① $x^2y'' + xy' = 1$ $\quad y(1) = 0, \quad y(1.4) = 0.0566$

$h = 0.1$

$x_0 = 1 \quad x_1 = 1.1, \; x_2 = 1.2, \; x_3 = 1.3, \; x_4 = 1.4$

$y_0 = 0. \quad y_4 = 0.0566$

Unknowns → y_1, y_2, y_3

Now, let's discretize the ODE using B.C's.

$$x_i^2 y_i'' + x_i y_i' = 1$$

$$\Rightarrow x_i^2 \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) + x_i \left(\frac{y_{i+1} - y_{i-1}}{2h}\right) = 1.$$

$i = 1, 2, 3$

$$\Rightarrow x_1^2 \left(\frac{y_2 - 2y_1 + y_0}{h^2}\right) + x_1 \left(\frac{y_2 - y_0}{2h}\right) = 1. \quad \text{(I)}$$

$$x_2^2 \left(\frac{y_3 - 2y_2 + y_1}{h^2}\right) + x_2 \left(\frac{y_3 - y_1}{2h}\right) = 1. \quad \text{(II)}$$

$$x_3^2 \left(\frac{y_4 - 2y_3 + y_2}{h^2}\right) + x_3 \left(\frac{y_4 - y_2}{2h}\right) = 1 \quad \text{(III)}$$

$$\left(\frac{-2x_1^2}{h^2}\right) y_1 + \left(\frac{x_1^2}{h^2} + \frac{x_1}{2h}\right) y_2 = 1 - \left(\frac{x_1^2}{h^2} - \frac{x_1}{2h}\right) y_0$$

(b_1, c_1, a_1)

$$\left(\frac{x_2^2}{h^2} - \frac{x_2}{2h}\right) y_1 + \left(\frac{-2x_2^2}{h^2}\right) y_2 + \left(\frac{x_2^2}{h^2} + \frac{x_2}{2h}\right)$$

(a_2, b_2, c_2)

$$a_i = \frac{x_i^2}{h^2} - \frac{x_i}{2h}, \quad b_i = \frac{-2x_i^2}{h^2}, \quad c_i = \frac{x_i^2}{h^2} + \frac{x_i}{2h}.$$

Now, we have,

$$b_1 y_1 + c_1 y_2 = 1 - a_1 y_0 \quad \text{(A)}$$

$$a_2 y_1 + b_2 y_2 + c_2 y_3 = 1 \quad \text{(B)}$$

$$a_3 y_2 + b_3 y_3 = 1 - a_3 y_4 \quad \text{(C)}$$

$AX = d$

$$A = \begin{bmatrix} b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 \\ 0 & a_3 & b_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1-a_1y_0 \\ 1 \\ 1-c_3y_4 \end{bmatrix}$$

Now, we can write (A) (B) & (C) as

$$y_1 + \frac{c_1}{b_1} y_2 = \frac{(1-a_1y_0)}{b_1}$$

So, $c_1' = \frac{c_1}{b_1}$ $\quad d_1' = \frac{(1-a_1y_0)}{b_1}$

Now, $\quad a_2y_1 + b_2y_2 + c_2y_3 = 1.$

$$a_2y_1 + \frac{a_2c_1}{b_1} y_2 = \frac{a_2}{b_1}$$

$$\left(b_2 - \frac{a_2c_1}{b_1}\right) y_2 + c_2y_3 = \left(1 - \frac{a_2}{b_1}\right)$$

$$y_2 + \frac{c_2y_3}{(b_2 - a_2c_1')} = \frac{(1 - a_2d_1')}{(b_2 - a_2c_1')}$$

$$c_2' = \frac{c_2}{b_2 - a_2c_1'}, \quad d_2' = \frac{1 - a_2d_1'}{b_2 - a_2c_1'}$$

