Simplex Method:

$$\max: Z = x_1 + 3x_2$$

s.to
$$x_1 + x_2 \le 100$$
$$x_1 + 2x_2 \le 110$$
$$x_1 + 4x_2 \le 160$$
$$x_1, x_2 \geq 0$$

Introduce slack variables:

$$x_1 + x_2 + x_3 = 100$$
$$x_1 + 2x_2 + x_4 = 110$$
$$x_1 + 4x_2 + x_5 = 160$$
$$x_1, x_2 \geq 0, \quad x_3, x_4, x_5 \geq 0$$

only slack variables are introduced.

$$\max: Z = x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

This LPP has only one optimal solution.

(See the Simplex Tableau)

	C_N	1	3	
C_B	B \ N	x_1	x_2	X_B
0	x_3	1	1	100
0	x_4	1	2	110
0	x_5	1	$\boxed{4}$	160
		-1	-3	0

	C_N	1	0	
C_B	B \ N	x_1	x_5	x_B
0	x_3	$\frac{3}{4}$	$-\frac{1}{4}$	60
0	x_4	$\boxed{\frac{2}{4}}$	$-\frac{2}{4}$	30
3	x_2	$\frac{1}{4}$	$\frac{1}{4}$	40
		$-\frac{1}{4}$	$\frac{3}{4}$	120

	C_N	0	0	
C_B	B \ N	x_4	x_5	X_B
0	x_3	$-3/2$	$\frac{1}{2}$	15
1	x_1	2	-1	60
3	x_2	$-1/2$	$\frac{1}{2}$	25
		$\frac{1}{2}$	$\frac{1}{2}$	135

$x_1 = 60, \; x_2 = 25, \; x_3 = 15$

$Z = 135$

Simplex Method:

$$\max: z = x_1 + 4x_2$$

s.to $\quad x_1 + x_2 \leq 100$

$$x_1 + 2x_2 \leq 110$$

$$x_1 + 4x_2 \leq 160$$

$$x_1, x_2 \geq 0$$

$$x_1 + x_2 + x_3 = 100$$

$$x_1 + 2x_2 + x_4 = 110$$

$$x_1 + 4x_2 + x_5 = 160$$

$x_1, x_2 \geq 0$, $x_3, x_4, x_5 \geq 0$

Slack variables are introduced.

$$\max: z = x_1 + 4x_2 + 0x_3 + 0x_4 + 0x_5$$

This LPP has infinitely many solutions.

	C_N	1	4	
C_B	$B \backslash N$	x_1	x_2	X_B
0	x_3	1	1	100
0	x_4	1	2	110
0	x_5	1	$\boxed{4}$	160
		-1	-4	0

	C_N	1	0	
C_B	$B \backslash N$	x_1	x_5	X_B
0	x_3	$\frac{3}{4}$	$-\frac{1}{4}$	60
0	x_4	$\boxed{\frac{2}{4}}$	$-\frac{2}{4}$	30
4	x_2	$\frac{1}{4}$	$\frac{1}{4}$	40
(optimal)		0	1	160^*

$x_1 = 0$
$x_2 = 40$

	C_N	0	0	
C_B	$B \backslash N$	x_4	x_5	X_B
0	x_3	$-\frac{3}{2}$	$\frac{1}{2}$	15
1	x_1	2	-1	60
4	x_2	$-\frac{1}{2}$	$\frac{1}{2}$	25
		0	1	160^*

$x_1 = 60$
$x_2 = 25$

$$X^* = \lambda \begin{pmatrix} 0 \\ 40 \end{pmatrix} + (1-\lambda) \begin{pmatrix} 60 \\ 25 \end{pmatrix}$$

$$0 \leq \lambda \leq 1$$

(infinite solutions)

Big-M Method: (I)

max: $Z = 2x_1 + 4x_2 + 6x_3$

Subject to
$$x_1 + 2x_2 + 3x_3 = 6$$
$$x_1 + 2x_2 + 5x_3 = 10$$
$$x_1, x_2, x_3 \geqslant 0$$

We introduce two artificial variables: $x_4, x_5 \geqslant 0$

$$x_1 + 2x_2 + 3x_3 + x_4 = 6$$
$$x_1 + 2x_2 + 5x_3 + x_5 = 10$$

max: $Z = 2x_1 + 4x_2 + 6x_3 - Mx_4 - Mx_5$

where M is very large +ve number.

M is used to drive out the artificial variables.

	C_N	2	4	6	
C_B	B \ N	x_1	x_2	x_3	X_B
$-M$	x_4	1	2	$\boxed{3}$	6
$-M$	x_5	1	2	5	10
		$-2M-2$	$-4M-4$	$-8M-6$	$-16M$

	C_N	2	4	$-M$	
C_B	B \ N	x_1	x_2	x_4	X_B
6	x_3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	2
$-M$	x_5	$-2/3$	$-4/3$	$-5/3$	0
	$Z=12$	$\frac{2M}{3}$	$\frac{4M}{3}$	$\frac{8M+6}{3}$	12

$x_1=0,\ x_2=0,\ x_3=2,\ x_4=0,\ x_5=0$

<u>optimal</u>

(b)

