Aerodynamics

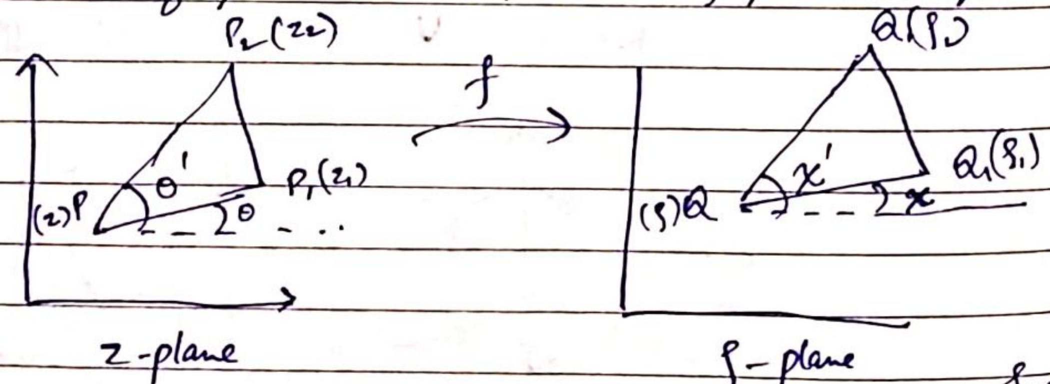
1. **Aerofoils & Kutta-Joukowski Theorem :-**

Conformal mapping : A mapping that preserves angle in both magnitude & orientation.

$$w = \phi + i\psi = f(z) \xrightarrow{\Gamma} g(s) = \underbrace{\cancel{\alpha + i\beta}}_{\text{simpler form}} \quad \xi + i\eta$$

conformal mapping

Let $f(z)$ be a single valued diff. complex function within a closed contour C in xy plane. Let $\zeta = \xi + i\eta$ be another complex variable such that $\zeta = f(z)$. Then corresponding ~~function~~ to each point in Z-plane or in C. There will be a point in ζ-plane or in C'. The necessary condition for such map to exist is $f'(z) \neq 0$ at any pt. in Z-plane or in C. This also means that $d\zeta/dZ$ must exist independently of the direction of δZ. Let P, P_1, P_2 & Q, Q_1, Q_2 be the '2' sets of points in Z-plane & ζ-plane, respectively



ζ: zeta

Now,
$$\frac{\zeta_1 - \zeta}{z_1 - z} = \frac{f(z_1) - f(z)}{z_1 - z}$$

$$\frac{\zeta_2 - \zeta}{z_2 - z} = \frac{f(z_2) - f(z)}{z_2 - z}$$

When $P_1 \to P$, $P_2 \to P$

$$\frac{\zeta_1 - \zeta}{z_1 - z} = f'(z) \quad ; \quad \frac{\zeta_2 - \zeta}{z_2 - z} = f'(z)$$

$$\Rightarrow \frac{\zeta_1 - \zeta}{z_1 - z} = \frac{\zeta_2 - \zeta}{z_2 - z} = f'(z) = \frac{d\zeta}{dz} \quad \text{--- (1)}$$

Next,

$$\frac{QQ_1 e^{i\chi}}{PP_1 e^{i\theta}} = \frac{QQ_2 e^{i\chi'}}{PP_2 e^{i\theta'}}$$

$$\Rightarrow \frac{QQ_1}{PP_1} e^{i(\chi - \theta)} = \frac{QQ_2}{PP_2} e^{i(\chi' - \theta')}$$

When $P_1 \to P$ & $P_2 \to P$

$$\Rightarrow e^{i(\chi - \theta)} = e^{i(\chi' - \theta')}$$

$$\Rightarrow \chi - \theta = \chi' - \theta' \Rightarrow \chi' - \chi = \theta' - \theta$$

i.e $\angle Q_1QQ_2 = \angle P_1PP_2$

Also, $$\frac{QQ_1}{PP_1} = \frac{QQ_2}{PP_2} = |f'(z)| = \frac{d\zeta}{dz}$$

$\Rightarrow \Delta P_1PP_2$ & ΔQ_1QQ_2 are similar

∴ This establishes the similarity of the corresponding infinitesimal element of the 2 planes. Such a relation between the 2 planes is called conformal representation of either one of the other.

