HILLSLOPES
ESCI 4701: GEOMORPHOLOGY

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UPDATED (SIGNIFICANTLY) 2019
SLIGHT EDITS 2020

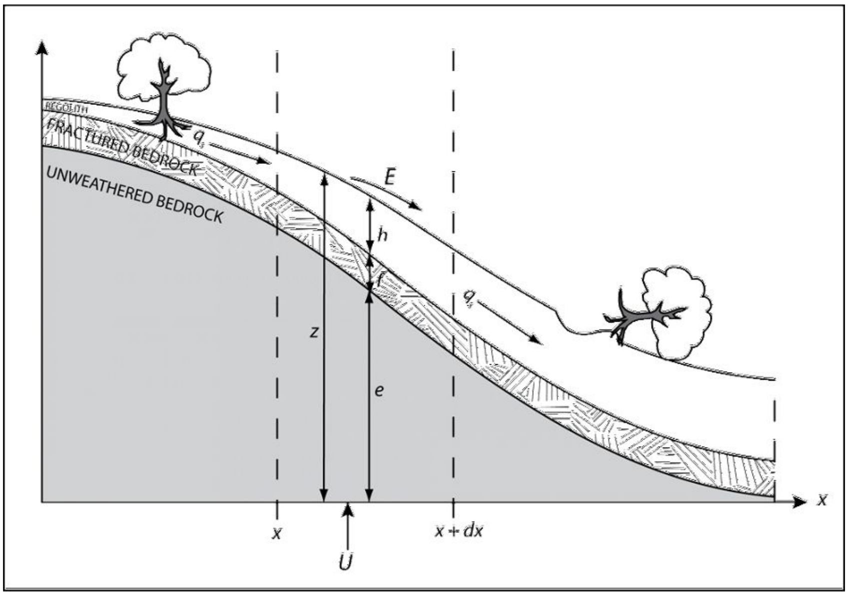


FIGURE 1. A schematic hillslope, from the Critical Zone Observatory.

Learning objectives:

- Learn the state of the established science in how masses move down hillslopes, alongside some of its limitations
  - Gradually (soil creep: diffusive)
  - Suddenly (mass wasting: advective)
- Derive the diffusion equation, which is a near-universal partial-differential equation that applies whenever a gradient-dependent process is combined with a conservation equation. (In this case: steeper hillslopes transport material more quickly, and volume of mobile material is conserved.)
- Learn why many hills have rounded tops
- Learn how to use Mohr-Coulomb failure to characterize slopes for potential landslides

Important definitions (for this class segment and beyond)

- **Steady**: process is constant in time (all time derivatives go to 0)
- **Uniform**: process is constant in space (all spatial derivatives go to 0)

1. Causes for downslope soil (or mobile-material) motion

In class, we discussed the reasons for downslope soil motion (or motion of any loose, small, mobile material), and there were many. Some ideas that we discussed (or the 2016 class came up with), listed here with my commentary and some additional or changed items, are:

- Rainsplash

- Overland flow ("sheetwash")
- Burrowing mammals (*MINNESOTA GOPHERS!*)
- Burrowing invertibrates (poofing up the soil)
- Soil type (can be connected to weathering processes and source material)
- Lithology, including bedrock type and fracture spacing
- Vegetation
  - Stabilizing hillslopes (with roots)
  - Tree throw (root ball displacement when trees tip over) causing erosion
  - Roots lifting soil around the plant
- Freeze/thaw and the growth of ice lenses

- Tillage

- Wind abrasion of hilltops (in arid regions – less important in humid regions)

2. Hillslope evolution by linear diffusion

2.1. **Slope-dependent sediment discharge (in 1D).** Almost all of the processes discussed in Section 1 are related to slope in some way.

- Burrowing animals throw regolith downhill.
- Overland flow can move regolith downslope and produce rills (small channels)
- Trees and frost lift soil perpendicularly to the hillslope, but gravity then pulls this material straight down, leading to a net downslope velocity vector.
- Rain splash likewise is able to transport material less far uphill (ejected material intersects the hillslope sooner) and farther downhill (ejected material can fly farther before hitting the lower slope)
- Tillage on hillslopes acts similarly: a plow that will distribute material uniformly on a flat slope will, on a hillside send more material downslope than it will upslope. This is being increasingly recognized as a major source of erosion in agricultural regions that is exposing the subsoil (B horizon) and reducing agricultural productivity.

Figure 2. Use this space to sketch how a uniform distribution of grains away from a rain impact or a plow might cause **(a)** uniform sediment redistribution on a flat surface and **(b)** more sediment to be sent downhill on a slope.

All of these processes move material downslope. Many can move more material downslope if the slope is steeper. As a result, the rate of sediment transport alone is not a good indicator of the individual processes. Because the resulting downslope sediment transport is process-agnostic, we can write a general rule of proportionality:

$$q_m \propto S \tag{1}$$

Where q_m is volumetric discharge of mobile hillslope material (m – sometimes called "sediment", "colluvium", or "mobile regolith") per unit width of the hillslope (i.e., through a hillslope cross-section) in units of [length^2 time^{-1}].

Geomorphologists use this basic proportionality to lump all of these hillslope-evolution processes together into a single constant, k_{hs}. We then redefine slope in terms of a local derivative in a coordinate system in which z is vertical and x is horizontal:

$$q_m = -k_{\text{hs}} \frac{\mathrm{d}z}{\mathrm{d}x} \tag{2}$$

The "$-$" sign is because the direction of transport is doownslope. k_{hs} is, effectively, a diffusivity, with units of [length^2 time^{-1}].

As noted above, this k_{hs} is really important! It's a black box – an admission of defeat before we even start. It's geomorphologists saying, *It is really incredibly difficult to decide whether this rate of downslope sediment transport resulted from worms or gophers or frost or trees tipping over or mineral breakdown or rainsplash or feral pigs digging (this is a real thing!) or a combination of many of these*. Because these processes move material downslope faster when the slope becomes steeper, geomorphologists can construct this effective **hillslope diffusivity** term, k_{hs}, and find out what it is locally, in different environments.

Question: what climatic, lithologic, or other factors might we expect to impact k_{hs}, and therefore the rate of mass transport down the side of a hillslope?

2.2. **Conservation of Mass.** The above provides a simple relationship to describe how hillslope material moves. However, this does not provide immediate information about the form of the landscape. In other words, how do our physically-based rules about material motion through the landscape eventually build the landforms that we know?

To connect fluxes or discharges with landscape form, we must hearken back to a well-known concept: *conservation of mass*. When properly accounting for material density, we can also refer to this as *conservation of volume*. Here, we will derive what this means quantitatively, within a control volume along a hillslope (Figure 3).

In the sections below, I will first discuss how the size of the box of mobile material shown in Figure 3 changes as a function of inputs and outputs (q_m). I will then solve this equation in combination with a transport rule (Equation 2) for a hillslope that comprises only mobile material – a scenario that is quite common with the thick till and outwash deposits that blanket much of Minnesota and the upper Midwest.[1] I will then expand this solution to incorporate aeolian deposition or erosion, which can act as a local source or sink of material. Following this, I will move into weathering and soil formation, and how to link the depth to bedrock (upper right) and thickness of the mobile layer to hillslope evolution in bedrock-dominated

[1]This approach is also that commonly used in landscape-evolution models, even though they are meant to simulate landscapes that include significant bedrock! As a field, we have some steps to take towards realism – and this could be another final-project topic.

landscapes. Finally, I will discuss how the gradient-based hillslope mass transport approach neglects important effects of the thickness of the soil.

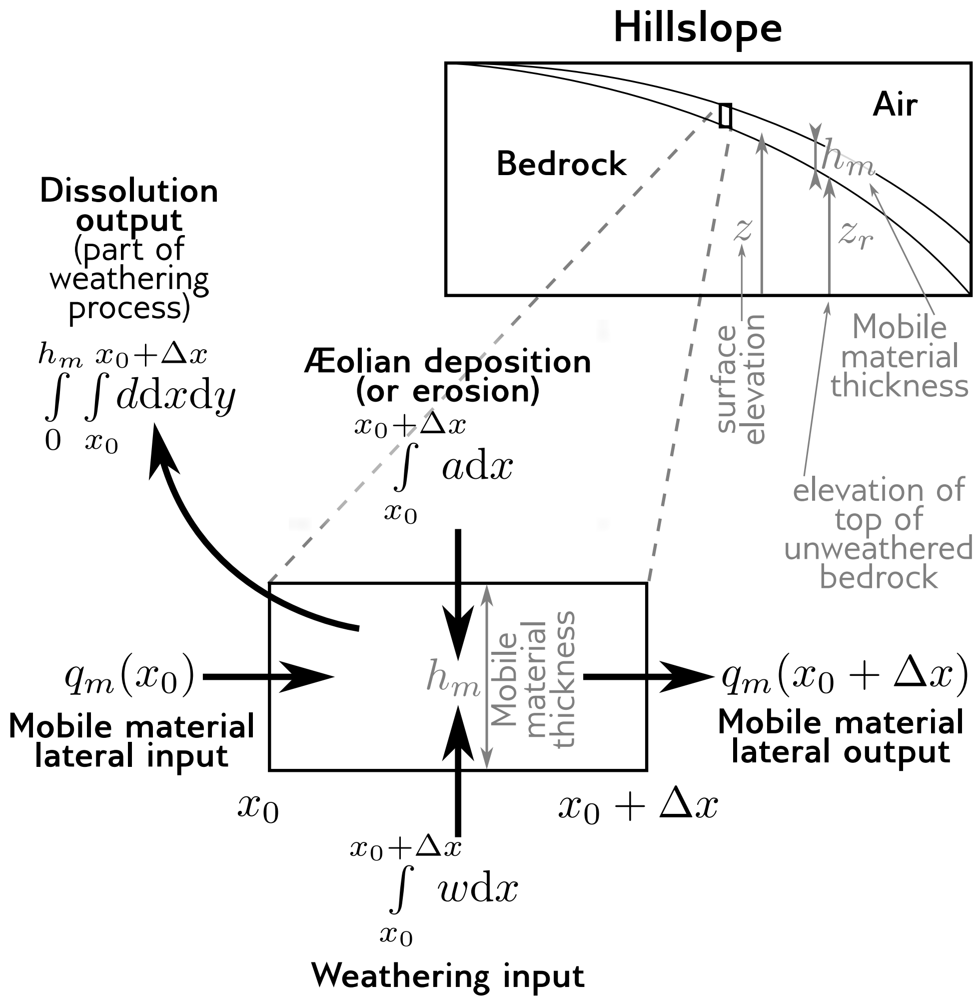


FIGURE 3. Mass balance across an imaginary box of soil of width Δx within a hillslope cross section. Terms include Q_m, mass discharge per unit hillslope width; a, deposition of airborne material – or removal of material by the wind; and w, addition of new mobile material via weathering.

