Assignment 2 Question 7

Monday, September 28, 2020 1:54 PM

i) $M_{kX}(t) = E[e^{(tk)X}] = M_X(kt)$

ii) $M_{X+k}(t) = E[e^{t(X+k)}] = E[e^{tk}e^{tX}] = e^{tk}E[e^{tX}] = e^{tk}M_X(t)$

iii) $M_Y(t) = E[e^{t(X_1+X_2+\cdots+X_n)}] = E[e^{tX_1}]E[e^{tX_2}]\cdots E[e^{tX_n}] = M_X(t)^n$ (independent)

iv) $M_Y(t) = E[e^{tY}] = \int e^{ty}p(y)\,dy = \int e^{tx}p(x+k)\,dx$ Let $x+k=z$, $x=z-k$

$= \int e^{t(z-k)}p(z)\,dz = e^{-kt}M_Z(t)$

v) $M_Y(t) = E[e^{tY}] = \int e^{ty}p(y)\,dy = \int e^{tx}p(2x)\,dx$ Let $z=2x$, $dz=2dx$

$= \int e^{t\frac{z}{2}}p(z)\cdot\frac{1}{2}dz = \frac{1}{2}M_Z(t/2)$