive)

$$= \sum_{i=0}^{4} P(i \text{ trains occur in 2 days})$$

$$= \sum_{i=0}^{4} P(X=i)$$

$$= \frac{(6)^0 e^{-6}}{0!} + \frac{(6)^1 e^{-6}}{1!} + \cdots + \frac{(6)^4 e^{-6}}{4!}$$

Day 1 | Day 2 | 5th