2.3. **Lateral conservation of volume in a hillslope element.** At location x_0, material enters the box at a rate $q_m(x)$. Likewise, at location $x_0+\Delta x$, material leaves the box at a rate $q_m(x_0+\Delta x)$. Pretty simple so far!

Second, let's make a couple basic points.

- If more material enters the box than exits it, and the material inside the box remains the same density, then the box must become thicker.
- If more material leaves the box than enters it, then the box must become thinner.

Let's call this thickness h_m: thickness of mobile material.[2]

How much thicker or thinner will the box become? Well that depends on the width of the box. What is this, and how will this impact the rate of change in h_m?

(3)

The rate of increasing or decreasing thickness of our box of mobile hillslope material should therefore depend on the box width and the net amount of material either entering or leaving the box. Symbolically, this is:

$$\frac{\Delta h_m}{\Delta t} = \frac{q_m(x_0) - q_m(x_0 + \Delta x)}{\Delta x} \tag{4}$$

Here, t is time.

At this point, I would like to draw your attention to the right-hand side of Equation 4. Does it remind you of anything? Let us rewrite it below, flipping its sign first, and changing the location-specific "x_0" terms to "x", which can then represent any point along the hillslope:

$$\frac{q_m(x + \Delta x) - q_m(x)}{\Delta x} \tag{5}$$

[2]One might argue that the box could also change its length, but this is not reasonable because each box, representing a segment of hillslope, must abut the prior and following hillslope segments. If I shorten this box, I must lengthen others or create new boxes to ensure that the full hillslope remains represented in the solution.

Using a particular definition that you may remember from your first calculus class (i.e., the definition of a derivative), one may write:

$$\frac{\mathrm{d}q_m}{\mathrm{d}x} = \lim_{\Delta x \to 0} \frac{q_m(x+\Delta x) - q_m(x)}{\Delta x} \tag{6}$$

So there we have it: the definition of a derivative. Taking $\lim(\Delta t \to 0)$ on the left-hand side of Equation 4 and putting the whole mess back together then gives you:

$$\frac{\partial h_m}{\partial t} = -\frac{\partial q_m}{\partial x} \tag{7}$$

The change in elevation of a thin imaginary box of material on the hillslope is equal to the amount of stuff that goes into it minus the amount of stuff coming out of it, divided by the width. Therefore, the rate of change in the elevation at this imaginary box is equal to the rate of material entering, minus the rate of material leaving, divided by its width.

2.4. **Combining a gradient-based transport rule with conservation of volume.** Equation 2 states that mobile-material transport rate is linearly dependent on slope. Equation 7 states that the change in mobile-material thickness per unit time is linearly proportional to the negative *divergence* (here taken in one dimension) of material through a particular point in space.

Plugging Eq. 2 into Eq. 7 gives us an equation for hillslope evolution due to linearly-diffusive soil creep. I've left you some space to work it out – this is important, as this is one of the most common forms of differential equations, ever. It's called the **diffusion equation**, and has analogies in heat transport (thermal diffusion via Fourier's Law), transport of chemicals through a membrane (chemical diffusion: Fick's Laws), and more.

$$\frac{\partial h_m}{\partial t} = \frac{\partial}{\partial x}\left(k_m \frac{\partial z}{\partial x}\right) \tag{8}$$

This above equation is written with k_m inside one of the spatial derivatives, meaning that the hillslope diffusivity may changes over the course of the hillside. You might expect that to occur because of lithology – a massive dolostone will generally have a different (and lower) diffusivity than a shale, meaning that the dolostone is more difficult to erode. Likewise, differences in hydrologic forcings and vegetation feedbacks (e.g., on equatorial- vs. polar-facing slopes: Figure 4) can cause differences in hillslope evolution. Changes in elevation in steep areas can impact (local along the hillslope), and larger hills that impact orographic precipitation can also have different diffusivities (k_m) on their wet and dry sides. Animal communities may also alter hillslope diffusivity – hooved animals can cause significant soil creep. Tilling hillsides increases diffusivity as well, and over the past century, this has lead to significant exposure of the less-agriculturally-productive **B horizon**, or "subsoil", in the upper Midwest.

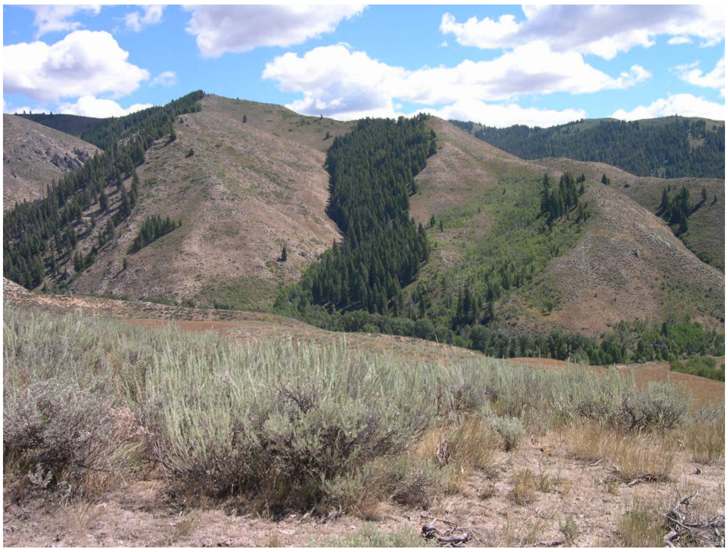


FIGURE 4. Differences in vegetation as a function of hillslope aspect – the azimuth that it faces. Poleward-facing hillslopes generally retain more moisture and can support a larger vegetation community. Here, this is represented by the trees (as opposed to the grasses). Photo from southwest Idaho by Thayne Tuason, `https://commons.wikimedia.org/wiki/File:Effects_of_aspect_on_vegetation-_SW_Idaho.JPG`. (The author, A. Wickert, has taken several similar photos in Wyoming and southeastern Minnesota, but those are on his office computer, and he is writing this from home on a Sunday.)

On a single hill, and especially a small one with uniform land cover and lithology, it is also possible to have a a single value for hillslope diffusivity (k_m). If we can therefore assume that k_m is uniform, then it becomes a constant that can be pulled out of the derivative.[3] The resultant equation is the most common form of the linear **diffusion equation**.

$$\frac{\partial h_m}{\partial t} = k_m \frac{\partial^2 z}{\partial x^2} \tag{9}$$

[3]If you need to prove this to yourself: use the chain rule. You will find that it is possible to move the term outside the derivative because the derivative of a constant is always zero.

2.5. **Hillslopes formed entirely of mobile material with no external inputs.** If a hillslope is formed entirely of mobile material, as is common with the extensive unconsolidated Quaternary-age sediments that blanket much of the upper Midwest, we can ignore the bedrock and any weathering (Figure 3). We also ignore any erosion or deposition by the wind: this is commonly the case in the humid portions of the upper Midwest, though was not always the case in the past (extensive wind-blown sediments blanket this region.) Therefore, any change in the thickness of the mobile material is consequently a direct change in the elevation of the hillslope, with no additional equations required.[4] Based on this, we can substitute the land-surface elevation, z, for h_m, and write:

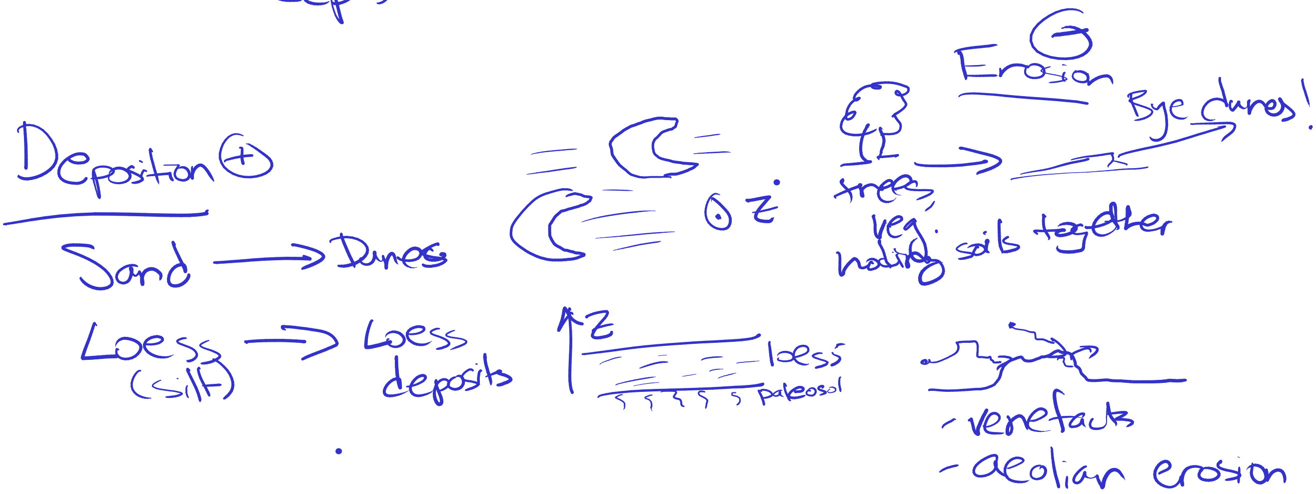
$$\frac{\partial z}{\partial t} = k_m \frac{\partial^2 z}{\partial x^2} \tag{10}$$

This is the **most common form used for hillslope processes in geomorphology** and in landscape-evolution models (models that simulate how rivers, slopes, and other processes change over time).[5]

2.6. **Adding erosion or deposition via wind.** Let us return to our box and Figure 3. What if there are local additions or subtractions of material to or from the box, for example because of aeolian erosion or deposition (whose rate is given by a in units of [length/time])? This can act as what is called a local **source** or **sink** term.

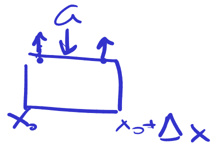
Before we move on, let's brainstorm a few mechanisms for erosion and/or deposition via wind:

+ deposition − erosion



[4]Any change in h_m is also a change in hillslope elevation when bedrock and weathering are involved, but this requires additional solutions for weathering rate and the production of mobile material.

[5]This is the most common even though most landscapes are underlain by bedrock! Geomorphologists who study hillslope processes and landscape evolution often simplify or ignore weathering and soil-forming processes. The validity of these assumptions remains unclear to me, but the diffusive model does have significant explanatory power for the shape of hills.



