Hypothesis Testing

Date 4

- Prob → The way events occur in Nature
  ↓
  makes us think that they are occuring coz of a reason
  ↓
  But the underlying cause is randomness

- Solⁿ → HT

  removes the element of randomness
  ↓
  proves that Observed Result is coz of a Reason and not Randomness.

① Alternate Hypothesis [H_A] → The claim we want to prove → opp of H_0

② Null Hypothesis [H_0] → established fact
→ assumed to be True until declared False
→ Opp of H_A.

eg → $H_0 : \mu = 500\,ml$ [= ≥ ≤]

Then

$H_a : \mu \neq 500\,ml$ [Two tailed Test] → $\alpha/2$

or

$H_a : \mu > 500\,ml$ [Right one-tailed Test] → α

or

$H_a : \mu < 500\,ml$ [Left one-tailed Test] → α

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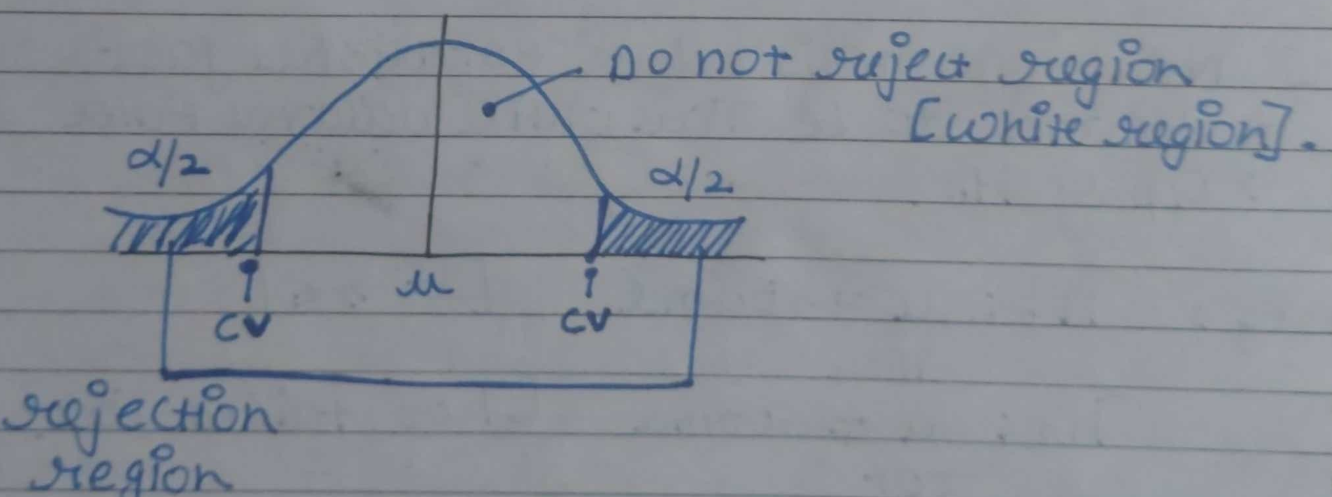
Outcomes of HT

1. Reject H_0
2. Do not Reject H_0

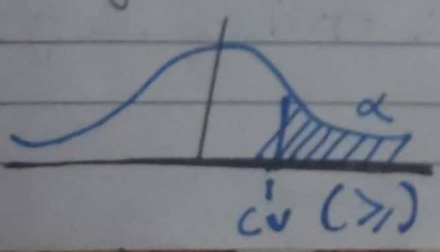
STEPS

1. Based on Ha → identify the type of Test [2 or 1 tail]
2. Perform Test of significance
3. Compare with critical value or level of significance (α)
4. Check for Rejection region.

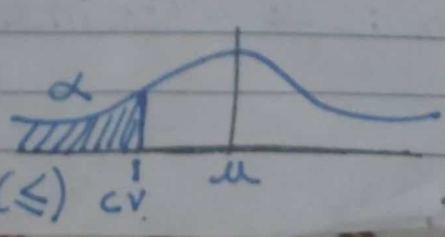
2-tail Test ($\alpha = \alpha$)



Right-1-tail Test



left-1-tail-test



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Level of significance (α)

'prob of error' in accepting or rejecting H_0.

Set value → 5% [0.05] → General value.
1% [0.01]

Confidence level → [C] → $\boxed{1-\alpha}$.

- Test of significance

① Z-Test ② T-test ③ Chi-square Test
④ F-test ⑤ ANOVA

① Z-Test

$$Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{N}} \text{ or } \frac{\bar{X}-\mu}{s/\sqrt{N}} \quad [N > 30]$$

either find p-value	either find critical value
p value from z score value ↓ compare with α ($p \le \alpha$, [redacted]) ↳ rejection	using α find critical value.

$\alpha = 0.05$	$\alpha = 0.01$
CV (2 tail) → ± 1.96	CV (2 tail) → ± 2.58
CV (1 tail) → 1.65	CV (1 tail) → 2.33
• [L → −1.65]	• [L → −2.33]
• [R → 1.65]	• [R → 2.33]

Spiral

chi-square as parametric → Test pop var on the Basis of sample var $\left[\chi^2 = \frac{\sigma_s^2 (n-1)}{\sigma_p^2}\right]$

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② T-test

When [N < 30] → formula → $\frac{\bar{X}-\mu}{\sigma/\sqrt{N}}$ | $\frac{\bar{X}-\mu}{S/\sqrt{N}}$

→ using α and degree of freedom [d.f = n-1] find C.V from t-test Table.

→ Check for Rejection Region.

③ Chi-square Test → Goodness of fit (→ Diff of observed v/s Expected) | Test of Independence of 2 variables

$\chi^2 = \frac{\Sigma(O-E)^2}{E}$ → χ^2 calculated

→ using α and d.f → find critical value from chi-square Table.

