Page No:

Date: / /

A] • Lagrange's formula → $\sum_{i=0}^{n} y_i \left[\prod_{j=0 \,\&\, j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)} \right]$

B] • $f(x+h) \rightarrow f(x+h) \rightarrow f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \ldots +$

C] • Newton forward → $P = \frac{x - x_0}{h}$

$y_0 + P \Delta y_0 + \frac{P(P-1)}{2!} \Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!} \Delta^3 y_0 + \ldots +$

D] • Newton Backward → $P = \frac{x - x_n}{h}$

$y_n + P \nabla y_n + \frac{P(P+1)}{2!} \nabla^2 y_n + \frac{P(P+1)(P+2)}{3!} \nabla^3 y_n + \ldots +$

E] • Hermite Interpolation

→ $\sum_{i=0}^{n} A_i(x) y_i + \sum_{i=0}^{n} B_i(x) y'_i$

→ $A_i(x) = [1 - 2(x - x_i) l'_i(x_i)] l_i^2(x_i)$

$B_i(x) = (x - x_i) l_i^2(x_i)$

→ $l_i(x_i) \rightarrow \prod_{j=0 \,\&\, j \neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}$

F. Spline Interpolation

→ Natural spline → $\boxed{M_0 = M_n = 0}$

→ $M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left(f_{i-1} - 2f_i + f_{i+1}\right)$

NOTE: Check the Intervals → check the given value and see in which interval it lies in, only use those splies (m).

→ $$P(x) = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right]$$
$$+ \frac{1}{h}\left[(x_i - x)\left(f_{i-1} - \frac{h^2}{6} M_{i-1}\right)\right] +$$
$$\frac{1}{h}\left[(x - x_{i-1})\left(f_i - \frac{h^2}{6} M_i\right)\right]$$

G. Simpson's 1/3 rule [$x \to x_n$ $y \to y_n$]

→ $$\frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2[y_2 + y_4 + y_6 \dots]\right]$$

H. Simpson's 3/8 rule

$\rightarrow$ $$\frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8\ldots)+2(y_3+y_6+y_9+y_{12}+\ldots)\right]$$

I. Trapezoidal rule

$\rightarrow$ $$\frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\ldots y_{n-1})\right]$$

J. Tri-diagonal System & Thomas Algorithm.

$\rightarrow$ $$\begin{bmatrix} b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 \\ 0 & a_3 & b_3 \end{bmatrix}$$

$\rightarrow$ Convert $b_n \rightarrow$ unit

$\rightarrow$ Convert $a_n \rightarrow 0$ using b_n

X ———— X ———— X ————

K. Gauss Quadrature Rule

$\rightarrow$ Convert $[b,a]$ to $[-1,1]$ using

$$\boxed{x = \frac{b-a}{2}t + \frac{b+a}{2}}$$ then $\boxed{dx = -dt}$

$\rightarrow$ $$\int_{-1}^{1} f(t)\,dt$$

Page No:
Date: / /

→ 1 Point → $\boxed{2f(0)}$

2 Point → $f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

3 Point → $\frac{1}{9}\left[5f\left(\sqrt{\frac{3}{5}}\right) + 8f(0) + 5f\left(-\sqrt{\frac{3}{5}}\right)\right]$

L. Least square approximation

→ $y = a_0 + a_1 x$ → $\Sigma y = n a_0 + a_1 \Sigma x$

→ $xy = x a_0 + a_1 x^2$ → $\Sigma xy = \Sigma x a_0 + a_1 \Sigma x^2$

M. Eigen value - Power Method

→ $Ax = B$ → $\boxed{Ax = \lambda V}$

NOTE → Multiply with a colum vector and take out the largest element out of the Resut.

N. Quadratic form $Q(x)$

→ $\boxed{Ax = B}$ → $\boxed{\frac{1}{2} X^T A X - X^T B}$

Page No:
Date: / /

O. LU Decomposition

$$L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

→ $\boxed{AX = B} \rightarrow \text{let } A = LU \rightarrow \boxed{LUX = B}$

let $\boxed{UX = Y} \rightarrow \boxed{LY = B}$

find Y and then find X using $\boxed{UX = Y}$

$A = LU \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

$$A = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & [L_{21}U_{12} + U_{22}] & [L_{21}U_{13} + U_{23}] \\ L_{31}U_{11} & [L_{31}U_{12} + L_{32}U_{22}] & [L_{31}U_{13} + L_{32}U_{23} + U_{33}] \end{bmatrix}.$$

P. Gauss elimination

→ $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \rightarrow \begin{bmatrix} a_1 & b_1 & c_1 \\ 0 & b_2 & c_2 \\ 0 & 0 & c_3 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}.$

Diagonal Dominance PROPERTY

Page No:

Date: / /

Q. <u>Gauss Jacobi</u>

$$\rightarrow \quad x_1 = \frac{1}{a_{21}}\left[d_1 - a_{12}y_0 - a_{13}z_0\right]$$

$$y_1 = \frac{1}{a_{22}}\left[d_2 - a_{21}x_0 - a_{23}z_0\right]$$

$$z_1 = \frac{1}{a_{33}}\left[d_3 - a_{31}x_0 - a_{32}y_0\right]$$

R. <u>Gauss Seidal Iteration</u>

$$x_1 = \frac{1}{a_{11}}\left[d_1 - a_{12}y_0 - a_{13}z_0\right]$$

$$y_1 = \frac{1}{a_{22}}\left[d_2 - a_{21}x_1 - a_{23}z_0\right]$$

$$z_1 = \frac{1}{a_{33}}\left[a_3 - a_{31}x_1 - a_{32}y_1\right].$$

S. $\boxed{\text{SOR}}$ → same as Gauss seidal But with addⁿ → $(1-\omega)x^{n-1} + \frac{\omega}{\text{Digonal}}\left[\quad\right]$

