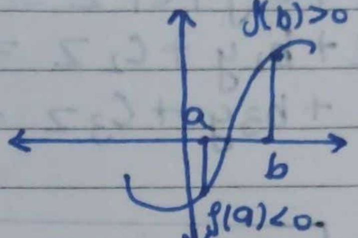
Finding roots of eq ← Numeric Methods

① Bisection Method [mid-point]

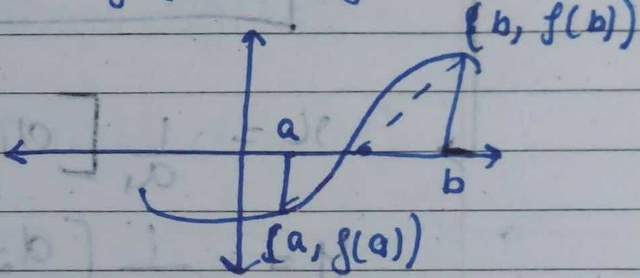
→ N.P = $\boxed{\frac{a+b}{2}}$



② regula Falsi Method

[Joing 2 points instead of finding the mid point]

→ N.P = $\frac{a f(b) - b f(a)}{f(b) - f(a)}$



③ Newton's Raphson Method [using slope]

[making Tangent through the point]

To find that one point [Starting point]

→ Find a and b → a+b/2 → a+b/4 → x_0

→ N.P = $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ [RoC = 2]

[$\mathcal{E}_{n+1} \propto \mathcal{E}_n^2$]

[error is dec]

④ finding roots of system of eq.

$\boxed{4.1}$ Gauss Jacobi iteration

eq

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\}$$

here

$$\boxed{\begin{aligned} |a_1| &> |b_1| + |c_1| \\ |b_2| &> |a_2| + |c_2| \\ |c_3| &> |a_3| + |b_3| \end{aligned}}$$

Condⁿ

$$x = \frac{1}{a_1}\left[d_1 - b_1y - c_1z\right] \quad -①$$

$$y = \frac{1}{b_2}\left[d_2 - a_2x - c_2z\right] \quad -②$$

$$z = \frac{1}{c_3}\left[d_3 - a_3x - b_3y\right] \quad -③$$

① → x coeff was Bigger [Condⁿ]
② y coeff was Bigger
③ z coeff was Bigger

* Iteration - 1

① isme sab ki values ek saath daalte hai nahi ek ki value dosri mai.

① Put x = 0, y = 0, z = 0 in ① ② ③

↳ $\boxed{x_1, \; y_1, \; z_1}$

iteration - 2

→ Put $x = x_1$ | $y = y_1$ | $z = z_1$ | we get (①, ②, ③)

↳ $\boxed{x_2, y_2, z_2}$

→ Such iteration will go on.

till in 2 iterations values of x, y, z are same to n decimal digits.

Tip → calculate ($\boxed{n+2}$ decimal digits).

1.2) Gauss Seidal

→ taking eq ① ② ③ with cod^n same.

· First step

① Put $y = z = 0$ into ① and find $\boxed{x = x_1}$

↳ isme ek ki value dosre mai dalte hai.

↳ we find reference point

→ we put $x = x_1$ and $z = 0$ in ② to find $\boxed{y = y_1}$

→ Then put $x = x_1$ and $y = y_1$ in ③ to find $\boxed{z = z_1}$.

- Second step
- Now we have ref point
1. in ① put $y=y_1$ and $z=z_1$ to find $\boxed{x=x_2}$
- in ② put $\boxed{x=x_2}$ and $z=z_1$ to find $\boxed{y=y_2}$
- in ③ put $x=x_2$ and $y=y_2$ to find $\boxed{z=z_2}$

→ Repeat untill in 2 iteration values of x, y, z are same to n-decimal places.

—— x —— x —— x ——

Interpolation

→ $y=f(x)$ → any func^n
h → fixed value

→ points are equidistant with interval 'h'.

→ $x_1 = x_0 + h$
$x_2 = x_1 + h \rightarrow x_0 + 2h$
$x_3 = x_2 + h \rightarrow x_0 + 3h$ ...

