Revised Simplex Method: Prob-1

$$\max: Z = x_1 + 2x_2 + 3x_3$$

subject to
$$x_1 + 5x_2 + 6x_3 \le 30$$
$$x_1 + 3x_2 + 2x_3 \le 18$$
$$x_1, x_2, x_3 \ge 0$$

$$\max: Z = x_1 + 2x_2 + 3x_3 + 0x_4 + 0x_5$$

s.to
$$x_1 + 5x_2 + 6x_3 + x_4 = 30$$
$$x_1 + 3x_2 + 2x_3 + x_5 = 18$$

$\underline{x_4, x_5} \ge 0$ Basic variables

$\underline{x_1, x_2, x_3} \ge 0$ Non-Basic variables

(a) $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $B_{New} = \begin{pmatrix} 6 & 0 \\ 2 & 1 \end{pmatrix}$, $B^{-1}_{New} = \begin{pmatrix} \frac{1}{6} & 0 \\ -\frac{2}{6} & 1 \end{pmatrix}$

(c) $B_{New} = \begin{pmatrix} 6 & 1 \\ 2 & 1 \end{pmatrix}$, $B^{-1}_{new} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{2}{4} & \frac{6}{4} \end{pmatrix}$

Extended Tableau

	C_V	1	2	3	0	0	
C_B	$B \backslash V$	x_1	x_2	x_3	x_4	x_5	b
0	x_4	1	5	6	1	0	30
0	x_5	1	3	2	0	1	18
		-1	-2	-3 ✓	0	0	0

(I) $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $C_B = (0 \quad 0)$, $C_B B^{-1} = (0 \quad 0) = Y$

$Y P_1 - 1 = (0 \quad 0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 1 = -1$, $(0 \quad 0)\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 2 = -2$

$(0 \quad 0)\begin{pmatrix} 6 \\ 2 \end{pmatrix} - 3 = -3$ (most negative)

$X_B = B^{-1} b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 18 \end{pmatrix} = \begin{pmatrix} 30 \\ 18 \end{pmatrix}$

$B^{-1} P_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} = \alpha_3$

min. Ratio $\left\{ \frac{30}{6}, \frac{18}{2} \right\} = 5$

$=$ 1st element

x_3 is the entering variable.

x_4 is the Departing variable.

(II) Basic variable: x_3, x_5, $C_B = (3 \quad 0)$

Non-Basic variable x_1, x_2, x_4

$$B_{new} = \begin{pmatrix} 6 & 0 \\ 2 & 1 \end{pmatrix}, \quad B_{new}^{-1} = \begin{pmatrix} \frac{1}{6} & 0 \\ -\frac{2}{6} & 1 \end{pmatrix}$$

$$Y = C_B B_{new}^{-1} = (3, 0) \begin{pmatrix} \frac{1}{6} & 0 \\ -\frac{2}{6} & 1 \end{pmatrix} = \left(\frac{1}{2}, 0\right)$$

For Non-Basic variables: x_1, x_2, x_4

$$YP_1 - 1 = \left(\frac{1}{2}, 0\right)\begin{pmatrix} 1 \\ 1 \end{pmatrix} - 1 = -\frac{1}{2} \quad (\text{-ve}) \text{ only}$$

$$YP_2 - 2 = \left(\frac{1}{2}, 0\right)\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 2 = \frac{1}{2} > 0$$

$$YP_4 - 0 = \left(\frac{1}{2}, 0\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = \frac{1}{2} > 0$$

x_1 is the entering variable

$$X_B = \begin{pmatrix} \frac{1}{6} & 0 \\ -\frac{2}{6} & 1 \end{pmatrix}\begin{pmatrix} 30 \\ 18 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$\alpha_1 = \begin{pmatrix} \frac{1}{6} & 0 \\ -\frac{2}{6} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{6} \\ \frac{4}{6} \end{pmatrix}$$

$$\text{min Ratio} = \left\{\frac{5}{1/6}, \frac{8}{4/6}\right\} = \{30, 12\}$$

2nd element

x_5 is the Departing variable.