To start with what will be a very straightforward set of math (but that can be good practice), let's recall that we have allowed the width of the box, Δx to shrink to an infinitesimal width, dx. Therefore, this aeolian term – which in principle is spread across the full box, simplifies as follows:

(11) $$\text{Areal rate of aeolian erosion or deposition} = \int_{x_0}^{x_0+\Delta x} a\,dx$$

(12) $$= ax\Big|_{x_0}^{x_0+\Delta x}$$

$= a(\cancel{x_0} + \Delta x) - \cancel{a(x_0)}$

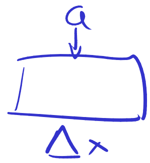
(13) $$= a\Delta x$$

$\frac{d}{dt}(S) = \dot{S}$

To determine the rate of vertical elevation change due to aeolian processes $\dot{z}_a$, we must (as before) divide by the width of the box. In so doing, we convert this areal rate of change into a vertical rate of change:

(14) $$\text{Rate of aeolian erosion or deposition} = \frac{a\Delta x}{\Delta x}$$

(15) $$= a$$



Therefore:

(16) $$\dot{z}_a = a$$

RATE OF ELEV CHANGE FROM AEOLIAN DEP.

Let us rewrite Equation 10 similarly, as:

(17) $$\dot{z}_q = k_m \frac{\partial^2 z}{\partial x^2}$$

where $\dot{z}_q$ signifies the portion of the change in mobile-material-column thickness that is due to downslope transport of hillslope material.

Now, by simply summing these aeolian ($\dot{z}_a$) and downslope-transport (i.e., creep) ($\dot{z}_q$) components of hillslope mobile-material transport, we can write an equation that combines aeolian inputs/outputs with soil-creep processes:

(18) $$\frac{\partial z}{\partial t} = \dot{z}_q + \dot{z}_a$$

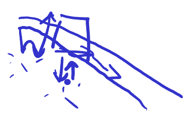
(19) $$= \boxed{k_m \frac{\partial^2 z}{\partial x^2} + a}$$

creep + aeolian

Here, a is an aeolian source/sink term that can arbitrarily and locally modify hillslope elevation. One could imagine inserting in its place a function that relates to the physics of aeolian sediment transport (to be covered later in the semester) and/or a data-driven parameter function based on the rates of aeolian-material accumulation observed in the field (via, for example, dated profiles of **loess** – wind-blown silt – deposition).

2.7. **Inputs of mobile material from weathering.** In the above cases, we have assumed an infinite supply of mobile material. But what if this is not the case? We must then return to our concept of a box in which h_m is changing (Figure 3).

By analogy to Equations 16 and 17, we may write:



(20) $$\cancel{\dot{h}_{ma} = a\dot{h}_{mq} = \frac{\partial^2 z}{\partial x^2}}$$

line break.

$\dot{h}_{ma} = a$

$\dot{h}_{mq} = k_m \frac{\partial^2 z}{\partial x^2}$

Along the same lines as those given in the aeolian section, above, we can write an expression for the weathering input, w:

(21)
$$\dot{h}_{mw} = w$$

and therefore:

creep aeolian weathering

(22)
$$\frac{\partial h_m}{\partial t} = \dot{z}_q + \dot{z}_a + \dot{z}_w$$

(23)
$$= k_m \frac{\partial^2 z}{\partial x^2} + a + w$$

This seems straightforward, right? But of course, there are complications:

- 2.7.1 We are solving for h_w. How do we convert this to solve for the elevation of the surface, z, and therefore compute the shape of the full hillslope?
- 2.7.2 Do we have any way of knowing what controls weathering rate, and including it in the equation? —> Soil-production function

Hence two sub-sub-sections, below:

2.7.1. *Weathering and land-surface elevation.* Mobile material on hillslopes typically has a porosity that ranges from a few percent to several tens of percent. Solid, unweathered rock typically has little to no porosity, and this is often what underlies hillslopes, though a wide range of porosities are possible (screenshot table held in Figure 5).

Table 2.4 Range of Values of Porosity

	n(%)
Unconsolidated deposits	
Gravel	25–40
Sand	25–50
Silt	35–50
Clay	40–70
Rocks	
Fractured basalt	5–50
Karst limestone	5–50
Sandstone	5–30
Limestone, dolomite	0–20
Shale	0–10
Fractured crystalline rock	0–10
Dense crystalline rock	0–5

Mountain core

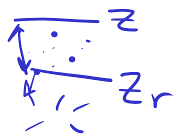
Freeze and Cherry, Table 2.4

Figure 5. Here, n is used for porosity as a percentage. In this class, I will use it as a value from 0 (entirely solid) to 1 (entirely void space, possibly filled with fluids). Table "borrowed" gratuitously from the wonderful *Groundwater* textbook by *Freeze and Cherry* (1979).

Through conservation of mass, we can evaluate whether weathering processes will cause the thickness of the mobile-material layer, h_m, to increase or decrease with respect to the rate at which the rock itself is weathering. We have defined w as an input rate of weathered material, meaning that any corrections that we create here will be applied to the rate of change in the unweathered-rock-surface elevation, z_r. To do so, we first define a bulk density of hillslope material (that is, mass of solids, therefore negating fluids that fill the pores) for both the rock (ρ_r)

and the mobile hillslope material (ρ_m). If mass is conserved within the rock–mobile-material continuum, then:

Δ thickness mobile material:



$$\dot{h}_{mw} = w \tag{24}$$

$$\dot{z}_r = -\frac{\rho_m}{\rho_r} w \tag{25}$$

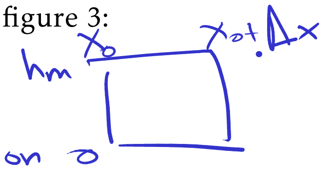
$$\dot{z} = z_r + h_m \tag{26}$$

$$= w\left(1 - \frac{\rho_m}{\rho_r}\right) \tag{27}$$

Therefore, if the density of the mobile hillslope material is less than that of the bedrock – as is typically the case – this equation states that the elevation of the hill surface may often rise due to weathering processes!

But not so fast! We haven't considered another component of weathering, beloved of low-temperature geochemists: dissolution. This results in material being removed from the hillslope. Here we consider the effects of dissolution on the layers of mobile material.[6]

The rate of dissolution is calculated across the full box, as shown in figure 3:



$$\text{rate of dissolution} = \int_0^{h_m}\int_{x_0}^{x_0+\Delta x} d\,\mathrm{d}x\mathrm{d}y \tag{28}$$

dissolution

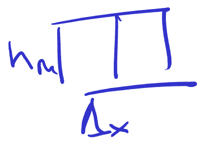
Here, d (for "dissolution") gives the volumetric rate of dissolution within the column.[7] While dissolution is really part of the weathering process, I find it convenient to break out. For now, we will assume that d is a constant – though in reality, it depends on water residence time in the hillslope and other factors that you may learn about in ESCI 4702: Hydrogeology.

Using the same reasoning as above, we can integrate this equation across the box and then divide by the box width to find the vertical increase (or decrease) in elevation with time.[8] This results in an equation for the change in elevation due to dissolution, $\dot{z}_d$:

$$\int_0^{h_m}\int_{x_0}^{x_0+\Delta x} d\,dx\,dy$$

$$dx\Big|_{x_0}^{x_0+\Delta x} = d(x_0+\Delta x - x_0) = d\Delta x$$

$$d\Delta x\, y\Big|_0^{h_m} = \frac{d\Delta x h_m}{\Delta x}$$



Convention: loss

$$\dot{z}_d = -h_m d \tag{29}$$

The – sign indicates that all of this material is lost.

Now this is interesting! Because we are only differentiating with respect to x, the height of the box remains in the equation. And this makes sense: the greater the length over which the

[6]Dissolution can also remove material from bedrock – notably in karstic landscapes – but these rates are commonly much slower than those in the mobile material, where high porosity and permeability coupled with a large surface area contribute to more rapid leeching and removal of mass.

[7]Take care to not confused this with the non-italicized ordinary differential operator "d"!

[8]Of course, it might be that the elevation remains constant and the density changes, but since we prescribe the density, this is a bit of physical reality that you can address freely in this set of equations.

dissolution reaction can occur, the faster it will occur. Putting this together with conservative[9] weathering processes (w) and other processes related to hillslope evolution results in:

$$\frac{\partial z}{\partial t} = \dot{z}_q + \dot{z}_a + \dot{z}_w + \dot{z}_d \tag{30}$$

dissolution

$$= k_m \frac{\partial^2 z}{\partial x^2} + a + w - h_m d \tag{31}$$

This equation is **trickier to solve** because you need to know the thickness of the mobile-material column! Of course, this very thickness may change as the hillslope evolves. If the evolution of the mobile-material thickness is coupled to the evolution of the hillslope, this creates the potential for a **feedback** in the equation that makes the a rate-controlling variable be a function of the hillslope state and evolution. As a result, unless we decide that the mobile-material layer stays constant through time, we now need to know how it evolves in a way that is independent of the total evolution of the hillslope profile.

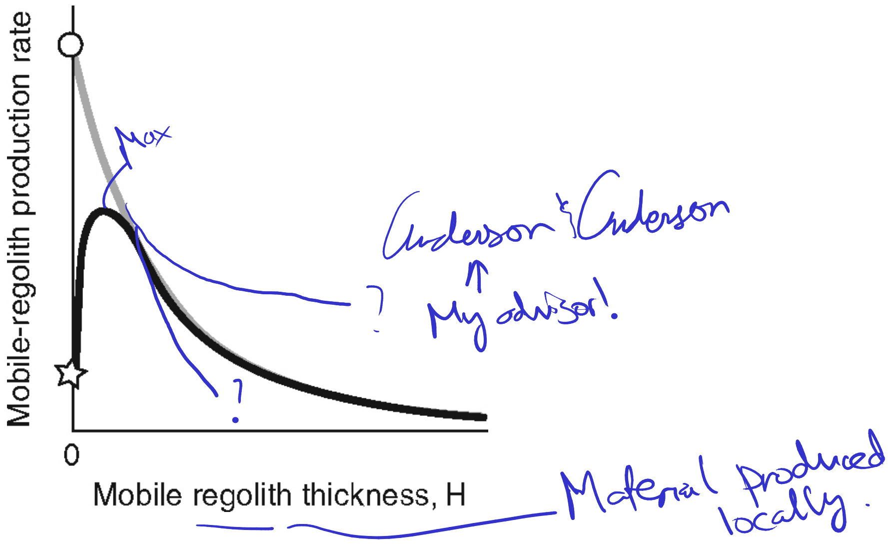


Figure 6. The "humped" and "exponential" functions for soil formation, from *Anderson and Anderson* (2010) (Chapter 7, Fig. 33). *Mobile regolith* – material produced in place on the hillslope – is a subset of my "mobile material", which also includes aeolian deposits and mobile material transported earlier by glaciers.