① Shifting operator (E)

→ $E^1 f(x) = f(x+h)$

$E^2 f(x) = E(Ef(x)) \to E(f(x+h)) \to f(x+2h)$.

$E^{-1} f(x) = \boxed{f(x-h)}$.

eg. $E^{-1}(y_1) \to E^{-1}(f(x_1)) \to (f(x_1-h)) \to f(x_0) = y_0$.

② Forward Difference (Δ)

→ $\Delta f(x) = \boxed{f(x+h) - f(x)}$. $\left[\begin{array}{l}\Delta y_0 = y_1 - y_0 \\ \Delta y_1 = y_2 - y_1\end{array}\right]$

$\Delta^2 f(x) = \Delta(\Delta f(x)) \to \Delta(f(x+h) - f(x))$ $\boxed{\Delta^2 y_1 = \Delta y_2 - \Delta y_1}$.

→ $f(x+2h) - f(x+h) - f(x+h) + f(x)$

$f(x+2h) - 2f(x+h) + f(x)$.

③ Backward Diff (∇)

$\nabla f(x) = f(x) - f(x-h)$

$\nabla y_1 = y_1 - y_0$ $\nabla^2 y_2 = \nabla y_2 - \nabla y_1$

$\nabla y_2 = y_2 - y_1$

(4) Centeral Diff operator (δ)

→ $\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$

(5) Averging operator (μ)

$$\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

(1) $\Delta = E - 1$

→ Let $\boxed{y = f(x)}$

→ $\Delta f(x) = f(x+h) - f(x)$

$\Delta f(x) = E f(x) - f(x)$

$$\boxed{\Delta = E - 1}$$

(2) $\nabla = 1 - E^{-1}$

→ Let $\boxed{y = f(x)}$

$\nabla f(x) = f(x) - f(x-h)$

$\nabla f(x) = f(x) - E^{-1} f(x)$

$$\boxed{\nabla = 1 - E^{-1}}.$$

(3) $\delta = E^{1/2} - E^{-1/2}$

$\delta(f(x)) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$

$= \delta(f(x)) = E^{1/2} f(x) - E^{-1/2} f(x)$

$\boxed{\delta = E^{1/2} - E^{-1/2}}$.

(4) $\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$.

$\rightarrow \mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$

$= \mu f(x) = \frac{1}{2}\left[E^{1/2} f(x) + E^{-1/2} f(x)\right]$

$\mu = \frac{1}{2}\left[E^{1/2} + E^{-1/2}\right]$.

(5) $\left[E^{1/2} + E^{-1/2}\right]\left[1+\Delta\right]^{1/2} = 2+\Delta$

$\rightarrow \left[E^{1/2} + E^{-1/2}\right]\left[E\right]^{1/2} \rightarrow E^1 + 1 \quad [E = \Delta + 1]$

$\rightarrow E^1 + 1 \rightarrow \boxed{\Delta + 2}$.

(6). $E = e^{hD} \quad \left[D = \frac{d}{dx}\right]$

$\rightarrow E f(x) = \boxed{f(x+h)}$. [Taylor's Theorem].

$f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \ldots\ldots$

$= \left[1 + Dh + \frac{h^2 D^2}{2!} + \frac{h^3 D^3}{3!} + + + \ldots\right] f(x) \quad \left[D = \frac{d}{dx}\right]$

$\left[E = 1 + Dh + \frac{h^2 D^2}{2!} + \frac{h^3 D^3}{3!} + \ldots\right] \rightarrow \boxed{E = e^{hD}}$.

$\boxed{\nabla = 1-E^{-1}}$ Page No.

