



Tutorial 6

Probability and Statistics

Question 1: Binomial MGF using Bernoulli

Derive the MGF for Binomial distribution indirectly, i.e., using the MGF for Bernoulli.

Start by deriving the MGF of Bernoulli from definition.

Question 2: Poisson Distribution

A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

- (a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.
- (b) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.
- (c) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.
- (d) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge

Question 3: Exponential Distribution

Beginning at time $t = 0$, we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a type-A bulb and a type-B bulb. The lifetime, X , of any particular bulb of a particular type is a random variable, independent of everything else, with the following PDF:

$$\begin{aligned} \text{for type-A Bulbs: } f_X(x) &= \begin{cases} e^{-x}, & x \geq 0, \\ 0, & \text{otherwise;} \end{cases} \\ \text{for type-B Bulbs: } f_X(x) &= \begin{cases} 3e^{-3x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases} \end{aligned}$$

- (a) Find the expected time until the first failure.
- (b) Find the probability that there are no bulb failures before time t .
- (c) Given that there are no failures until time t , determine the conditional probability that the first bulb used is a type-A bulb.