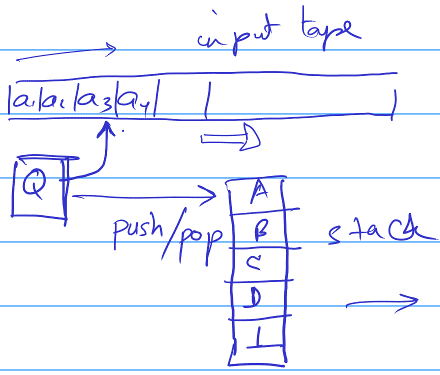
Pushdown Automata
(nondeterministic)



Reads from left to right, just once.

→ push / pop at the first / top position
(LIFO last in first out)

Formally, an NPDA is a 7 tuple

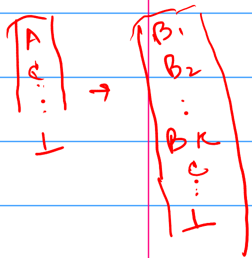
$$M = (Q, \Sigma, \Gamma, \delta, s, \underbrace{\perp}_{\text{stack bottom}}, F)$$

$\perp \in \Gamma$

where $Q \to$ (finite) set of states
$\Sigma \to$ (finite) input alphabet
$\Gamma \to$ stack alphabet
$\delta \subseteq (Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma) \times (Q \times \Gamma^*)$
$s \in Q$ (start state)
$F \subseteq Q$ the final or accept state

empty stack occurs when the stack bottom $(\perp)$ is popped and there is nothing in the stack.

$$((p, a, A), (q, B_1 B_2 \cdots B_k)) \in \delta$$



If M is in state p, reading a (on input tape), and the stack has A as the top element, it can pop A off the stack, push $\overline{B_1 B_2 \ldots B_k}$ in the stack, move the reader one place to the right, and assume state q.

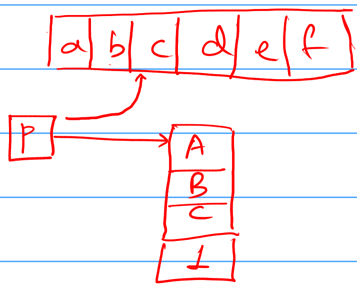
$((p, \varepsilon, A), (q, B_1 B_2 \ldots B_k))$
reader remains in its current place.

A configuration

$Q \times \Sigma^* \times \Gamma^* \rightarrow$ contents of stack

↳ portion of input UNREAD till now

↳ current state



$\Downarrow$

$$(p, cdef, ABC\perp)$$

Start configuration on input x?

$$(s, x, \perp)$$

next configuration relation: $\xrightarrow[M]{1}$

if $((p, a, A), (q, \gamma)) \in \delta$

then

for any $y \in \Sigma^*$ and $\beta \in \Gamma^*$

$$(p, ay, A\beta) \xrightarrow[M]{1} (q, y, \gamma\beta)$$

and if

$$((p, \epsilon, A), (q, \gamma)) \in \delta$$

then we can have

$$(p, y, \underline{A}\beta) \xrightarrow[M]{1} (q, y, \gamma\beta)$$

Analogously $\xrightarrow[M]{n}$ after n steps and $\xrightarrow[M]{*}$

$C \xrightarrow[M]{0} D \iff C = D$

$C \xrightarrow[M]{n+1} D \iff \exists E$ s.t. $C \xrightarrow[M]{n} E$ and $E \xrightarrow[M]{1} D$

$C \xrightarrow[M]{*} D \iff \exists n \geq 0$ s.t. $C \xrightarrow[M]{n} D$

Acceptance

1) Acceptance by empty stack
2) Acceptance by final state

} equivalent

Design an NPDA to accept all balanced parentheses