(7) $hD = \log(1+\Delta) = -\log(1-\nabla).$

$$\boxed{hD = \log(1+\Delta)}$$

$$e^{hD} = e^{\log(1+\Delta)}$$

$$E = 1+\Delta$$

$$\boxed{E-1 = \Delta}.$$

$$e^{hD} = e^{-\log(1-\nabla)} \rightarrow e^{hD} = e^{\log\left(\frac{1}{1-\nabla}\right)}$$

$$E = \frac{1}{1-\nabla}$$

$$= 1-\nabla = \frac{1}{E}$$

$$1-\nabla = E^{-1}$$

$$\boxed{1-E^{-1} = \nabla}.$$

(8) $(1+\Delta)(1-\nabla) = 1$

$$E \cdot E^{-1} = 1$$

$$E^{0} = 1$$

$$1 = 1.$$

x ———— x

$$\sin A + \sin B \rightarrow 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\rightarrow 2\sin\left(\frac{2x+4h}{2}\right)\cos\left(\frac{2h}{2}\right)$$

$$2\sin(x+2h)\cos(h) - 2\sin(x+2h)$$

$$\rightarrow (2\cos h - 2). \rightarrow 2(\cos h - 1)$$

① $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$

$\rightarrow \left(\frac{E^2-2E+1}{E}\right)e^x \rightarrow e^{x+h} - 2e^x + e^{x-h}$

$\frac{Ee^x}{\Delta^2 e^x} \rightarrow \frac{e^{x+h}}{e^{x+2h} - 2e^{x+h} + e^x}$

$\rightarrow \frac{e^{2x+2h} - 2e^{2x+h} + e^{2x}}{e^{x+2h} - 2e^{x+h} + e^x} \rightarrow \frac{\cancel{e^{2x}}\cancel{(e^{2h} - 2e^h + 1)}}{\cancel{e^x}\cancel{(e^{2h} - 2e^h + 1)}} \rightarrow e^x.$

② $\left(\frac{\Delta^2}{E}\right)\sin(x+h) + \frac{\Delta^2 \sin(x+h)}{E\sin(x+h)}.$

$\rightarrow \left(\frac{E^2-2E+1}{E}\right)\sin(x+h) \rightarrow \sin(x+2h) - 2\sin(x+h) + \sin x$

$\sin A + \sin B \rightarrow 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\rightarrow 2\sin(x+h)\cosh - 2\sin(x+h)$

$\sin(x+h)[2\cosh - 2] \rightarrow 2\sin(x+h)[\cosh - 1].$

$\rightarrow \frac{\Delta^2\sin(x+h)}{E\sin(x+h)} \rightarrow \frac{\sin(x+3h) - 2\sin(x+2h) + \sin(x+h)}{\sin(x+2h)}$

$\rightarrow 2\sin(x+2h)\cosh - 2\sin(x+2h) + \frac{2\cancel{\sin(x+2h)}(\cosh-1)}{\cancel{\sin(x+2h)}}$

$\rightarrow 2\sin(x+h)(\cosh-1) + 2(\cosh-1)$

$\rightarrow 2(\cosh-1)(\sin(x+h)+1)$ Ans

Date

Page No.

Imp

- $\Delta = E - 1$
- $\nabla = 1 - E^{-1}$
- $E = e^{hD}$

L Taylor's series $\rightarrow f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots$

$\rightarrow hD = \log(1+\Delta) = -\log(1-\nabla)$

Trigo

$\rightarrow \sin A + \sin B$

$\quad \hookrightarrow 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$\rightarrow \cos A + \cos B$

$\quad \hookrightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$.

$\rightarrow \sin A - \sin B$

$\quad \hookrightarrow 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\rightarrow \cos A - \cos B$

$\quad \hookrightarrow -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$.

x is closer / nearer to x_0

so work with y_0.

Newton Forward [we choose → x_0]

→ $$y = f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \ldots\ldots$$

where $\boxed{p = \frac{x - x_0}{h}}$.

Newton Backward [we choose → x_n]

↳ so work with y_n

x is nearer / closer to x_n

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots$$

where $\boxed{p = \frac{x - x_n}{h}}$.

Gauss Forward Central Diff.

Isme $\boxed{x_0 \to \text{Beech}}$ [if x lies in B/w].