2.7.2. *Mobile material thickness and mobile-regolith production rates.* The rate of production of mobile material is related to weathering at the bedrock–mobile-material interface.[10] Most commonly, this is described using one of two common functions: a **humped** function (like a **gamma** function) and an **exponential** function (Figure 6). Both functions indicate that mobile-material production rates (i.e., weathering rates) increase as one approaches the surface from great depth. This is sensible: porosity and permeability are higher near the surface, and water residence times are lower – leading to more flushing of the system with water that is not yet saturated in the chemical species that constitute the weathering bedrock. This means that chemical gradients are higher, and the dissolution reaction proceeds more quickly. Near the

[9] no mass entering or leaving – **not** including dissolution

[10] I use the phrase "mobile material" because this has not necessarily been altered to form a soil. The term "mobile regolith" is also used, but "regolith" is not universally defined, and so I find it to be, at present, not a very useful piece of jargon. "Soil" is often used for all of these, but this again is defined differently in different fields, and ambiguity causes major problems in science, so I avoid it here except when writing unambiguously about material that has been altered by soil-forming processes. *Of course, bedrock may be mobilized too, in large landslides or rock falls, but I'm starting to run out of possible / useful terms.*

surface, bare rock cannot hold onto water and create a microenvironment capable of supporting much weathering. In reality, the thickness of material required to create such an environment is very small (mm? cm?) (Figure 7), and therefore the hump likely lies very near to the surface. As a result of this – and for mathematical expediency – I will use the exponential weathering function in these notes.

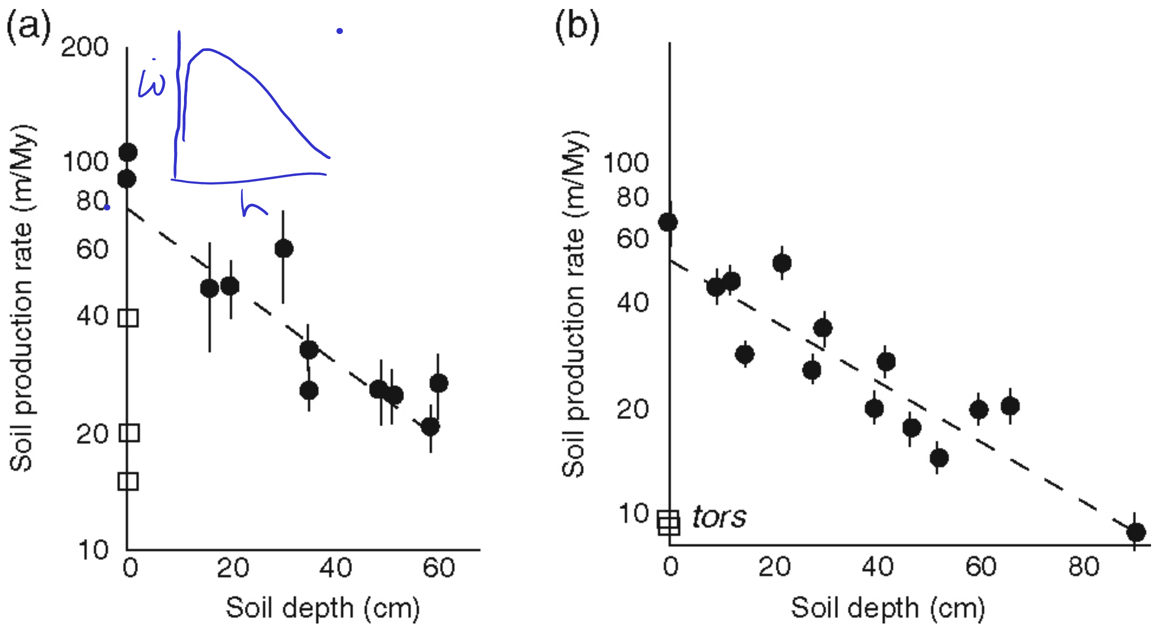


FIGURE 7. Data and exponential fits for rates of bedrock weathering, from *Anderson and Anderson* (2010) (Chapter 7, Fig. 34). Note that it uses "mobile regolith" and "soil" where I use "mobile material".

Based on the above discussion, the rate of weathering, w, is a function of the thickness of the layer of mobile material, h_m. Because the function is an exponential, it is related to h_m by a particular length scale... I will call this length scale δz_w, and it is often on the order of a few tens of centimeters.

(32) $$w = w_0 e^{-h_m/\delta z_w}$$ decay length.

Here, w_0 is a defined surface weathering rate, and $e \approx 2.718$ is Euler's number.

This now adds a second **feedback** between mobile-material thickness and the rate of land-surface-elevation change. However, solving it requires a simultaneous solution for the thickness of the mobile-material layer, which we can now define. Our resultant system of differential equations is:

creep aeolian weathering dissolution

(33) Mob. mat. $$\frac{\partial h_m}{\partial t} = k_m \frac{\partial^2 z}{\partial x^2} + a + w_0 e^{-h_m/\delta z_w} - h_m d$$

(34) Rock $$\frac{\partial z_r}{\partial t} = -\frac{\rho_m}{\rho_r} w_0 e^{-h_m/\delta z_w}$$

(35) $$\frac{\partial z}{\partial t} = \frac{\partial h_m}{\partial t} + \frac{\partial z_r}{\partial t}$$

(36) $$= k_m \frac{\partial^2 z}{\partial x^2} + a + \left(1 - \frac{\rho_m}{\rho_r}\right) w_0 e^{-h_m/\delta z_w} - h_m d$$

Using equation 36, we can:

(1) Prescribe an initial topography ($z(x)$)
(2) Prescribe an initial mobile-material thickness distribution ($h_m(x)$)
(3) Allow these to evolve

2.8. **Hilltop curvature with linear-diffusive hillslopes.** The above differential equations define linear-diffusive hillslopes. Although I spent a long time wading into complexities, the core equation used in landscape-evolution modeling is Equation 10. When couched in terms of erosion, and all of the weathering/dust/etc. portions are removed, this equation becomes

$$\dot{\varepsilon} = -k_m \frac{\mathrm{d}^2 z}{\mathrm{d}x^2} \tag{37}$$

where $\dot{\varepsilon}$ is erosion rate, which will be uniform across the landscape. This erosion rate term is going to be used througout the class, and erosion rate is always defined in the negative direction (for convenience, because erosion removes material). Hence, a "minus" sign has appeared on the right-hand side. An additional change from Equation 10 is that I return the ∂ symbols to simple "d"s. This is because I have defined this landscape to exist at **steady state**, such that the erosion rate on the left is simply a constant.[11]

In differential form, it is somewhat difficult to see how one should connect the form of the landscape to the implicit hillslope transport and transport rates involved. So let's integrate Equation 37 to get a spatial form of its steady-state profile.

Just from looking at this equation, we can see that the **curvature should increase with higher erosion rate and lower erodibility**. In other words, if it's harder to move material, or if material needs to be moved more quickly, then a steeper slope is required!

Here is your space to integrate and solve:

... and the answer is:

$$z = -\frac{1}{2}\frac{\dot{\varepsilon}}{k_m}x^2 + z_0 \tag{38}$$

[11]This **steady** form of the hillslope is important, as it implies that something (river erosion? sea level? tectonics? a combination?) is either causing the edges of the hillslope to fall or the center of the hill to rise at a rate that equals the mean (and **uniform**) erosion rate. Keep this idea in your back pockets for now: later we will discuss how rivers and hillslopes can be coupled. A quick teaser? Rivers set the boundaries of the hillslopes, but hillslopes processes set the sediment inputs to the rivers.

z_0 is the elevation of the ridge crest. What this equation states is that **hilltops are parabolic**! This parabolic form is quite common in nature; Figure 8 shows data (circles) and results from a simple diffusional model based on a linear diffusion equation (lines), and how well they match. This is a big take-away, and one you should test in the field – and see whether it is true or not.

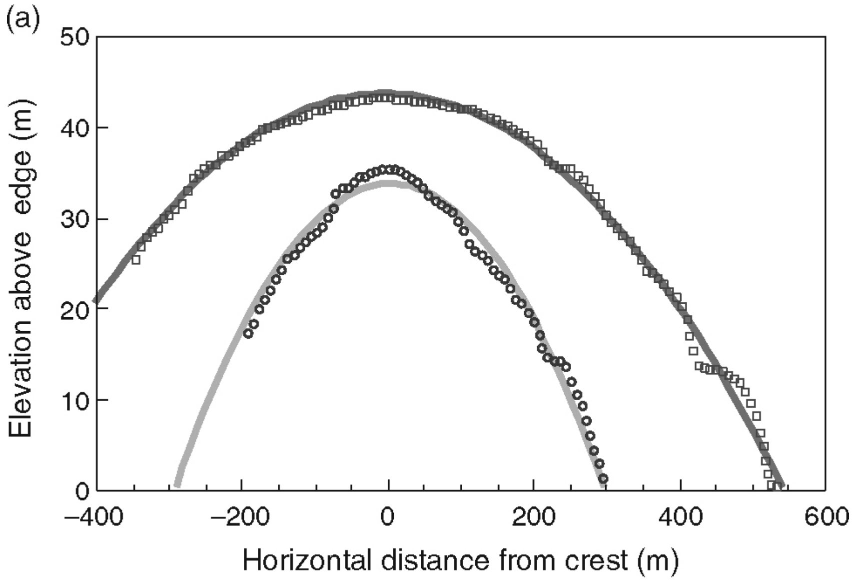


FIGURE 8. Data and parabolic fits for hillslope profiles, from *Anderson and Anderson* (2010) (Chapter 10, Fig. 4a).

Remember that linear-diffusive assumption holds only for hilltops controlled by uniform erosion rates all around, and typically by incising rivers at either side: it does not account for plateau surfaces, hilltops that have large deposited piles of glacial till, etc. (unless that till has adjusted ot the interglacial geomorphic processes, which usually it hasn't – too little time). It also can't account for significant aeolian activity or time variability in boundary conditions (e.g., erosion, uplift). Nevertheless, it fits a wide range of hillslopes, indication that Equation 37 and its in-built assumptions are broadly valid despite their simplicity.

