Arpan

C.S. ineq. – $|\langle X, Y\rangle|^2 \leq \langle X,X\rangle\langle Y,Y\rangle$ (In LA)

$\|X\cdot Y\|^2 \leq \|X\|\,\|Y\|$

Q2

To prove – $E[XY]^2 \leq E[X]\,E[Y]$

Let $W = (X - \alpha Y)^2 \geq 0$

$$E[(X-\alpha Y)^2] = E[X^2 - 2\alpha XY + \alpha^2 Y^2]$$

$$= E[X^2] - 2\alpha E[XY] + \alpha^2 E[Y^2]$$

$$E[(X-\alpha Y)^2] \geq 0$$

$$\Rightarrow E[X^2] - 2\alpha E[XY] + \alpha^2 E[Y^2] \geq 0$$

putting $\alpha = \dfrac{E[XY]}{E[Y^2]}$

$$\Rightarrow E[X^2] - 2\frac{(E[XY])^2}{E[Y^2]} + \frac{(E[XY])^2}{E[Y^2]} \geq 0$$

$$\Rightarrow E[X^2]\,E[Y^2] \geq (E[XY])^2$$

Equality holds when $E[(X-\alpha Y)^2] = 0$

$$\Rightarrow X = \alpha Y$$

$$\Rightarrow X = \frac{E[XY]}{E[Y^2]}\cdot Y \quad \text{always}$$

Q3.

$X = N(\mu, \sigma^2)$. $\mu = 60, \sigma = 20$

$X = N(60, 20^2)$

$P(X \geq 80)$?

The value of $\phi(k)$ is the CDF of $N(0,1)$
ie. $P(X \leq k) = \phi(k)$ where $X = N(0,1)$

So, we need to scale down our problem when using ϕ. $\left(\frac{X-\mu}{\sigma}\right)$

$\Rightarrow P(X \geq 80) = 1 - P(X \leq 80)$ (Equality doesnt matter for continuous)

$= 1 - P\left(\frac{X-60}{20} \leq \frac{80-60}{20}\right)$

$= 1 - P(Z \leq 1)$ $Z \approx N(0,1)$

$= 1 - \phi(1)$

$= 0.1587$ (from ϕ table)