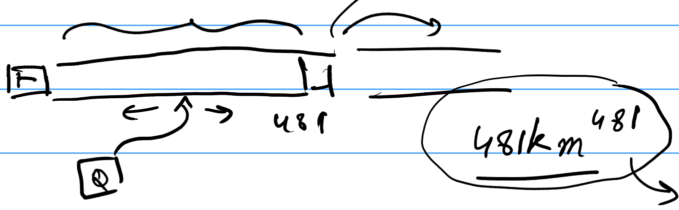
Given a TM M, does M

e) ever move its head more than 481 tape cells away from the left endmarker, on input ε?



$|\Gamma| = m \qquad |Q| = k$

✓ Study the finite no of configuration of M on a bounded length of the tape and decide whether M is looping or not.

f) accept the null string ε? → undecidable

Suppose that we can decide whether M accepts ε or not

Given TM A and input x, does A halt on x?

↓

TM B

s.t. Given any input y on B, B does the following

i) erases input y
ii) writes x on its tape
iii) runs A on x
iv) accepts if A halts on x.

A halts on x ⟺ B accepts all strings ⟺ $L(B) = \Sigma^*$

A loops on x ⟺ $L(B) = \phi$

We can decide $\Longleftrightarrow$ if B accepts ε or not

We can decide $\Longleftrightarrow$ if $L(B) = \Sigma^*$ or not

We can decide if A halts on x or not.

g) does it accept any string at all?
h) accept every string?
i) accept a finite set?
j) accept a regular set?
k) accept a CFL?
l) accept a recursive set?

} solvable by reductions

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diagonalization → Proves if a problem is undecidable directly

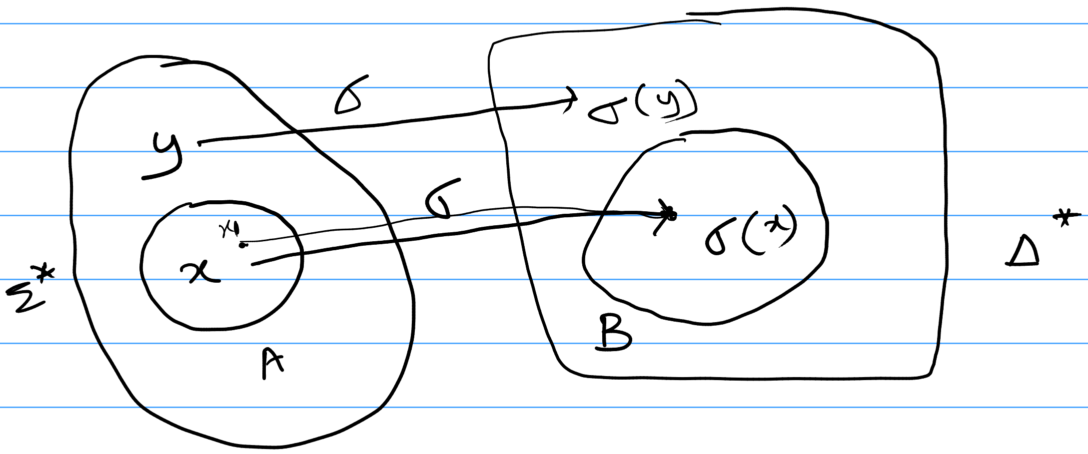
reduction → transforms an undecidable problem A to B, and concludes that B is also undecidable (many-one reduction)

Given sets $A \subseteq \Sigma^*$ and $B \subseteq \Delta^*$ a many-one reduction of A to B is a computable function

$\sigma : \Sigma^* \to \Delta^*$

s.t. $\forall x \in \Sigma^*$

$$x \in A \Longleftrightarrow \sigma(x) \in B$$



Theorem i) If $A \le_m B$ and B is r.e. then so is A.

$\Leftrightarrow$ If $A \le_m B$ and A is not r.e. then neither is B

ii) If $A \le_m B$ and B is recursive then so is A.

$\Updownarrow$

If $A \le_m B$ and A is not recursive, then neither is B.

Proof: i) Suppose $A \le_m B$ via σ and B is r.e.

M is a TM s.t. $B = L(M)$.

$$\underset{N}{A} \xrightarrow{\sigma} \underset{M}{B}$$

Let N be a TM for A s.t.

$\downarrow$ build N

given x, compute $\sigma(x)$, run M on $\sigma(x)$

accept if M accepts.

N accepts $x \Leftrightarrow M$ accepts $\sigma(x)$ — by construction

$\Leftrightarrow \sigma(x) \in B$ — defn. of M

$\Leftrightarrow x \in A$

ii) H.W.