Steady state *For a steady-state hillslope, one can draw a box around the soil profile and declare its volume to be constant. This means that the upslope colluvium delivery into the box, minus the colluvium export out of the box, plus additional material from weathering of the bedrock, must equal zero.*

Tectonic application: *For a given k_m in a landscape, hilltop curvature should give you information on relative erosion (exhumation) rates aross that landscape. If we think of a special case, which we call a steady-state landscape, uplift equals erosion. If this can be assumed, then you can actually* *obtain a spatial estimate of relative uplift rates from the shapes of hilltops alone, provided that the assumptions of linear diffusion are met. More-curved hilltops indicate faster uplift, smoother hilltops indicate slower uplift. Not too bad, huh? To read more about this and how it relates to landscape evolution, you can read about the method in Hurst et al. (2012).*

2.9. **Dating by landform diffusion.** Rivers, the sea, and faults all create sharp scarps. These then diffuse over time via hillslope processes. Therefore, older scarps should have smoother and more diffuse profiles. If k_m is uniform, and you have a few dates – either from the historical record or from Quaternary dating techniques (e.g., radiocarbon, luminescence, cosmogenic radionculide), then it becomes possible to calibrate a diffusional model, using the equations above, for the appropriate value of k_m. Using this calibrated value, one can then model the ages of other features – perhaps those that are more difficult to date using direct means. An example below is a set of marine terraces along the Santa Cruz coastline in California (Figure 9).

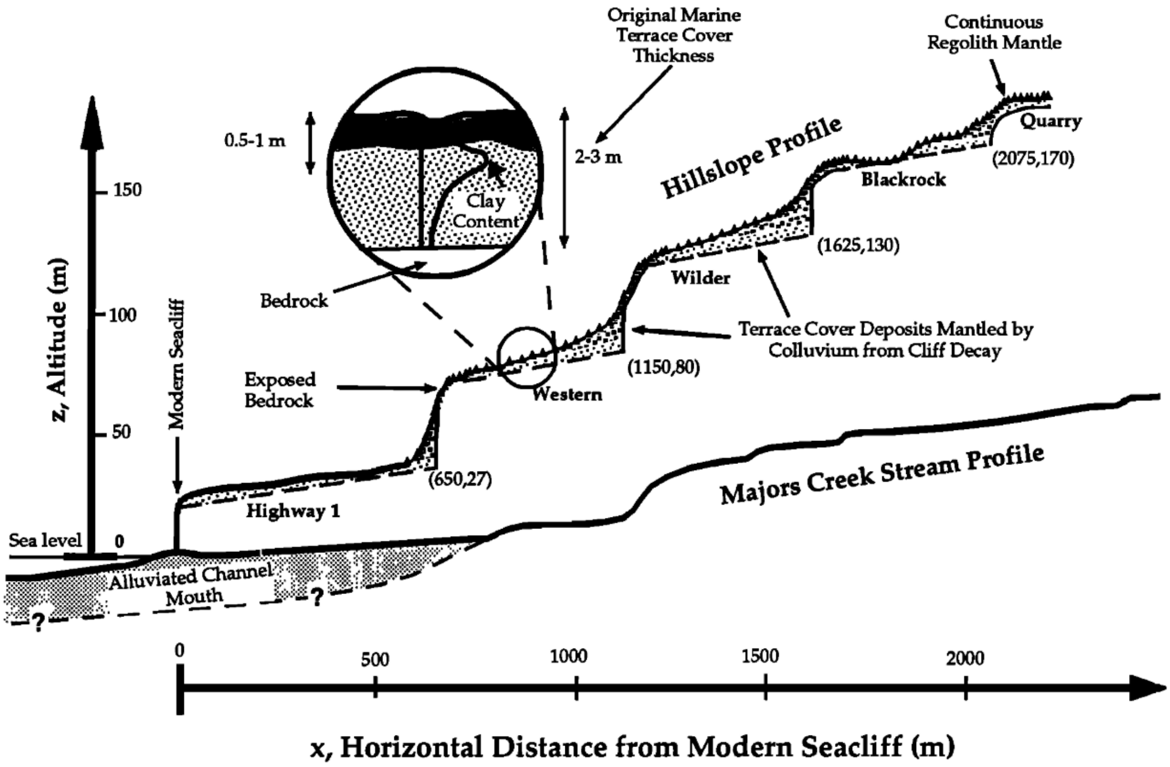


Figure 9. Marine terrace and stream profiles from near Majors Creek, Santa Cruz, California. Each terrace is a wave-cut platform recording a particular past sea-level highstand, and abandoned during the subsequent sea-level fall. The continued uplift of the coast is fast enough to preserve these past highstand-related terraces, creating a marine–tectonic "strip chart" recorder of Earth's past. These data are from **hand leveling** (think back to your surveying lab). Here, the eroding bedrock hillslopes are mantled by increasing amounts of colluvium – material from erosion of hillslopes – as they become older (i.e., higher, farther from the coast), and the overall profile shape of the older terraces becomes more rounded and diffuse. From *Rosenbloom and Anderson* (1994), Figure 4.

2.10. **Additional applications of linear-diffusive hillslope theory.**

- Relative ages of fault scarps
- Relative ages of marine terraces

- Relative ages of glacial moraines
- Sediment supply to rivers
- Landscape evolution modeling
- Broad inferences of geologic-time-scale continental evolution (very primitive early geophysical models used diffusion exclusively for rates of sediment delivery from the continents to the sea)

3. Reality check-in: Not all hills are these perfectly smooth parabolas

Yes! This is a theoretical shape that is achieved fully only in areas with incising rivers around eroding hills made of a uniform material. It is the dominant process in a great many places, but we must recognize that in others, there may be other processes active. And in much of Minnesota, even if these diffusive processes are occurring, the overall form of the landscape bears so strong a glacial imprint that the hillslope signature is not as easily visible.

3.1. **Explicit consideration of soils in hillslope transport.** Our idea of diffusive hillslopes is all well and good, but we're taking a really zoomed-out, coarse view of the landscape. First off, let's make one thing clear: the above equation doesn't "care" where the material is that moves. In reality, however, the vast majority of it is at the top. Here's a figure from Roger Hooke's measurements at Bevens Creek, in the Minnesota River Valley. It shows higher rates of creep in the top layers of soil.

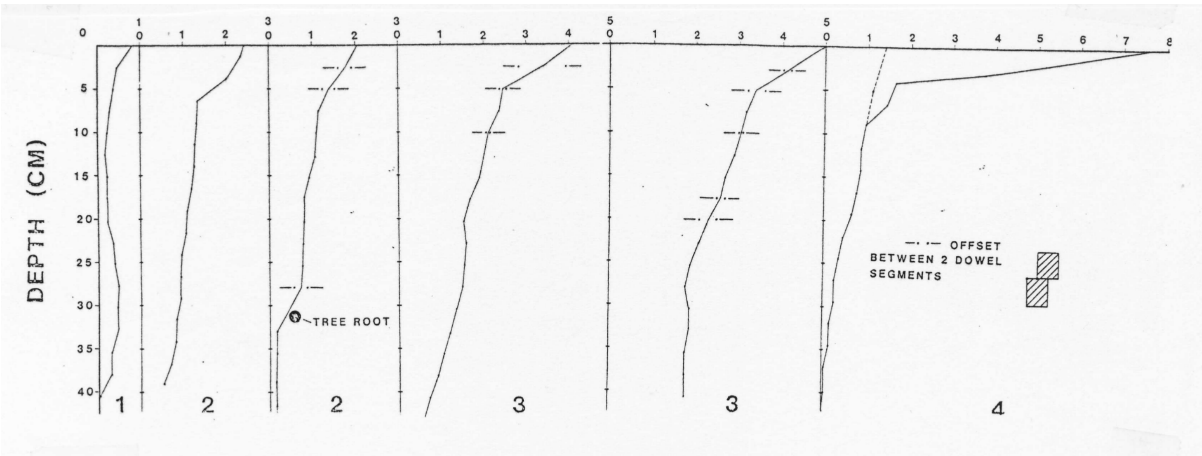


Figure 10. (From Roger Hooke): Deformation profiles measured at the Bevens Creek creep stations. Profiles were measured by inserting segmented wooden rods in vertical holes made in the soil in 1968 and then excavating the rods 9.5 years later and recording the displacement form vertical. Bold numbers at bottom indicate creep station where profile was obtained. Units on x-axis are creep in total centimeters moved over the 9.5-year observation period.

So if the topsoil keeps eroding, how does the hillslope keep from losing all of its soil? Well, it has to produce that soil as well. The mobile-material production curves (Figure 6) provide some idea about how this works.

Even more importantly, the fact that the the above diffusion-based approach is insensitive to the depth at which the material moves means that it can move mass even if no mobile material is present. YIKES!

In answering this, I stray beyond most published literature, but find this to be a helpful – and honestly, cathartic, process. My goal in this is to start to think about a bit more realism – if we get enough time in class, or if you're interested in your final project. For now, I will start with two key, initial, and hopefully-obvious statements:

- If there is no mobile material, nothing moves.
- The amount of mobile material in motion increases as soil depth increases.

The data in Figure 10 indicate that most of the displacement is typically towards the top of the soil profile, and it diminishes with depth. The function to be used to describe it isn't immediately obvious, so I choose to use an exponential because of its mathematical ease-of-use andits ability to approximately match these data. Within the zone of mobile material, the velocity (u) of a soil parcel is a function of its elevation (z') below the Earth surface (z):

$$u(z) \propto e^{(z'-z)/(\Delta z_u)} \tag{39}$$

Here, Δz_u is a length scale associated with a $1/e$ drop-off in velocity of hillslope material.

In order to compute the total amount of material moved on a hillslope, I must integrate $u(z)$ across the zone of mobile material:

$$q_m \propto \int_{z_r}^{z} = u(z) \int_{z_r}^{z} e^{(z'-z)/(\Delta z_u)} \mathrm{d}z' \tag{40}$$

We can solve this equation using u substitution:

$$q_m \propto \Delta z_u \left(1 - e^{(z_r - z)/(\Delta z_u)}\right) \tag{41}$$

Based on the geometry in Figure 3, we know that $z_r - z = -h_m$. Substituting this in the above equation gives us:

$$q_m \propto \Delta z_u \left(1 - e^{h_m/(\Delta z_u)}\right) \tag{42}$$

The next step is to merge this with the other terms in Equation 2. The resultant expression relates to slope, a constant of proportionality, and **the thickness of the mobile material**:

$$q_m = -k_{\mathrm{hs}} \Delta z_u \left(1 - e^{h_m/(\Delta z_u)}\right) \frac{\mathrm{d}z}{\mathrm{d}x} \tag{43}$$

FIGURE 11. Use this space to draw a continuum of hillslopes with slope as an x axis. Around 30-35 degrees, hills transition from being dominated by gradual downslope creep to being dominated by mass-wasting events.

