$n(S_1) = 3$ (3 heads in sample 1).

Probab and statistics

↱ connecting sample space point with Real Number.

Random Variable → numerical desc of the outcome of a statistical experiment.

① Discrete RV → That assume only a finite number or an infinite seq of values.

omial
ssion

eg → Tossing coins 2 times → SS → {HH, HT, TH, TT}

Probab Distribution of RV (X) [No. of Heads] → finite no. description of a exp.

X (No. of Head)	0	1	2
P(X)	1/4	2/4	1/4

Distribution

here we can't say about X at 0.5, only fixed finite values so it is a DRV

② Continuous RV → assume any value in some interval on the real number line.

└ infinite values

→ Normal

→ Covering every point

→ Large sample SPACE

→ Distribution of weigth of 1K + students [Range → 50-60 kg] [will be covering every point]

→ PMF (Probability mass func^n).

└ Rep R.V with its probability.

① $P(X) \geq 0$

② $\Sigma P(X) = 1$

This is for DRV

Distribution func^n

→ rep P.m.f in a cumulative form.

$$F(x) = \begin{cases} 1/4 & x \le 0 \\ 3/4 & x \le 1 \\ 1 & x \le 2 \end{cases}$$

Q →

x	0	1	2	3	4	5	6	7
$P(X) \leftarrow f(x)$	0	K	2K	2K	3K	$2K^2$	$2K^2$	$7K^2+K$
	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

① K

→ $P(X) = 1$

$8K + K + 10K^2$

$10K^2 + 9K - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$(10K - 1) = 0 \rightarrow K = 1/10 = 0.1$ ✓

$K + 1 = 0 \rightarrow K = -ve$ (Not possible)

② $P(x<6) = 0.81$

$P(x \geq 6) = 0.19$

$P(0 < x < 5) = 0.8$

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③ Distribution func^n

$$F(x) = \begin{cases} 0.0 & x \le 0 \\ 0.1 & x \le 1 \\ 0.3 & x < 2 \\ 0.5 & x < 3 \\ 0.8 & x \le 4 \\ 0.81 & x \le 5 \\ 0.83 & x \le 6 \\ 1 & x \le 7 \end{cases}$$

0.5

④ if $P(x \le c) > \frac{1}{2}$, find val of c.

→ $P(x \le 4) = 0.8 > 0.5$

$\boxed{c = 4}$.

$\frac{0.5}{0.7} \rightarrow \boxed{\frac{5}{7}}$ ← $\frac{P(x=3)+P(x=4)}{P(x>2)}$

⑤ $P\left(\frac{1.5 < x < 4.5}{x > 2}\right) \rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \rightarrow \frac{P(1.5 < x < 4.5 \cap x > 2)}{P(x > 2)}$

- Cond^n Probability → $P(B|A) \rightarrow \frac{P(B \cap A)}{P(A)}$

  $P(A|B) \rightarrow \frac{P(A \cap B)}{P(B)}$

- Baye's Theorem → calc probab based on it's proir

evidence, likelihood

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$ → Before evidence probab. prior

Hypothesis

posterior

$P(B)$ → marginal (given Data probab).

$P(B|A)$ → probab of evidence given that Hypothesis is True.

$\boxed{CRV}$.

☆→

ex→ if x is a <u>CRV</u>

$$\begin{cases} \alpha(2x-x^2) & 0<x<2 \\ 0 & \text{otherwise.} \end{cases}$$

① <u>α</u>

→ in CRV → $\int_{-\infty}^{\infty} f(x)\,dx = 1$

→ $\cancel{\int_{-\infty}^{0} f(x)\,dx}_{0} + \int_{0}^{2} f(x)\,dx + \cancel{\int_{2}^{\infty} f(x)\,dx}_{0} = 1$

$= \alpha\int_0^2 (2x-x^2)\,dx = 1 \rightarrow \alpha\left[x^2-\frac{x^3}{3}\right]_0^2 = 1$

$\alpha\left[4-\frac{8}{3}\right] = 1 \Rightarrow \boxed{\alpha = \frac{3}{4}}$

② $P(x>1) \rightarrow \int_1^{\infty} f(x)\,dx$

→ $\int_1^2 f(x)\,dx + \cancel{\int_2^{\infty} f(x)\,dx} \leftarrow 0$

→ $\frac{3}{4}\int_1^2 2x-x^2\,dx \rightarrow \frac{3}{4}\left[x^2-\frac{x^3}{3}\right]_1^2$

→ $\frac{3}{4}\left[4-\frac{8}{3}-1+\frac{1}{3}\right] \rightarrow \frac{3}{4}\left[3-\frac{7}{3}\right]$

$\frac{3}{4}\left[\frac{2}{3}\right] = \frac{1}{2}$.

Q2 $X \to CRV$

$$f(x) = \begin{cases} kx^2 & \boxed{-3 < x < 3} \\ 0 & \text{otherwise} \end{cases}$$

(a) K

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \to \int_{-3}^{3} Kx^2\,dx$$

$$\to K\left[\frac{x^3}{3}\right]_{-3}^{3} \to K\left[\frac{27}{3} + \frac{27}{3}\right] = 1$$

$$\boxed{K = \frac{1}{18}} \leftarrow$$

7

(b) $P(1 \le x \le 2) \to \frac{1}{18}\int_{1}^{2} x^2\,dx \to \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{2}$

$$\frac{1}{18}\left[\frac{8}{3} - \frac{1}{3}\right] \to \boxed{\frac{7}{54}}$$

$$\boxed{\int_{\infty}^{1} f(x)\,dx = 0}$$

(c) $P(X \le 2) \to \int_{-\infty}^{2} f(x)\,dx$

$$\cancel{\int_{-\infty}^{-3} f(x)\,dx} \;0 + \int_{-3}^{2} f(x)\,dx$$

$$\to \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} \to \frac{1}{18}\left[\frac{8}{3} + \frac{27}{3}\right] \cdot \frac{1}{18}\left[\frac{+35}{3}\right] = \frac{+35}{54}$$

$\cancel{9} - \frac{19}{54}$

$\Rightarrow \boxed{\cancel{\frac{35}{54}}}$

$\boxed{f(x) = \text{Pdf}}$

$\rightarrow F_x(x)$ $\boxed{F(x) = \text{Cdf}}$

Distribution $\boxed{\text{C.d.f}}$ (Cummulative).

↳ cummulative form of pdf.

$$F_x(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx.$$

* Just like Distribution funch of PMF

* Relⁿ B/w Distribution funchⁿ and Density funchⁿ.

$\rightarrow$ $\dfrac{d}{dx} F(x) = f(x).$

Q→ $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 < x < 2 \\ 0 & \boxed{\text{O/w}} \end{cases}$

$P(x \geq 1.5)$

$\rightarrow \int_{1.5}^{\infty} f(x)\,dx \rightarrow \int_{1.5}^{2} f(x)\,dx$

$\left[2x - \dfrac{x^2}{2}\right]_{1.5}^{2} \rightarrow 4 - 2 - \left[3 - \dfrac{2.25}{2}\right]$

$2 - \left[\dfrac{6 - 2.25}{2}\right]$

$2 - \left[\dfrac{3.75}{2}\right] = \dfrac{0.25}{2} = 0.125$

$\begin{array}{r} 6.00 \\ 2.25 \\ \hline 3.75 \end{array}$

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② CDF of PDF

→ $F(X \le x) \to \int_{-\infty}^{x} f(x)\,dx$

* $\boxed{x \le 0} \to \int_{-\infty}^{x} f(x)\,dx = 0$

* $\boxed{x \le 1} \to$

$\int_{-\infty}^{1} f(x)\,dx \to \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{0} + \int_{0}^{x} f(x)\,dx$

$\left[\frac{x^2}{2}\right]_0^x = \boxed{\frac{x^2}{2}}$.

* $\boxed{x \le 2} \to \int_{-\infty}^{2} f(x)\,dx$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{x} f(x)\,dx$

$\underbrace{\int_{-\infty}^{0}}_{0}$ $\frac{x^2}{2} \to \boxed{1/2}$ 1

$\cancel{\left[2x - \frac{x^2}{2}\right]_1^2}$

$\cancel{4 - 2}$

$\cancel{2 - \left[2 - \frac{1}{2}\right]}$

$\cancel{2 - 2 + \frac{1}{2}}$

① .

* < PdP break karna hai

$\left[2x - \frac{x^2}{2}\right]_1^x \to 2x - \frac{x^2}{2} - 2 + \frac{1}{2}$

$2x - \frac{x^2}{2} - \frac{3}{2}$.

$\boxed{2x - \frac{x^2}{2} - 1}$.

$x > 2 \to \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{0} + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \underbrace{\int_{2}^{\infty} f(x)\,dx}_{0}$

$= \underline{1}$

$F(X) = \begin{cases} 0 & -\infty < x < 0 \\ x^2/2 & 0 < x < 1 \\ 2x - x^2/2 - 1 & 1 < x < 2 \\ 1 & \text{otherwise } (x > 2) \end{cases}$

$4 - 2 \quad ① \quad 2 - \frac{1}{2} - 1$

$1 - 1 \quad \frac{1}{2} \quad 1 - \frac{1}{2} = \frac{1}{2}$

Mathematical Expectation

→ Let x be any random variable and $\phi(x)$ any funcⁿ of x.

Then Expection of $\phi(x)$ is denoted by $E\left[\phi(x)\right]$.

$$E\left[\phi(x)\right]$$

DRV: $\boxed{\sum \phi(x)\, P(x)}$

CRV: $\int_{-\infty}^{\infty} \phi(x) f(x)\, dx$.