Date 24/10/19

$$\zeta = f(z) \quad , \quad z = x + iy$$
$$= \xi + i\eta$$

$$\frac{d\zeta}{dz} = f'(z) = \frac{d}{dz}(\xi + i\eta)$$

$$= \frac{d}{dx}(\xi + i\eta)\frac{dx}{dz}$$

$$= \frac{d\xi}{dx} + i\frac{d\eta}{dx} \quad — (1)$$

Also, $$\frac{d\zeta}{dz} = \frac{d}{dy}(\xi + i\eta)\frac{dy}{dz}$$

$$= \frac{d\xi}{dy} + i\frac{d\eta}{dy} \quad — (2)$$

24/10/19

Conformal Mapping :-

$\zeta = f(z) = f(x+iy) = \xi + i\eta$

From complex analysis :-

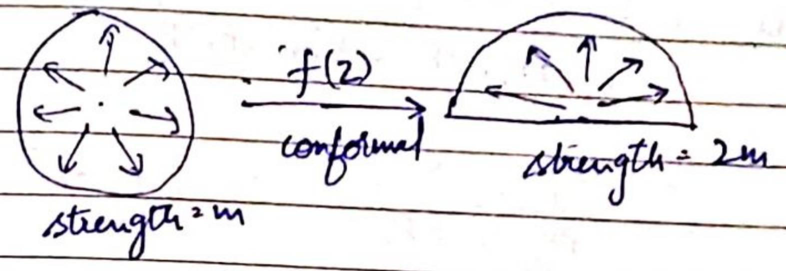
$$\frac{d\zeta}{dz} = \xi_x + i\eta_x$$

From CR-equation :-

$$\frac{\partial \xi}{\partial x} = \frac{\partial \eta}{\partial y} \quad , \quad \frac{\partial \xi}{\partial y} = -\frac{\partial \eta}{\partial x}$$

$$\frac{d\zeta}{dz} = \frac{\partial \eta}{\partial y} + i\,\eta_x = i\left(\frac{\partial \eta}{\partial x} - i\frac{\partial \eta}{\partial y}\right)$$

$$\rightarrow f'(z) = \sqrt{(\partial\eta/\partial x)^2 + (\partial\eta/\partial y)^2}$$



$w = F_1(z) = \phi + i\psi$, where ϕ & ψ are velocity & current functions of any motion within the contour C' in ζ-plane.

Then within C',

$$\phi + i\psi = F_1(\xi + i\eta) = f_1(\xi,\eta) + i\bar{f_1}(\xi,\eta)$$

& C' given by,

$$\psi = \bar{f_1}(\xi,\eta)$$

Similarly,

$$w = F_1(\zeta)$$
$$= F_1(\xi + i\eta) = F_1(\xi(x,y) + i\eta(x,y))$$

$$\Rightarrow \phi + i\psi = F_2(x,y) = f_2(x,y) + i\bar{f_2}(x,y)$$

In this case the contour is :-

$$\psi(x,y) = \bar{f_2}(x,y) = \text{constant}$$

This shows that ψ & ϕ are same in both the planes.

$$q_1 = dw/dz \quad , \quad q_2 = \cancel{\frac{d\zeta}{dz}}\ dw/d\zeta$$

$$|q_1|^2 = \left|dw/dz\right|^2 \quad , \quad |q_2|^2 = \left|\frac{\cancel{d\zeta}/dz}{dw/d\zeta}\right|^2$$

$$\Rightarrow |q_1|^2 = |q_2|^2$$

$$\left|dw/dz\right|^2 = \left|d\zeta/dz\right|^2 \quad -(1)$$

classmate

Date

Page

$$\frac{\Delta Q_1 Q Q_2}{\Delta P_1 P P_2} = \frac{\frac{1}{2} QQ_1 \cdot QQ_2 \cdot \sin \angle Q_1 Q Q_2}{\frac{1}{2} \cdot PP_1 \cdot PP_2 \cdot \sin \angle P_1 P P_2}$$

$$= \frac{QQ_1 \cdot QQ_2}{PP_1 \cdot PP_2} = |f'(z)|^2$$

$$= (\partial\eta/\partial x)^2 + (\partial\eta/\partial y)^2$$

$$=: h^2$$

$$\Rightarrow \Delta Q_1 Q Q_2 = h^2 \, \Delta P_1 P P_2$$

Area of the Δ^{le} in ζ & Z planes are of ratio h^2 to 1

$$\Rightarrow d\zeta \, d\eta = h^2 dx\, dy$$

From ①, $|q_2|^2 = |dw/d\zeta|^2 = |dw/dz|^2 \, |dz/d\zeta|^2$

$$= q_1^2/h^2$$

$$\Rightarrow |q_2|^2 d\zeta d\eta = \frac{|q_1|^2}{h^2} \cdot h^2 dx\, dy$$

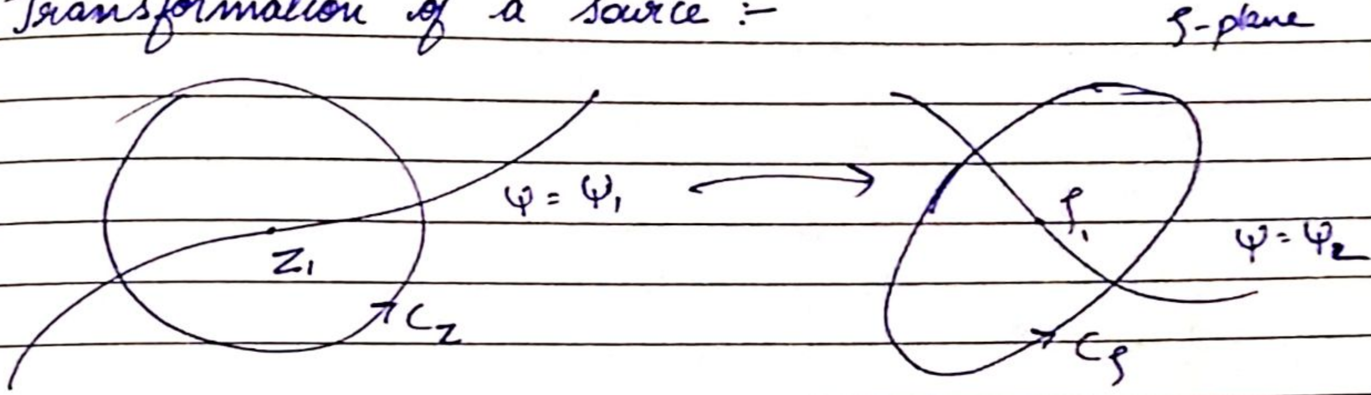
$$\Rightarrow |q_2|^2 d\zeta d\eta = |q_1^2| \, dx\, dy$$