$$y_2 + c_2'y_3 = d_2'$$

Now,

Now $\quad a_3y_2 + b_3y_3 = (1 - c_3y_4)$

$$a_3y_2 + a_3c_2'y_3 = a_3d_2'$$

$$y_3(b_3 - a_3c_2') = (1 - c_3y_4) - a_3d_2'$$

$$y_3 = \frac{\{(1 - c_3y_4) - a_3d_2'\}}{(b_3 - a_3c_2')}$$

$\Rightarrow y_3 = 0.034437\ 45167$

Now, $y_2 = d_2' - c_2'y_3$

$= 0.016655\ 74562$

Now,

$$y_1 = \frac{1}{b_1} - \frac{c_1}{b_1} y_2$$

$= 4.574181079 \times 10^{-3}$

x_0	x_1	x_2	x_3	x_4	x_5
0	0.2	0.4	0.6	0.8	1.0

(2) $y'' = x + y$, $y(0) = 0$, $y(1) = 0$ $\quad h = 0.2$

$y_i'' = x_i + y_i$

$$\Rightarrow \frac{(y_{i+1} - 2y_i + y_{i-1})}{h^2} = x_i + y_i$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 1\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2}\right)}_{c_i} y_{i+1} = \underbrace{x_i}_{d_i}$$

$y_0 = 0$, $y_5 = 0$

$$\left.\begin{aligned} a_1y_0 + b_1y_1 + c_1y_2 &= d_1 \\ a_2y_1 + b_2y_2 + c_2y_3 &= d_2 \\ a_3y_2 + b_3y_3 + c_3y_4 &= d_3 \\ a_4y_3 + b_4y_4 + c_4y_5 &= d_4 \\ y_1 + c_1'y_2 &= d_1' \end{aligned}\right\}$$ Solve for y_1, y_2, y_3, y_4

$$c_1' = \frac{c_1}{b_1}$$

$$d_1' = \frac{(d_1 - a_1y_0)}{b_1}$$

$y_2 + c_2'y_3 = d_2'$,

$y_3 + c_3'y_4 = d_3'$

$y_4 + c_4'y_5 = d_4'$ $\quad c_i' = \frac{c_i}{b_i - a_ic_{i-1}'}$, $\quad d_i' = \frac{d_i - a_id_{i-1}'}{b_i - a_ic_{i-1}'}$

$y_4 = d_4' - c_4'y_5$

Solving this, we get

$y(0) = 0$ $\quad y(0.2) = -0.028994$, $\quad y(0.4) = -0.050312\ 94$

$y(0.6) = -0.05880$ $\quad y(0.8) = -0.05441$

$\Rightarrow y_3 = 0.034437\ 45167$

Now, $y_2 = d_2' - c_2' y_3$

$= 0.016655\ 14562$

Now,

$y_1 = \frac{1}{b_1} - \frac{c_1 y_2}{b_1}$

$= 4.5\ 74181079 \times 10^{-3}$

$x_0\ x_1\ x_2\ x_3\ x_4\ x_5$

$0\ 0.2\ 0.4\ 0.6\ 0.8\ 1.0$

(2) $y'' = x + y$, $y(0) = 0$, $y(1) = 0$, $h = 0.2$

$y_i'' = x_i + y_i$

$\Rightarrow \frac{(y_{i+1} - 2y_i + y_{i-1})}{h^2} = x_i + y_i$

$\Rightarrow \left(\frac{1}{h^2}\right) y_{i-1} + \left(\frac{-2}{h^2} - 1\right) y_i + \left(\frac{1}{h^2}\right) y_{i+1} = x_i$

$a_i \quad b_i \quad c_i \quad d_i$

$a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$

$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2$

$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3$

$a_4 y_3 + b_4 y_4 + c_4 y_5 = d_4$

$y_0 = 0$

$y_5 = 0$

Solve for y_1, y_2, y_3, y_4

$y_1 + c_1' y_2 = d_1'$

$c_1' = \frac{c_1}{b_1}$

$d_1' = \frac{(d_1 - a_1 y_0)}{b_1}$

$y_2 + c_2' y_3 = d_2'$,

$y_3 + c_3' y_4 = d_3'$

$y_4 + c_4' y_5 = d_4'$

$y_4 = d_4' - c_4' y_5$

$c_i' = \frac{c_i}{b_i - a_i c_{i-1}'}$, $d_i' = \frac{d_i - a_i y_{i-1}'}{b_i - a_i c_{i-1}'}$

