Let's X, Y be jointly gaussian.

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp \tag{1}$$

$$\left\{ - \frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2} - 2\rho\frac{(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y} + \frac{(y-\mu_Y)^2}{\sigma_Y^2}\right]\right\}$$

Now, recalling that X and Y are independent if and only if $f_{XY}(x,y) = f_X(x)f_Y(y)$, if you set $\rho = 0$ in (1) you get:

$$f_{XY}(x,y) = f_X(x)f_Y(y)$$

and we are done.