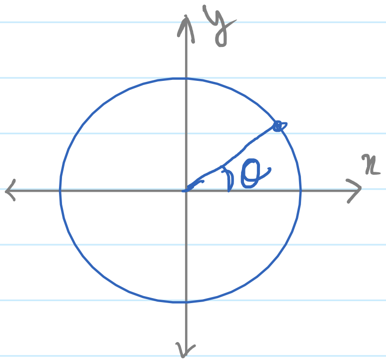
# Assignment 3 | Q 10

30 October 2020 13:20




Since P1 lies on the circumference, let it be $(\cos\theta, \sin\theta)$.

Any rectangle will be entirely in/on the circle iff all its vertices lie on/in the circle.

Let the P2 be $(x, y)$

then the 4 verticles will be:

$(x, y)$
$(\cos\theta, \sin\theta)$
$(x, \sin\theta)$
$(\cos\theta, y)$

For the last 2 vertices,

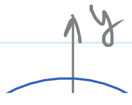
$$x^2 + \sin^2\theta \leq 1$$

$$= \quad x^2 \leq \cos^2\theta \Rightarrow |x| \leq |\cos\theta| \quad ①$$

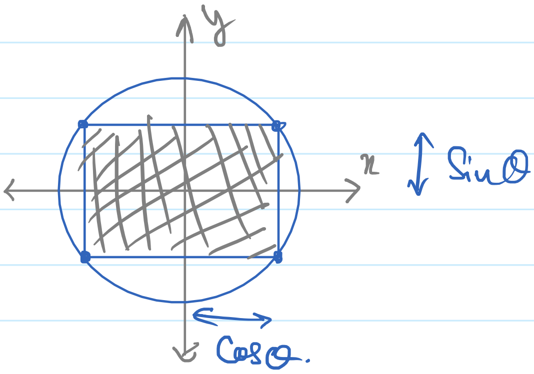
$$y^2 + \cos^2\theta \leq 1.$$

$$\Rightarrow \quad |y| \leq |\sin\theta| \quad ②$$

What is the prob of both Condition 1 and 2 being satisfied?




Prob: $(2\sin\theta) \cdot (2\cos\theta)$




Prob: $\dfrac{(2\sin\theta)\cdot(2\cos\theta)}{\pi r^2}$

$r = 1.$

$= 2\cdot\dfrac{2|\sin\theta|\cdot|\cos\theta|}{\pi}$

Total Probability: $\dfrac{1}{\cancel{2}\pi}\displaystyle\int_0^{2\pi}\dfrac{\cancel{2}\,2|\sin\theta||\cos\theta|\,d\theta}{\pi}.$

$\Rightarrow \dfrac{4}{\pi^2}\displaystyle\int_0^{\pi/2}\sin 2\theta\,d\theta = \dfrac{4}{\pi^2\cdot 2}\left.-\cos 2\theta\right|_0^{\pi/2}$

$= \dfrac{4}{\pi^2}\cdot\dfrac{\cancel{2}}{\cancel{2}} = \boxed{\dfrac{4}{\pi^2}}$