Solving this, we get

$y(0) = 0$ $y(0.2) = -0.028954$, $y(0.4) = -0.050312\ 99$

$y(0.6) = -0.05880$ $y(0.8) = -0.04941$

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Q(3) $y'' - 2y = 0$, $y(0) = 1$, $y'(1) = 0$.
$h = 0.2$

$$y_i'' - 2y_i = 0$$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) - 2y_i = 0$$

$$\Rightarrow \left(\frac{1}{h^2}\right) y_{i-1} - \left(\frac{2}{h^2} + 2\right) y_i + \frac{1}{h^2} y_{i+1} = 0$$

a_i b_i c_i d_i

Now, $y_0 = 0$, $y_n' = 0$

$$\Rightarrow \frac{(y_{n+1} - y_{n-1})}{2h} = 0 \Rightarrow y_{n+1} = y_{n-1}$$

Now, we have

$$a_i y_{i-1} + b_i y_i + c_i y_{i+1} = 0 \qquad i = 1, 2, \ldots, n$$

(I) $a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$ → we have to solve these (5) eqⁿˢ
(II) $a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2$
(III) $a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3$
(IV) $a_4 y_3 + b_4 y_4 + c_4 y_5 = d_4$
(V) $a_5 y_4 + b_5 y_5 + c_5 y_6 = d_5$

$y_4 = y_6$

$(a_5 + c_5) y_4 + b_5 y_5 = d_5$

x_0 x_1 x_2 x_3 x_4 x_5
0 0.2 0.4 0.6 0.8 1.0 1.2

Solving these we get

$y(0.2) = 0.786509$ | $y(0.6) = 0.536243$ | $y(1) = 0.461007$
$y(0.4) = 0.635938$ | $y(0.8) = 0.479447$

x_1 x_0 x_4 x_2 x_3 x_4 x_5

-0.25 0 0.25 0.5 0.75 1 1.25

Q4 $y'' - 2xy' - 2y = -4x$, $y(0) = y'(0)$.

$2y(1) - y'(1) = 1$.

$y_i'' - 2x_i y_i' - 2y_i = -4x_i$

$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) - 2x_i\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) - 2y_i = -4x_i$

$\Rightarrow \underbrace{\left(\frac{1}{h^2} + \frac{x_i}{h}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} - 2\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2} - \frac{x_i}{h}\right)}_{c_i} y_{i+1} = \underbrace{-4x_i}_{d_i}$

So we will have

$a_0 y_{-1} + b_0 y_0 + c_0 y_1 = d_0$

$a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$

$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2$ — (x)

$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_4$

$a_4 y_3 + b_4 y_4 + c_4 y_5 = d_5$

Now, from the B.C, we have

$y(0) = y'(0) \Rightarrow y_0 = y_0' \Rightarrow y_0 = \frac{y_1 - y_{-1}}{2h}$

$\Rightarrow 2h y_0 = y_1 - y_{-1}$

$\Rightarrow y_{-1} = y_1 - 2h y_0$

$2y(1) - y'(1) = 1$.

$\Rightarrow 2y_4 - y_4' = 1$

$\Rightarrow 2y_4 - \left(\frac{y_5 - y_3}{2h}\right) = 1$.