$$\Rightarrow \tfrac{1}{2} \iint \rho |q_2|^2 d\zeta\, d\eta = \tfrac{1}{2} \iint \rho |q_1|^2 dx\, dy$$

$$\Rightarrow T_\zeta = T_Z$$

K.E is same in both planes.

Transformation of a source :-



Let there be a source of strength m at z_1 & ζ_1 be the corresponding pt in ζ-plane. Let these be regular points of transformation. Then a small curve C_Z may be drawn to exclude z_1 & similarly for C_ζ to exclude ζ_1. Since we know the value of stream function is independent of the domain considered, we have

$$\int_{C_Z} d\psi = \int_{C_\zeta} d\psi$$

But, $\displaystyle\int_{C_Z} d\psi = \int_{C_Z} \left(\frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy \right) = \int_{C_Z} (-v\,dx + u\,dy)$

= the total flow across the contour C_Z

= sum of strength of sources " " C_Z.

4\.
Theorem: Under conformal ~~definition~~ representation a uniform line source maps into another uniform line source of the same strength.

Proof:-

Let there be a uniform line source of strength 'm' per unit length through the pt. $z = z_0$ & suppose the conformal transformation $\zeta = f(z)$. Let C_{z_0} be the curve around z_0 in z-plane & C_{z_0} is mapped into C_{ζ_0} in ζ-plane. Then $\zeta = \zeta_0$ lies in C_{ζ_0}.

The complex potential in z-plane & ζ-plane is same and has the form

$$w = \phi + i\psi, \text{ in z-plane}$$
$$w = \phi' + i\psi', \text{ in } \zeta\text{-plane}$$
$$\Rightarrow \quad \phi = \phi', \quad \psi = \psi'$$

Since 'ψ' is the same at the corresponding pts. $z = z_0$ & $\zeta = \zeta_0$ we have $\int_{C_{z_0}} d\psi = \int_{C_{\zeta_0}} d\psi'$ —(1)

But in z-plane,

$$w = -m\log(z - z_0)$$

$$\Rightarrow dw = -m\,dz/(z - z_0)$$

$$\Rightarrow \int_{C_{z_0}} dw = -\int_{C_{z_0}} m\,dz/(z - z_0)$$

$$\Rightarrow \int_{C_{z_0}} (d\phi + d\psi) = -m \cdot 2\pi i \quad \text{by Cauchy's residue Thm}$$

$$\Rightarrow \int_{C_{z_0}} d\psi = -2\pi m \quad —(2)$$

(2) indicates the value of fluid crossing unit thickness of C_{z_0} per unit time

$$\Rightarrow \int_{C_{\zeta_0}} d\psi' = -2\pi m$$

$\leftarrow$ indicates

$\Rightarrow$ the volume of the fluid crossing unit thickness of C_{ζ_0} per unit time

$\therefore$ The conformal mapping preserves the strength of the simple source.

5. Kutta - Joukowskii theorem :-

When a cylinder of any shape is placed in a uniform stream of speed U, the resultant ~~thrust~~ thrust on the cylinder is a lift of magnitude $K\rho U$ per unit length & at right angles to the stream. K is the circulation around cylinder.

Proof :- Let there be a fixed cylinder of some form in the finite region of the plane, its cross-section containing origin. The disturbance of the stream caused by the cylinder can be represented at a great distance in the form:

$$w = Az + B/z^2 + \ldots\ldots,$$ where A, B depend on U & K.

Let the direction of the stream makes an angle α with x-axis. Then the complex potential due to uniform stream velocity 'U' is

$$w_2 = Ue^{-i\alpha}z \quad — (2)$$

And due to circulation,

$$w_3 = \frac{ik}{2\pi} \log z \quad — (3)$$

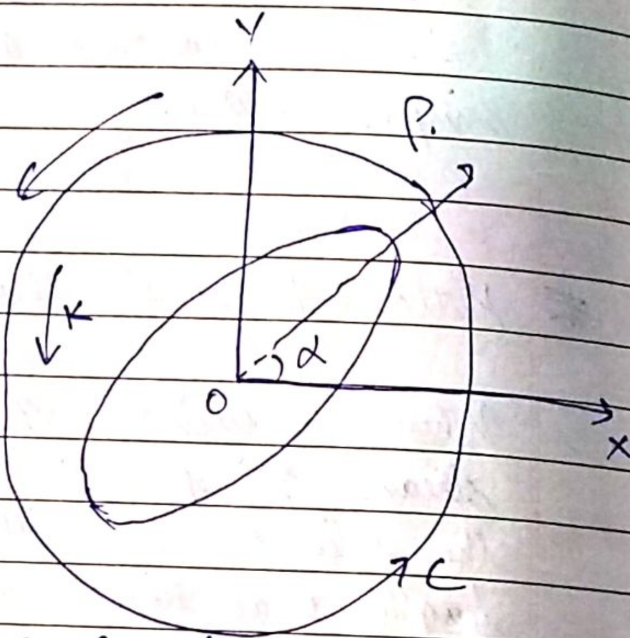
$\therefore$ The complete complex potential is

$$w = w_1 + w_2 + w_3$$

$$= Ue^{-i\alpha z} + \frac{iK}{2\pi}\log z + Az + \frac{B}{z^2} + \dots \quad -(4)$$