This equation now has the proper behavior in the limits. As $h_m \to 0$, $1 - e^{h_m/(\Delta z_u)} \to 0$ as well, and hence, $q_m \to 0$. As $h_m \to \infty$, $1 - e^{h_m/(\Delta z_u)} \to 1$, removing the exponential term from the right-hand side of the equation, and approaching a form analogous to the standard form for hillslope-diffusion. This, as I noted in Section 2.5, is the proper solution for a hillslope made entirely of mobile material. All other hillslope solutions should lie between the two.[12]

(Important Aside.) ***Soils geomorphology*** *is an important branch of geomorphology that we are not going to discuss in detail here, beyond some simple definitions of soil horizons in the field. Kyungsoo Yoo, professor in the department of Soil, Water, and Climate, is an expert in both soils and geomorphology, and is an excellent resource for those of you wanting to learn more about both.*

4. When hills are steep

The above sections on gradual hillslope diffusion works for most gradual slopes significantly below the **angle of repose**.[13] However, steeper hills tend to fail in large mass-wasting events instead of by slow soil creep and associated processes listed at the beginning of this chapter of notes. Here we discuss how to compute stability and form of steep hillslopes.

4.1. **Nonlinear diffusion and hillslopes.** Let's start out by assuming that you're interested in the long-term evolution of hillslopes, and the overall shape of the landscape. We're going to assume this becasue (1) this is what is useful to understand landscape-forming processes in this class, and (2) landslide science is a rich and complex field of its own that we will only cover briefly below. So in the meantime, just wear a helmet when you're working in steep areas and don't ski 38-degree slopes in avalanche conditions.

As geomorphologists, we have a basic modification of the linear-diffusion equation for hillslope evolution that can be applied to this steep terrain. In order to do so we set a critical slope, S_c, that is the asymptotic maximum slope attainable by a hillside. As the slope of the hillside increases towards the critical slope, the erosion rate increases as well to ensure that the slope remains below S_c. This simulates the effect of mass-wasting events, like landslides.

[12] Author's note: I should publish this somewhere, or check if someone else has, or perhaps you (the student) and I should chat about somehow involving it in a class project and/or publishing it.

[13] The angle of repose is the angle of a critically-steep slope, which will fail if steepened further, maintained as a function of the material's internal friction.

We construct the hillslope nonlinear-diffusion equation based on the work of *Roering et al.* (1999), by modifying Equation 2 to include the nonlinear terms:

(44)
$$q_m = \frac{k_m \dfrac{\mathrm{d}z}{\mathrm{d}x}}{1 - \left(\dfrac{\left| \frac{\partial z}{\partial x} \right|}{S_c} \right)^2}$$

What happens when $\mathrm{d}z/\mathrm{d}x \to 0$ (a flat surface)? What happens when $\mathrm{d}z\mathrm{d}x \to S_c$?

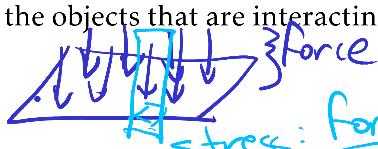
Because of the nonlinearity in the slope dependence of q_m, it becomes very difficult to write an equation similar to 10. This integration typically must be completed numerically – by a computer.

4.2. **Hillslope stability and mass wasting.** Soil doesn't just gently move down hillslopes – sometimes it fails in discrete events. Such discrete **mass-wasting events** (Figure 12) dramatically modify hillslopes and pose a significant natural hazard.

To understand landsliding, a type of mass wasting, I turn to **continuum mechanics** – the study of how continuous materials deform.[14] More specifically, I consider **brittle failure**. This is the same principle that is applied to understand shallow faults in the Earth, fractures in buildings and other structures, and so on. It is based on a very simple problem that you may have learned about in physics: a block on an inclined plane.

In this example, the location and orientation of the plane is defined. Inside a continuous body, on the other hand, **any arbitrary plane may be the slip surface**!

Because we are working within a continuum, it no longer makes as much sense to discuss discrete forces, because these require that we know the sizes of the objects that are interacting. Rather, we can define **stresses** as a force per unit area:



(45)
$$\text{stress} \longrightarrow \sigma = \frac{F}{A}$$

stress: $\dfrac{\text{force}}{\text{area}}$

Here, σ is a stress, F is a force, and A is an area over which that force is applied.

[14]Continuum mechanics is an extraordinarily useful field, and was one of my favorite classes as an undergraduate. I am not doing it much service here: I am jumping past all of the basics and theorems and complexities to provide a relatively straightforward applications to landsliding. I would encourage anyone interested in further learning about this to find a full class or to follow Brad Hager's course, 12.005, at MIT (https://ocw.mit.edu/courses/earth-atmospheric-and-planetary-sciences/12-005-applications-of-continuum-mechanics-to-earth-atmospheric-and-planetary-sciences-spring-2006/)

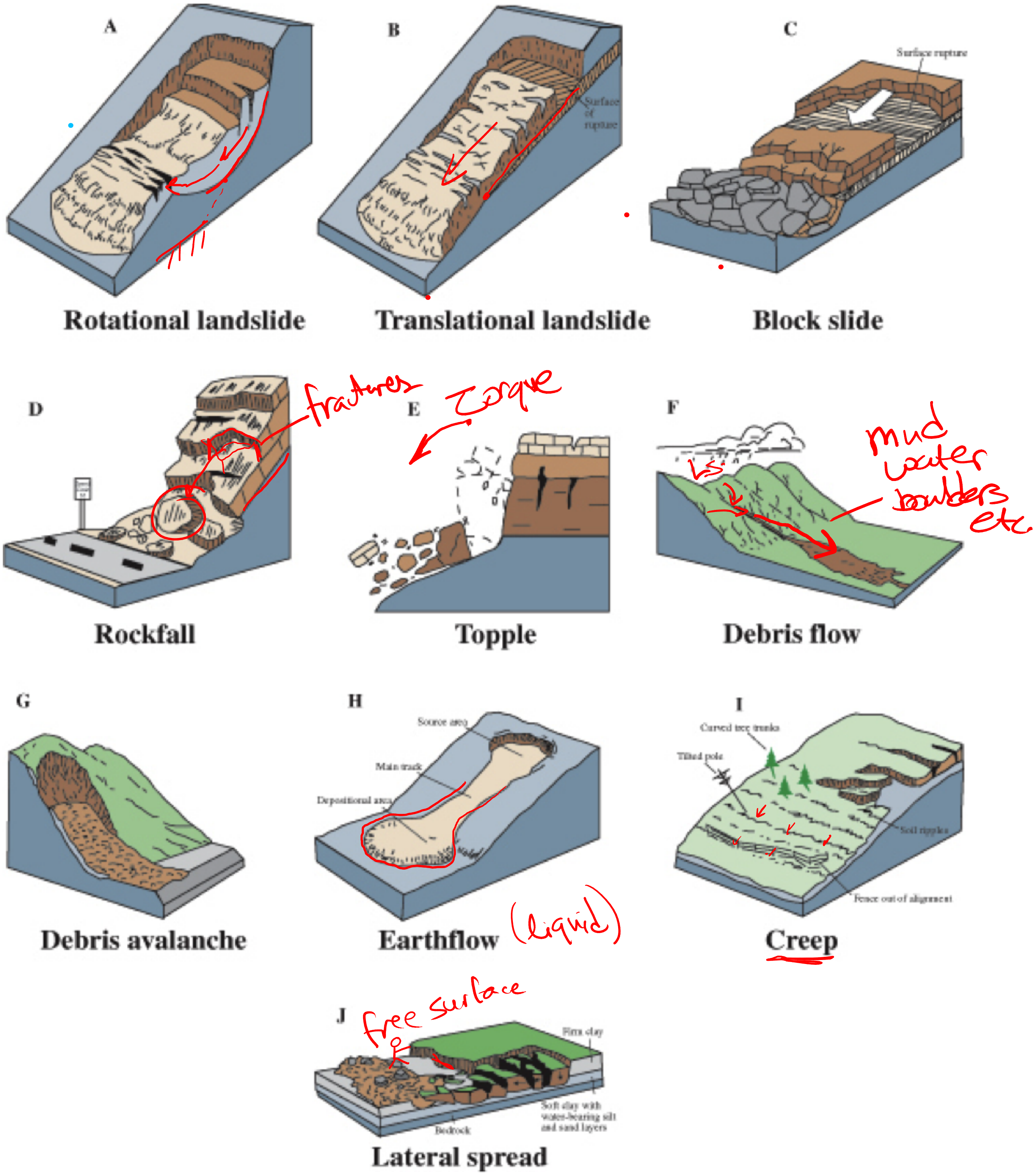


Figure 12. Rapid mass-wasting events – all except **I** – are discrete instances in which significant material moves down hillslopes. (Figure from the internet; original source unknown.)

σ is the symbol that is used generically for stress, and is also used for **normal stress** – that stress that contributes to friction and is perpendicular to the plane. This is synonymous with the normal force.

τ is often used for the **shear stress** – the stress that is parallel to the plane and drives motion.

Following Figure 13, we can set up a balance between the driving stresses for failure and the resisting stresses that inhibit failure within a **continuum**. By "continuum", I mean that rather

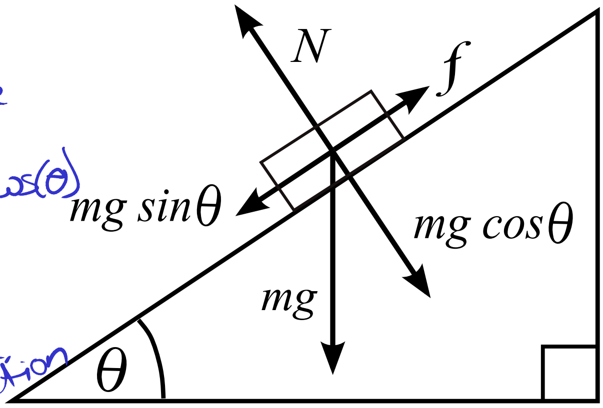
TRANSLATIONAL

@ incipient failure

$\cancel{mg}\sin(\theta) = f\,\cancel{mg}\cdot\cos(\theta)$

$\sin(\theta) = \mu\cos(\theta)$ $\quad f = \mu$ coeff friction

$\mu = \tan(\theta)$



Driving

$mg\sin(\theta)$

Resisting

$f\,mg\cos(\theta)$

$\theta = \tan^{-1}(\mu)$

Angle of internal friction

FIGURE 13. Free-body diagram for the classic problem of a block on an inclined plane. Figure drafted by Wikimedia user Mets501.

than a discrete block that is separate from the plane across which it moves, that the body itself is deforming: in other words, both the "block" and the "plane" are part of the same material.