→ $$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_{-1} + \frac{(u+1)u(u-1)}{3!}\Delta^3 y_{-1} + \frac{(u+1)u(u-1)(u-2)}{4!}\Delta^4 y_{-2} + \frac{(u+2)(u+1)u(u-1)(u-2)}{5!}\Delta^5 y_{-2} + \ldots$$

$u = \boxed{\frac{x - x_0}{h}}$ $(0 < u < 1)$.

at $\boxed{x=30}$.

x	y		Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
21	18	$\rightarrow y_{-2}$	Δy_{-2}	$\Delta^2 y_{-2}$	$\Delta^3 y_{-2}$	$\Delta^4 y_{-2}$
25	17	$\rightarrow y_{-1}$	Δy_{-1}	$\Delta^2 y_{-1}$	$\Delta^3 y_{-1}$	
$\boxed{x_0} \rightarrow 29$	17	$\rightarrow y_0$	Δy_0	$\Delta^2 y_0$		
33	16	$\rightarrow y_1$	Δy_1			
37	15	$\rightarrow y_2$				

—— x —— x —— x ——

- **Gauss Backward**

$$\rightarrow y = y_0 + u\Delta y_{-1} + \frac{u(u+1)}{2!}\Delta^2 y_{-1} + \frac{(u-1)u(u+1)}{3!}\Delta^3 y_{-2} + \frac{(u-1)u(u+1)(u+2)}{4!}\Delta^4 y_{-2} + \frac{(u-2)(u-1)u(u+1)(u+2)}{5!}\Delta^5 y_{-3} + \dots$$

where $\boxed{u = \frac{x-x_0}{h}}$ $(-1 < u < 0)$.

—— x —— x —— x ——

- **Unequal interval.**

① Langrange's

② Newton Divided Difference

upar x substract except x_0 ($y_n \to x_n$)
niche x_0 subtract
(y_a $y_n \to x_n$) ↑

Date			
Page No.			

① Langrange's

$$\begin{bmatrix} x_0 & x_1 & x_2 & x_3 & x_4 \\ y_0 & y_1 & y_2 & y_3 & y_4 \end{bmatrix}.$$

→ $$\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 +$$

$$\left[\Delta^3 y_1 = \frac{\Delta^2 y_2 - \Delta^2 y_1}{x_4 - x_1}\right.$$

$$\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

—— x —— x —— x —— x ——

② Newton Divided Difference

$$\left[\Delta^3 y_0 = \frac{\Delta^2 y_1 - \Delta^2 y_0}{x_3 - x_0}\right]$$

→ Divided Diff operator (Δ)

- └ Divided Bi hota hai. (forward/div)

$$\Delta y_0 = \frac{y_1 - y_0}{x_1 - x_0} \qquad \Delta^2 y_0 = \frac{\Delta y_1 - \Delta y_0}{x_2 - x_0}$$

$$\Delta y_1 = \frac{y_2 - y_1}{x_2 - x_1} \qquad \Delta^2 y_1 = \frac{\Delta y_2 - \Delta y_1}{x_3 - x_1}$$

$$\Delta y_2 = \frac{y_3 - y_2}{x_3 - x_2} \qquad \Delta^2 y_2 = \frac{\Delta y_3 - \Delta y_2}{x_4 - x_2}$$

$$y = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 + (x-x_0)(x-x_1)(x-x_2)(x-x_3)\Delta^4 y_0 + \dots$$

$\boxed{\text{Imp}}$ ↓

☆ if q mai → $\underline{f(n)}$ or $\underline{f'(n)}$

→ $\boxed{x = x}$ → $\boxed{\text{Phele} \rightarrow y=f(x) \text{ ki eq in terms of } x}$

and then put $\boxed{x = n}$.

For CRV → Distribution (cumulative)

P(x) → F(x) or f(x).

called → P.D.F (Probab Density func

Pdf → rep CRV with its probabilit

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$.

For DRV → mean → $E(x) \rightarrow \sum x P(x)$

For CRV → mean → $E(x) \rightarrow \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

Numeric Differentiation

→ finding values of $y'(x_i)$, $y''(x_i)$... $[x = x_P]$

→ equal intervals → Newton's forward and Backward

→ unequal intervals → Langrange's and Divided Diff.

Imp ✱ In unequal → we don't have specific formulas
① → we find $f(x)$ using any method in terms of x.
② Then we find $f'(x)$ and $f''(x)$ from $f(x)$
③ Put $x = x_i$ in $f'(x)$ and $f''(x)$.