By Blasius Theorem:-

The force exerted on the cylinder is



$$X - iY = \frac{1}{2} i\rho \int_C \left(\frac{dw}{dz}\right)^2$$

$$= \frac{1}{2} i\rho \int_C \left[-i\alpha Ue^{-i\alpha z} + \frac{iK}{2\pi z} + A - \frac{B}{z^2} - \frac{C}{z^3}\right]^2$$

$$= \frac{1}{2} i\rho \int_C \left[-i\alpha Ue^{-i\alpha z}\right] dz + \cancel{\frac{1}{2} i\rho \int_C} \left[\frac{iK}{2\pi z} + A - \frac{B}{z^2} - \frac{C}{z^3} \dots\right]^2 dz$$

$$= \frac{1}{2} i\rho \int_C \left[\left(-i\alpha Ue^{-i\alpha z}\right)^2 dz + \left(\frac{iK}{2\pi} + A - \frac{B}{z^2} - \frac{C}{z^3}\right)^2 dz + 2\left(-i\alpha Ue^{-i\alpha z}\right)\left(\frac{iK}{2\pi z} + A - \frac{B}{z^2} \dots\right) dz\right]$$

$$\boxed{dw/dz = Ue^{-i\alpha} + iK/2\pi z - \dots}$$

29/10/19

$X - iY = \frac{1}{2} i\rho \cdot 2\pi i$ (Sum of the residues at $z=0$)

Since $z=0$ is the pole of dw/dz inside C, therefore the residue at $z=0 = \frac{iK\,Ue^{-i\alpha}}{\pi}$

$$X - iY = \frac{1}{2} \cdot i\rho \cdot 2\pi i \cdot \frac{iK\,Ue^{-i\alpha}}{\pi}$$

$$= -i\rho KUe^{-i\alpha}$$

$$\Rightarrow X - iY = -i\rho KU(\cos\alpha - i\sin\alpha)$$

$$X = -KU\rho\sin\alpha$$

$$Y = \rho KU\cos\alpha$$

$$\text{Resultant lift} = \sqrt{X^2 + Y^2}$$

$$= \sqrt{(\rho K U)^2 (\cos^2\alpha + \sin^2\alpha)}$$

$$= \rho K U$$

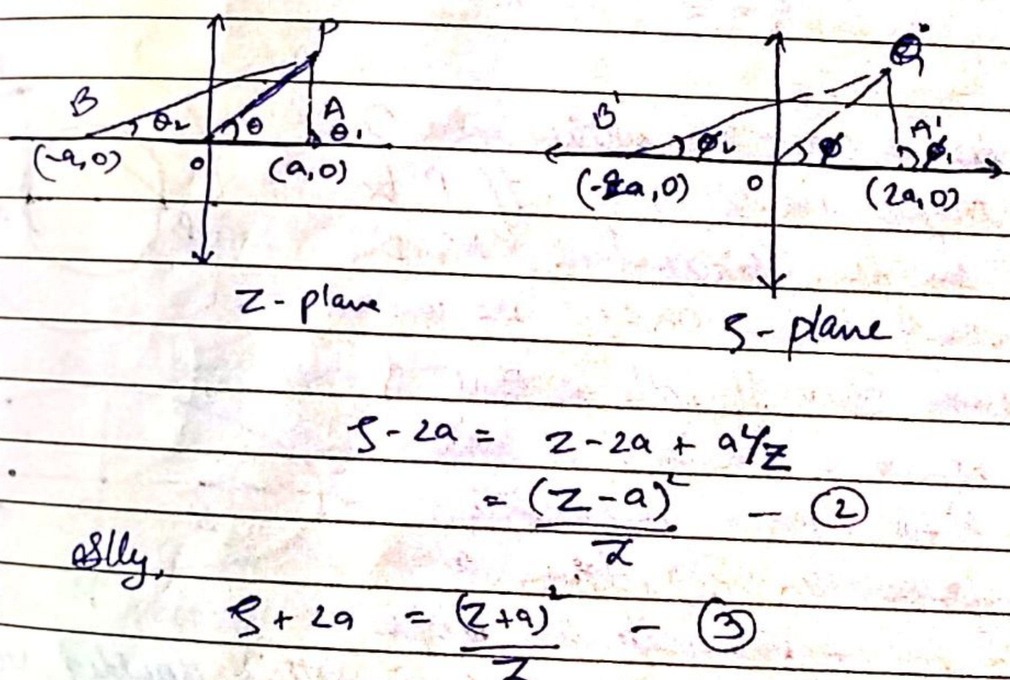
which always acts right angle to cylinder.

