Matching Problem of Preference Model

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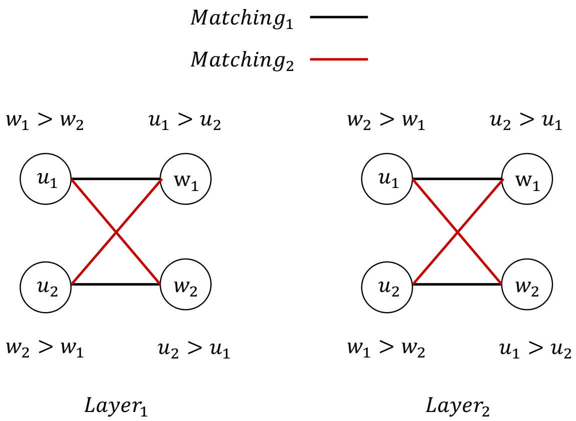
Outline

Outline

Matching Problem with Multi-Layer



- Matching
- Preference List
- Multi-Layer
- Cost-$rank_{u_1}^{(1)}(w_1)$

Global Layer and α-Layer Minimum Cost

- Global Layer
  For the global layer cost, the goal is to find a matching M whose sum of cost in each layer is less than D.

- α-Layer
  In addition, in terms of the α-layer cost, the goal is to find a matching M whose sum of cost in certain α layers chosen from the total l layers is less than D.

Four Evaluation Criteria

- $egal\text{-}cost(M) := \sum_{\{u,w\} \in M} (rank_u(w) + rank_w(u))$
- $regret\text{-}cost(M) := \max_{i \in V(M)} rank_i(M(i))$
- $equal\text{-}cost(M) := \sum_{(u,w) \in M} |rank_u(w) - rank_w(u)|$
- $balance\text{-}cost(M) := max\{\sum_{(u,w) \in M} rank_u(w), \sum_{(u,w) \in M} rank_w(u)\}$

Cumulative Extension

In order to make the research contents more meaningful, we adopt three extension methods to extend the original four evaluation criteria to eighteen listed below. Firstly, we use the cumulative extension method to acquire six evaluation criteria :

Table: Six Evaluation Criteria of Cumulative Extension

Criterion	Formula
sum-cost-1(M)	$\sum_{i \in V(M)} rank_i(M(i))$
sum-cost-2(M)	$\sum_{(u,w) \in M} (rank_u(w) * rank_w(u))$
sum-cost-3(M)	$\sum_{(u,w) \in M} \lvert rank_u(w) - rank_w(u) \rvert$
sum-cost-4(M)	$\sum_{(u,w) \in M} \max(rank_u(w), rank_w(u))$
sum-cost-5(M)	$\max_{(u,w) \in M} rank_u(w) + \max_{(w,u) \in M} rank_w(u)$
sum-cost-6(M)	$\prod_{(u,w) \in M} rank_u(w) + \prod_{(u,w) \in M} rank_w(u)$

Cumulative Extension

Then, we employ the multiplicative extension method to obtain six evaluation criteria:

Table: Six Evaluation Criteria of Multiplicative Extension

Criterion	Formula
mul-cost-1(M)	$\prod_{i \in V(M)} rank_i(M(i))$
mul-cost-2(M)	$\prod_{(u,w)\in M}(rank_u(w) + rank_w(u))$
mul-cost-3(M)	$\prod_{(u,w)\in M} \lvert rank_u(w) - rank_w(u)\rvert$
mul-cost-4(M)	$\prod_{(u,w)\in M} \max(rank_u(w), rank_w(u))$
mul-cost-5(M)	$\max_{(u,w)\in M} rank_u(w) * \max_{(w,u)\in M} rank_w(u)$
mul-cost-6(M)	$\sum_{(u,w)\in M} rank_u(w) * \sum_{(u,w)\in M} rank_w(u)$

Cumulative Extension

Finally, we utilize the maximum extension method to acquire six evaluation criteria:

Table: Six Evaluation Criteria of Maximum Extension

Criterion	Formula
max-cost-1(M)	$\max_{i \in V(M)} rank_i(M(i))$
max-cost-2(M)	$\max_{(u,w)\in M}(rank_u(w) + rank_w(u))$
max-cost-3(M)	$\max_{(u,w)\in M}(rank_u(w) * rank_w(u))$
max-cost-4(M)	$\max_{(u,w)\in M} \lvert rank_u(w) - rank_w(u)\rvert$
max-cost-5(M)	$\max\{\sum_{(u,w)\in M} rank_u(w), \sum_{(u,w)\in M} rank_w(u)\}$
max-cost-6(M)	$\max\{\prod_{(u,w)\in M} rank_u(w), \prod_{(u,w)\in M} rank_w(u)\}$

Outline

Contribution of this Semester

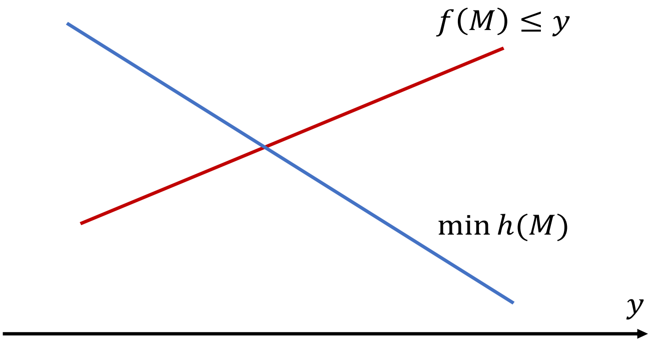
Through further research, we have achieved a breakthrough based on the results of the previous semester.

