Matching Problem of Preference Model

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Outline

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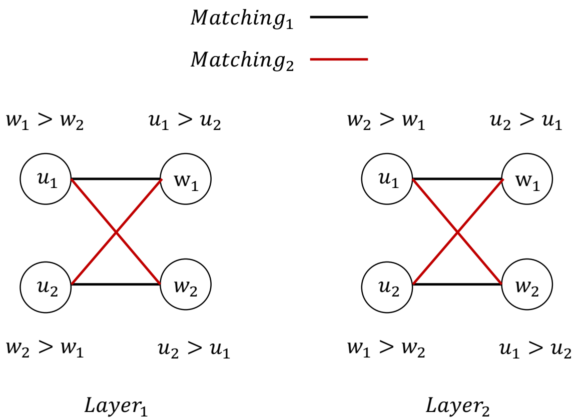
Matching Problem with Multi-Layer



- Matching
- Preference List
- Multi-Layer
- Cost-$rank_{u_1}^{(1)}(w_1)$

Four Evaluation Criteria

- $egal\text{-}cost(M) := \sum_{\{u,w\} \in M} (rank_u(w) + rank_w(u))$
- $regret\text{-}cost(M) := \max_{i \in V(M)} rank_i(M(i))$
- $equal\text{-}cost(M) := \sum_{(u,w) \in M} |rank_u(w) - rank_w(u)|$
- $balance\text{-}cost(M) := max\{\sum_{(u,w) \in M} rank_u(w), \sum_{(u,w) \in M} rank_w(u)\}$

Global Layer and α-Layer Minimum Cost

- Global Layer
  For the global layer cost, the goal is to find a matching M whose sum of cost in each layer is less than D.

- α-Layer
  In addition, in terms of the α-layer cost, the goal is to find a matching M whose sum of cost in certain α layers chosen from the total l layers is less than D.

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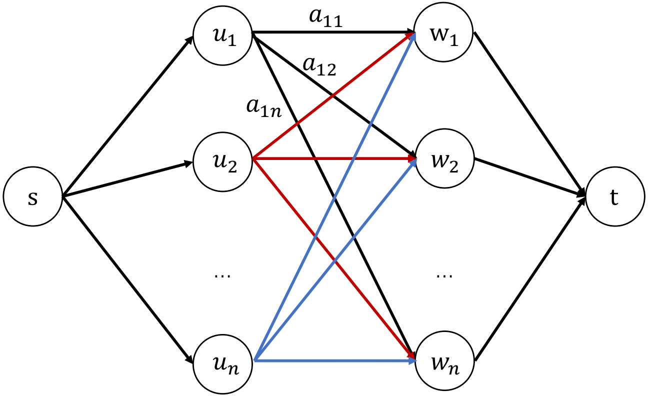
Egalitarian Cost with Global Layer



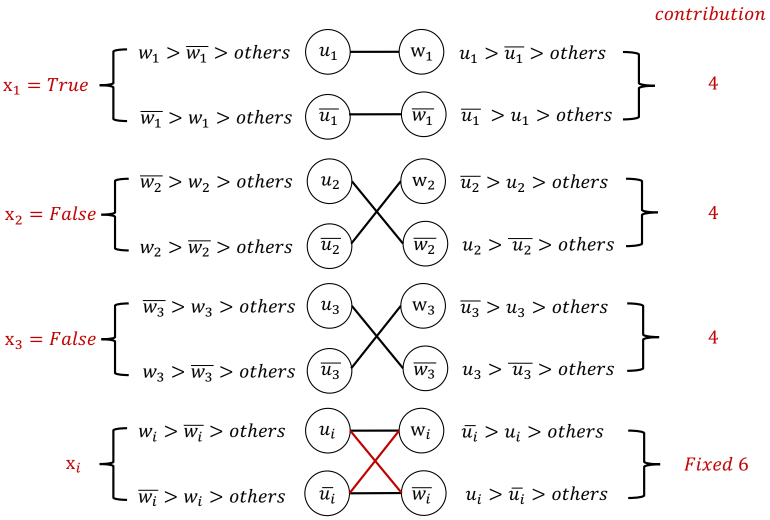
$$a_{ij} = \sum_{k=1}^{k \le l} (rank_{u_i}^{k}(w_j) + rank_{w_j}^{k}(u_i))$$

- Cost Flow Algorithm
- No Negative Loop
- $O(n^3 logn)$

1-IN-3SAT

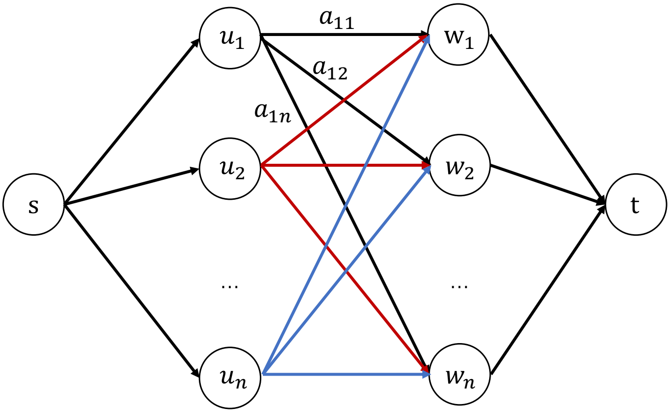
- INSTANCE
  A collection of clauses $C_1, ..., C_m, m > 1$; each C_i is a disjunction of exactly three literals.
- QUESTION
  Is there a truth assignment to the variables occuring so that exactly one literal is true in each C_i?
- Example
  $X = \{x_1, ..., x_5\}$, $C = \{C_1, C_2, C_3\}$
  $C_1 = \{\overline{x_1}, \overline{x_2}, x_3\}$, $C_2 = \{\overline{x_1}, x_4, x_5\}$, $C_3 = \{\overline{x_2}, x_4, x_5\}$

Egalitarian Cost with α-Layer



- NP-hard

Regret Cost with Global Layer



- Maximum Flow
- $O(n^3)$

$$a_{ij} = \max\left(rank_{u_i}^{k}(w_j), rank_{w_j}^{k}(u_i)\right), k \in [1, l]$$

Balance Cost with Global Layer

- Failed attempts on bipartite graph
- Reduce the partitioning problem to the generalized bipartite graph problem
- However, the generalized bipartite graph problem is NP-hard while the original may be not
- Still studying in the original problem

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Summary of the Four Matching Model

According to the four models described above, we could summarize our contributions to this Matching Problem of Preference Model.

Table: Complexity Analysis of the Four Matching Models

criterion	Global Layer	α-Layer
Egalitarian Cost	$O(n^3 logn)$	NP-hard
Regret Cost	$O(n^3)$	NP-hard
Equal Cost	$O(n^3 logn)$	Studying
Balance Cost	Studying	NP-hard

Q & A session

Q & A

Thank you!