Joukowskii Transformation, Joukowskii aerofoils & Hypothesis

Consider the transformation, :-

$$\zeta = z + a^2/z \quad -(1)$$

Now, let A & B be the points at $z = -a, a$. These points are getting mapped to A' & B' resp. i.e. $\zeta = 2a$ & $\zeta = -2a$



$$\zeta - 2a = z - 2a + a^2/z$$
$$= \frac{(z-a)^2}{z} \quad -(2)$$

Slly,
$$\zeta + 2a = \frac{(z+a)^2}{z} \quad -(3)$$

So from (3) :-

$$A'Q e^{i\phi} = \frac{(AP e^{i\theta_1})^2}{OP e^{i\theta}}$$

$$\Rightarrow A'Q = \frac{AP^2}{OP} \quad \& \quad e^{i\phi_1} = e^{i(2\theta_1 - \theta)} \Rightarrow \phi_1 = 2\theta_1 - \theta$$
$$-(4)$$

Again from (2) :-

$$B'Q = BP^2/OP \quad \& \quad \phi_2 = 2\theta_2 - \theta \quad -(5)$$

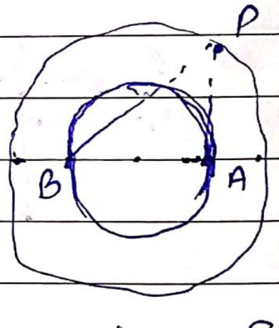
$$\therefore A'QB' = \phi_1 - \phi_2 = 2(\theta_1 - \theta_2) = 2\angle APB \quad -(6)$$

From (4) ; (5) :-

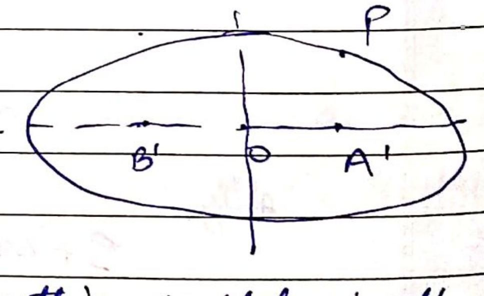
$$A'Q + B'Q = \frac{AP^2 + BP^2}{OP} = \frac{2(OP^2 + OA^2)}{OP},$$

since OA is median of ΔAPB, i.e. $AP^2 + BP^2 = 2(OP^2 + OA^2)$

When $|z|$ is very large, then $\xi = z$

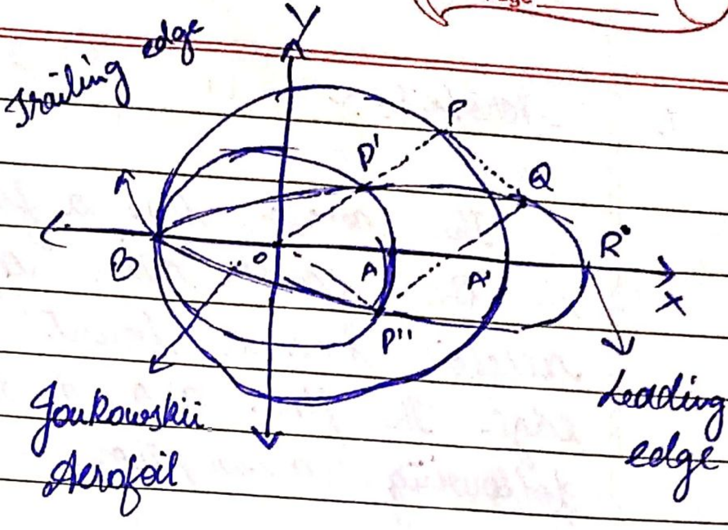


Consider a circle C with centre O at origin in z-plane. If P be a pt on this circle then, $A'Q + B'Q$ = constant because OA & OP are constant from (vi). It follows that Q will describe an ellipse with foci A' & B'. Hence Joukowskii transformation (i) transforms a circle in z-plane to an ellipse in ξ-plane.



Also the angle is getting doubled via the transformation. i.e if P be a point on the smaller circle with $\angle APB = \pi/2$ then $\angle A'QB' = \pi$ i.e. Q lies on $A'B'$!

Let P be the pt. "z" on the bigger circle & P' be it's inverse point w.r.t circle AB & P'' be the reflection of P' on x-axis, then P'' be the pt. a^2/z. Then we draw a parallelogram with OP, OP'



as adjacent sides & OQ be the diagonal s.t Q be the pt. $\zeta = z + a^2/z$. The locus of the point Q is a fish shaped contour which touches the line BA on both sides & such ~~colour~~ contour is known as Joukowskii's aerofoils where B is the trailing edge & R is the leading edge.

Also,

$$\frac{d\zeta}{dz} = 1 - a^2/z^2$$

So, $d\zeta/dz = 0 \Rightarrow z = \pm a$

And $z = a$ is mapped $\zeta = 2a$ &

$\zeta = -2a$

But we need to know

$\frac{d\zeta}{dz}$ at B so either $\frac{d\zeta}{dz} = 0$ (or) $\frac{d\zeta}{dz} = \infty$

If $\vec{q}$ be the velocity at B for the circle & $\vec{q}'$ be the velocity at B' of the aerofoil, then

$$|q'| = |dw/d\zeta| = |dw/dz| \cdot |dz/d\zeta| = q\,|dz/d\zeta|$$

If $d\zeta/dz = 0$ then $dz/d\zeta = \infty \Rightarrow \vec{q}' = \infty$ at B. → not suitable

To avoid infinite velocity at trailing edge. $\frac{d\zeta}{dz} = 0$ at trailing edge B.

