Pushdown automata

1) Accept by final state :

   i) whole input string is scanned

   ii) after scaning the whole input, M enters a state in F.

—x—

$$(s, x, \perp) \xrightarrow[M]{*} (q, \epsilon, \gamma)$$

this can be any leftover string. (γ)

$q \in F$, $\gamma \in \Gamma^*$

2) Accept by empty stack :

   i) whole input string is scanned

   ii) the last element (maybe $\perp$ or anything else) is popped off the stack without pushing anything else.

—x—

$$(s, x, \perp) \xrightarrow[M]{*} (q, \epsilon, \epsilon)$$

The stackbottom ($\perp$) can be popped and pushed.

If only $\perp$ remains in the stack, it is still <u>nonempty</u>. It is just like any other character in Γ.

If the stack becomes empty after a pop, but immediately something else is pushed, and the whole input string is scanned then it is still <u>not</u> empty.

Acceptance by empty stack is equivalent to acceptance by final state.

M accepts by empty stack (resp, by accept state)

$M = (Q, \Sigma, \Gamma, \delta, s, \perp, F)$

$M' = (Q \cup \{u, t\}, \Sigma, \Gamma \cup \{\perp\!\!\perp\}, \delta', u, \perp\!\!\perp, \{t\})$

two extra states

$\delta' = \delta \cup$

$\{ (u, \epsilon, \perp\!\!\perp), (s, \perp \perp\!\!\perp)),$

$((q, \epsilon, A), (t, A))$, $q \in G$, $A \in \Delta$

where

$G = \cancel{Q}F$, $\Delta = \{\perp\!\!\perp\} \cup \Gamma$

$((t, \epsilon, A), (t, \epsilon))$ where $A \in \Gamma \cup \{\perp\!\!\perp\}$

$\}$

Suppose M accepts x (by ~~empty stack~~ accept state)

$(s, x, \perp) \xrightarrow[M]{n} (q, \epsilon, \epsilon)$

$(u, x, \perp\!\!\perp) \xrightarrow[M']{1} (s, x, \perp\perp\!\!\perp) \xrightarrow[M']{n} (q, \epsilon, \perp\!\!\perp)$

$(t, \epsilon, \epsilon) \xleftarrow[M']{1} (t, \epsilon, \perp\!\!\perp) \xleftarrow[M']{1}$

accepted

Suppose, M' accepts x. (through final state)

$(s, x, \perp) \xrightarrow[M]{n} (q, \epsilon, \gamma)$, $q \in F$

$(u, x, \perp\!\!\perp) \xrightarrow[M']{1} (s, x, \perp\perp\!\!\perp) \xrightarrow[M']{n} (q, \epsilon, \gamma\perp\!\!\perp) \xrightarrow[M']{1} (t, \epsilon, \gamma\perp\!\!\perp) \xrightarrow[M]{*} (t, \epsilon, \epsilon)$

M' accepts x by empty stack

Now suppose M' accepts x by empty stack.
We show that M accepts x by final state.

$$(u, x, \perp\!\!\perp) \xrightarrow[M']{1} (s, x, \perp\perp\!\!\perp) \xrightarrow[M']{n} (q, y, \gamma\perp\!\!\perp) \xrightarrow[M']{1} (t, y, \gamma\perp\!\!\perp) \xrightarrow[M']{*} (t, \varepsilon, \varepsilon)$$

$q \in G$ — still holds.

once M' enters state t, it cannot read anymore symbols (by definition of δ')

$\Rightarrow \; y = \varepsilon$

$q \in F$?

$$\boxed{(s, x, \perp) \xrightarrow[M]{n} (q, \varepsilon, \gamma)}$$ still holds — is $\gamma = \varepsilon$?

If γ is ε, we are done, otherwise there is no rule in δ' for this transition

$$(q, \varepsilon, \gamma\perp\!\!\perp) \xrightarrow[M']{1} (t, \varepsilon, \gamma\perp\!\!\perp)$$

$\therefore \gamma = \varepsilon$

$\therefore$ M accepts x.

Part 2 — M accepts through final state.
We construct equivalent M' which accepts through stack.

Only Rule 2 in $\delta' \setminus \delta$ could facilitate

$$(q, \varepsilon, \gamma\perp\!\!\perp) \xrightarrow[M']{1} (t, \varepsilon, \gamma\perp\!\!\perp)$$

But then $q \in G = F$

So q is accepted through final state by M.