If $\phi(x) = x$

① Mean → $\bar{x}$ or $\boxed{E(x)}$

- DRV → $\boxed{\sum x\, P(x)}$
- CRV → $\int_{-\infty}^{\infty} x f(x)\, dx$

② Variance → $E(x-\bar{x})^2$
↓
$E(x^2) - \left[E(x)\right]^2$

Std dev $= \sqrt{Var}$

x ——— x ——— x

①

X	1	2	3	4	5
$P(x)$	0.2	0.35	0.25	0.15	0.05
$x^2 =$	1	4	9	16	25

$E(x) = \sum x P(x) \rightarrow 0.2 + 0.70 + 0.75 + 0.60 + 0.25$
$= \underline{2.5}$

$Var = \boxed{E(x^2) - \left[E(x)\right]^2}$

Q → 13 cards are drawn simultanously from a pack of 52 cards if ace count → 1, face card → 10

Other → denomination

Find Expectation of Total score in 13 cards. [mean.

Face cards → 10

→

X	1	2	3	4	5	6	7	8	9	10	10	10	10
$P(X)$	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13	1/13

$\sum x_i P(x) = E(x)$

Var → $E(x^2) - [E(x)]^2$.

— x — x — x — x —

- Continous

$$f(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & o/w \end{cases}$$

→ mean = $\int_{-\infty}^{\infty} x f(x)\, dx = E(x)$

var → $E(x^2) - (E(x))^2$.

$$\boxed{\int_{-\infty}^{\infty} x^2 f(x)\, dx}$$

$E(X) \rightarrow \int_0^{\infty} x\, 2e^{-2x}\, dx$ $\qquad \Gamma 2 = 1$

$2\int_0^{\infty} x e^{-2x}\, dx.$

<u>Gamma</u>

$$\int_0^{\infty} x^{n-1} e^{-ax}\, dx = \frac{\Gamma n}{a^n}$$

$$2\left(\frac{\Gamma 2}{4}\right) \rightarrow \underline{\underline{1}} \rightarrow \boxed{\frac{1}{2}}$$

Var $\rightarrow$ $E(X^2) = \int_{-\infty}^{\infty} x^2\, 2e^{-2x}\, dx$

$$2\int_0^{\infty} x^2 e^{-2x}\, dx \rightarrow \cancel{2}\cdot\frac{\Gamma 3}{\cancel{8}\,4} \rightarrow \boxed{\frac{\Gamma 3 \cdot 2}{4} - \left(\frac{1}{2}\right)^2}$$

$$\rightarrow \cancel{\frac{2}{3} - \frac{1}{2}} \quad \cancel{\frac{4-3}{6}} = \boxed{\frac{1}{6}} \quad . \frac{2}{3} - \frac{1}{4} \quad \frac{8-}{12}$$

$$\frac{1}{2} - \frac{1}{4} \rightarrow \frac{2-1}{4} = \boxed{\frac{1}{4}}.$$

X ——— X ——— X

RV

DRV
↓ PD
Pmf
└ $P(X) \geq 0$ | $\sum P(X) = 1$
↓ cumulative way of Pmf ($x < n$, $x < n_1$)
cdf
↓
mean → $\boxed{\sum x P(X)}$ $\underline{\underline{(E(X))}}$

CRV
↓ PD
Pdf
└ $f(x) \geq 0$
$\int_{-\infty}^{\infty} f(x) = 1$
↓ cum way of Pdf
cdf $[F(X)]$ → $\int_{-\infty}^{x} f(x)\, dx$
↓
mean ↳ $\int_{-\infty}^{\infty} x f(x)\, dx$ $\quad E(X)$

variance $E(X^2) - [E(X)]^2$.

$$\boxed{\sum_{x=0}^{n} e^{xt}\, nc_x\, p^x q^{n-x}}$$

$E(e^{xt})$

e^{xt}

z^x

$$\sum_{n=0}^{x} nc_x (e^t \cdot p)^x \cdot q^{n-x}$$

$\rightarrow (e^t p + q)^n$

☆ $M_x(t) = (pe^t + q)^n$

<u>Characteristic Func^n</u>

☆ $\phi_x(t) = E(e^{ixt})$

$\rightarrow \phi_x(t) = (pe^{it} + q)^n$

<u>Probability Generating Func^n</u>

$Z_x(t) = E(z^x)$

$\rightarrow Z_x(t) = (pz+q)^n$

☆• If the exp is repeated 'N' times, the fre- of r- success (x) → N. $nc_r\, p^r q^{n-r}$.

→ when $\boxed{n}$ is small and finite
→ [redacted] Here Diff B/w p and q is small

Binomial Dist

→ probab of getting 'x' success in 'n' independent trials of a Binomial experiment.
either success or failure.

$n \rightarrow$ no. of repeated independent trails
$p \rightarrow$ probab of success
$q \rightarrow$ probab of failure
$\boxed{p+q=1}$

→ $P(r) \rightarrow$ probab of getting a success

$P(r) \rightarrow {}^nC_r (p)^r (q)^{n-r} \rightarrow {}^nC_r (p)^r (1-p)^{n-r}$

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

[r or x]

- mean of BD → np
- variance of BD → npq.

$\sum_{x=0}^{n} {}^nC_x p^x q^{n-x}$

* $P(r)$ is a pmf [PD of a DRV → $P(x) \geq 0$ | $\sum P(x) = 1$

$\boxed{(p+q)^n}$

M.G.F of a BD

→ Moment Generating func^n of BD

→ $M_x(t) = E(e^{xt})$.

$$= \sum_{x=0}^{n} e^{xt}\, {}^nC_x p^x q^{n-x}$$

[$E(X) = \sum x P(x)$

np

mean → $E(X) \to BD + DRV \to \sum_{x=0}^{n} x P(x) \to np$

$$\to \sum_{x=0}^{n} \cancel{x}\, {}^nC_x\, p^x q^{n-x}$$

$$\sum_{x=0}^{n} x \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$\sum_{x=0}^{n} x \frac{n!}{x(x-1)!(n-x)!} p^x q^{n-x}$$

$$\sum_{x=0}^{n} \frac{\cancel{n}(n-1)!}{(x-1)!(n-x)!} p \cdot p^{x-1} q^{n-x}$$

$$np \sum_{x=0}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} q^{n-x} \quad \left[\sum_{x=0}^{n} {}^nC_x p^x q^{n-x} = 1 \right.$$

$$np \boxed{\sum_{x=0}^{n} \frac{(n-1)!}{(x-1)!((n-1)-(x-1))!} p^{x-1} q^{(n-1)-(x-1)}} \to \underline{\underline{1}}$$

mean of BD = np.

$n \rightarrow$ $n \rightarrow$ p q

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Variance $\rightarrow$ $E(x^2) - [E(x)]^2$

$\rightarrow$ $E(x^2) = \sum_{x=0}^{n} x^2 P(x) - (np)^2$

$$\sum_{x=0}^{n} x^2 \, nC_x \, p^x q^{n-x}$$

$$\sum_{x=0}^{n} x^2 \frac{n!}{(n-x)!\,x!}$$

$E(x(x-1))$

$E(x^2) - E(x)$

$\Rightarrow$

$E(x(x-1))$

$$E(x^2) = E[x(x-1)] + E(x)$$

$$\sum_{x=0}^{n} x(x-1) \frac{n!}{x!\,(n-x)!} p^x q^{n-x}$$

$$\sum_{x=0}^{n} x\cancel{(x-1)} \frac{n(n-1)(n-2)!}{x\cancel{(x-1)}(x-2)!\,((n-2)-(x-2))!} p^2 \cdot p^{x-2} q^{n-2-x-2}$$

$$n(n-1)p^2 \cdot \sum_{x=0}^{n} \frac{(n-2)!}{(x-2)!\,((n-2)-(x-2))!} p^{x-2} q^{n-2-x-2}$$

$E(x(x-1)) = \boxed{n(n-1).p^2}$

$E(x^2) = n(n-1)p^2 + np \rightarrow n^2p^2 - np^2 + np$

<u>Var</u> $np - np^2 \rightarrow np(1-p) = \boxed{npq}$.

Q1) → The Probability that man aged 60 will live upto 70 is 0.65. Out of 10 men, now aged 60

① atleast 7 will live up to 70

① → $p = 0.65$ | $n = 10$
$q = 0.35$ | $x \geq 7, 8, 9, 10$

→ $P(X) = P(X \geq 7)$

$10C_7 (0.65)^7 (0.35)^3 + 10C_8 (0.65)^8 (0.35)^2 + 10C_9 (0.65)^9 (0.35)^1 + 10C_{10} (0.65)^{10} (0.35)^0$

$= \underline{\underline{0.5139}}$.

② exactly 9 will live up to 70
$P(X) = P(X = 9)$

→ $10C_9 (0.65)^9 (0.35)^1 = \underline{\underline{0.0725}}$.

③ atmost 9 will live up to 7
$P(X) = P(X \leq 9) = 1 - P(x = 10)$ or $\underline{\underline{1 - P(X > 9)}}$

→ $1 - 10C_{10} (0.65)^{10} (0.35)^0$

$= \underline{0.9865}$.

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Q → Out of 800 families with 5 children each How many families could be expected to have

① 3 Boys ② 5 girls

③ either 2 or 3 boys ④ at least 2 Girls.

→ $N = 800$, $n = 5$ | $p = 1/2$ (probab of getting a boy), $q = 1/2$

① 3 Boys

→ $800 . {}^5C_3 (1/2)^3 (1/2)^2$

② $800 . {}^5C_0 (1/2)^0 (1/2)^5$

③ $800 . [{}^5C_2 (1/2)^2 (1/2)^3 + {}^5C_3 (1/2)^3 (1/2)^2]$

④ atmost 3 Boys → $P(x) = P(X \leq 3)$

$1 - P(X \geq 3)$

$800 \times [1 - [{}^5C_4 (1/2)^4 (1/2)^1 + {}^5C_5 (1/2)^5 (1/2)^0]]$.

Q → Fitting of Binomial Distribution.

fitting frequency of x by BD.