The velocity at B is taken as 0, i.e taken as stagnation pt. of flow in z-plane. This is Joukowskill Hypothesis.

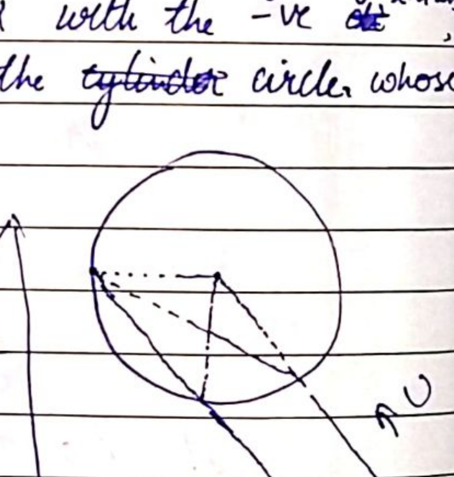
7. ## Aerofoils :-

The aerofoil has a fish type profile. It is employed in the construction of modern airplanes. Such an aerofoil has a blunt leading edge & a sharp leading edge. The flow around the aerofoil depends on the following assumptions.

(i) The air behaves as an incompressible fluid
(ii) The aerofoil is a cylinder whose cross-section is a curve of fish type.
(iii) The flow is 2D irrotation cyclic motion

Flow past a circle :-

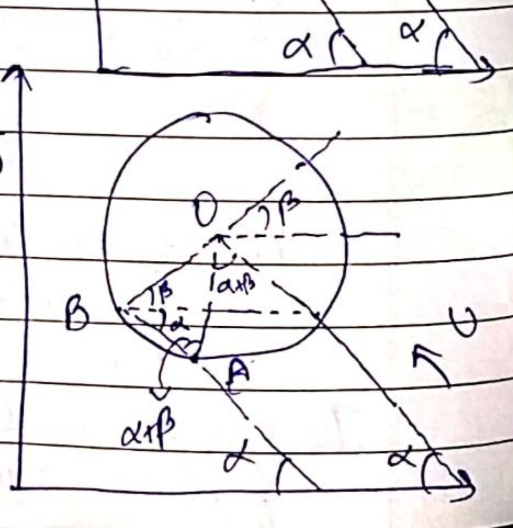
Let U be the velocity of the stream at infinity. Its direction making an angle α with the $-ve$ ~~x~~ x-axis, K be the circulation around the ~~cylinder~~ circle, whose center is at z_0 & radius b. Then,



$$w = Ue^{i\alpha}(z-z_0) + \frac{Ub^2e^{-i\alpha}}{z-z_0} + \frac{ik}{2\pi}\log(z-z_0) \text{, } b \text{ is the volume}$$

To find the stagnation point

$$\frac{dw}{dz} = 0 \Rightarrow Ue^{i\alpha} - \frac{Ub^2e^{-i\alpha}}{(z-z_0)^2} + \frac{ik}{2\pi(z-z_0)} = 0 \quad -(1)$$



classmate
Date 4/11/19
Page

Taking the stagnation pt. as $z = z_0 + be^{i(\pi+\beta)}$
$= z_0 - be^{i\beta}$.

Then, (1) reduces to,
$$U\left(e^{i\alpha} - e^{-i(\alpha+2\beta)}\right) - \frac{ik}{2\pi b} e^{-i\beta} = 0$$

$$\Rightarrow \quad K = 4\pi b U \sin(\alpha+\beta)$$
$$\Rightarrow K \leq 4\pi b U \quad \text{since } \sin(\alpha+\beta) \leq 1 \quad \text{— (2)}$$

4/11/19

* Flow around a circle :-

$$w = Ue^{i\alpha}(z-z_0) + Ub^2e^{-i\alpha}/_{z-z_0} + {}^{ik}/_{2\pi} \log(z-z_0)$$

$$\Rightarrow {}^{dw}/_{dz} = Ue^{i\alpha} - Ub^2e^{-i\alpha}/_{(z-z_0)^2} + {}^{ik}/_{2\pi} \log(z-z_0)$$

For stagnation points, ${}^{dw}/_{dz} = 0$. Let us take the stagnation point as:-
$$z = z_0 + be^{i(\pi+\beta)} = z_0 - be^{i\beta}$$
Eq. 1 reduces to,
$$U\left[e^{i\alpha} - e^{-i(\alpha+2\beta)}\right] - {}^{ik}/_{2\pi b}\, e^{-i\beta} = 0$$
$$\Rightarrow 2\pi b U\left[e^{i(\alpha+\beta)} - e^{-i(\alpha+\beta)}\right] = ik$$
$$\Rightarrow 2\pi b U \, 2i \sin(\alpha+\beta) = ik$$
$$\Rightarrow k = 4\pi b U \sin(\alpha+\beta)$$
$$\Rightarrow k < 4\pi b U \quad , \text{ since } b, U > 0$$