$\Rightarrow 4hy_4 - y_5 + y_3 = 2h$

$\Rightarrow y_5 = (4hy_4 + y_3 - 2h)$

Putting these in the eqn (x)

$a_0(y_1 - 2h y_2) + b_0 y_0 + c_0 y_1 = d_0$ — (c₀)

$(b_0 - 2a_0 h) y_0 + (c_0 + a_0) y_1 = d_0$ — (i)

(b₀) $a_1 y_0 + b_1 y_1 + c_1 y_2 = d_1$ — (ii)

$a_2 y_1 + b_2 y_2 + c_2 y_3 = d_2$ — (iii)

$a_3 y_2 + b_3 y_3 + c_3 y_4 = d_3$ — (iv)

$a_4 y_3 + b_4 y_4 + c_4(4h y_4 + y_3 - 2h) = d_5$

$\Rightarrow \underbrace{(a_4 + c_4)}_{(a_4)} y_3 + \underbrace{(b_4 + c_4 \cdot 4h)}_{(b_4)} y_4 = \underbrace{d_5 + 2hc_4}_{(d_4)}$ — (v)

i	a_i	b_i	c_i	
0	16	-34	16	$b_0 - 2a_0 h = -42$
1	17	-34	15	$c_0 + a_0 = 32$
2	18	-34	14	$a_4 + c_4 = 32$
3	19	-34	13	$b_4 + c_4 \times 4h = -22$
4	20	-34	12	$d_4 + 2hc_4 = 2$

Now, solving these eqn using thomas meth

$$y_0 + \left(\frac{c_0}{b_0}\right) y_1 = \left(\frac{d_0}{b_0}\right)$$

($c_0 \to c_0'$, $d_0 \to d_0'$)

$$y_0 + c_0' y_1 = d_0'$$

$$y_1 + c_1' y_2 = d_1'$$

$$y_2 + c_2' y_3 = d_2'$$

$$y_3 + c_3' y_4 = d_3'$$

$$y_4 + c_4' y_5 = d_4'$$

$c_4' = 0$

Solving these, we get,

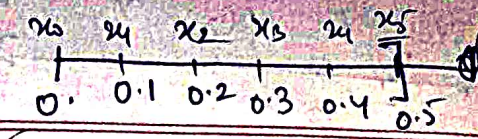
$y(0) = 0.731802$

$y(0.25) = 0.96049$

$y(0.5) = 1.28107$

$y(0.75) = 1.73339$

$y(1) = 2.43039$



B C

Q4 A $y'' + 2xy' + 2y = 4x$, $y(0) = 1$, $y(0.5) = 1.2791$,

$y_0 = 1$ $y_5 = 1.2791$

$$y_i'' + 2x_i y_i' + 2y_i = 4x_i$$

$$\Rightarrow \left(\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}\right) + 2x_i\left(\frac{y_{i+1} - y_{i-1}}{2h}\right) + 2y_i = 4x_i$$

$$\Rightarrow \underbrace{\left(\frac{1}{h^2} - \frac{2x_i}{2h}\right)}_{a_i} y_{i-1} + \underbrace{\left(\frac{-2}{h^2} + 2\right)}_{b_i} y_i + \underbrace{\left(\frac{1}{h^2} + \frac{2x_i}{2h}\right)}_{c_i} y_{i+1} = \underbrace{4x_i}_{d_i}$$

So, we have

$$\left.\begin{aligned} a_1 y_0 + b_1 y_1 + c_1 y_2 &= d_1 \\ a_2 y_1 + b_2 y_2 + c_2 y_3 &= d_2 \\ a_3 y_2 + b_3 y_3 + c_3 y_4 &= d_3 \\ a_4 y_3 + b_4 y_4 + c_4 y_5 &= d_4 \end{aligned}\right\}$$ we have to solve for y_1, y_2, y_3 & y_4

Now solving them using thomes meth.

$$y_1 + c_1' y_2 = d_1'$$
$$y_2 + c_2' y_3 = d_2'$$
$$y_3 + c_3' y_4 = d_3'$$
$$y_4 + c_4' y_5 = d_4'$$

we get, $y(0.1) = 1.09029$
$y(0.2) = 1.16117$
$y(0.3) = 1.21434$
$y(0.4) = 1.25249$