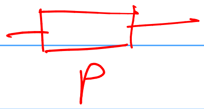
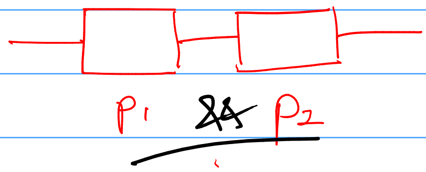
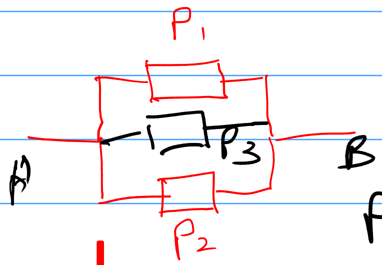
Tutorial 3

Question 2





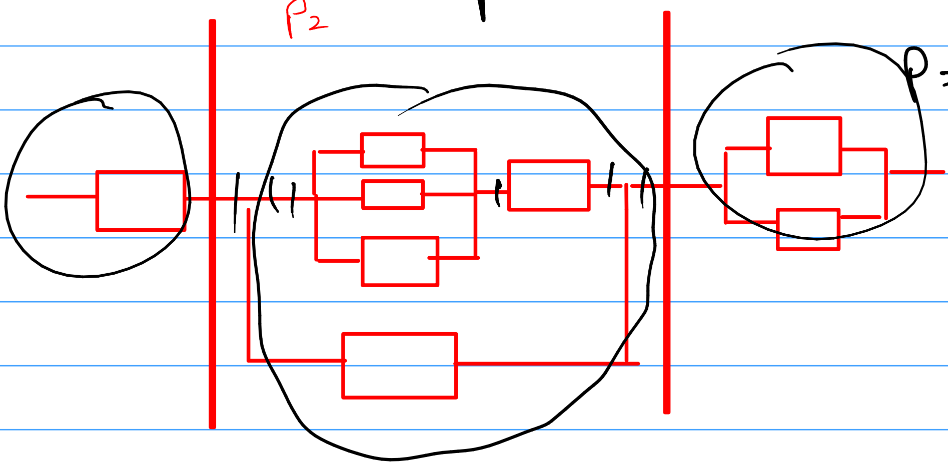
$P = \underline{p_1 p_2}$



P

$(1-P) = (1-P_1)(1-P_2)(1-P_3)$

P =



Question 3

A v/s B		B v/s C		A v/s C	
0.5	0.5	0.6	0.4	0.7	0.3

Round 1 - 2 games (both to be won)
Round 2 - 2 games (both to be won by challenger to topple A)

(a) A priori prob. - Prob. before an inference
A posteriori prob. - Prob. after an observation (looking back)

1. Second round req -

$$P(\text{either B or C win both}) = \underset{B}{(0.6)^2} + \underset{C}{(0.4)^2}$$

2. B wins First round

$$P(\text{B wins twice}) = (0.6)^2$$

3. A retains championship -

$$P(\text{A wins}) = 1 - \underbrace{P(\text{B wins})}_{(0.6)^2(0.5)^2} - P(\text{C wins})$$

$$= 1 - (0.6)^2(0.5)^2 - (0.4)^2(0.3)^2$$

$$=$$

2

(b) (i) P(B surviving challenges | Second round req)

$= \dfrac{P(B \text{ wins twice})}{P(B \text{ wins twice}) + P(C \text{ wins twice})}$ ✓ $\dfrac{(0.6)^2}{(0.6)^2 + (0.4)^2}$

P(second round | B wins twice) = 1

(ii) P(A retains championship | 2nd rou)

= P(A wins at least one | B surviving) P(B surviving | 2nd round)
+ P(A " " " | C ") P(C surviving | 2nd round)

$= P(A|B) \cdot P(B|2) + P($

$(1-(0.5)^2)$. $(1-(0.3)^2)$

2nd

A ✓

(c) P(B won First round | A won match 1 of 2nd round)

$= \dfrac{P(A \text{ won match 1 vs } B \mid B \text{ won round 1})\, P(B \text{ won round 1} \mid 2\text{nd r.})}{P(A \text{ won match 1 of 2nd round})}$

$= \dfrac{\dfrac{(0.6)^2}{(0.6)^2+(0.4)^2} \times (0.5)}{\dfrac{(0.6)^2}{(0.6)^2+(0.4)^2} \times (0.5) + \dfrac{(0.4)^2}{(0.6)^2+(0.4)^2} \times (0.7)}$ ✓

Question 6

$$P(k \text{ boys} \mid n \text{ children}) = {}^nC_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k} = {}^nC_k \frac{1}{2^n}$$

$n \geq k$

$$P_0 = 1 - \alpha p(1 + p + p^2 + \cdots)$$

$P_0 + P_1 + \cdots$

$$P_1 = \alpha p$$
$$P_2 = \alpha p^2$$
$$\vdots$$

-1

p_0 $\quad$ p_1

$$P(k \text{ boys}) = P(k \text{ boys} \mid 0 \text{ children}) + P(k \text{ boys} \mid 1 \text{ child}) + \cdots$$

$$= \sum_{n=1}^{\infty} \left(\frac{{}^nC_k}{2^n}\right)(\alpha p^n) = \alpha \sum_{n=1}^{\infty} {}^nC_k \left(\frac{p}{2}\right)^n$$

$$= \alpha\left({}^kC_k \frac{p^k}{2^k} + {}^{k+1}C_k \frac{p^{k+1}}{2^{k+1}} + \cdots\right)$$

$$= \alpha \frac{p^k}{2^k}\left({}^kC_k + {}^{k+1}C_k\left(\frac{p}{2}\right) + {}^{k+2}C_k\left(\frac{p}{2}\right)^2 + \cdots\right)$$

$$= \alpha \frac{p^k}{2^k}\left({}^kC_0 + {}^{k+1}C_1\left(\frac{p}{2}\right) + {}^{k+2}C_2\left(\frac{p}{2}\right)^2 + \cdots\right)$$

$$\left(1 - \frac{p}{2}\right)^{-(k+1)}$$

$$(1-x)^{-n} = 1 + {}^nC_1 x + {}^{n+1}C_2 x^2 + \cdots$$

$$= \frac{\alpha p^k}{2^k} \frac{2^{k+1}}{(2-p)^{(k+1)}} = \frac{2\alpha p^k}{(2-p)^{k+1}}$$

https://math.stackexchange.com/questions/85733/probability-that-a-family-with-n-children-has-exactly-k-boys