→ 4 coins are tossed 100 times

→. Fit a Binomial Distribution for Data and calc Theortical frequency

$\boxed{\Sigma f_i / \Sigma y = N}$

x	f	xf	from (4)	100·(4)
No. of Head	freq			
0	5	0	0.0676	6.76
1	29	29	0.2599	25.4
2	36	72	0.3747	37.4
3	25	75	0.2400	24
4	5	200	0.0576	[illegible]
	$\Sigma f = \underline{\underline{100}}$	$\Sigma fx = \underline{\underline{196}}$		5.76
				6
				$= \underline{\underline{10}}$

(1) $\text{mean} = \underline{1.96}$ $\quad \underline{\underline{(n=4)}}$

(2) fitting it to $\underline{BD}$

(3) $np = 1.96 \leftarrow$ mean of B.D

(4) $p = \frac{1.96}{4} \Rightarrow \underline{\underline{0.49}}$

(5) $q = \boxed{0.51}$.

$BD \to P(x) = {}^nC_x\, p^x q^{n-x}$

$\to {}^4C_x (0.49)^x (0.51)^{4-x}$ — (4)

we will count the P(x) for every value of x from Table.

at $x = 0, 1, 2, 3, 4$

$\boxed{N=100} \to 100 \times (4)$

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Q1) The probab that a pen manufactured by a Company will be defective is $\boxed{1/10}$. If 12 such pens are manf.

a) Exactly 2 will be defective
b) None will be defective
c) Atleast two will be defective.

→ $n=12$

p (prob of succ → prob of defective) $= 1/10$

$q = 9/10$

a) $x=2 \rightarrow {}^{12}C_2 (1/10)^2 (9/10)^{10}$.

b) $x=0 \rightarrow {}^{12}C_0 (1/10)^0 (9/10)^{12}$

c) $\boxed{x \geq 2} \rightarrow 1-(x<2) \rightarrow 1-\left[{}^{12}C_0 (1/10)^0 (9/10)^{12} + {}^{12}C_1 (1/10)^1 (9/10)^{11} \right]$.

Q2) → In a sampling, a large no. of parts manf by a machine, the mean no. of defectives in a sample of 20 is 2. Out of 1000 such samples how many would be expected to contain atleast 3 defective parts.

2) $E(x) = 2. \rightarrow np = 2 \rightarrow 20p = 2 \rightarrow \boxed{p = 1/10}$

$P(x) \rightarrow P(X \geq 3) = 1 - P(X<3)$ $\boxed{q = 9/10}$.

$1 - [P(0) + P(1) + P(2)]$.

$\underline{N=1000}$ $\underline{n=20}$

$1000 * [1 - (P(0) + P(1) + P(2)]$.

Q3) → In 256 sets of 12 tosses of a coin. How many cases are one can expect 8 heads and 4 tails.

→ $N = 256$
$n = 12$
$p = 1/2$ (probab of head)
$q = 1/2$.

8 heads means the rest would be 4 tails and here p is in ref with Head
$X = 8$

→ $P(8) = 12C_8 (1/2)^8 (1/2)^4 . \to 12C_8 (1/2)^{12}$

→ $\frac{12!}{8!(4)!} \to \frac{\cancel{12}^{3} \times 11 \times \cancel{10}^{5} \times \cancel{9}^{3}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} \to 55 \times 9 \times (1/2)^{12}$.

Q4) → If the mean and Variance of BD are 4 and 2 resp. Find the probab of

a) Exactly 2 success
b) less than 2 success
c) atleast 2 success.

→ $np = 4$
$npq = 2 \to 4q = 2 \to q = 0.5 \ (1/2)$

→ $\frac{n}{2} = 4 \to \boxed{n = 8}$ $p = 1/2$

a) → $^8C_2 (1/2)^2 (1/2)^6 \rightarrow {}^8C_2 (1/2)^8$

b) $P(X) \rightarrow P(X<2) \rightarrow P(0) + P(1)$

c) $P(X) \rightarrow P(X \geq 2) = 1 - P(X<2) = \boxed{1-[P(0)+P(1)]}$.

Q5] → The probab that a Bomb dropped from a plane will strike the target is 1/5. If 6 Bombs are dropped, find the probability that

a) Exactly 2 strike the Target.
b) Atleast two will strike the Target

p (prob of strike) $= 1/5 \rightarrow q = 4/5$
$n = 6$

a) $X=2 \rightarrow {}^6C_2 (1/5)^2 (4/5)^4$.

b) $P(X) \rightarrow P(X \geq 2) \rightarrow 1 - P(X<2) \rightarrow 1 - [P(0) + P(1)]$.

Q6] → If the chance that one of the ten telephone lines is Busy at an instant is 0.2.

(a) → what is the chance that 5 of the lines are Busy

(b) what is the probab that all the lines are Busy.

→ $n = 10$ | p (prob of being Busy) $= 0.2$ | $q = 0.8$

a) → $^{10}C_5 (0.2)^5 (0.8)^5$

b) → $^{10}C_{10} (0.2)^{10} (0.8)^0$.

$106 + 68$
166 174

$\boxed{\text{Imp}}$.

Q7) → Fitting Binomial Distribution.

→ Data below provides the Data of seed germinating out of 10 on damp filter paper for 80 sets of seeds. Fit a Binomial Distribution

x	0	1	2	3	4	5	6	7	8	9	10
y	6	20	28	12	8	6	0	0	0	0	0

40 4.

xy → 0 20 56 36 32 30 0 0 0 0 0

$\Sigma xy \rightarrow 174 \rightarrow$ mean $= \frac{174}{80} = \underline{\underline{2.18}}$

$\Sigma y \rightarrow 80$

$E(x) \rightarrow np = 2.18 \rightarrow n = 10$

$p = \underline{\underline{0.22}}$

$q = \underline{\underline{0.78}}$

fitted BD → $\boxed{N(p+q)^n}$

$80(0.22+0.78)^{10}$

Compare

$\boxed{\text{approx}}$

x	y (f)	${}^{10}C_x (0.22)^x (0.78)^{10-x} = n$	$80 \times n$
0	6		
1	20		
2	28		
3	12		
4	8		BD freq
5	6		
6	0		
7	0		
8	0		
9	0		
10	0		

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Q8 Fitting BD and Comparing Theortical with actual

x	0	1	2	3	4	5
y →	2	14	20	34	22	8
xy	0	14	40	102	88	40

$\Sigma xy \to 284$ → mean → $\underline{2.84}$ $(n=5)$

$\boxed{\Sigma y \to 100}$
$\parallel$
$\underline{\underline{N}}$

$np = 2.84 \to p = \frac{2.84}{5} = 0.56$

$q = \underline{\underline{0.44}}$.

fitted BD $= N(p+q)^n$ ~~(N → not given)~~

$(p+q)^n \to (0.56+0.44)^5$.

$\to \boxed{100\ (0.56+0.44)^5}$.

compare

x	f	${}^5C_x (0.56)^x (0.44)^{5-x} = n$	approx $100 \times n$
0	2		↓
1	14		Theortical
2	20		freq
3	34		by
4	22		$\underline{\underline{BD}}$.
5	8		

$$\lim_{n\to\infty} \frac{[n(n-1)(n-2)\dots(n-(x-1))].\cancel{(n-x)!}}{\cancel{(n-x)!}\,x!} \cdot a$$

$$\lim_{n\to\infty} \frac{n^x\left[1-\frac{1}{n}\right]\left[1-\frac{2}{n}\right]\dots\left[1-\frac{x-1}{n}\right]}{x!} \cdot a$$

$$\lim_{a\to\infty} \frac{1}{a} = 0$$

$$\lim_{n\to\infty} \frac{\cancel{n^x}}{x!}\left(\frac{m^x}{\cancel{n^x}}\right)\left(1-\frac{m}{x}\right)^n\left(1-\frac{m}{x}\right)^{-x}$$

$$\rightarrow e^{-m}$$

$$\lim_{n\to\infty} \frac{m^x}{x!}\left(1-\frac{m}{x}\right)^n\left(1-\frac{m}{x}\right)^{-x}$$

$$0$$

$$\rightarrow \boxed{\frac{m^x e^{-m}}{x!}}.$$

☆→ Mean is $\boxed{m}$ But how?

Mean of DRV or Pmf → $E(x) = \Sigma x P(x)$

$$\Sigma x \frac{m^x e^{-m}}{x!} \rightarrow e^{-m} \Sigma x \frac{m^x}{x!}$$

$$\rightarrow e^{-m} \Sigma \cancel{x} \frac{m^x}{\cancel{x}(x-1)!} \rightarrow e^{-m} \Sigma\, m.\frac{m^{x-1}}{(x-1)!}$$

$$me^{-m} \Sigma \frac{m^{x-1}}{(x-1)!} \rightarrow m\cancel{e^{-m}}.\cancel{e^{m}} = \underline{\underline{m}}$$

Variance → $E(x^2) - (E(x))^2$.

$$E(x^2) = E(x(x-1)) + E(x).$$

(DRV)

Poisson Distribution

- probab of getting of 'x' success.

$$\boxed{P(x) = \frac{m^x e^{-m}}{\lfloor x}}$$ (m = mean → E(x) → np] (variance = mean).

$$P(x) = \frac{m^x e^{-m}}{x!}$$ $\left[P(x) = \frac{m^x e^{-m}}{x!}\right]$.

- → when $\boxed{n}$ is $\boxed{\text{large}}$.
- → Diff B/w p and q is Big (so p might be small). ↳ $p \to 0$

⭐ Prove that poisson Distribution is a limiting case of BD, under following condn

1. $n \to \infty$
2. $p \to 0$
3. $np = \lambda\ (m) \to$ finite → $p = \boxed{\frac{m}{n}}$.

