

8. For the i^{th} person, we introduce a random variable x_i that takes the value 1 if the person selects his/her own hat and takes the value 0 otherwise. Since

$$P(x_i = 1) = \frac{1}{n} \text{ and } P(x_i = 0) = 1 - \frac{1}{n},$$

mean of x_i is,

$$E[x_i] = 1 \cdot \frac{1}{n} + 0 \cdot \left(1 - \frac{1}{n}\right) = \frac{1}{n}$$

and

$$X = x_1 + x_2 + x_3 + \dots + x_n$$

$$\begin{aligned} \Rightarrow E[X] &= E[x_1] + E[x_2] + \dots + E[x_n] \\ &= n \times \frac{1}{n} \end{aligned}$$

$$E[X] = 1$$