✱ Newton's forward Difference

→ $$y(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \ldots + \ldots$$

→ $P = \frac{x - x_0}{h}$ $= \boxed{Ph + x_0 = x}$

$P^2 - P$ $\quad$ $P^3 - 3P^2 + 2P$

→ $$y(Ph + x_0) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\Delta^3 y_0 +$$

$P^4 - 6P^3 + 11P^2 - 6P$ ← $\frac{P(P-1)(P-2)(P-3)}{4!}\Delta^4 y_0 + \ldots$

$y'(x)$ → $$\left[\frac{dy}{dx}\right] = \frac{1}{h}\left[\Delta y_0 + \frac{(2P-1)}{2!}\Delta^2 y_0 + \frac{(3P^2 - 6P + 2)}{3!}\Delta^3 y_0 + \frac{4P^3 - 18P^2 + 22P - 6}{4!}\Delta^4 y_0 + \ldots\right]$$

$$y''(x) = \left[\frac{d^2 y}{dx^2}\right] = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6P-6}{3!}\Delta^3 y_0 + \frac{12P^2 - 36P + 22}{4!}\Delta^4 y_0 + \ldots\right].$$

$$\left[\frac{d^2 y}{dx^2}\right] = \frac{1}{h^2}\left[\Delta^2 y_0 + (P-1)\Delta^3 y_0 + \frac{12P^2 - 36P + 22}{4!}\Delta^4 y_0 + \ldots\right].$$

$$\boxed{p = \frac{x - x_n}{h}}$$

when $x = x_n$ | $p = 0$

$\rightarrow \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left(\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \dots\right)$

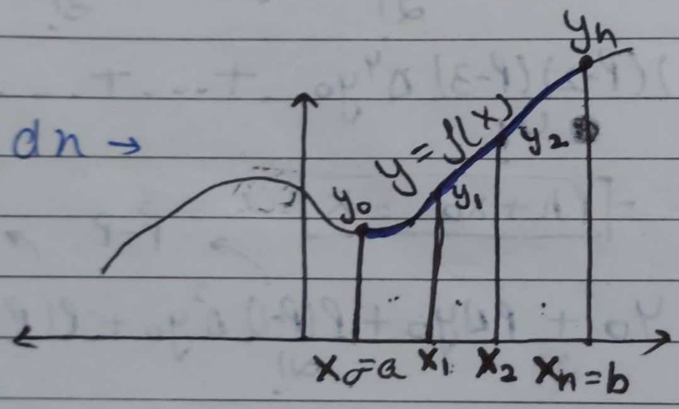
$\rightarrow \left(\frac{d^2y}{dx^2}\right)_{x=0} = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{22}{24}\nabla^4 y_n\right)$

x ——— x ——— x

Numeric Integration

→ finding → $\int_a^b f(x)\,dx$ → $\int_a^b y\,dx$ → in a diff way (divide and sum)

- $\int_a^b y\,dx$ → area under curve.



1. we divide the area into n parts with equal distance - **h**.
2. We then find $x_0 \rightarrow x_n$
3. find corresponding $y_0 \rightarrow y_n$
4. summission of $y_0 \rightarrow y_n$ using rules
   - ↳ Trapezodial Rule
   - ↳ Simpson's 1/3 and 3/8 Rule.

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Date | Page No.

① $h = \frac{b-a}{n}$

② $x_0 = a$
$x_1 = x_0 + h$
$x_2 = x_0 + 2h$
$x_3 = x_0 + 3h = b$.

③ $x_0 \rightarrow y_0$
$x_1 \rightarrow y_1$
$x_2 \rightarrow y_2$
$x_3 \rightarrow y_3$

Whole inside process of rules.

④ TRAPEZODIAL RULE

④ → $$\int_{a=x_0}^{b=x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2\underbrace{(y_1+y_2+y_3+\dots+y_{n-1})}_{\text{middle values}}\right]$$

→ any value of n.

⑤ Simpon's 1/3 Rule

⑤ → $$\int_{a=x_0}^{b=x_n} y\,dn = \frac{h}{3}\left[(y_0+y_n) + 4\underbrace{(y_1+y_3+y_5+\dots y_{n-1})}_{①} + \overbrace{2(y_2+y_4+y_6+\dots+y_{n-2})}^{②}\right]$$

① ↓ middle values which are odd/ not multiple of 2.

