18. We can write $X$ as a geometric mean with PM

$$P_X(k) = (1-p)^{k-1} p \qquad k = 1, 2, \ldots$$

The mean and variance of $X$ are given by

$$E[X] = \sum_{k=1}^{\infty} k(1-p)^{k-1} p$$

$$var[X] = \sum_{k=1}^{\infty} (k - E[X])^2 (1-p)^{k-1} p$$

Instead of evaluating the infinite sums, we can apply the total expectation theorm with

$A_1 = \{X = 1\} = ($first try is a success$)$

$A_2 = \{X > 1\} = ($first try is a failure$)$

If the first try is successful, we have $X = 1$ and

$$E[X \mid X = 1] = 1$$

If the first try fails $(X > 1)$. we have wasted one try, and we are back where we started. So, the expected number of remaining tries $E[X]$ and

$$E[X \mid X > 1] = 1 + E[X].$$

Thus,

$$E[X] = P(X=1) E[X \mid X = 1] + P(X > 1) E[X \mid X > 1]$$

$$E[X] = p \cdot 1 + (1-p) \cdot (1 + E[X])$$

$$\Rightarrow E[X] = 1/p$$

With similar reasoning, we also have

$$E[X^2 \mid X = 1] = 1, \quad E[X^2 \mid X > 1] = E[(1+X)^2] = 1 + 2E[X] + E[X^2]$$

So

$$E[X^2] = p \cdot 1 + (1-p)(1 + 2E[X] + E[X^2]) \Rightarrow E[X^2] = \frac{2}{p^2} - \frac{1}{p}$$

$$var(X) = E[X^2] - (E[X])^2 = \left(\frac{2}{p^2} - \frac{1}{p}\right) - 1/p^2 = \frac{1-p}{p^2}$$