Table: Complexity Analysis of the Four Matching Models

	Old		New	
criterion	Global Layer	α-Layer	Global Layer	α-Layer
Egalitarian Cost	$O(n^3 logn)$	NP-hard	$O(n^3)$	NP-hard
Regret Cost	$O(n^3)$	NP-hard	$O(n^2\sqrt{n})$	NP-hard
Equal Cost	$O(n^3 logn)$	Studying	$O(n^3)$	NP-hard
Balance Cost	Studying	NP-hard	Dichotomy with 01-IP	NP-hard

Balance Cost on Global Layer

- $\max\{\sum_{(u,w)\in M} rank_u(w), \sum_{(u,w)\in M} rank_w(u)\}$
- Let $f(M) = \sum_{(u,w)\in M} rank_u(w)$, $h(M) = \sum_{(u,w)\in M} rank_w(u)$
- If we let $f(M) \le y$ to find the minimum value of $h(M)$, it is obvious that as y increases, $f(M)$ is increasing, while $h(M)$ is decreasing shown as below.



Balance Cost on Global Layer

$$
\begin{cases}
\min \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} rank_{w_j}(u_i) * x_{ij} \\
s.t. \quad \sum_{j=1}^{n} x_{ij} = 1, \ i = 1, 2, ..., n \\
\qquad \sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, ..., n \\
\qquad \sum_{i=1}^{n} \sum_{j=1}^{n} rank_{u_i}(w_j) * x_{ij} \leq y \\
\qquad x_{i,j} \in [0, 1], \ \text{integer}, \ i, j = 1, 2, ..., n
\end{cases}
$$

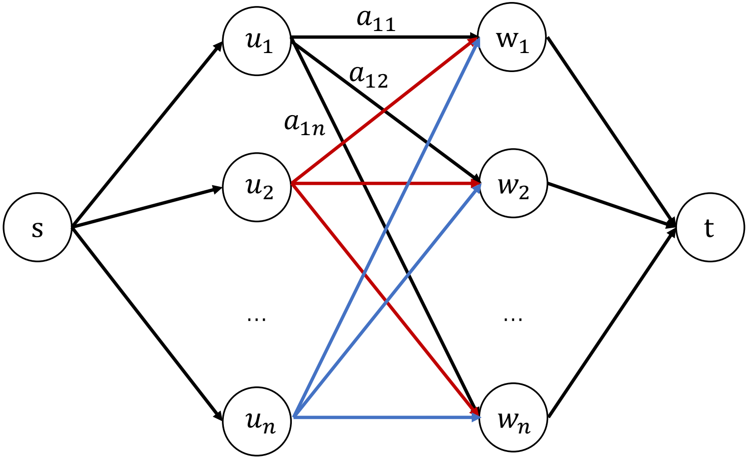
- 01 IP
- cutting-plane method
- branch and bound method
- Approximate method
- Parameter complexity

Outline

sum-cost-5(M) on Global Layer

- $\max_{(u,w)\in M} rank_u(w) + \max_{(w,u)\in M} rank_w(u) \iff f(M) + h(M)$
- If we let $f(M) \leq y_1$ to find the minimum value of $h(M)$, it is obvious dichotomy is helpful.
- Let $f(M) \leq y_1$ and $h(M) \leq y_2$, enumerate y_1, divide y_2
- Remove all edges (i, j) where $rank_{u_i}(w_j) > y_1$ or $rank_{w_j}(u_i) > y_2$

sum-cost-5(M) on Global Layer



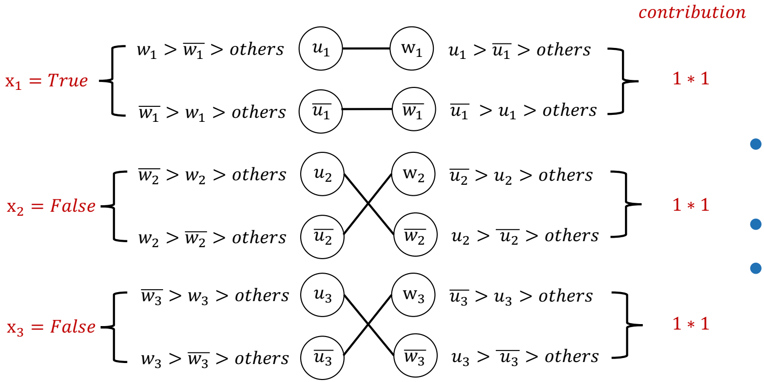
- Maximum Flow - Dinic
- $O(n^3 log(n)\sqrt{n})$