- ② → middle values which are even/ multiple of 2.

→ (n should be multiple of 2).

Date
Page No.

⑥ Simpson's 3/8 Rule

→ $$\int_{a=x_0}^{x_n=b} f(x)\,dx \to \frac{3h}{8}\Big[(y_0+y_n)+3\underbrace{(y_1+y_2+y_4+y_5+y_7+\dots+y_{n-1})}_{①}+2\underbrace{(y_3+y_6+y_9+\dots+y_{n-3})}_{②}\Big]$$

- ① → Not multiple of 3 → values
- ② → multiple of 3 → values.

→ n should be multiple of 3.

—— X —— X —— X ——

Q → $\int_0^6 \frac{dx}{1+x^2}$ by ① Trapezodial Rule
② Simpsons 1/3 Rule
③ Simpson's 3/8 Rule.

→ how to choose that value of n which statisfies all ③ rules

n can be ~~6, 12~~, 6, 12, 18 . . .

Ans → $n = 6$ | $a = 0 = x_0$ | $b = 6 = x_6$

$h = \frac{b-a}{n} \to \frac{6-0}{6} = \underline{\underline{h = 1}}$

$x_0 = 0 = \underline{a}$
$x_1 = 1$
$x_2 = x_0 + 2h = 2$
$x_3 = x_0 + 3h = 3$
$x_4 = x_0 + 4h = 4$
$x_5 = x_0 + 5h = 5$
$x_6 = x_0 + 6h = \underline{6 = b}$

$x_n = x_0 + nh$	$y_n = 1/1+(x_n)^2$
$x_0 = 0 = a$	$y_0 = 1$
$x_1 = 1$	$y_1 = 1/2$
$x_2 = 2$	$y_2 = 1/5$
$x_3 = 3$	$y_3 = 1/10$
$x_4 = 4$	$y_4 = 1/17$
$x_5 = 5$	$y_5 = 1/26$
$x_6 = 6 = b$	$y_6 = 1/37$

① Trapezodial

$\rightarrow \int_a^b f(x)dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \dots y_n)\right]$

$\int_a^b f(x)dx = \frac{h}{2}\left[(1 + 1/37) + 2(1/2 + 1/5 + 1/10 + 1/17 + 1/26)\right]$

$\rightarrow \int_a^b \frac{1}{1+x^2} dx = \frac{1}{2}\left[1.02 + 1.79\right]$

$\Rightarrow \boxed{1.405}$

Simpson's 1/3 Rule

$$\rightarrow \int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_n) + 3(y_1+y_3+y_5+\dots y_{n-1}) + 2(y_2+y_4+y_6+\dots y_{n-2})\right]$$

$$\rightarrow \frac{1}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$\rightarrow \frac{1}{3}\left[\left(1+\frac{1}{37}\right) + 4\left(1/2 + 1/10 + 1/26\right) + 2\left(1/5 + 1/17\right)\right]$$

$$\rightarrow \frac{1}{3}\left[1.02 + \overset{2.55}{\cancel{1.91}} + 0.51\right] = \boxed{1.36}.$$

Simpson's 3/8 Rule

$$\int_0^6 \frac{dx}{1+x^2}\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+y_5+y_7+\dots+y_{n-1}) + 2(y_3+y_6+y_9+\dots y_{n-3})\right].$$

$$\rightarrow \frac{3}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$\frac{3}{8}\left[\left(1+\frac{1}{37}\right) + 3\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{17}+\frac{1}{26}\right) + 2\left(\frac{1}{10}\right)\right].$$