In the figure below, let's set up a stress balance on a an arbitrary plane within a continuous body.[15] This will be analogous to the block on an inclined plane, with the mass above the plane creating a driving stress that increases with the angle of the plane, and friction along the plane itself – proportional to the cosine of the angle of the plane with the horizontal – generating a resisting force. Because we are considering parcels of a continuum instead of discrete masses and objects, we will couch our solution in terms of **stresses** – forces per unit area. This removes the dependence on the size of the object, and allows us to write an analogous solution to that shown in Figure 13.

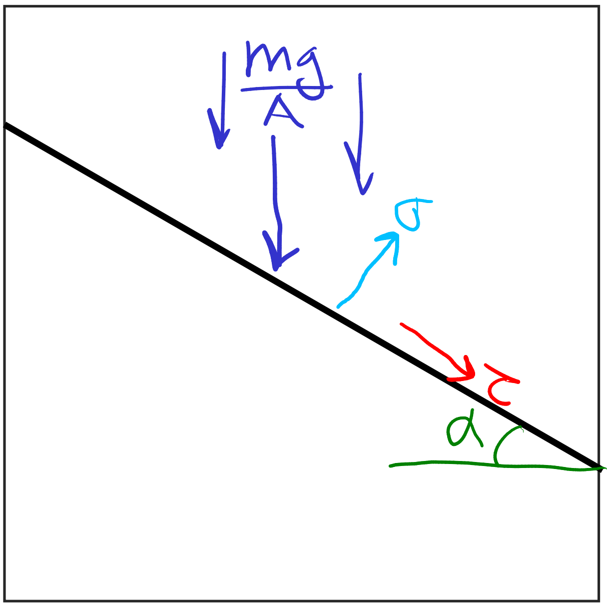


FIGURE 14. Failure along a plane (thick black line) in an imaginary block of material – let's say it's within a hillslope. I want you to draw arrows indicating the direction of failure along this plane, should it occur. After this, mark the angle from the horizontal, as well as the orientations of the normal stress (σ) and the shear stress (τ).

[15]We haven't yet defined the failure angle or plane, but for the moment, we will assume that one exists. Even if a failure plane does not exist initially, one will form in the case of a mass failure.

Before we continue much farther, let's define a few variables. Some of these will be new to you, and defined in the following paragraphs.

- σ_d Driving stress, pushing the hillside towards failure
- σ_r Resisting stress, pushing the hillside towards stability
- σ Normal stress (normal force per unit area)
- τ Shear stress (force parallel to the plane of potential failure per unit area)
- P_f Pore-fluid pressure (pressure
- θ Angle of the plane of potential failure
- σ_{eff} The effective normal stress ($\sigma_{\text{eff}} = \sigma - P_f$
- σ_c Cohesion: a stress that holds hillslope material together
- ρ Density, generically. Subscripts are often added to show what material is being considered (e.g., ρ_w: water density; ρ_r: rock density).
- g Acceleration due to the force of gravity
- h Thickness of block that is failing; this may vary with the distance along the block, but we will focus on a simple example in which this is constant.
- σ_g Stress oriented parallel to the orientation of the gravitational acceleration
- μ Coefficient of internal friction: analogous to the coefficient of friction between discrete bodies, but between elements within the continuous body (e.g, parcels of sand or clay)
- λ_p Porosity: the fraction of the volume that comprises voids (pore space between grains), as opposed to material
- f_w The fraction of the pore space taken up by water

The P_f term represents a rapidly deforming fluid – in our case, water – within the pore space between grains.[16] In the same way that you feel lighter when you are underwater because of buoyancy, water partially buoys up material on hillslopes. This reduces the effective normal stress – often written as σ_{eff} – and therefore the friction. Because the **pressure** of water is equal in all directions, it can only affect the normal stresses, and has no effect on the shear stresses. Thus, the addition of water can make a hillslope more likely to fail.

Cohesion, σ_c is a stress that corresponds to the strength of the material: how much stress can it experience before it starts to fail? An example of a cohesionless material is sand: it fails and forms an angle of repose instantly. A material with cohesion, on the other hand, is clay: a block of clay can support steep walls.

Let us consider a planar landslide (Figure **??**) with a constant thickness to explain each of these terms and find what sets σ_d and σ_r

This potential landslide block will be at the point of incipient failure – that is, the point at which the driving stresses equal the resisting stresses, when (by definition):

(46) driving stress $$\sigma_d = \sigma_r$$ resisting stress

Based on our free-body diagram, we can start to define the stresses involved in landsliding. The vertical stress due to the weight of the block – and therefore aligned with the orientation of gravitational acceleration – is:

density thickness

(47) $$\sigma_g = \rho g h$$

grav accel $\simeq 9.8$ m/s^2

This is simply the density of the material, times acceleration due to gravity, times the thickness of the material. We can check the units of the right-hand side to see how they compare to a force. In SI units, and in the same order (ρ, g, h):

Force: [N] ↓ $\frac{kg\,m}{s^2}$

(48) $$\left[\frac{\text{kg}}{\text{m}^{\cancel{3}}}\right]\left[\frac{\cancel{\text{m}}}{\text{s}^2}\right][\cancel{\text{m}}] \qquad \frac{kg}{m\,s^2}$$

ρ g h

[16]We neglect air because it is not only in the pores, but is also occupies the space outside the hillslope – hence helping to hold it up. As a result, all air-based pressure terms cancel out. Even neglecting this, the density of air is so low compared to rock, soil, sediment, or water, that it is negligible in this problem.

stress $\frac{N}{m^2}$ → $\frac{\text{force}}{\text{area}}$

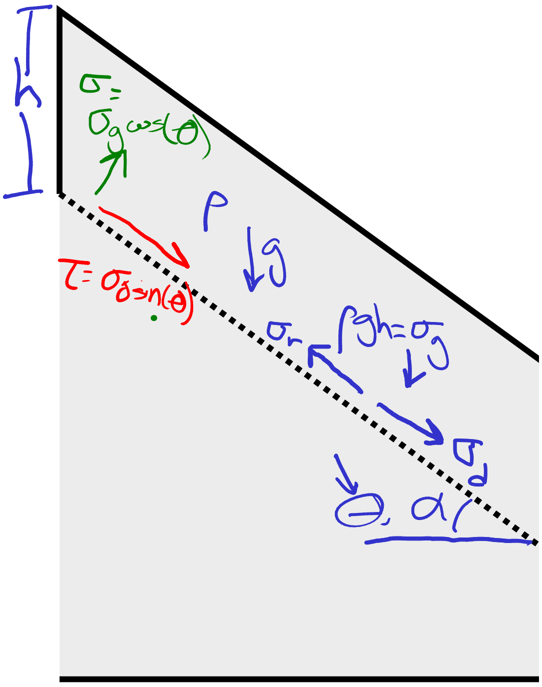


FIGURE 15. Draw in each of the above variables on this planar landslide.

All together, this gives us:

$$\left[\frac{\text{kg}}{\text{m s}^2}\right] \tag{49}$$

STRESS

These are the units of a force per unit area, which is defined as a stress.

What is ρ? This depends on the solid material that makes up the hillslope, plus the amount of additional fluid (water weight) that in the hillslope.

DENSITY porosity fract. of pore space flooded w/ water

$$\rho = (1-\lambda_p)\rho_r + \lambda_p f_w \rho_w \; + \lambda_p(1-f_w) \tag{50}$$

Rock water density air? water?

The more water that is in the hillside, the heavier the material is.

Next, let's split σ_g between the shear stress and the normal stress:

Air negligible

$$\sigma = \sigma_g \cos\theta \text{ [normal stress]} \quad \tau = \sigma_g \sin\theta \text{ [shear stress]} \tag{51}$$

Normal Shear

$\rho_{granite} = 2{,}700$ kg/m³
$\rho_{water} = 1000$ kg/m³

This is precisely analogous to the normal force and the slope-parallel force for the block on the inclined plane (Figure 13).

At this point, we have fully characterized the **driving stress**:

$$\sigma_d = \tau = \big((1-\lambda_p)\rho_r + \lambda_p f_w \rho_w\big) g h \sin\theta \tag{52}$$

What remains to be done is to add modifications to the resisting forces based on 1 **friction**, 2 **pore-fluid pressure**, 3 **cohesion**.

In a cohesionless block of material with no water content, the resisting force is just analogous to the frictional force from the block on the inclined plane:

≈0.6 (Byerlee's Law) negligible

$$\sigma_{r,\text{ no water, no cohesion}} = \mu\big((1-\lambda_p)\rho_r + \lambda_p f_w \rho_w\big) g h \cos\theta \tag{53}$$

coefficient of friction

$\sigma_d = \sigma_r$
$\sin(\theta) = \mu \cos(\theta)$
$\mu = \tan(\theta)$
Angle of repose

In other words, the normal stress times the friction defines the resisting stress in this simplified case.

Next, we add the effect of pore-fluid pressure. While a tiny amount of fluid in the pores actually increases the cohesion between the grains, any additional fluid just decreases the resisting stress by reducing the effective weight of the particles on the hillslope by buoyantly

Angle of repose ← Angle of internal friction τ = same as above, w/ water

① Pore-fluid press.
② Weight.

supporting them. Through this, we can define an effective normal stress,

ρ_{mineral} ρ_{water} P_f: pore fluid pressure. Buoyancy.

$$\sigma_{\text{eff}} = \sigma - P_f \tag{54}$$

$$= \Big((1-\lambda_p)\rho_r + \cancel{\lambda_p f_w \rho_w} - \cancel{\lambda_p f_w \rho_w}\Big) gh \tag{55}$$

$$= (1-\lambda_p)\rho_r g h \tag{56}$$

The pore-fluid pressure term therefore exactly negates the additional weight of the block when computing the frictional resisting forces, but not when calculating the driving forces. wet → failure.

The last component to add is a **cohesion** term, σ_c. This represents stresses that can hold the material together. This can include the "stickiness" of clays, the strength of root fibers, and the effects of interlocking clasts of rock. This term is added to the resisting forces, as it needs to be overcome before any motion can happen.

The resulting full equation for the resisting forces is:

cohesion.

$$\sigma_r = \mu \sigma_{\text{eff}} \cos\theta + \sigma_c \tag{57}$$

By combining this with the driving forces, we can obtain the point at which a hillslope is at the point of incipient failure ($\sigma_d = \sigma_r$):

$$\underbrace{\tau}_{\text{shear (driving) stress}} = \overbrace{\mu \sigma_{\text{eff}} \cos\theta + \sigma_c}^{\text{Resisting stress}} \tag{58}$$

5. The link between hillslopes and channels: coupled components of landscape evolution

Hillslope processes dominate over fluvial processes where drainage areas are too small for enough water to collect to start eroding channels. As drainage areas grow, the landscape transitions from one whose form is dominated by hillslope processes to one whose form is dominated by **fluvial** (or river-related) processes.