The stagnation points are given by $z - z_0 = be^{i(\pi+\beta)} = t$
From (1),
$$Ue^{i\alpha} - {}^{1}/_{t^2}\, U^2b^2e^{-i\alpha} + \frac{ik}{2\pi t} = 0$$
$$\Rightarrow t = be^{-i\alpha}\left[-i\sin(\alpha+\beta) \pm \cos(\alpha+\beta)\right], \text{ putting } k = 4\pi b U \sin(\alpha+\beta)$$

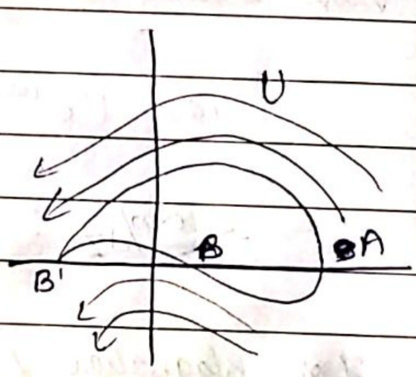
Blasius
Exam point of view

$$= be^{-i\alpha} e^{-i(\alpha+\beta)} \quad \text{or}, -be^{+i\beta}$$
$$= be^{-i(2\alpha+\beta)} \quad \text{or} \ -be^{i\beta}$$

∴ the points $t_1 = be^{-i(2\alpha+\beta)}$
$$= be^{-i[2\pi-(2\alpha+\beta)]}$$
& $t_2 = -be^{+i\beta} = be^{i(\pi+\beta)}$

are points B & A, respectively where $dw/dz = 0$. i.e, A & B are the stagnation pts.

9. Flow past an aerofoil :-



From Joukowski transformation,
(1) $\zeta = z + a^2/z$ transforms a circle into an aerofoil. Thus the stagnation point P will be transformed into trailing edge B' and the other stagnation point A will be transformed to A''. B' will lie on the -ve side of ζ-axis s.t OB' = -2a. Let U be the velocity of the stream at infinity. its direction making an angle α with ζ-axis. We take the origins of both z & ζ planes to coincide then B & B' will lie on the left side of origin O.

By Blasius Theorem,
$$X - iY = \tfrac{1}{2} i \rho \int_{C_\zeta} \left(dw/d\zeta\right)^2 d\zeta \quad \text{— (2)}$$

From (1) :-
$$dw/d\zeta = dw/dz \cdot dz/d\zeta = \frac{dw/dz}{d\zeta/dz} = \frac{dw/dz}{1-a^2/z^2}$$
$$\left(dw/d\zeta\right)^2 = dw/d\zeta \cdot dw/d\zeta = \frac{(dw/dz)^2}{\ldots} \quad \text{— (3)}$$

Cauchy Residual Theorem

Exam point of view

Now,

$$w = Ue^{i\alpha}(z-z_0) + Ub^2 e^{-i\alpha}/z + iK/2\pi \log(z-z_0)$$

$$dw/dz = Ue^{i\alpha} - Ub^2 e^{-i\alpha}/(z-z_0)^2 + iK/2\pi(z-z_0)$$

$$\frac{1}{(1-a^2/z^2)}\left(dw/dz\right)^2 = (1-a^2/z^2)^{-1}\left[Ue^{i\alpha} - \frac{Ub^2e^{-i\alpha}}{z^2}(1-z_0/z)^{-2} + \frac{iK}{2\pi z}(1-z_0/z)^{-1}\right]^2$$

$$= \left(1+2a^2/z^2+\dots\right)\left[Ue^{-i\alpha} - \frac{Ub^2e^{-i\alpha}}{z^2}\left(1+2z_0/z+\dots\right) + \frac{iK}{2\pi z}\left(1+z_0/z+\dots\right)\right]^2 \quad -(4)$$

By, (2)

$$X - iY = i\rho/2 \int_{C_2} \frac{(dw/dz)^2}{(1-a^2/z^2)}\, dz$$

$$= i\rho/2 \cdot 2\pi i \left[\text{sum of the residues of } \frac{(dw/dz)^2}{(1-a^2/z^2)}\right]$$

$$= \cancel{i\rho/2 \cdot 2\pi i}$$

$$= -\pi\rho \text{ coeff. of } 1/z \text{ in RHS of } (4)$$

$$= -\pi\rho\; 2Ue^{i\alpha}\, iK/2\pi = -i\rho K e^{i\alpha} U$$

$X = \rho K U \sin\alpha \quad Y = \rho K U \cos\alpha \Rightarrow F = \rho K U$

Lift acting on the aerofoil $= \rho K U = 4\pi b U^2 \sin(\alpha+\beta)$

10. Moment of the force acting on aerofoil

$$(dw/d\zeta)^2 \; \zeta \, d\zeta = \frac{z + a^4/z}{1 - a^4/z^2} \times \left(\frac{dw}{dz}\right)^2 \times dz$$