→ Binomial converges poisson

→ $\lim\limits_{n\to\infty} P(x) \to \lim\limits_{n\to\infty} {}^nC_x (p)^x (1-p)^{n-x}$

$$\lim_{n\to\infty} {}^nC_x \left(\frac{m}{n}\right)^x \left(1-\frac{m}{x}\right)^{n-x}$$

$$\lim_{n\to\infty} {}^nC_x \left[\left(\frac{m}{n}\right)^x \underbrace{\left(1-\frac{m}{x}\right)^n}_{\to a} \cdot \left(1-\frac{m}{x}\right)^{-x}\right]$$

$$\lim_{n\to\infty} \frac{n!}{(n-x)!\,x!} \cdot a$$

$E[x(x-1)] = \sum_x x(x-1)\,p(x)$

$\sum_x x(x-1)P(x) \rightarrow \sum_x x(x-1)\dfrac{m^x e^{-m}}{x!}$

$e^{-m}\sum_x \cancel{x(x-1)}\dfrac{m^x}{\cancel{x(x-1)}(x-2)!}$

$e^{-m}\sum_x m^2 \dfrac{m^{x-2}}{(x-2)!} \rightarrow e^{-m}.m^2 \sum \dfrac{m^{x-2}}{(x-2)!}$

$e^{-m}.m^2.e^m = m^2$

$E(x^2) = \underline{\underline{m^2+m}}$

$\underline{\underline{\text{var}}} \rightarrow \cancel{m^2}+m-\cancel{m^2} = \underline{m}$

Moment Generating Funch (MGF).

(1) Prove that $P(x)$ is a Pmf

To be Pmf $\rightarrow \sum P(x) = 1$

$\sum \dfrac{m^x e^{-m}}{x!} \rightarrow e^{-m}\sum \dfrac{m^x}{x!} \rightarrow e^{-m}.e^m = e^0 = \underline{\underline{1}}$

MGF $\rightarrow M_x(t) \rightarrow E(e^{xt})$

Mean of Pmf $\rightarrow \underline{\underline{\sum xP(x)}} \rightarrow \sum e^{xt}\dfrac{m^x e^{-m}}{x!}$

$$e^{-m} \sum e^{xt} \frac{m^x}{x!} \rightarrow e^{-m} \sum \frac{(e^t m)^x}{x!}$$

$$e^{-m} \cdot e^{me^t}$$

$$\rightarrow e^{-m+me^t} \rightarrow \boxed{e^{m(e^t-1)}}.$$

② Characterstic func

$\rightarrow$ $\phi_x(t) \Rightarrow E(e^{itx})$

$\rightarrow$ $\phi_x(t) \Rightarrow \boxed{e^{m(e^{it}-1)}}$.

③ Probability Generating funch.

$\rightarrow$ $Z_x(t) \rightarrow E(z^x)$.

$Z_x(t) \rightarrow e^{zm-m} \rightarrow \boxed{e^{m(z-1)}}$

—x—x—x—x—

$E(X)$ in terms of MGF

$$E(X) = \frac{d}{dt}\left(m_x(t)\right)_{t=0} \cdot \left(E(X^2) = \frac{d^2}{dt^2}\left(m_x(t)\right)_{t=0}\right.$$

—x—x—x—x—

Q1) → Given that 2% of the fuses manufactured by a firm are defective. Find probab that a box containing 200 fuses has

a) Atleast 1 defective fuses.

a) $P(X) \to x \geq 1$ or $1 - P(x<1) = \boxed{1 - P(0)}$

$p = 0.02$ | $\underline{\underline{n = 200}}$
$q = 0.98$ |

$\underline{\underline{m = np \Rightarrow 4}}$

$P(0) \to \frac{m^x e^{-m}}{x!} \to \frac{4^0 e^{-4}}{0!} = e^{-4} = \frac{1}{e^4} = \boxed{0.018}$
$(x=0)$

$1 - 0.018 = \underline{\underline{0.082}}$

b) 3 or more defective fuses

$P(X) \to P(x \geq 3) = 1 - P(x<3) = 1 - [P(0) + P(1) + P(2)]$

→ $P(0) = \cancel{0.018}\ 0.082$

154

$P(1) = 4^1 e^{-4} \Rightarrow 0.072$

$P(2) \to \frac{\cancel{16}^{8} e^{-4}}{2} \to \underline{0.144}$

$1 - [0.298] = 0.702$

$1 - [\cancel{0.090}_{0.154} + 0.144) = \cancel{1 - [0.230]} = \cancel{0.770}$

c) No defective fuses

$= 0.018 = \underline{\underline{P(0)}}$

Q2) In a certain fac creating Blades there is a small chance at 0.002 for any Blade to be defective. The blades are supplied in packets at 10. Using PD,

a) No defective blades
b) One Defective Blades.

→ In consignment of 10000 packets.

→ $N = 10000$
$n = 10$ → $np = m = \underline{0.02}$
$p = 0.002$

a) → $x = 0 \rightarrow P(0) = \dfrac{(0.02)^0 e^{-0.02}}{0!}$ $\left[\dfrac{m^x e^{-m}}{x!}\right]$

$= e^{-0.02} = \underline{0.98} \rightarrow 10000 \times 0.98 \rightarrow \underline{9800}$

b) $x = 1 \rightarrow P(1) = \dfrac{(0.02)^1 \cdot 0.98}{1} \rightarrow \underline{0.019}$.

$\Rightarrow 10000 \times 0.019$
$= \underline{190}$

Q3) If Probab of a Bad reacth from a certain injection is 0.01. Find the chance that out of 200 individuals more than 2 will get Bad reacth

= $n = 200$ → $m = np = 200 \times 0.01$
$p = 0.01$ [Bad result] $\Rightarrow \underline{2}$

= $P(X) = P(x > 2) = 1 - P(x \leq 2) = \boxed{1 - [P(0) + P(1) + P(2)]}$

$P(0) = \frac{m^x e^{-m}}{x!} = \frac{(2)^0 e^{-2}}{0!} = e^{-2} = \underline{\underline{0.13}}$

$P(1) = \frac{(2)^1 e^{-2}}{1!} = \underline{\underline{0.26}}$

$P(2) = \frac{\cancel{4}^{2} e^{-2}}{\cancel{2}} = \underline{\underline{0.26}}$

$\Rightarrow 1 - 0.65 = \boxed{0.35}$.

Q4) Fitting of Poisson Dist (similar to BD but P(x) of Poisson Dist) $[\Sigma fi / \Sigma y = N]$.

→ A skilled type on routine work kept a record of mistake made in a day during 300 working days.

x	0	1	2	3	4	5	6
f	143	90	42	12	9	3	1
xf	0	90	84	36	36	15	6

$\Sigma f = 300 \rightarrow N$

$n = 6$

$\Sigma f = 300$

$\Sigma xf = 267$

$\rightarrow m = \frac{267}{300} = \underline{0.89}$

$np \rightarrow \underline{0.89} \rightarrow p = \boxed{0.14}$

X	f	$P(x) = \frac{m^x e^{-m}}{x!}$	N × P(x) (approx)
0	143	$P(0) = \frac{(0.89)^0 e^{-0.89}}{0!}$	Theortical probability.
1	90	$P(1) = \frac{(0.89)^1 e^{-0.89}}{1!}$	
2	42	$P(2) = \frac{(0.89)^2 e^{-0.89}}{2!}$	
3	12	$P(3) = \frac{(0.89)^3 e^{-0.89}}{3!}$	
4	9	$P(4) = \frac{(0.89)^4 e^{-0.89}}{4!}$	
5	3	$P(5) = \frac{(0.89)^5 e^{-0.89}}{5!}$	
6	1	$P(6) = \frac{(0.89)^6 e^{-0.89}}{6!}$	

X ——— X ——— X

Q →

X	0	1	2	3	4
f	122	60	15	2	1
xf	0	60	30	6	4

Let's solve this properly.

→ $\Sigma f \Rightarrow \boxed{200 = N}$ $\boxed{n = 4}$

$\Sigma xf \Rightarrow 100 \rightarrow m = \frac{100}{200} = \boxed{0.5}$

$e^{-m} = \text{constant}$

$np = 0.5 \rightarrow p = \frac{0.5}{4} = 0.125$

$= e^{-0.5} = 0.606$

x	f	$P(X) \Rightarrow m^x e^{-m}/x!$	N × P(X) (approx)
0	122	$(0.5)^0 e^{-0.5}/0!$ $= 0.606$	200×0.606 = 121.2 = 121
1	60	$(0.5)^1 \cdot 0.606/1!$ $= 0.303$	200×0.303 = 60.6 = 61
2	15	$= (0.5)^2 \cdot 0.606/2$ $= 0.07575$	200×0.07575 15.15 = 15
3	2	$(0.5)^3 \cdot 0.606/6$ $= 0.012625$	200×0.012625 2.525 3
4	1	$= (0.5)^4 \cdot 0.606/24$ $= 0.0015$	200×0.0015 = 0.3 = 0
	200		

Q→ A manufacturer knows that the condensor he makes contain on an avg 17% defective. He packs them in box of 100. What is the probab that a box picked at random will contain 3 or more defective —

= $n = 100$ | $m = np = \underline{\underline{17}}$
$p = 0.17$

$P(x) = P(X \geq 3) = 1 - P(X < 3)$
$1 - [P(0) + P(1) + P(2)]$

$\Rightarrow P(0) = \frac{(17)^0 e^{-17}}{0!} = e^{-17} =$

$P(1) = 17 \cdot e^{-17}$

$P(2) = \frac{(17)^2 e^{-17}}{2} = \underline{\underline{144.5 e^{-17}}}$

$1 - [e^{-17}(1 + 17 + 144.5)]$

$= 1 - 162.5 e^{-17} = 1 - 0.0000068$
$= 0.99$.

$\boxed{\text{if } p = 1\% = 0.01}$

= $np \Rightarrow 100 \times 0.01 = 1 = m$

= $P(0) = \frac{(1)^0 e^{-1}}{0!} = e^{-1} = \underline{\underline{0.36}}$

$1 - 0.90$
$= \underline{\underline{0.10}}$

$P(1) = 0.36$ $\Rightarrow \underline{\underline{0.90}}$.
$P(2) = \underline{\underline{0.18}}$

$\frac{\mu_3^2}{\mu_2^3}$ $\frac{\mu_4}{\mu_2^2}$

Q→ A car hire firm has 2 cars which it hires out day by day. The no. of demands for a car on each day is distributed as an PD with mean 1.5. Calculate the probability of day

1. on which there is no demand
2. on which demand is refused.

