Date
11/09/2017

Lecture 16

If f is discontinuous at some points, then at those points FS is

$$\frac{f(x+)+f(x-)}{2} = a_0+\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$$

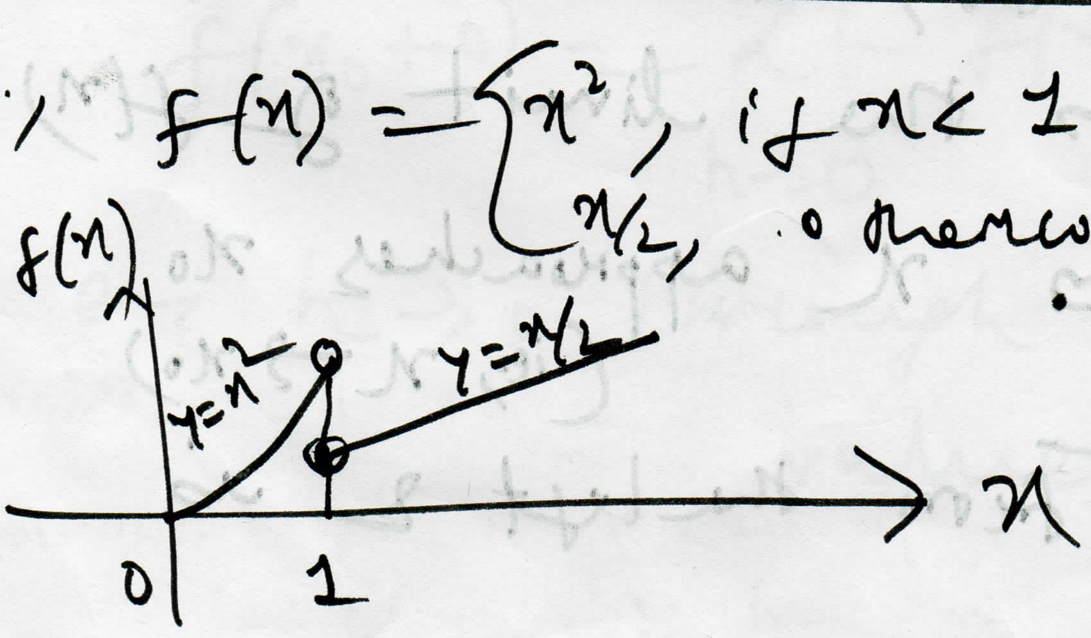
At the end points,
$[-\pi,\pi]$, at $x=\pm\pi$

$$\frac{f(\pi-)+f(-\pi)+}{2} = \boxed{a_0+\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)}$$

at $x=\pm\pi$

---

eg.) $f(x) = \begin{cases} x^2, & \text{if } x<1 \\ x/2, & \text{otherwise.}\end{cases}$



∴ left hand & right hand limits of the f^n $f(x)$ is

$$f(1-0) = 1$$

$$\& \ f(1+0) = 1/2.$$

The left-hand limit $\underline{\text{(L.H.L)}}$ of $f(x)$ at x_0 is defined as the limit of $f(x)$ as x approaches x_0 (ie, $x \to x_0$) $\overline{\underset{x}{\to} x_0}$ from the left & is frequently denoted by

$$f(x_0 - 0)$$

Thus,

$$f(x_0 - 0) = \lim_{h \to 0} f(x_0 - h)$$

as $h \to 0$ through positive values.

$\therefore$

Slly, the right hand limit is denoted by $f(x_0 + 0)$

&

$$f(x_0 + 0) = \lim_{h \to 0} f(x_0 + h)$$

as $h \to 0$ through positive values.

The left-hand & right-hand derivatives of $f(x)$ at x_0 are defined as the limits of

$$L.H.D = \lim_{h \to 0} \frac{f(x_0 - h) - f(x_0 - 0)}{-h}$$

$$\& \quad R.H.D = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0 + 0)}{h}$$

resly as $h \to 0$ through positive values.

If $f(x)$ is continuous at x_0, the last term in both numerators is simply $f(x_0)$.

Functions of any period of length $p = 2L$

The fns considered so far had period 2π for simplicity. Of course in applications, periodic fns will generally have other periods.

We show that the transition from period $p = 2\pi$ to period $p = 2L$ is quite simple. (how?) It amounts to a stretch (or contraction) of scale on the axis.

If a function of period $p = 2L$ has a Fourier series, we claim that this series is

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right) \quad \longrightarrow (1)$$

with the Fourier co-efficients of $f(x)$ given by the Euler formulas

(a) $$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx$$

(b) $$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx,$$

$n = 1, 2, \cdots$

(c) $$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx,$$

The series in (1) $\quad n = 1, 2, \cdots$

with arbitrary co-efficients is called a trigonometric (2) series Σ $\underline{m-1}$ extends

to any period p.

proof:- Eq^n (1) & (2) follow

by a change of scale, say,

Let $v = \frac{\pi x}{L}$.

Then $x = \frac{Lv}{\pi}$.

Also, $x = \pm L$ corresponds

to $v = \pm \pi$.

Thus, f, regarded as a

function of v, say, $g(v)$

i.e., $f(x) = g(v)$

has period 2π.

Accordingly, by (8) & (7) $\underline{\text{(where? is it)}}$

[i.e., $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \to (8)$

where, $a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx,\ n = 1, 2, \ldots$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx,\ n = 1, 2, \ldots \Big\} \to (7)$]

with ϑ instead of x,

this 2π-periodic f^n

$g(\vartheta)$ has the Fourier series

$$g(\vartheta) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\vartheta + b_n \sin n\vartheta) \to (3)$$

$\left[f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)\right.$

with coefficients

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(\vartheta)\,d\vartheta$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\vartheta)\cos n\vartheta\,d\vartheta$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} g(\vartheta)\sin n\vartheta\,d\vartheta \to (4)$$