$$a_{ij} = (rank^k_{u_i}(w_j) \leq y_1 \,\&\&\, rank^k_{w_j}(u_i) \leq y_2)$$

mul-cost-6(M) on α-Layer

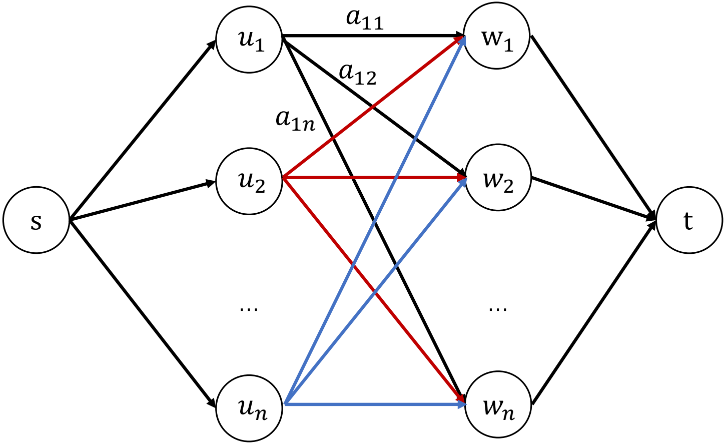
- 1-IN-3SAT
- INSTANCE
  A collection of clauses $C_1, ..., C_m$, $m > 1$; each C_i is a disjunction of exactly three literals.
- QUESTION
  Is there a truth assignment to the variables occuring so that exactly one literal is true in each C_i?
- Example
  $X = \{x_1, ..., x_5\}$, $C = \{C_1, C_2, C_3\}$
  $C_1 = \{\overline{x_1}, \overline{x_2}, x_3\}$, $C_2 = \{\overline{x_1}, x_4, x_5\}$, $C_3 = \{\overline{x_2}, x_4, x_5\}$

mul-cost-6(M) on α-Layer



- $\sum rank_u(w) * \sum rank_w(u)$
- $D = 9m^2$
- NP-hard

max-cost-1(M) on Global Layer



- max $rank_i(M(i))$
- $O(n^2 log(n)\sqrt{n})$

$$a_{ij} = (rank^k_{u_i}(w_j) \leq y \,\&\&\, rank^k_{w_j}(u_i) \leq y)$$

Outline

Summary of the Basic Four Matching Model

According to the four models described above, we could summarize our analysis of time complexity on the four basic evaluation criteria.

Table: Complexity Analysis of the Four Basic Matching Models

Criterion	Global Layer	α-Layer
Egalitarian Cost	$O(n^3)$	NP-hard
Regret Cost	$O(n^2\sqrt{n})$	NP-hard
Equal Cost	$O(n^3)$	NP-hard
Balance Cost	Dichotomy with 01-IP	NP-hard

Summary of the Cumulative Extension Model

According to the six models extended by cumulative method described above, we could summarize our analysis of time complexity on the cumulative extension models.

Table: Complexity Analysis of the Six Matching Models through Cumulative Extension

Criterion	Global Layer	α-Layer
sum-cost-1(M)	$O(n^3)$	NP-hard
sum-cost-2(M)	$O(n^3)$	NP-hard
sum-cost-3(M)	$O(n^3)$	NP-hard
sum-cost-4(M)	$O(n^3)$	NP-hard
sum-cost-5(M)	$O(n^3 log(n)\sqrt{n})$	NP-hard
sum-cost-6(M)	Enumerate with 01-IP	NP-hard

Summary of the Multiplicative Extension Model

For the six models extended by multiplicative method described above, we could summarize our analysis of time complexity on the multiplicative extension models.

Table: Complexity Analysis of the Six Matching Models through Multiplicative Extension

Criterion	Global Layer	α-Layer
mul-cost-1(M)	$O(n^3)$	NP-hard
mul-cost-2(M)	$O(n^3)$	NP-hard
mul-cost-3(M)	$O(n^3)$	NP-hard
mul-cost-4(M)	$O(n^3)$	NP-hard
mul-cost-5(M)	$O(n^3 log(n)\sqrt{n})$	NP-hard
mul-cost-6(M)	Enumerate with 01-IP	NP-hard

Summary of the Maximum Extension Model

In terms of the six models extended by maximum method described above, we could summarize our analysis of time complexity on the maximum extension models.

Table: Complexity Analysis of the Six Matching Models through Maximum Extension

Criterion	Global Layer	α-Layer
max-cost-1(M)	$O(n^2 log(n)\sqrt{n})$	NP-hard
max-cost-2(M)	$O(n^2 log(n)\sqrt{n})$	NP-hard
max-cost-3(M)	$O(n^2 log(n)\sqrt{n})$	NP-hard
max-cost-4(M)	$O(n^2 log(n)\sqrt{n})$	NP-hard
max-cost-5(M)	Dichotomy with 01-IP	NP-hard
max-cost-6(M)	Dichotomy with 01-IP	NP-hard

Q & A session

Q & A

Thank you!