① → $m = 1.5$

$np = \boxed{1.5}$

① $P(0) \to \frac{m^0 e^{-1.5}}{0!} = e^{-1.5} = \boxed{0.2231}$

—— x —— x —— x ——

② → demand is refused

↳ some demand was made which was not possible

$P(x) = P(x > 2)$

$1 - P(x \le 2) \to \boxed{1 - [P(0) + P(1) + P(2)]}$

—— x —— x —— x ——

$P(x) = \frac{m^x e^{-m}}{x!}$ (when $n \to$ large | $p \to$ small).

→ mean = $m = np$ = variance

→ MGF = $\boxed{E(e^{xt})}$ → $\boxed{\text{imp} \to \sum \frac{m^x}{x!} = e^m}$

→ CF → $\boxed{E(e^{ixt})}$

→ PGF → $\boxed{E(z^x)}$

→ fitting of BD = same procedure for PD

Denoted by → $X \sim N(\mu, \sigma)$.

Mean

first → $\underline{f(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$\boxed{z\sigma+\mu} \leftarrow \boxed{Z = \frac{X-\mu}{\sigma}}$ (z-score) → $\frac{dz}{dx} = \frac{1}{\sigma}(1)$

↳ x

↳ $dz = \sigma^{-1} dx$

$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2 z^2}}$ $\boxed{\sigma dz = dx}$

$E(X) = \int_{-\infty}^{\infty} x f(x) dx \rightarrow \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2 z^2} dx$

$\rightarrow \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-1/2 z^2} dx \rightarrow \frac{\cancel{\sigma} 1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2} z^2} dz$

$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (z\sigma + \mu) e^{-1/2 z^2} dz$

$\frac{1}{\sqrt{2\pi}} \left[\underbrace{\int_{-\infty}^{\infty} z\sigma e^{-1/2 z^2} dz}_{\text{odd}} + \int_{-\infty}^{\infty} \mu e^{-1/2 z^2} dz \right]$

$\boxed{\frac{2\mu}{\sqrt{2\pi}} \left[\int_0^{\infty} e^{-z^2/2} dz \right]}$

let $z^2/2 = p \rightarrow z = \sqrt{2p}$

$= 2z dz = 2 dp$

$dz = \frac{2dp}{2z} \rightarrow \frac{dp}{z}$

$\rightarrow \frac{2\mu}{\sqrt{2\pi}} \left[\int_0^{\infty} \frac{e^{-p}}{\sqrt{2p}} dp \right] \rightarrow \frac{\cancel{2}\mu}{\cancel{2}\sqrt{\pi}} \int_0^{\infty} \frac{e^{-p}}{\sqrt{p}} dp$

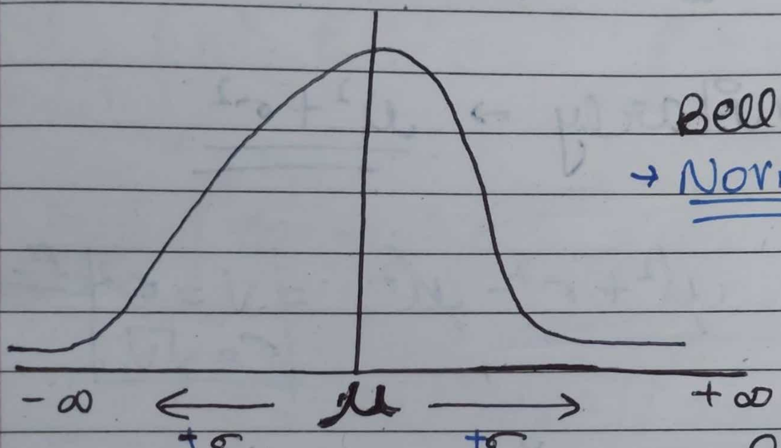
[CRV] ↓

Normal Distribution

- A cont^n rand var X is said to follow normal Dist with mean(μ) and stand dev (σ), if it's pdf is

→ $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

→ $\frac{1}{\sigma\sqrt{2\pi}} e^{-a}$ $\left[a = \frac{(x-\mu)^2}{2\sigma^2}\right]$



Bell-shaped
→ Normal curve

95% of Data $\pm 2.5\sigma$
99% of Data $\pm 3\sigma$

$$\boxed{\begin{gathered} -\infty < X < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{gathered}}$$

- symmetric about it's mean
- if not symmetric → skewness

→ $f(x)$ of pdf $= \int_{-\infty}^{\infty} f(x)\,dx$ [Property of pdf = 1]

[Instead of Σ we use $\int$]

☆ → $$\boxed{P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx}$$

$$\left[\int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma n}{a^n}\right]$$

$n-1 = -1/2$
$n = 1 - 1/2$ $n = 1/2$ $a = 1$

$$\rightarrow \frac{\mu}{\sqrt{\pi}} \int_0^\infty p^{-1/2} e^{-p} dx$$

$$\frac{\mu}{\sqrt{\pi}} \frac{\Gamma 1/2}{1^{1/2}} \rightarrow \frac{\mu}{\cancel{\sqrt{\pi}}} \frac{\cancel{\sqrt{\pi}}}{1} = \boxed{\mu}.$$

Variance $\boxed{E(x^2) - (E(x))^2}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dx$$

similarly $\rightarrow$ $\mu^2 + \sigma^2$

$$= V = (\sigma^2) = \cancel{\mu^2} + \sigma^2 - \cancel{\mu^2} = \boxed{\begin{array}{l} V = \sigma^2 \\ \sigma = \sqrt{V} \end{array}}$$

Imp

- $\displaystyle\int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma n}{a^n}$ | $n \rightarrow$ frac $\rightarrow \Gamma n+1 = n\Gamma n$; $n \rightarrow$ integer $\rightarrow \Gamma n+1 = n!$; $\Gamma 1/2 = \sqrt{\pi}$
- $\displaystyle\sum_{x=0}^{n} {}^nc_r\, p^r q^{n-r} = (p+q)^n$
- $\displaystyle\sum_x \frac{m^x}{x!} = \boxed{e^m}.$
- $\boxed{Z = \frac{x-\mu}{\sigma}}.$

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- $E(X^n) = \frac{d^n}{dt^n}\left(M_X(t)\right)_{t=0}$

- $F(x) \rightarrow$ cumulative distribution of pdf $= \int_{-\infty}^{x} f(x)\,dx$

  $\frac{d}{dx} F(x) = f(x).$

→ Variance → $E(X^2) - (E(X))^2$ ($E(X) = \mu$)

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx + \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dx$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2\, e^{-\frac{1}{2}z^2}\,dx \qquad \left[z = \frac{x-\mu}{\sigma} \rightarrow \sigma dz = dx \quad z\sigma + \mu = x \right]$$

$$\frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (z\sigma + \mu)^2 e^{-\frac{1}{2}z^2} \cancel{\sigma}\, dz$$

$$\frac{1}{\sqrt{2\pi}} \left[\underbrace{\int_{-\infty}^{\infty} z^2\sigma^2 e^{-\frac{1}{2}z^2} dz}_{1} + \underbrace{\boxed{\int_{-\infty}^{\infty} 2z\sigma\mu\, e^{-\frac{1}{2}z^2} dz}}_{\text{odd} \rightarrow 0} + \underbrace{\int_{-\infty}^{\infty} \mu^2 e^{-\frac{1}{2}z^2} dz}_{2} \right]$$

(1) = Let $\frac{z^2}{2} = p \rightarrow z^2 = 2p \rightarrow z = \sqrt{2p}$

$2z\,dz = 2\,dp \rightarrow dz = \frac{dp}{\sqrt{2p}}$ — (3)

$$2\sigma^2 \int_0^{\infty} \frac{2 \cdot p\, e^{-p}\,dp}{\sqrt{2p}} \Rightarrow 2\sqrt{2}\sigma^2 \int_0^{\infty} p^{1/2} e^{-p}\,dp$$

$$= \cancel{2\sqrt{2}}\; 2\sqrt{2}\sigma^2\, \Gamma(3/2) \rightarrow \cancel{2}\sqrt{2}\sigma^2 \frac{\sqrt{\pi}}{\cancel{2}}$$

$$= \boxed{\sqrt{2\pi}\sigma^2}.$$

(2) similarly for (2) using (3)

we get $\to \boxed{\sqrt{2\pi}\,\mu^2}$.

$$\to \frac{1}{\cancel{\sqrt{2\pi}}}\left[\cancel{\sqrt{2\pi}}(\sigma^2+\mu^2)\right]$$

$$E(x^2) = \sigma^2+\mu^2 \quad \Bigg| \quad V = \sigma^2 + \cancel{\mu^2} - \cancel{\mu^2} \quad \boxed{V=\sigma^2} \to \boxed{\sigma = \sqrt{V}} \Bigg|.$$

—— x —— x —— x ——

Moment Generating funcn

$$M_x(t) = E(e^{xt})$$

$$E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} f(x)\,dx \to \int_{-\infty}^{\infty} e^{xt} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dx \longrightarrow$$

—— x —— x —— x ——

$$\cancel{\frac{2e^{\mu t+\sigma^2t^2/2}}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\theta}\,d\theta}{\sqrt{2}\sqrt{\theta}}}$$

$$\to \frac{e^{\mu t+\sigma^2t^2/2}}{\cancel{\sqrt{\pi}}} \int_0^\infty \theta^{-1/2} e^{-\theta}\,d\theta \to \Gamma 1/2 = \cancel{\sqrt{\pi}}$$

$$\boxed{e^{\mu t+\sigma^2t^2/2}} = M_x(t) \text{ of ND}$$

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$$\rightarrow \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{xt} e^{-z^2/2} \, dx \qquad z = \frac{x-\mu}{\sigma}$$

$$\rightarrow \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{(z\sigma+\mu)t} \, e^{-z^2/2} \, \cancel{\sigma} dz$$

$$\rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{(z\sigma+\mu)t} e^{-z^2/2} \, dz$$

$$\rightarrow \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\mu t} e^{z\sigma t} e^{-z^2/2} \, dz$$

$$\frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{z\sigma t} e^{-z^2/2} \, dz$$

$$\frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \cancel{e^{z\sigma t}} \times e^{-\frac{1}{2}[z^2 - 2z\sigma t + (\sigma t)^2 - (\sigma t)^2]}$$

$$\frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}[(z-\sigma t)^2 - (\sigma t)^2]}$$

$$\frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} e^{\sigma^2 t^2/2} \, dt$$

$$\frac{e^{\mu t + \sigma^2 t^2/2}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} \, dt$$

let $\frac{1}{2}(z-\sigma t)^2 = \theta$

$(z-\sigma t)\,dz = d\theta$

$dz = \frac{d\theta}{z-\sigma t}$

$$\boxed{(z - \sigma t) = \frac{d\theta}{\sqrt{2\theta}}}$$

$$\leftarrow \frac{2 e^{\mu t + \sigma^2 t^2/2}}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-\frac{1}{2}(z-\sigma t)^2} \, dt$$

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$M_X(t)$ of ND $\rightarrow$ $\boxed{e^{\mu t + \sigma^2 t^2/2}}$

* MGF of $Z \rightarrow M_Z(t) \rightarrow E(e^{Zt})$

$\rightarrow Z = \frac{X-\mu}{\sigma} \rightarrow E\left(e^{\frac{X-\mu}{\sigma} t}\right)$.

