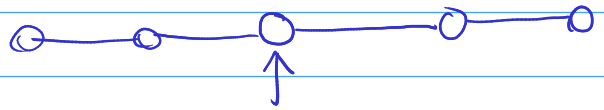
# Tutorial 4

Two steps

(a)



~~FF~~
① FB
② BF
~~BB~~

} Sample space

$P(1) = p(1-p)$

$P(2) = (1-p)p$

∵ Exclusive ⇒ $P(\text{ans}) = 2p(1-p)$

(b) 2 steps fwd, 1 step bwd

FFB, BFF, FBF

✓ | ↓

$ppp(1-p)$ $(1-p)ppp$ $p(1-p)p$

∵ Exclusive ⇒ $P(\text{ans}) = 3p^2(1-p)$

(c)

① ~~FF~~ B ✓
② BFF ✓
③ FBF ✓

} same probability

$$P(\text{ans}) = \frac{2}{3} \leftarrow \text{sample space}$$

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## Question 4

Event A → he has n coins

$P(i)$ = probability starts with 'i' coins and ends with n coins

Event win let $W$

$$P(W) = p \quad | \quad P(W^c) = 1 - p$$

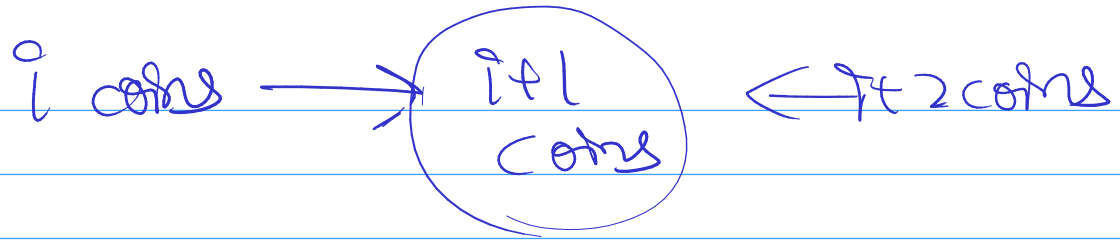
Total probability

$$P(i) = P(i+1 \mid W)P(W) + P(i-1 \mid W^c)P(W^c)$$

① $(0 < i < n)$

gain a coin ← win

lose a coin ← lose




② $p_i = p_{i+1} \times p + p_{i-1} \times (1-p)$

↓ split

③ $p_i p + p_i (1-p) = p_{i+1} p + p_{i-1} (1-p)$

④ $(p_i - p_{i-1})(1-p) = p(p_{i+1} - p_i)$

⑤ $q = \frac{1-p}{p}$ | $(p_i - p_{i-1})\, q = p_{i+1} - p_i$

⑥ $(p_i - p_{i-1})\, q = p_{i+1} - p_i$

(i) $(p_{i+1} - p_i)\, q = p_{i+2} - p_{i+1}$

Multiply

cancelled

(ii) $(p_{i+2} - p_{i+1})\, q = p_{i+3} - p_{i+2}$

Mult.

cancelled

Extremities

$$(p_{n-1} - p_{n-2})q = p_n - p_{n-1}$$

$$(p_2 - p_1)q = p_3 - p_2 \quad \checkmark$$

(7) $p_{a+1} - p_a = q^a p_1$ ✓ some value of $a < n$

- $p_{a+1} = p_a + q^a p_1$ ✓
- $p_a = p_{a-1} + q^{a-1} p_1$ ✓
- $p_{a-1} = p_{a-2} + q^{a-2} p_1$ ✓

Add

$$p_{a+1} = p_1 + qp_1 + q^2 p_1 + \cdots + q^a p_1$$

(8)
$$p_{a+1} = \begin{cases} \dfrac{1 - q^{a+1}}{1-q} p_1 & \text{if } q \neq 1 \\ (a+1)\, p_1 & \text{if } q = 1 \end{cases}$$

$a\left(\dfrac{1-q^n}{1-q}\right)$ if $q \neq 1$

if $q = 1$

(9) $p_n = 1$ ⟶ put 9 in 8

(10) $p_n = \begin{cases} \left(\dfrac{1-q^n}{1-q}\right) p_1 & \\ n\,p_1 & \end{cases}$ ⟵ get

get $p_1 \Rightarrow p_k$

(11) $p_1 = \begin{cases} (1-q)/(1-q^n) & q \neq 1 \\ 1/n & q = 1 \end{cases}$

put (11) in (8)

(12) $p_k = \begin{cases} \dfrac{(1-q^k)}{(1-q)} \dfrac{(1-q)}{(1-q^n)} & q \neq 1 \\ k/n & q = 1 \end{cases}$