→

Page No:
Date: / /

SOR → Successive over Relaxation

$$X_1 = (1-\omega) X_0 + \frac{\omega}{a_{11}} \left[d_1 - a_{12} Y_0 - a_{13} Z_0 \right]$$

$$Y_1 = (1-\omega) Y_0 + \frac{\omega}{a_{22}} \left[d_2 - a_{21} X_1 - a_{23} Z_0 \right]$$

$$Z_1 = (1-\omega) Z_0 + \frac{\omega}{a_{33}} \left[d_3 - a_{31} X_1 - a_{32} Y_1 \right].$$

T. <u>Steepest Descent Method</u>

→ $X_{i+1} = X_i + \lambda S_i$

$$S_i = - \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}_{(x_0, y_0)}$$

$$\lambda = \frac{S^T . S}{S^T . H . S} \quad \Bigg| \quad H = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial f}{\partial y} \end{bmatrix}_{(x_0, y_0)}.$$

Explicit scheme → using $y_{n-1}, y_{n-2}, y_{n-3} \ldots$

Page No:

Implicit scheme → y_n, y_{n-1}, y_{n-2}

Date: / /

U. Euler Method

→ $$y_n = y_{n-1} + h f(x_{n-1}, y_{n-1})$$

LTE → $O(h^2)$, GTE → $O(h)$

V. Runge-Kutta order

2nd order

→ $$y_n = y_{n-1} + \frac{1}{2}(K_1 + K_2)$$

$K_1 = h f(x_{n-1}, y_{n-1})$

$K_2 = h f(x_{n-1} + h, y_{n-1} + K_1)$

LTE → $O(h^3)$ → GTE → $O(h^2)$

4th order

→ $$y_n = y_{n-1} + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$K_1 = h f(x_{n-1}, y_{n-1})$

$K_2 = h f(x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{K_1}{2})$

$K_3 = h f(x_{n-1} + \frac{h}{2}, y_{n-1} + \frac{K_2}{2})$

$K_4 = h f(x_{n-1} + h, y_{n-1} + K_3)$.

LTE → $O(h^5)$

GTE → $O(h^4)$

Page No:
Date: / /

W. **Milne's method** $i = 4, 5, \ldots$

$$P \to y_i = y_{i-4} + \frac{4h}{3}\left(2f_{i-3} + f_{i-2} + 2f_{i-1}\right)$$

NOTE → $f(x_i, y_i) = f_i$

$$C \to y_i = y_{i-2} + \frac{h}{3}\left[f_{i-2} + 4f_{i-1} + f_i\right]$$

—X————X———X—

X. FINITE DIFFERENCES

→ Boundary Values problems

→ $y'' + y' + y = C(x) \;\Big|\; y'' + A(x)y' + B(x)y = C(x)$

→ $y' \to \frac{y_{i+1} - y_i}{h}$ $(O(h))$ forward Diff

→ $y' \to \frac{y_{i+1} - y_{i-1}}{2h}$ $O(h^2)$ Centeral Difference

→ $y'' \to \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$ $O(h^2)$ Centeral Diff

$$\boxed{y_i^{K+1} = y_i^{K} + \Delta y_i}$$

Page No:

Date: / /

Y. Transfer Equations

$$\rightarrow \frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2} \quad \left[\frac{\partial v}{\partial t} + c\frac{\partial u}{\partial x} = \mu\frac{\partial^2 u}{\partial x^2}\right]$$

Given $u(x,0)$ | U_j^0

$u(0,t)$ | U_0^n

$u(l,t)$ | U_l^n

Burger's Equation [non linear] [$c=0$]

$$\rightarrow \frac{\partial v}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2}$$

$\rightarrow U_j^n$ known as $U(x_j, t_n)$.

- FTCS $\rightarrow \dfrac{U_j^{n+1} - U_j^n}{\Delta t} = \mu\left[\dfrac{U_{j+1}^n - 2U_j^n + U_{j-1}^n}{(\Delta x)^2}\right]$

$$U_j^{n+1} - U_j^n = r\left[U_{j+1}^n - 2U_j^n + U_{j-1}^n\right] \quad O(\Delta t, (\Delta x)^2)$$

$r = \dfrac{\mu\Delta t}{(\Delta x)^2}$ and is stable $\rightarrow$ $\boxed{r \le 1/2}$

- **Crack-Nicolson**

$$U_j^{n+1} - U_j^{n} = \frac{r}{2}\left[\left[U_{j+1}^{n} - 2U_j^{n} + U_{j-1}^{n}\right] + \left[U_{j+1}^{n+1} - 2U_j^{n+1} + U_{j-1}^{n+1}\right]\right]$$

$\rightarrow O\left((\Delta t)^2, (\Delta x)^2\right)$.

X —— X —— X

2. Linear Hyperbolic PDE

$\rightarrow$ $\boxed{\dfrac{\partial u}{\partial t} + c\dfrac{\partial u}{\partial x} = 0}$ $\boxed{U_j^n}$

FTCS $\rightarrow$ $\dfrac{U_j^{n+1} - U_j^{n}}{\Delta t} + c\left[\dfrac{U_{j+1}^{n} - U_{j-1}^{n}}{2\Delta x}\right] = 0$

FTBS $\rightarrow$ $\dfrac{U_j^{n+1} - U_j^{n}}{\Delta t} + c\left[\dfrac{U_{j}^{n} - U_{j-1}^{n}}{\Delta x}\right] = 0$

Upwind scheme

if $c > 0 \rightarrow$ FTBS

if $c < 0 \rightarrow$ FTFS

$\hookrightarrow c\left[\dfrac{U_{j+1}^{n} - U_j^{n}}{\Delta x}\right]$