(III)

$$B_{new} = \begin{pmatrix} 6 & 1 \\ 2 & 1 \end{pmatrix}, \quad B_{new}^{-1} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{2}{4} & \frac{6}{4} \end{pmatrix}$$

<u>Basic Variables</u> : $x_3, x_1 \Rightarrow c_B = (3 \quad 1)$

<u>Non-Basic Variables</u>: x_2, x_4, x_5

Now $y = c_B \bar{B}^{1} = (3 \quad 1)\begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{2}{4} & \frac{6}{4} \end{pmatrix}$

$= \left(\frac{1}{4}, \frac{3}{4}\right)$

For x_2: $\quad yP_2 - 2 = \left(\frac{1}{4}, \frac{3}{4}\right)\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 2$

$= \frac{14}{4} - 2 = \frac{6}{4} \quad (+ve) >$

For x_4 : $yP_4 - 0 = \left(\frac{1}{4}, \frac{3}{4}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = \frac{1}{4} > 0$

For x_5 : $yP_5 - 0 = \left(\frac{1}{4}, \frac{3}{4}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 = \frac{3}{4} > 0$

<u>Present sol^n is optimal</u>:

$X_B = \begin{pmatrix} x_3 \\ x_1 \end{pmatrix} = \bar{B}^{1} b = \begin{pmatrix} \frac{1}{4} & -\frac{1}{4} \\ -\frac{2}{4} & \frac{6}{4} \end{pmatrix}\begin{pmatrix} 30 \\ 18 \end{pmatrix} = \begin{pmatrix} 3 \\ 12 \end{pmatrix}$

$\Rightarrow \boxed{\begin{matrix} x_1 = 12 \\ x_3 = 3 \end{matrix}} \qquad z^* = 21$

$z = c_B X_B = (3, 1)\begin{pmatrix} 3 \\ 12 \end{pmatrix} = 21$

<u>(Optimal Solution)</u>

Revised Simplex Method: Prob: 2

$$\max: Z = 4x_1 + 5x_2 + 2x_3$$

s.to $\quad 2x_1 + 5x_2 + 2x_3 \leq 60$

$\quad 4x_1 + 3x_2 + 2x_3 \leq 50$

$\quad x_1, x_2, x_3 \geq 0$

$$\max: Z = 4x_1 + 5x_2 + 2x_3 + 0x_4 + 0x_5$$

s.to $\quad 2x_1 + 5x_2 + 2x_3 + x_4 = 60$

$\quad 4x_1 + 3x_2 + 2x_3 + x_5 = 50$

$\underline{x_4, x_5} \geq 0$, Basic Variables

$\underline{x_1, x_2, x_3 \geq 0}$ Non-Basic Variables

(a) $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $B_{new} = \begin{pmatrix} 5 & 0 \\ 3 & 1 \end{pmatrix} \qquad B_{new}^{-1} = \begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{5} & 1 \end{pmatrix}$

(c) $B_{new} = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix} \qquad B_{new}^{-1} = \begin{pmatrix} \frac{2}{7} & -\frac{2}{14} \\ -\frac{3}{14} & \frac{5}{14} \end{pmatrix}$

(6) Extended Tableau

	Cv	4	5	2	0	0	
C_B	B \ V	x_1	x_2	x_3	x_4	x_5	b
0	x_4	2	5	2	1	0	60
0	x_5	4	3	2	0	1	50
		-4	-5	-2	0	0	0

(I) $B = B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $C_B = (0\ 0)$, $C_B B^{-1} = Y = (0, 0)$

$$YP_1 - 4 = (0\ 0)\begin{pmatrix} 2 \\ 4 \end{pmatrix} - 4 = -4$$

$$YP_2 - 5 = (0\ 0)\begin{pmatrix} 5 \\ 3 \end{pmatrix} - 5 = -5 \quad (\text{most} -\text{ve})$$

2nd Col^n

$$YP_3 - 2 = (0, 0)\begin{pmatrix} 2 \\ 2 \end{pmatrix} - 2 = -2$$

$$X_B = B^{-1} b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 60 \\ 50 \end{pmatrix} = \begin{pmatrix} 60 \\ 50 \end{pmatrix}$$

$$\alpha_2 = B^{-1} P_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

min: Ratio : $\left\{ \frac{60}{5}, \frac{50}{3} \right\} = 12$ 1st element

$= \{12, 50/3\} = 12$

x_2 is the entering variable

x_4 is the Departing variable

(II) Basic variables: x_2, x_5 , $C_B = (5, 0)$

Non-Basic variables: x_1, x_3, x_4

$$B_{new} = \begin{pmatrix} 5 & 0 \\ 3 & 1 \end{pmatrix}, \quad B_{new}^{-1} = \begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{5} & 1 \end{pmatrix}$$

$$Y = C_B B_{new}^{-1} = (5 \quad 0)\begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{5} & 1 \end{pmatrix} = (1 \quad 0)$$