$$\frac{z + a^4/z}{1 - a^4/z^2}\left(dw/dz\right)^2 = \left(z + a^4/z\right)\left(1 + a^4/z^2 + \cdots\right)\left[Ue^{i\alpha} - \frac{Ub^2 e^{-i\alpha}}{z^2}\left(1 + \frac{2z_0}{z} \cdots\right) + \frac{ik}{2\pi z}\left(1 + \frac{z_0}{z} \cdots\right)\right]$$

$$= \left(z + \frac{2a^2}{z} + \cdots\right)\left(\cdots\cdots\right)$$

$\Rightarrow$ coeff. of $\frac{1}{z} = -2U^2b^2 - \frac{K^2}{4\pi^2} + \frac{iUe^{i\alpha} z_0}{\pi} + 2a^2U^2e^{2i\alpha}$

$$M = \text{Real part of } \left\{ -\frac{\rho}{2} \times \text{coeff. of } 1/z \times 2\pi i \right\}$$

$$= Re\left\{ -\frac{\rho}{2}\left\{ -2b^2U^2 - \frac{K^2}{4\pi^2} + \frac{iUe^{i\alpha} z_0}{\pi} + 2a^2U^2e^{2i\alpha} \right\} \times 2\pi i \right\}$$

Taking $z_0 = ce^{i\lambda}$, $c > 0$ then, $M = Re\left[-\frac{\rho}{2}\left\{ -2b^2U^2 - \frac{K^2}{4\pi^2} + \frac{iUe^{i\alpha}ce^{i\lambda}}{\pi} + 2a^2U^2e^{2i\alpha} \right\} 2\pi i \right]$

$$M = 2\pi\rho U^2\left[2bc\sin(\alpha+\beta)\cos(\alpha+\lambda) + a^2\sin 2\alpha\right]$$

This is the amount at z_0. The moment at the trailing edge is $aY + M$

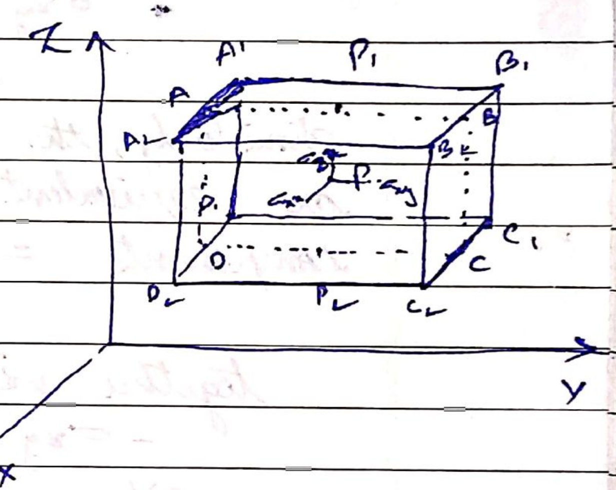
Required moment at B

$$= a\rho K U \cos\alpha + 2\pi\rho U^2 \left[2bc \sin(\alpha+\beta)\cos(\alpha+\eta) + a^2 \sin 2\alpha\right]$$

$$= 4a\pi b U^2 \rho \cos\alpha + 2\pi\rho U^2 \left[2bc \sin(\alpha+\beta)\cos(\alpha+\eta) + a^2 \sin 2\alpha\right]$$

Navier-Stokes Equation

With $P(x,y,z)$ as center and edges of length δx, δy, δz parallel to coordinates axes, we construct an elementary rectangular parallelopiped. Let us consider the fluid motion is viscous. We assume that the fluid element is moving & its mass is $\rho\,\delta x\,\delta y\,\delta z$. Let the coordinate



of P_2 & P_1 be $(x+\frac{\delta x}{2}, y, z)$ & $(x-\frac{\delta x}{2}, y, z)$ respectively. At P the force components $\parallel$ to OX, OY, OZ on the rectangular surface $ABCD$ of area $\delta y\,\delta z$ through P & having $\hat{i}$ as the unit normal are $= (\sigma_{xx}\,\delta y\,\delta z,\ \sigma_{xy}\,\delta y\,\delta z,\ \sigma_{xz}\,\delta y\,\delta z)$ — (1)

At the point $P_2(x+\delta x/2, y, z)$ the components of the force are $A_2B_2C_2D_2$ is

$$= \left[\left(\sigma_{xx} + \frac{\delta x}{2}\frac{\partial \sigma_{xx}}{\partial x}\right)\delta y\,\delta z,\ \left(\sigma_{xy} + \frac{\delta x}{2}\frac{\partial \sigma_{xy}}{\partial x}\right)\delta y\,\delta z,\ \left(\sigma_{zx} + \frac{\delta x}{2}\frac{\partial \sigma_{zx}}{\partial x}\right)\delta y\,\delta z\right] \quad \text{— (2)}$$

Slly force on P_1 in $A_1B_1C_1D_1$,

$$= \left[-\left(\sigma_{xx} - \frac{\delta x}{2}\frac{\partial \sigma_{xx}}{\partial x}\right)\delta y\,\delta z,\ -\left(\sigma_{xy} - \frac{\delta x}{2}\frac{\partial \sigma_{xy}}{\partial x}\right)\delta y\,\delta z,\ -\left(\sigma_{xz} - \frac{\delta x}{2}\frac{\partial \sigma_{xz}}{\partial x}\right)\delta y\,\delta z\right] \quad \text{— (3)}$$