$\rightarrow E\left(e^{\frac{X-\mu}{\sigma}t}\right) = \int_{-\infty}^{\infty} e^{\frac{x-\mu}{\sigma}t} f(x)\,dx$

$\rightarrow E\left[e^{\frac{xt - t\mu}{\sigma}}\right] = E\left[e^{\frac{xt}{\sigma}} \cdot e^{-\frac{\mu t}{\sigma}}\right]$

$e^{-\frac{\mu t}{\sigma}}\left[E\left(e^{\frac{xt}{\sigma}}\right)\right]$ $\qquad \frac{\sigma^2 t^2}{2}$

$e^{-\frac{\mu t}{\sigma}} M_X(t/\sigma)$ $\qquad \frac{\cancel{\sigma^2}\left(\frac{t^2}{\cancel{\sigma^2}}\right)}{2}$

$e^{-\frac{\mu t}{\sigma}}\left(e^{\mu(t/\sigma) + \sigma^2 t^2/\sigma^2/2}\right)$

$e^{-\frac{\mu t}{\sigma}}\left(e^{\frac{\mu t}{\sigma} + t^2/2}\right)$ $\quad e^{-\frac{\mu t}{\sigma}}\left(e^{\frac{\mu t}{\sigma}} \cdot e^{t^2/2}\right)$

$\cancel{e^{-\frac{\mu t}{\sigma}} e^{\frac{\mu t}{\sigma}} + d^{-\frac{\mu t}{\sigma}} e}$

<u>Ans</u> $M_Z(t) = \boxed{e^{t^2/2}}$.

* $\boxed{E(X) = \frac{d}{dt}\left(M_X(t)\right)_{t=0}}$

$\boxed{E(X^2) = \frac{d^2}{dt^2}\left[M_X(t)\right]_{t=0}.}$

→ Value of x and value of z

L Conversion to get Distributed result

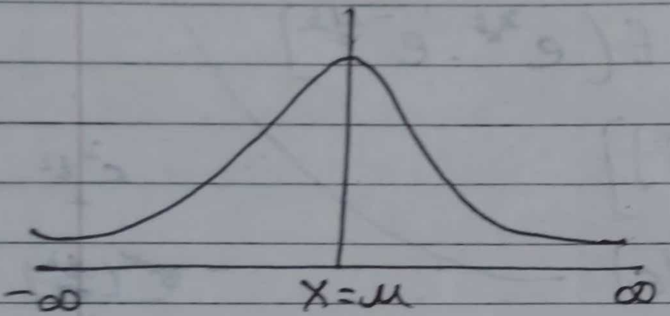
→ Area Under the Curve

→ $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$ $\left(z = \frac{x-\mu}{\sigma}\right)$

L $-\infty < x < \infty$

$-\infty < \mu < \infty$

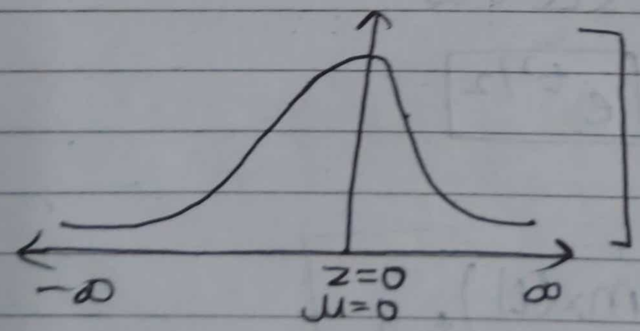
$\sigma > 0$



→ When we put $\mu = 0$ and $\sigma = 1$

$z = \frac{x-\mu}{\sigma}$ → z approaches x

$\boxed{-\infty < z < \infty}$



This tells us that this is symmetrical

① → Convert x into z using μ and σ to get symmetry.

1 feet = 12 inch

Q1) → height of 300 students are ND with μ = 64.5 In and σ = 3.3 In.

① less than 5 feet
② B/w 5 feet and 5 feet and 9 inch

① $\mu = 64.5$ $\left[z = \frac{x-\mu}{\sigma} \right]$

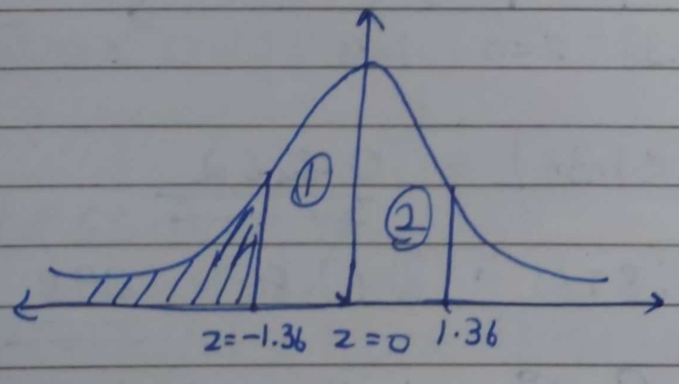
$\sigma = 3.3$

$f(x) \rightarrow f(X < 60)$

$f(x) \rightarrow f(x<60) \rightarrow f\left(z < \frac{60-64.5}{3.3}\right]$

$P(z < -1.36)$.

$\boxed{1 = 2}$



req area = $0.5 - P(-1.36 < z < 0)$.

$0.5 - P(0 < z < 1.36)$ $(1=2)$.

$0.5 - 0.4131$

$= 0.0869 \rightarrow f(x) \rightarrow$ Probab

$\rightarrow 300 \times 0.0869 \rightarrow 26.07 \simeq 26$.

B> B/w 5 feet and 5 feet 9 inch.

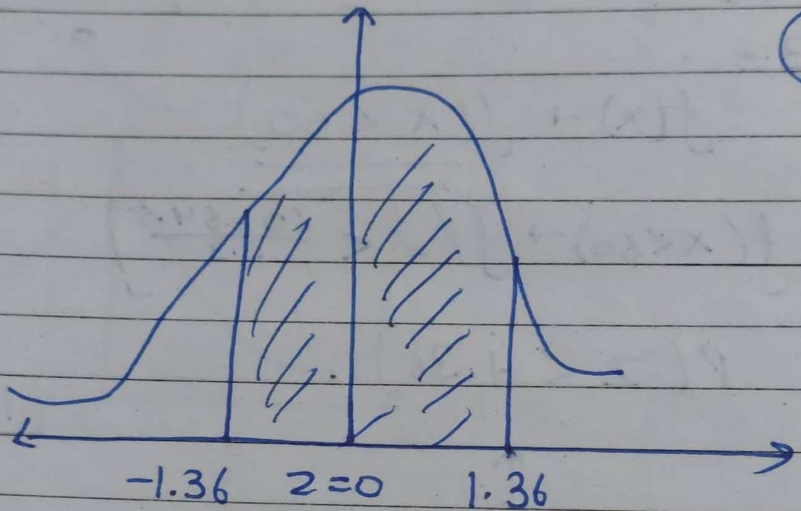
B] $\rightarrow f(x) \rightarrow f(60 < x < 69) \rightarrow f(a < z < b)$

$a = \frac{60-64.5}{3.3} \rightarrow -1.36$

$b = \frac{69-64.5}{3.3} \rightarrow \frac{4.5}{3.3} \rightarrow \underline{\underline{1.36}}$

$\rightarrow f(-1.36 < z < 1.36)$

$1 = 2$



2. $f(z < 1.36) = \underline{\underline{0.8262}}$

$\rightarrow 300 \times 0.8262 \rightarrow 247.86 \sim \underline{\underline{248}}$

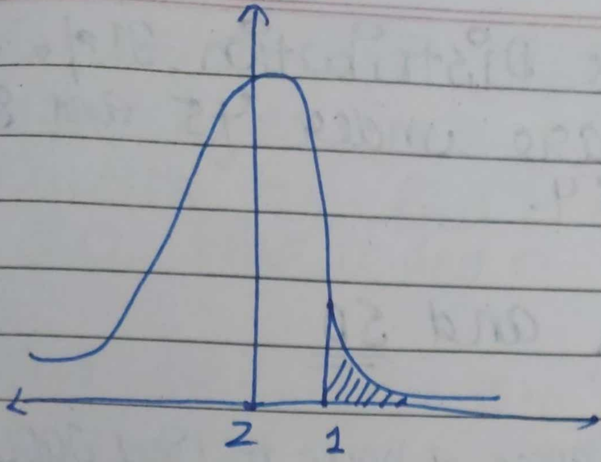
Q2) → The Distribution of 500 workers in a factory with $\mu = 75$ Rs and $\sigma = \underline{\underline{15}}$
No. of workers

① gets more than 90.