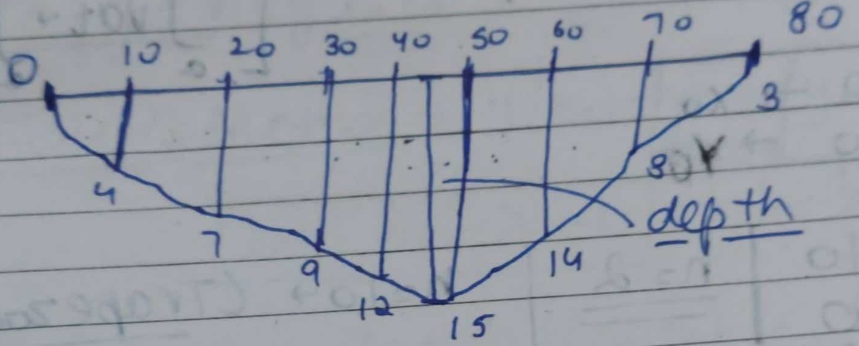
$$= \frac{3}{8}\left[1.02 + 2.39 + 0.20\right]. = \boxed{1.35}.$$

6! 2!

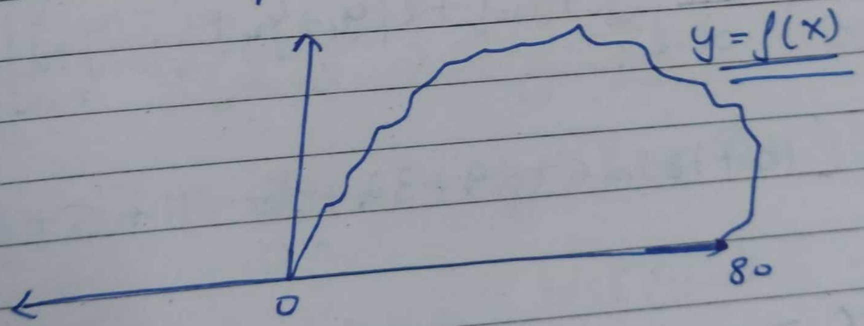
Q→ A River is 80m wide. The depth 'd' in meter at a distance x meter from one Bank is given → find area of cross-section

x (m)	0	10	20	30	40	50	60	70	80
d (m)	0	4	7	9	12	15	14	08	03.
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

→ let's understand the problem



= we can image this as x axis (Bank) and depth as curve.



→ a = 0, b = 80 | n = 8, h = 10 | for every $x_i \to y_i$ is given

h = 8 (we apply Trapezodial rule).

$$\int_0^{80} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+y_3+\dots+y_{n-1})\right].$$

$$= \frac{10}{2}\left[(0+3) + 2(4+7+9+12+15+14+8)\right]$$

Ans → $\boxed{705}$.

- T remains unchanged as ~~...~~ velocity is given in $\boxed{km/min}$.

Date
Page No.

Q→ The velocity v (km/min) of a moped which starts from rest is given at fixed interval of time t (min) as follow. Find approx distance.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x
t(min)	2	4	6	8	10	12	14	16	18	2
v → (km/min)	10	18	25	29	32	20	11	5	2	0
	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_1

At $t = 0$, $V = 0$.

$$\left[\int_a^b v\,dt \to \int_a^b \frac{ds}{dt}\,dt = s\Big|_a\right]$$

$t=0 \to x_0$
$V=0 \to y_0$

→ $n = 10$, $a = 0$, $b = 20$ | $h = 2$ | $n = 10 \to$ (Trapezodial)

$$\to \int_0^{20} v\,dt \to \frac{h}{2}\left[(y_0+y_{10}) + 2(y_1+y_2+\dots y_9)\right].$$

$$\to \left[2(10+18+25+29+32+20+11+5+2)\right] = 304$$

$\boxed{\text{NOTE}}$ $\boxed{\text{Imp}}$

~~we are~~ ~~...~~

- If we are given with 'x' ordinates then $\boxed{n = x-1}$

→ x ordinates → $\boxed{x \text{ no. of values of } y}$.

Numerical solⁿ of ODE

1 Euler's Method

To find solⁿ of ODE at given value of x

- Given → $\frac{dy}{dx} = f(x,y)$ and y at x_0 $[y(x_0) = y_0]$.
- To Find → y at x_n → $y(x_n) = y_n = ?$

Euler's Method → $y(x_n) = \boxed{y_n} = \boxed{y_{n-1} + h f(x_{n-1}, y_{n-1})}$

$\boxed{h = x_n - x_{n-1}}$ → $h = \boxed{\frac{x_n - x_{n-1}}{n}}$

$n=1$ → $y_1 = y_0 + h f(x_0, y_0)$ $[h = x - x_0]$.