<u>For Non-Basic variables:</u>

$$Y P_1 - 4 = (1 \quad 0)\begin{pmatrix} 2 \\ 4 \end{pmatrix} - 4 = -2 \quad (-ve) \checkmark$$

$$Y P_3 - 2 = (1 \quad 0)\begin{pmatrix} 2 \\ 2 \end{pmatrix} - 2 = 0 \qquad ?$$

$$Y P_4 - 0 = (1 \quad 0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = 1$$

$$X_B = \begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{5} & 1 \end{pmatrix}\begin{pmatrix} 60 \\ 50 \end{pmatrix} = \begin{pmatrix} 12 \\ 14 \end{pmatrix}$$

$$\alpha_1 = B^{-1} P_1 = \begin{pmatrix} \frac{1}{5} & 0 \\ -\frac{3}{5} & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ \frac{14}{5} \end{pmatrix}$$

$$\text{Min: Ratio} \left\{ \frac{12}{2/5}, \frac{14}{14/5} \right\} = 5 \checkmark$$

$$= \{30, 5\} \qquad \underline{\text{2nd element}}$$

x_1 is the entering variable

<u>x_5 is the Departing variable.</u>

(III) $$B_{new} = \begin{pmatrix} 5 & 2 \\ 3 & 4 \end{pmatrix}, \quad B_{new}^{-1} = \begin{pmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{3}{14} & \frac{5}{14} \end{pmatrix}$$

Basic variables : x_2, x_1, $C_B = (5, 4)$

Non-Basic variables: $\underline{x_3, x_4, x_5}$

$$Y = C_B B^{-1} = \begin{pmatrix} 5 & 4 \end{pmatrix} \begin{pmatrix} \frac{4}{14} & -\frac{2}{14} \\ -\frac{3}{14} & \frac{5}{14} \end{pmatrix}$$

$$= \left(\frac{8}{14}, \frac{10}{14} \right) = Y$$

$$Y P_3 - 2 = \left(\frac{8}{14}, \frac{10}{14} \right) \begin{pmatrix} 2 \\ 2 \end{pmatrix} - 2 = \frac{36}{14} - 2 = \frac{4}{7} > 0$$

$$Y P_4 - 0 = \left(\frac{8}{14}, \frac{10}{14} \right) \begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = \frac{4}{7} > 0$$

$$Y P_5 - 0 = \left(\frac{8}{14}, \frac{10}{14} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 = \frac{10}{14} > 0$$

$$X_B = B^{-1} b = \begin{pmatrix} \frac{4}{14} & -\frac{2}{14} \\ -\frac{3}{14} & \frac{5}{14} \end{pmatrix} \begin{pmatrix} 60 \\ 50 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$C_B = (5, 4), \quad X_B = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$Z = C_B X_B = (5, 4) \begin{pmatrix} 10 \\ 5 \end{pmatrix} = 70$$

$$x_2^* = 10, \quad x_1^* = 5, \quad x_3^* = 0, \quad Z^* = 70$$

Revised Simplex Method: Problem: 3

$$\max: z = x_1 + 3x_2 + 4x_3$$

s.to
$$4x_1 + 8x_2 + 5x_3 \le 80$$
$$4x_1 + 5x_2 + 10x_3 \le 105$$
$$x_1, x_2, x_3 \geq 0$$

$$\max: z = x_1 + 3x_2 + 4x_3 + 0x_4 + 0x_5$$

s.to
$$4x_1 + 8x_2 + 5x_3 + x_4 = 80$$
$$4x_1 + 5x_2 + 10x_3 + x_5 = 105$$
$$\underline{x_4, x_5 \geq 0}, \quad \underline{x_1, x_2, x_3 \geq 0}$$

(a) $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(b) $B_{new} = \begin{pmatrix} 1 & 5 \\ 0 & 10 \end{pmatrix}$, $B_{new}^{-1} = \begin{pmatrix} 1 & -\frac{5}{10} \\ 0 & \frac{1}{10} \end{pmatrix}$

(c) $B_{new} = \begin{pmatrix} 8 & 5 \\ 5 & 10 \end{pmatrix}$, $B_{new}^{-1} = \begin{pmatrix} \frac{10}{55} & -\frac{5}{55} \\ -\frac{5}{55} & \frac{8}{55} \end{pmatrix}$

Extended Tableau

	C_V	1	3	4	0	0	
C_B	B \ V	x_1	x_2	x_3	x_4	x_5	b
0	x_4	4	8	5	1	0	80
0	x_5	4	5	10	0	1	105
		-1	-3	-4	0	0	0

$$C_B = (0 \quad 0) \qquad X_B = \begin{pmatrix} x_4 \\ x_5 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(I) Basic variable: x_4, x_5