$\mu = 75$
$\sigma = 15$

$P(x) \rightarrow f(x) \rightarrow f(x > 90) \rightarrow f\left(z > \frac{90-75}{15}\right)$

$f(z > 1)$



→ $f(z>1) = 0.5 - f(z<1)$

$0.5 - f(0<z<1)$

$0.5 - 0.3413$

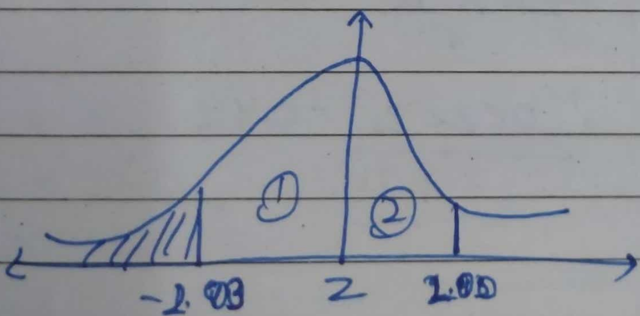
$= \underline{\underline{0.1587}}$.

$= 500 \times 0.1587 = 79.35 \simeq \underline{\underline{79}}$

ⓑ less than 46.

→ $P(x)\ f(x) \to f(x<46) \to f\left(z < \frac{46-75}{15}\right)$

$f(\underline{\underline{z<-2.00}})$



$= f(z<-1.93) = 0.5 - f(z<-2.00)$ $\quad$ $\underline{\underline{0.5-0.4772}}$

$0.5 - f(z<2.00)$ $\quad$ 0.0228

$0.5 - f(0<z<2.00)$ $\quad$ $\times$ 500

$= 11.4 \simeq \boxed{11}$.

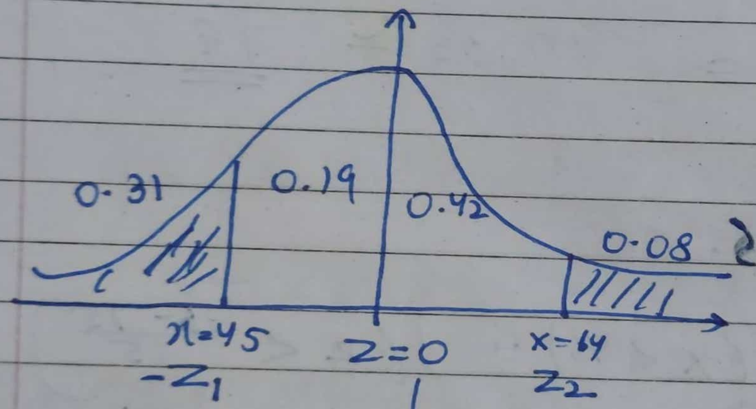
Q → In a Normal Distribution 31% of the items are under 45 and 8% are over 64.

Find <u>mean</u> and <u>SD</u>

① we will be given area + have to find value (find μ, σ eq in terms of z).

⇒

$x<45$	0.31
$x>64$	0.08
$45<x<64$	0.61



$$\boxed{+z_1 = \frac{x-\mu}{\sigma}}$$

$P(-z_1 < z < 0) = 0.19$

$= -z_1 = 0.5$

$\boxed{z_1 = -0.5}$

$-0.5 = \frac{45-\mu}{\sigma}$

→ $-0.5\sigma + \mu = 45$

$$z_2 = \frac{x-\mu}{\sigma}$$

$P(0<z<z_2) = 0.42$

$\boxed{z_2 = 1.41}$

$1.41 = \frac{64-\mu}{\sigma}$

$1.41\sigma + \mu = 64$

$\mu = 49.97$ | $\sigma = 9.95$

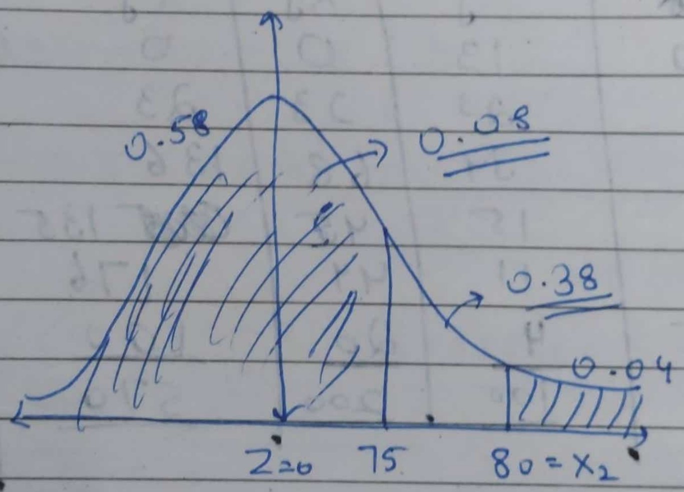
Q → If skulls are classified as A, B and C according as the L, B and index as under 75, b/w 75 and 80, or over 80, find μ and σ given

A → 58% | B → 38% | C → 4%

$f(0.20) = 0.08$ | $f(1.75) = 0.46$.

→

$X < 75$	0.58
$75 < X < 80$	0.38
$X > 80$	0.04



$Z_2 = \frac{80-\mu}{\sigma}$

$X_1 = 75$

$Z_1 = \frac{75-\mu}{\sigma}$

here $P(0 < Z < Z_1) \Rightarrow 0.08$

$Z_1 \Rightarrow 0.20$

$0.20 = \frac{75-\mu}{\sigma}$

$\boxed{0.20\sigma + \mu = 75}$

$X_2 = 80$

$Z_2 = \frac{80-\mu}{\sigma}$

$P(0 < Z < Z_2) = 0.46$

$Z_2 = 1.75$

$1.75\sigma + \mu = 80$

$\mu = 74.4$ || $\sigma = 3.5$

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Q→ Find the eq of normal probab curve that may fit to the following

x	0	1	2	3	4	5
f	13	23	34	15	11	4

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

$$\boxed{z = \frac{x-\mu}{\sigma}}$$

$\mu = E(x) = \frac{\sum xf}{\sum f}$ here → | $\sigma = \sqrt{\frac{\sum x^2 f}{\sum f} - \left(\frac{\sum xf}{\sum f}\right)^2}$

$V = E(x^2) - (E(x))^2$

x	x^2	f	xf	$x^2 f$
0	0	13	0	0
1	1	23	23	23
2	4	34	68	136
3	9	15	45	135
4	16	11	44	176
5	25	4	20	100
		100	200	570

$\mu = \frac{200}{100} = 2$

$S.D = \sqrt{\frac{570}{100} - 4}$

$\sqrt{5.7 - 4} = \sqrt{1.7} = \underline{1.3} = \sigma$

$$\frac{1}{1.3\sqrt{2\pi}}\, e^{-\frac{1}{2}z^2}$$

$$z = \frac{x-2}{1.3}$$

$$\boxed{\frac{1}{1.3\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-2}{1.3}\right)^2}}$$ · fitted eq.

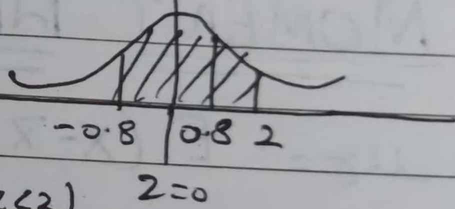
Q → $\boxed{\mu = 30,\ \sigma = 5}$ given

(1) $26 \le x \le 40$

→ $P(26 \le x \le 40) \rightarrow P(a \le z \le b)$

$a = -\frac{4}{5} = -0.8$ | $b = 2$

$P(-0.8 \le z \le 2) \Rightarrow$



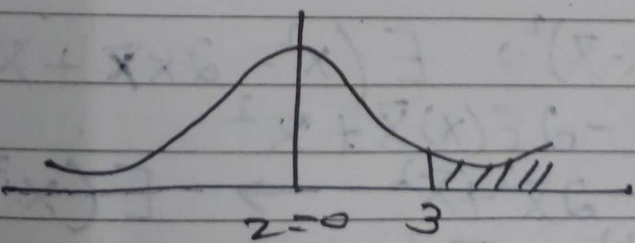
Ans → $P(0 < z < 0.8) + P(0 < z < 2)$

---

(2) $x \geq 45$

→ $P(x \geq 45) \rightarrow P(z \geq a)$

$a = \frac{45-30}{5} = \boxed{a = 3}$ → $P(z \geq 3)$



Ans → $\boxed{\frac{1}{2} - (P(0 < z < 3))}$

(3) $|x-30| \leq 5$

$\rightarrow -x+30 \leq 5$ | $x-30 \leq 5$

$x-30 \geq -5 \rightarrow x \geq 25$ | $x \leq 35$

$25 \leq x \leq 35$

$P(a \leq z \leq b) \rightarrow a = -1$ | $b = 1$

$P(-1 \leq z \leq 1)$

Ans $\rightarrow$ $\boxed{2 \cdot P(0 < z < 1)}$.

Imp ↓

(3) **MOMENT ABOUT Mean (μ)**

$\rightarrow \mu_r = E(x-\bar{x})^r$ $\left[\frac{1}{n}\Sigma(x-\bar{x})^r\right.$

$\mu_1 = E(x-\bar{x})^1 \rightarrow E(x) - E(\bar{x})$

[expectation of mean would always be mean itself]

- $\mu_1 = \bar{x} - \bar{x} = 0$ | $\boxed{\mu_1 = 0}$.

- $\mu_2 = E(x-\bar{x})^2 \rightarrow E(x^2 - 2x\bar{x} + \bar{x}^2)$

$\rightarrow E(x^2) - 2E(x)\bar{x} + \bar{x}^2$

$\rightarrow E(x^2) - 2\bar{x}^2 + \bar{x}^2 \longrightarrow E(x^2) - 1\bar{x}^2$

$\rightarrow E(x^2) - \bar{x}^2$

$\Rightarrow E(x^2) - [E(x)]^2 = \underline{Var}$

$\rightarrow \boxed{\mu_2' - (\mu_1')^2}$

Moments

(1) MOMENTS ABOUT ORIGIN

→ $\boxed{\mu'}$

$\mu'_r = \boxed{E(X-0)^r} \rightarrow \boxed{E(x)^r} \rightarrow \frac{\Sigma f(x)^r}{N}$

$\mu'_1 = E(x) = \text{mean} = \bar{x}$

$\mu'_2 = E(x^2)$

$\mu'_3 = E(x^3)$

$\mu'_4 = E(x^4)$

(2) MOMENTS ABOUT ANY POINT A

→ $\boxed{\mu''}$

$\mu''_r = E(X-A)^r \rightarrow \frac{1}{n}\Sigma f(X-A)^r$

$\mu''_1 = E(X-A)$

$\mu''_2 = E(X-A)^2$

$\mu''_3 = E(X-A)^3$

$\mu''_4 = E(X-A)^4$

→ ☆ similar to μ'_r But instead of origin here we have a ref point.