$n=2$ $y_2 = y_1 + h f(x_1, y_1)$ $[h = x_2 - x_1]$

Q→ find $y(2.2)$ using EM from eq

→ $\frac{dy}{dx} = -xy^2$ with $y(2) = 1$

Given → $x_0 = 2$, $y_0 = 1$ | $f(x,y) = -xy^2$ |

- To find $y(2.2)$.
- we will take $\boxed{n=4}$ [want to divide h in 4 parts to get more accurate ans]

→ $h = 2.2 - 2.0 = \frac{0.2}{4} = 0.05$

$0.05 \times 2 = 0.10$

$x_0 = 2$

$x_1 = x_0 + h = 2.05$

$x_2 = x_0 + 2h = 2.10$

$x_3 = x_0 + 3h = 2.15$

$x_4 = x_0 + 4h = 2.20$

- $\boxed{y_4}$ is our ans

→ $y_1 = y_0 + h f(x_0, y_0)$

→ $y_1 = 1 + 0.05[-2]$

$y_1 = 1 - 0.10$

- $\boxed{y_1 = 0.90}$

→ $y_2 = y_1 + h f(x_1, y_1)$

$y_2 = 0.90 + 0.05[-1.6605]$

$\boxed{y_2 = 0.81}$

→ $y_3 = y_2 + h f(x_2, y_2)$

$y_3 = 0.81 + 0.05[-1.37781]$

$\boxed{y_3 = 0.74}$

→ $y_4 = y_3 + h f(x_3, y_3)$

$y_4 = 0.74 + 0.05[-1.27734]$

Ans $\boxed{y_4 = 0.68}$

2 Runge Kutta's 4th order

Same Concept :

Given → $\frac{dy}{dx} = f(x,y)$ and $\boxed{y(x_0) = y_0}$

To find → $\boxed{y(x_n) \to y_n}$.

$h \to \underline{x_n - x_0}$ or $\boxed{h \to \frac{x_n - x_0}{h}}$.

eq → $y_{n+1} = y_n + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$k_1 = h f(x_n, y_n)$

$k_2 = h f(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$

$k_3 = h f(x_n + \frac{h}{2}, y_n + \frac{k_2}{2})$

$k_4 = h f(x_n + h + y_n + k_3)$.

---x---x---x---x---

→ $\frac{dy}{dx} = f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}$ | $y(0) = 1$
$y(0.4) = y_n = ?$

→ $f(x,y) = \frac{y^2 - x^2}{y^2 + x^2}$ | $x_0 = 0$
$y_0 = 1$

$x_n = 0.4$ → $h = 0.4 - 0$ → $h \to \frac{0.4}{2} = \underline{\underline{0.2}}$

$x_0 = 0 \longrightarrow y_0$

$x_1 = x_0 + h \to 0.2 \longrightarrow y_1$ [Answer]

$x_2 = x_0 + 2h \to 0.4 \longrightarrow y_2$

• $Y_1 = Y_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$

→

$K_1 = hf(x_0, y_0) \to hf(0, 1) \to 0.2 . 1 = 0.2$

$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = hf(0.1, 1.1) = 0.19$

$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right) = hf(0.1, 1.095) = 0.19$

$K_4 = hf(x_0 + h, y_0 + K_3) = hf(0.2, 1.19) = 0.18$

Hmm $Y_1 = 1 + \frac{1}{6}\left[0.20 + 0.38 + 0.38 + 0.18\right] = \boxed{1.19}$

• $Y_2 = Y_1 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$

$K_1 = hf(x_1, y_1) \to hf(0.2, 1.19) = 0.18$

$K_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) \to hf(0.3, 1.28) = 0.17$

$K_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right) \to hf(0.3, 1.27) = 0.17$

$K_4 = hf(x_1 + h, y_1 + K_3) \to hf(0.4, 1.44) = 0.18$

$\Rightarrow Y_2 = 1.19 + \frac{1}{6}\left[0.18 + 0.34 + 0.34 + 0.18\right]$

Ans $\boxed{Y_2 = 1.36}$.