Non-Basic variable: x_1, x_2, x_3

$$P_1 = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \qquad P_2 = \begin{pmatrix} 8 \\ 5 \end{pmatrix} \qquad P_3 = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad P_5 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad b = \begin{pmatrix} 80 \\ 105 \end{pmatrix}$$

$$Z = C_B X_B = 0 \Rightarrow (0 \quad 0)\begin{pmatrix} 80 \\ 105 \end{pmatrix} = 0$$

$$Y = C_B B^{-1} = (0, 0)$$

$Z_1 - C_1 = Y P_1 - C_1$, $Z_2 - C_2 = Y P_2 - C_2$, $Z_3 - C_3 = Y P_3 - C_3$

$Z_1 - C_1 = -1$, $Z_2 - C_2 = -3$ continue $Z_3 - C_3 = -4$

x_3 is entering variable. ✓

x_5 is departing variable ✓

Basic variable: x_4, x_3

$C_B = (0, 4)$ N.B. variable x_1, x_2, x_5

Now

$$B_{new} = \begin{pmatrix} 1 & 5 \\ 0 & 10 \end{pmatrix} \quad B_{new}^{-1} = \begin{pmatrix} 1 & -\frac{5}{10} \\ 0 & \frac{1}{10} \end{pmatrix}$$

$$Y = C_B B_{new}^{-1} = (0 \;\; 4)\begin{pmatrix} 1 & -5/10 \\ 0 & 1/10 \end{pmatrix} = \left(0, \frac{4}{10}\right)$$

$$YP_1 - c_1 = \left(0, \frac{4}{10}\right)\begin{pmatrix} 4 \\ 4 \end{pmatrix} - 1 = \frac{6}{10} \quad +ve$$

$$YP_2 - c_2 = \left(0, \frac{4}{10}\right)\begin{pmatrix} 8 \\ 5 \end{pmatrix} - 3 = -1 \;(-ve)$$

$$YP_5 - 0 = \left(0, \frac{4}{10}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 = \frac{4}{10} \;(+ve)$$

x_2 is entering variable.

$$X_B = B^{-1}b = \begin{pmatrix} 1 & -5/10 \\ 0 & \frac{1}{10} \end{pmatrix}\begin{pmatrix} 80 \\ 105 \end{pmatrix} = \begin{pmatrix} 80 - \frac{105}{2} \\ \frac{105}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{55}{2} \\ \frac{105}{2} \end{pmatrix}$$

$$B^{-1}P_2 = \begin{pmatrix} 1 & -5/10 \\ 0 & 1/10 \end{pmatrix}\begin{pmatrix} 8 \\ 5 \end{pmatrix} = \begin{pmatrix} \frac{11}{2} \\ \frac{5}{2} \end{pmatrix}$$

min: Ratio $\left(\frac{55}{2} \Big/ \frac{11}{2}, \frac{105}{2} \Big/ \frac{5}{2}\right) = \left(\frac{55}{11}, \frac{105}{5}\right)$

(II) $= (5, 21)$: 1st element

$\left.\begin{array}{l} x_2 \text{ is entering variable} \\ x_4 \text{ is departing variable} \end{array}\right\}$

$$C_B = (3, 4), \quad X_B = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$$

$$B_{new} = \begin{pmatrix} 8 & 5 \\ 5 & 10 \end{pmatrix}, \quad B_{new}^{-1} = \begin{pmatrix} \frac{10}{55} & -\frac{1}{11} \\ -\frac{1}{11} & \frac{8}{55} \end{pmatrix}$$

$C_B = (3, 4)$, $X_B = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}$: $\underline{\underline{NBV}}$ x_1, x_4, x_5

$$Y = C_B B^{-1} = (3, 4)\begin{pmatrix} \frac{10}{55} & -\frac{5}{55} \\ -\frac{5}{55} & \frac{8}{55} \end{pmatrix}$$

$$= \left(\frac{10}{55}, \frac{25}{55}\right)$$

(III) $$= \left(\frac{2}{11}, \frac{5}{11}\right)$$

$$YP_1 - 1 = \left(\frac{2}{11}, \frac{5}{11}\right)\begin{pmatrix} 4 \\ 4 \end{pmatrix} - 1 = \frac{28}{11} - 1 = \frac{18}{11} > 0$$

$$YP_4 - 0 = \left(\frac{2}{11}, \frac{5}{11}\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 0 = \frac{2}{11} > 0$$

$$YP_5 - 0 = \left(\frac{2}{11}, \frac{5}{11}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix} - 0 = \frac{5}{11} > 0$$

<u>optimal</u>

$$X_B = B^{-1} b = \begin{pmatrix} \frac{10}{55} & -\frac{5}{55} \\ -\frac{5}{55} & \frac{8}{55} \end{pmatrix}\begin{pmatrix} 80 \\ 105 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{800 - 525}{55} \\ \frac{-400 + 840}{55} \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$X_B = \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ $\quad C_B = (3, 4)$ $\quad$ | $x_2^* = 5$

$Z^* = C_B X_B = (3, 4)\begin{pmatrix} 5 \\ 8 \end{pmatrix} = 47^*$ $\quad$ | $x_3^* = 8$