→ ☆ So the reln B/w $\boxed{\text{MA Mean}}$ and $\boxed{\text{MA origin}}$ will be similar to reln B/w $\boxed{\text{MA mean}}$ and $\boxed{\text{MA Point}}$.

$$\mu_1' = \bar{x} = E(x)$$

- $\mu_3 \Rightarrow E(x-\bar{x})^3 \rightarrow E(x^3 - 3x^2\bar{x} + 3x(\bar{x})^2 - (\bar{x})^3]$

$$E(x^3) - 3E(x^2)\,\bar{x} + 3E(x)(\bar{x})^2 - (\bar{x})^3]$$

$$\mu_3' - 3\mu_2'\mu_1' + 3\mu_1'(\mu_1')^2 - (\mu_1')^3]$$

$$\mu_3' - 3\mu_2'\mu_1' + 3(\mu_1')^3 - (\mu_1')^3$$

$$\mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

- $\mu_4 \Rightarrow E(x-\bar{x})^4$

$\rightarrow$ $E(x-\bar{x})^4 \rightarrow x^4 - 4x^3\bar{x} + 6x^2(\bar{x})^2 - 4x(\bar{x})^3 + ($

$$E(x^4) - 4E(x^3)\bar{x} + 6E(x^2)(\bar{x})^2 - 4E(x)(\bar{x})^3 - (\bar{x})^4$$

$\rightarrow$ $\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 4\mu_1'(\mu_1')^3 + (\mu_1'$

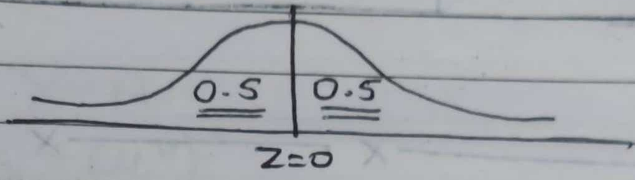
$\rightarrow$ $\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 4(\mu_1')^4 + (\mu_1')^4$

$\rightarrow$ $\boxed{\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4}$

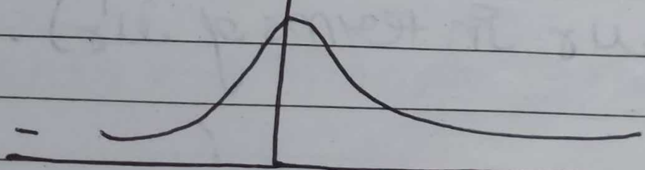
Skewness → lack of symmetry of Distribution w.r.t Normal Distance

→ $(\beta_1) = \boxed{\frac{\mu_3^2}{\mu_2^3}}$. | $\boxed{\gamma_1 = \pm\sqrt{\beta_1}}$.

$\beta_1 = 0$ → follows Normal Dist

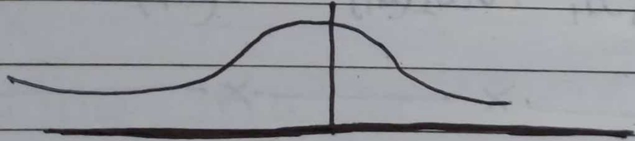


$\beta_1 > 0$ → unsymmetrical - positively skewed.



(more area on +ve side)

$\beta_1 < 0$ → unsymmetrical - negatively skewed

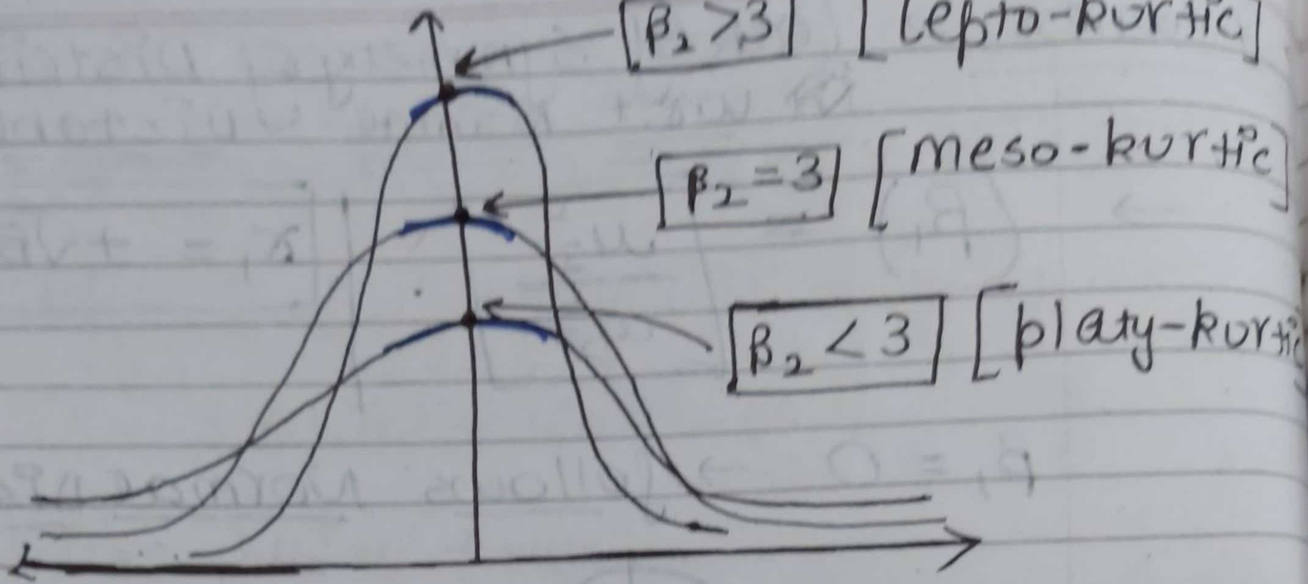


(more area on -ve side).

Kurtosis → flatness of the curve w.r.t to Normal Dist

$\beta_2 \Rightarrow \boxed{\frac{\mu_4}{\mu_2^2}}$ | $\gamma_2 = \boxed{\beta_2 - 3}$.

→ Tells us if the Distribution is above the Normal std dist or Below.



x ——— x ——— x

relⁿ / moment around mean in terms of moment around origin

(μ_r in terms of μ'_r).

$\mu_1 = 0$

$\mu_2 = \mu'_2 - (\mu'_1)^2$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$

$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4$

x ——— x ——— x

(μ_r in terms of μ''_r)

[$\mu'_r \rightarrow \mu''_r$]

$\mu_1 = 0$

$\mu_2 = \mu''_2 - (\mu''_1)^2$

$\mu_3 = \mu''_3 - 3\mu''_2\mu''_1 + 2(\mu''_1)^3$

$\mu_4 = \mu''_4 - 4\mu''_3\mu''_1 + 6\mu''_2(\mu''_1)^2 - 3(\mu''_1)^4$

μ'_r in terms of μ_r

$\mu'_1 = \bar{X} - A \quad (A=0) = \bar{X}$

$\mu'_2 = \mu_2 + (\mu'_1)^2$

$\mu'_3 = \mu_3 + 3\mu_2\mu'_1 + (\mu'_1)^3$

$\mu'_4 = \mu_4 + 4\mu_3\mu'_1 + 6\mu_2(\mu'_1)^2 + (\mu'_1)^4.$

μ''_r in terms of μ_r

$(\mu'_r \leftrightarrow \mu''_r)$

→ $\mu''_1 = \bar{X} - A$

$\mu''_2 = \mu_2 + (\mu''_1)^2$

$\mu''_3 = \mu_3 + 3\mu_2\mu''_1 + (\mu''_1)^3$

$\mu''_4 = \mu_4 + 4\mu_3\mu''_1 + 6\mu_2(\mu''_1)^2 + (\mu''_1)^4.$

Moments ⟷ Distance in Physics

↳ Describe the characteristics of a Distribution.

★ Fitting straight line (/) using MLS.

→ $\boxed{y = ax+b}$ [To find a and b from x, y]

→ $\boxed{① \ \Sigma y = a\Sigma x + nb}$

$[xy = ax^2 + bx]$

→ $\boxed{② \ \Sigma xy = a\Sigma x^2 + b\Sigma x}$

• Fitting 2nd degree Parabola using [redacted] m

$$y = ax^2 + bx + c$$

① $\Sigma y = a\Sigma x^2 + b\Sigma x + nc$

$[xy = ax^3 + bx^2 + cx]$

② $\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$

$[x^2y = ax^4 + bx^3 + cx^2]$

③ $\Sigma x^2y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$.

$\boxed{\text{END}}$.

— x — x — x —

• if $x = x_0$ then $p = 0$, so →

→ $$y'(x) = \left[\frac{dy}{dx}\right]_{x=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0\right] \ldots$$

→ $$y''(x) = \left[\frac{dy}{dx}\right]_{x=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{22}{24}\Delta^4 y_0 + \ldots\right].$$

— x — x — x —

Similarly for Backward

→ $$y'(x) = \left(\frac{dy}{dx}\right) = \frac{1}{h}\left[\nabla y_n + \frac{2p+1}{2!}\nabla^2 y_n + \frac{3p^2+6p+2}{3!}\nabla^3 y_n + \frac{4p^3+18p^2+22p+6}{4!}\nabla^4 y_n + \ldots\right.$$

→ $$y''(x) = \left(\frac{d^2y}{dx^2}\right) = \frac{1}{h^2}\left[\nabla^2 y_n + (p+1)\nabla^3 y_n + \frac{12p^2+36p+22}{4!}\right.$$