Big-M Method: (II)

$$\min: Z = 2x_1 + 4x_2 + 6x_3$$

subject to

$$x_1 + 2x_2 + 3x_3 \geq 6$$
$$x_1 + 2x_2 + 5x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

$$\max: -Z = -2x_1 - 4x_2 - 6x_3$$

subject to

$$x_1 + 2x_2 + 3x_3 - x_4 + x_5 = 6$$
$$x_1 + 2x_2 + 5x_3 - x_6 + x_7 = 10$$

$x_4, x_6 \geq 0$ surplus variables

x_5 and x_7 are the artificial

$$\max: -Z = -2x_1 - 4x_2 - 6x_3 - Mx_5 - Mx_7$$

where M is a large +ve number. It is used to drive out the artificials

	C_N	-2	-4	-6	0	0	
C_B	$B \backslash N$	x_1	x_2	x_3	x_4	x_6	X_B
$-M$	x_5	1	2	$\boxed{3}$	-1	0	6
$-M$	x_7	1	2	5	0	-1	10
		$-2M+2$	$-4M+4$	$-8M+6$	M	M	$-16M$

		-2	-4	$-M$	0	0	
C_B	$B \backslash N$	x_1	x_2	x_5	x_4	x_6	X_B
-6	x_3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	2
$-M$	x_7	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{5}{3}$	$\boxed{\frac{5}{3}}$	-1	0
		$\frac{2M}{3}$	$\frac{4M}{3}$	$\frac{8M-6}{3}$	$\frac{-5M}{3}+2$	M	-12

		-2	-4	$-M$	$-M$	0	
C_B	$B \backslash N$	x_1	x_2	x_5	x_7	x_6	X_B
-6	x_3	$\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{1}{5}$	$-\frac{1}{5}$	2
0	x_4	$-\frac{2}{5}$	$-\frac{4}{5}$	-1	$\frac{3}{5}$	$-\frac{3}{5}$	0
		$\frac{4}{5}$	$\frac{8}{5}$	M	$M+\frac{6}{5}$	$\frac{6}{5}$	-12

$$x_1 = 0,\ x_2 = 0,\ x_3 = 2$$

$$-z = -12,\quad z = 12$$

(d)

Two-Phase Simplex Method:

$$\min: Z = 2x_1 + 4x_2 + 6x_3$$

s.to
$$x_1 + 2x_2 + 3x_3 \geq 6$$
$$x_1 + 2x_2 + 5x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

$$\max: -Z = -2x_1 - 4x_2 - 6x_3$$

s.to
$$x_1 + 2x_2 + 3x_3 - x_4 + x_5 = 6$$
$$x_1 + 2x_2 + 5x_3 - x_6 + x_7 = 10$$

x_4 and x_6 are surplus var.

x_5 and x_7 are artificial var.

Phase-I : $\min: Z_1 = x_5 + x_7$

$$\max: -Z_1 = -x_5 - x_7$$

s.to
$$x_1 + 2x_2 + 3x_3 - x_4 + x_5 = 6$$
$$x_1 + 2x_2 + 5x_3 - x_6 + x_7 = 10$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

Phase-II : $\max: -Z = -2x_1 - 4x_2 - 6x_3$

	CN	0	0	0	0	0	
C_B	B \ N	x_1	x_2	x_3	x_4	x_6	X_B
-1	x_5	1	2	$\boxed{3}$	-1	0	6
-1	x_7	1	2	5	0	-1	10
		-2	-4	-8	1	1	-16

	CN	0	0	-1	0	0	
C_B	B \ N	x_1	x_2	x_5	x_4	x_6	X_B
0	x_3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	2
-1	x_7	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{5}{3}$	$\frac{5}{3}$	-1	0
		$\frac{2}{3}$	$\frac{4}{3}$	$\frac{8}{3}$	$-\frac{5}{3}$	1	0

	CN	0	0	-1	-1	0	
C_B	B \ N	x_1	x_2	x_5	x_7	x_6	X_B
0	x_3	$\frac{1}{5}$	$\frac{2}{5}$	0	$\frac{1}{5}$	$-\frac{1}{5}$	2
0	x_4	$-\frac{2}{5}$	$-\frac{4}{5}$	-1	$\frac{3}{5}$	$-\frac{3}{5}$	0
		0	0	1	1	0	0

<u>Phase- I</u> aptimal

$x_3 = 2, \ x_4 = 0, \ x_5 = x_7 = 0$

Phase-II

	c_N	-2	-4			0	
C_B	B \ N	x_1	x_2	x_5	x_7	x_6	X_B
-6	x_3	$\frac{1}{5}$	$\frac{2}{5}$			$-\frac{1}{5}$	2
0	x_4	$-\frac{2}{5}$	$-\frac{4}{5}$			$-\frac{3}{5}$	0
		$\frac{4}{5}$	$\frac{8}{5}$			$\frac{6}{5}$	-12

	c_N	-2	-4				
C_B	B \ N	x_1	x_2	x_5	x_7	x_6	X_B
-6	x_3	$\frac{1}{5}$	$\frac{2}{5}$			$-\frac{1}{5}$	2
0	x_4	$-\frac{2}{5}$	$-\frac{4}{5}$			$\frac{3}{5}$	0
		$\frac{4}{5}$	$\frac{8}{5}$			$\frac{6}{5}$	-12

$x_1 = 0, \ x_2 = 0, \ x_3 = 2$

$-z = -12 \qquad z = 12 \text{ (min)}$