→ If χ^2 calc > χ^2 cv → rejection.

—— X —— X —— X —— X —— X ——

Q1] → Z test when working with proportion

A] → Yes, $Z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{N}}} \rightarrow \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{N}}}$ [$\hat{p}$ = sample prop] [p = pop. prop]

—— X —— X —— X —— X ——

$\boxed{H_0}$	True	False
Rej	Type-1 err [FP]	OK
Not Rej	OK	Type-2 err [FN]

① Chi-Square Test

- Chi-square Test is a statistical Test used to determine if there is a '<u>significant difference</u>' Between the Observed frequencies and the Expected frequencies in one or more categories

- Commonly used to evaluate
  - → Goodness of fit of an observed dataset to a Theoretical Distribution
  - → Test the Independence B/w 2 variables | Also <u>correlation B/w them</u>

    Independence = weak correlation

- The Chi-square Test calculates the Difference B/w the Observed and Expected frequencies → and the Resulting chi-square Statistic is compared to a critical value to determine the level of significance

$$\chi^2 = \frac{\Sigma(O-E)^2}{E}$$

df = no. of observation − 1

* using α and d.f → find critical value from chi-square Table

If $\chi^2_c > \chi^2_{cv}$ → reject H_0

$\boxed{\text{In R}}$

```
library (tidyverse)
library (broom)

data <- data.frame(
    Observed = c(18, 20, 22, 10),
    Group = c("A", "B", "C", "D") )
```
Ⓐ

chi-square_test <- chisq.test(data$observed, data$expected)

- [If not specified, the expected frequencies will be calculated based on the Observed Data]

- Use the tidy function from the broom package To extract the relevant results of the chi-square test.

→ result = tidy(chi-square-test).

$\boxed{Ex2}$ [Table]

```
library(MASS)
print(str(survey)
① - stu_data = data.frame(survey$Smoke, survey$Exer)
② - stu_data = table(~~survey$Sm~~ stu-data)
test = chisq.test(smoke_excercise)
                   └ stu-data
result = tidy(test)
          └ from broom
```

———— × ———— × ———— × ———— × ————

T-Test

- Significance of the DIFFERENCE of the MEAN VALUES.
- when the sample size is small + population s.d is unknown

Assumptions

① population Distribution is Normal
↳ use Shapiro-wilk normality-test
[shapiro.test(Y)]

② Sample are Random and Independent

③ Sample size is small

④ population s.d (σ) is not known

* MANN-WHITNEY-U TEST :- Non parametric Counterpart of T-Test

1] One-sample T-Test [Ho vs Ha]

- Comparing Sample Mean with that of the population mean

$$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$$

```
t.test(y, mu=12, conf.level = 0.95,
       alternative = 'two.sided'
       'less' or 'greater')
```

Ho

1-α

2] Two-sample T-Test and Paired T-Test

Two-sample T-Test: Compare the means (significance) of Two Diff'n Samples which are totally Independent

Paired T-Test: Compare the means (significance) of Two Diff'n Samples which are Dependent

$$t = \frac{\bar{X_1} - \bar{X_2}}{\sqrt{\frac{s_1^2}{(\sqrt{n_1})^2} + \frac{s_2^2}{n_2}}} \rightarrow \boxed{\frac{\bar{X_1} - \bar{X_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

numeric, categorical

```
t.test(y~x, data= ,
       paired = FALSE,
       alternative =  ,
       conf.level =  )
```

```
t.test(x, y, paired = TRUE)
```

* ANOVA → Analysis of Variance

- Use to Test the significance of the Diff of The Mean values among more than 2 samples group.
- Extension of T-Test and Z-Test
- It uses F-Test to statiscally Test the Equality of Means and the relative variance B/w Them.

→ aov → $\dfrac{\text{Variance B/w The Sample Means}}{\text{Variance within the Samples}}$ [stock]

→ aov(numeric ~ $\overbrace{\text{categorical 1 + categorical 2}}^{\text{2 WAY ANOVA}}$ +..+., data=)

→ aov(numeric ~ $\overbrace{\text{categorical}}^{\text{1 WAY ANOVA}}$, data=)

ASSUMPTIONS

a) Population Distribution is Normal

b) Samples are random and Independent

c) Homogeneity of Sample variance

d) one-way | Two-way Anova

eg →

Gender	Scores	Age-group
(B,G)		(10,11,12)

[either Gender $\boxed{\text{or}}^{\text{1WAY}}$ Age-group affects Variance of Scores or $\boxed{\text{Both}}_{\text{2WAY}}$]

If Age Group

take Avg of Data of 10 year old → A_1 of G_1
take Avg of Data of 11 year old → A_2 of G_2
" " " " " 12 " " → A_3 of G_3

→ Stack $\boxed{\begin{matrix} G_1 \\ G_2 \\ G_3 \end{matrix}}$

A_4 → avg of stack

If Gender

take avg of Data of Boys → A_1 of G_1
take avg of Data of Girls → A_2 of G_2

→ Stack $\boxed{\begin{matrix} G_1 \\ G_2 \end{matrix}}$

A_3 → avg of stack

If Both

- take Avg of Data of 10 year old + who are Boys → A_1 → G_1
  " " " " " 11 " " + " " " → A_2 of G_2
  " " " " " 12 " " + " " " → A_3 of G_3

  Each Age Grp of Boys

- Avg of Boys = avg of [A_1, A_2, A_3] = A_8 or A_4

- Same method for Girls → A_5, A_6, A_7 Age-grp for girls
  - ↳ A_6 or A_8 = avg of [A_5, A_6, A_7]

———X———X———X———X———X———