Students whose sanitisation time $= K$

are let us say type-K students

so we can consider the overall arrival process of students as the merging of n poisson sub-processes :-

The K^{th} subprocess corresponds to arrivals of type-K customers, is independent of the other arrival subprocesses.

It has rate $= \lambda p_K$

where $p_K = P(X_i = k)$

$X_i =$ Sanitisation time of i^{th} Student

Now Let $N_t^K =$ number of type-K Student in the queue at time t.

$\therefore$ $N_t =$ no. of Students in queue at time t.

$$N_t = \sum_{K=1}^{n} N_t^K$$ (N_t^K random variables are independent)

Let us now determine the PMF of N_t^K.

Date: / /

→ A type-K Student is in the queue at time t if and only if that customer arrived b/w times t-K and t.

Hence N_t^K has a Poisson PMF with mean $\lambda K P_K$.

Now, since the sum of independent Poisson random variable is poisson, it follows that N_t has a Poisson PMF

$$E[N_t] = \sum_{K=1}^{n} \lambda K P_K$$

$$= \lambda \sum_{K=1}^{n} K P_n$$

$$= \cancel{\lambda E[X_i]} = \lambda E[X_i]$$

$\therefore$ N_t is Poisson PMF with rate $= \lambda E[X_i]$

$$P(N_t = K) = \frac{e^{-\lambda'} \lambda'^K}{K!} = \frac{\lambda'^K e^{-\lambda'}}{K!}$$

where $\lambda' = \lambda E[X_i]$

Scanned with CamScanner