Probability and Statistics – Tutorial

6th October 2020

We will start soon.

Until then if you have any questions or doubts, put in the chat.

Time → till 1307

Q1

- A coin is tossed repeatedly until you get a tail and then you win a^k dollars , where k is the number of heads before you got the tail. Where a>0 . How much is a fair amount to pay to play this game?

time till
13:15

Q2

Verify the Schwarz Inequality:

$$E(XY)^2 \leq E(X^2) \cdot E(Y^2)$$

Hint: Consider the function

$$f(t) = \mathbb{E}\left((tX - Y)^2\right)$$

What can you comment about $f(t)$?

Q3

The yearly snowfall at Mountain Rainier is modeled as a normal random variable with a mean of 60 inches and a standard deviation of 20 inches. What is the probability that this year's snowfall will be at least 80 inches? You can leave your answer in terms of phi.

Q4

Given an exponential random variable pdf$(X) = \lambda e^{-\lambda x}$. Find the expectation on:

- X (The mean)
- X^2 (Variance + Squared Mean)
- X^{35}

Try it out for Gaussian Random variable too.

6. [Application 🔥] One of the important features of a distribution is how heavy its tails are, especially for risk management in finance. If you recall the 2009 financial crisis, that was essentially the failure to address the possibility of rare events happening. Risk managers understated the kurtosis (kurtosis means 'bulge' in Greek) of many financial securities underlying the fund's trading positions. Sometimes seemingly random distributions with hypothetically smooth curves of risk can have hidden bulges in them. And we can detect those using MGF!

Just a thought!

You visit your friends. They have 2 children. They tell you one of them is male. What is the probability that the other is female. The answer is 66.66% ofc. Now a boy comes running to you. You see the boy, you know he is a boy, let's name him Atsi. Tell me what the probability is that the other child is Female. The answer is 50% ofc. I promise you that both my answers are correct. Still from getting no new information, just a name, (or looking at the boy, same thing) your probability changes. Why?

Let X be the snowfall this year: $X \sim \mathcal{N}(60, 20^2)$.

$$\begin{aligned}
(X \geq 80) &= (X < 80)^c \\
\Rightarrow \; P(X \geq 80) &= 1 - P(X < 80) \\
&= 1 - P\left(\frac{X - 60}{20} < \frac{80 - 60}{20}\right) \\
&= 1 - \Phi(1) \\
&= 1 - 0.8413 \\
&= 0.1587
\end{aligned}$$