Dual Simplex Method: Problem-1

$$\min: Z = x_1 + 3x_2$$

s.to

$$x_1 + x_2 \geq 10$$
$$x_1 + 2x_2 \geq 11$$
$$x_1 + 4x_2 \geq 16$$
$$x_1, x_2 \geq 0$$

$$\max: -Z = -x_1 - 3x_2$$

s.to

$$-x_1 - x_2 \leq -10$$
$$-x_1 - 2x_2 \leq -11$$
$$-x_1 - 4x_2 \leq -16$$
$$x_1, x_2 \geq 0$$

$$\max: -Z = -x_1 - 3x_2$$

s.to

$$-x_1 - x_2 + s_1 = -10$$
$$-x_1 - 2x_2 + s_2 = -11$$
$$-x_1 - 4x_2 + s_3 = -16$$

s_1, s_2, s_3 are slack variables

s_1, s_2, s_3 are also basic variables

Since s_1, s_2, s_3 are -ve, it is infeasible.

	−1	−3	
B \ N	x_1	x_2	X_B
s_1	−1	−1	−10
s_2	−1	−2	−11
s_3	−1	**−4**	−16 ✓ most −ve
	1	3	0

min Ratio $\left\{\left|\frac{1}{-1}\right|, \left|\frac{3}{-4}\right|\right\}$

$= \left\{1, \frac{3}{4}\right\} = \frac{3}{4}$

2nd element

s_3 is departing variable

x_2 is entering variable

B \ N	x_1	s_3	X_B	
s_1	$-\frac{3}{4}$	$-\frac{1}{4}$	−6	✓
s_2	$-\frac{2}{4}$	$-\frac{2}{4}$	−3	
x_2	$\frac{1}{4}$	$-\frac{1}{4}$	4	
	$\frac{1}{4}$	$\frac{3}{4}$	−12	

B.V	s_1	s_3	x_B
x_1	$-\frac{4}{3}$	$\frac{1}{3}$	8
s_2	$-\frac{2}{3}$	$\frac{1}{3}$	1
x_2	$\frac{1}{3}$	$-\frac{1}{3}$	2
	$\frac{1}{3}$	$\frac{8}{3}$	-14

min. Ratio $\left\{ \left| \frac{\frac{1}{4}}{-\frac{3}{4}} \right| , \left| \frac{\frac{3}{4}}{-\frac{1}{4}} \right| \right\} = \{\frac{1}{3}, 3\}$
$= \frac{1}{3}$

s_1 is the Departing variable

x_1 is the Entering variable.

optimal

$x_1^* = 8$

$x_2^* = 2$

$s_2^* = 1$

$-z^* = -14$

$z^* = 14$

(This has one optimal soln)

Dual Simplex Method: Problem: II

$$\min: Z = 2x_1 + 4x_2 + 6x_3$$

s.to
$$x_1 + 2x_2 + 3x_3 \geq 60$$
$$x_1 + 2x_2 + 5x_3 \geq 100$$
$$x_1, x_2, x_3 \geq 0$$

$$\max: -Z = -2x_1 - 4x_2 - 6x_3$$

s.to
$$-x_1 - 2x_2 - 3x_3 \leq -60$$
$$-x_1 - 2x_2 - 5x_3 \leq -100$$
$$x_1, x_2, x_3 \geq 0$$

$$-x_1 - 2x_2 - 3x_3 + S_1 = -60$$
$$-x_1 - 2x_2 - 5x_3 + S_2 = -100$$

	−2	−4	−6	
B \ N	x_1	x_2	x_3	X_B
S_1	−1	−2	−3	−60
S_2	−1	−2	(−5)	−100 ✓
	2	4	6	0

$$\min: \left\{ \left|\frac{2}{-1}\right|, \left|\frac{4}{-2}\right|, \left|\frac{6}{-5}\right| \right\}$$

$$\min: \left\{ 2, 2, \frac{6}{5} \right\} = \frac{6}{5}$$ 3rd element

s_2 is the Departing variable

x_3 is the entering variable

B	x_1	x_2	s_3	X_B
s_1	$-\frac{2}{5}$	$-\frac{4}{5}$	$-\frac{3}{5}$	0
x_3	$\frac{1}{5}$	$\frac{2}{5}$	$-\frac{1}{5}$	20
	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{6}{5}$	-120

$x_1^* = 0$, $x_2^* = 0$ $s_3^* = 0$

$s_1^* = 0$, $x_3^* = 20$

$-z^* = -120$, $z^* = 120$

only one optimal soln