Channel and hillslope processes interact closely. The bottoms of the hillslopes serve as the upper boundary condition on river channels, which are responsible for feeding them with water and sediment. Likewise, river channels serve as the lower boundary for the hillslopes: if the river aggrade (increase their bed elevations), they raise the level of the bottoms of the hillslopes, and raise the elevation of the bottom of hillslope deposition. Likewise, if they incise (decrease their bed elevations), they create a cliff at the toe of the hillslope, which may diffuse and/or landslide, eventually causing the rest of the hillslope to follow.

Let's use the space below to sketch this situation for both river aggradation and incision:

This two-way coupling between channel and hillslopes will become central to our understanding of **landscape evolution**.

Additional figures

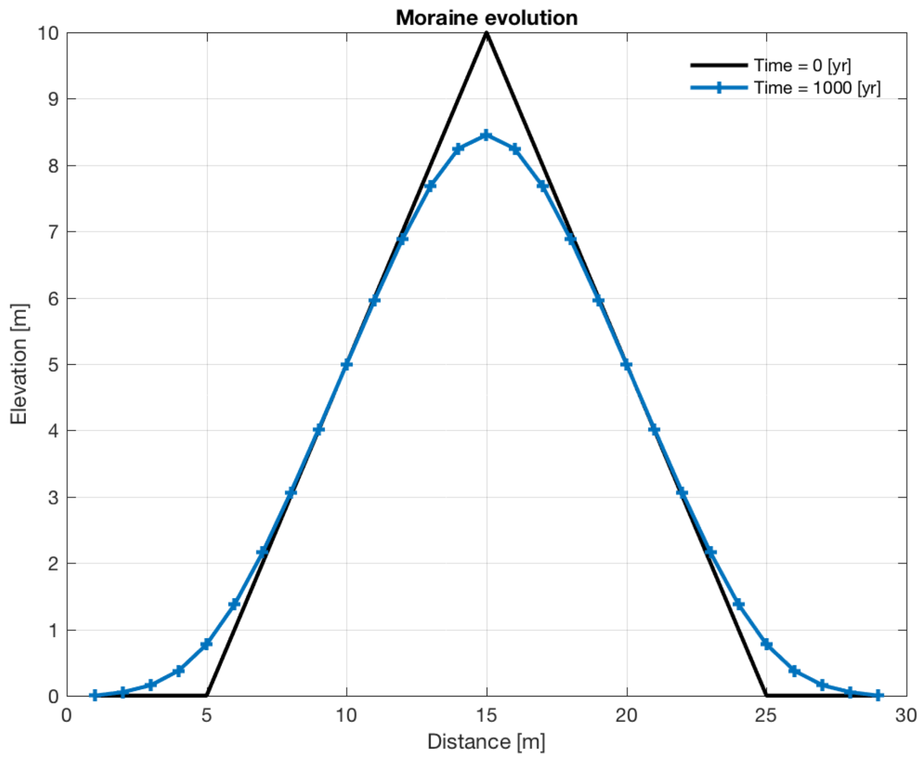


Figure 16. Diffusive evolution of an initially-steep-crested moraine. From Dylan Mikesell, Boise State University, SERC: `https://serc.carleton.edu/matlab_computation2016/activities/159830.html`.

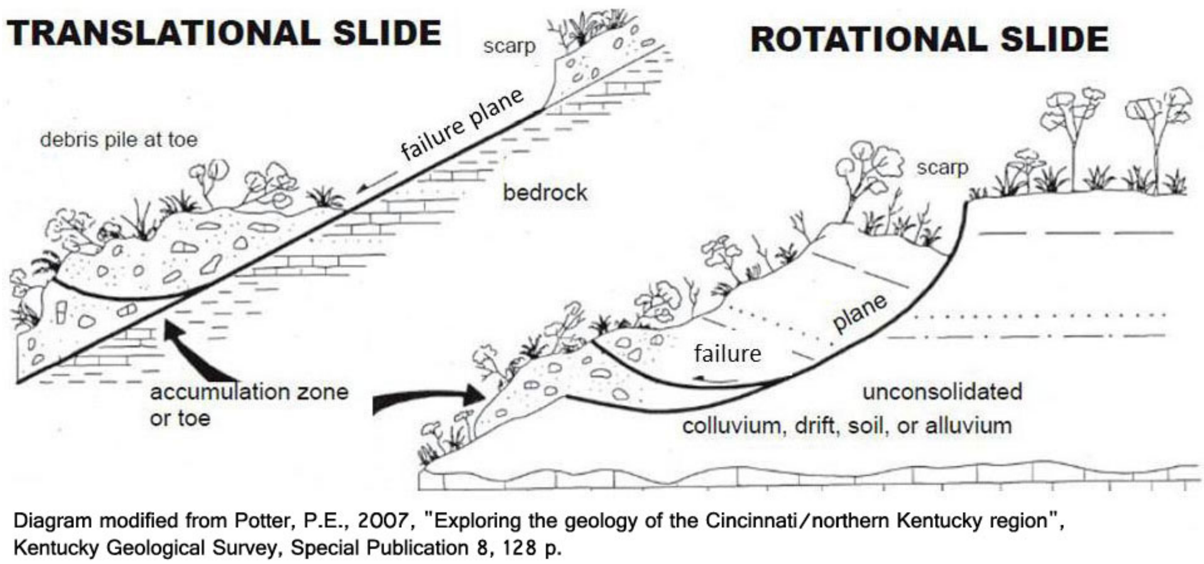


Figure 17. Translational landslides (like the one for which we solve above) and rotational landslides.

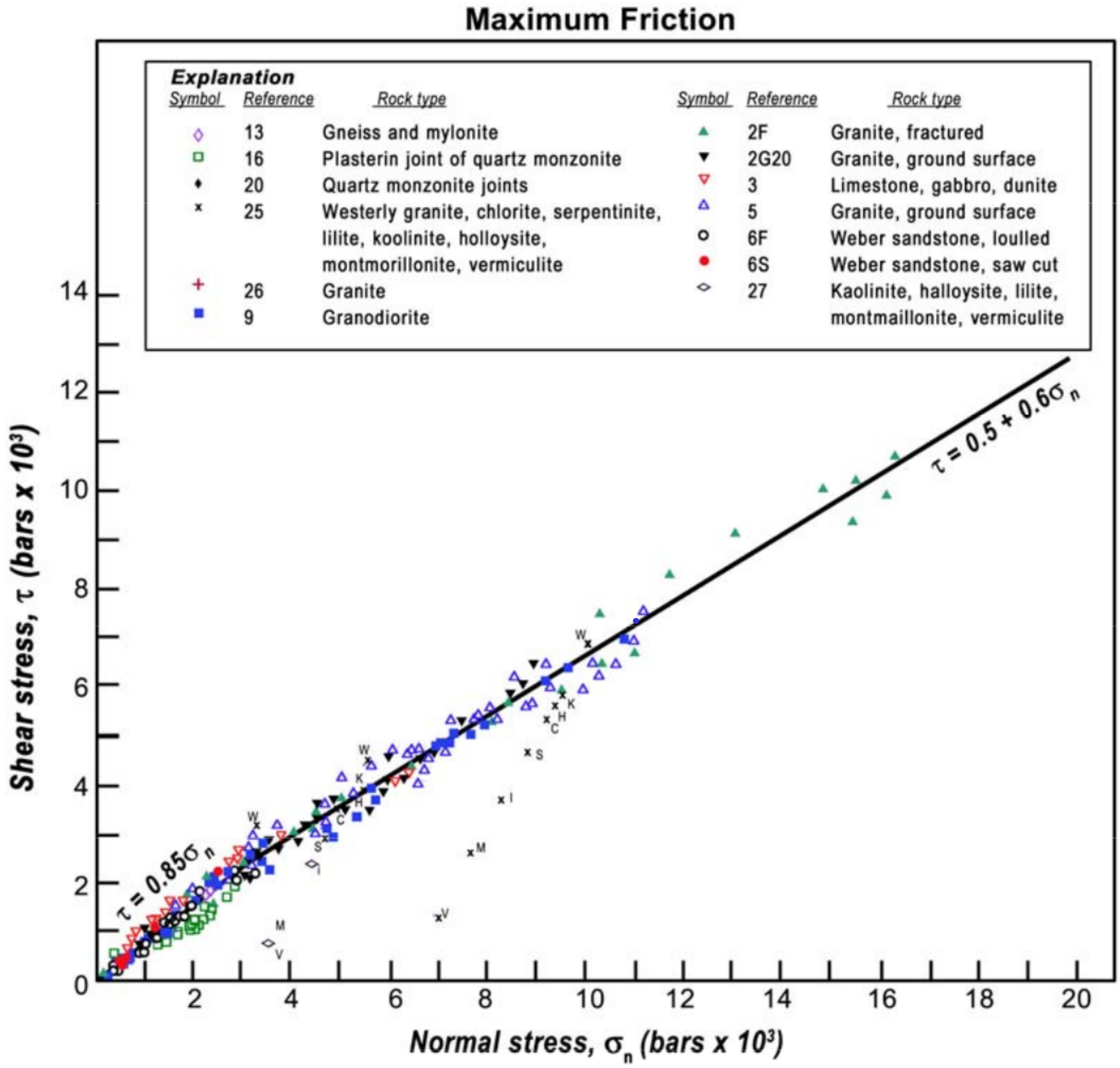


FIGURE 18. Byerlee's Law: almost all rocks have a **coefficient of internal friction** of 0.6. Even though these experiments were performed for conditions deep within Earth, this larger-scale relationship holds true for sand as well. Figure from Brad Hager (12.005, lecture 6) and MIT OCW.

REFERENCES

Anderson, R. S., and S. P. Anderson (2010), *Geomorphology: the mechanics and chemistry of landscapes*, 651 pp., Cambridge University Press.

Freeze, R. A., and J. A. Cherry (1979), *Groundwater*, Prentice-Hall, Inc., Englewood Cliffs, NJ, USA.

Hurst, M. D., S. M. Mudd, R. Walcott, M. Attal, and K. Yoo (2012), Using hilltop curvature to derive the spatial distribution of erosion rates, *Journal of Geophysical Research*, *117*(F2), F02,017, doi:10.1029/2011JF002057.

Roering, J. J., J. W. Kirchner, and W. E. Dietrich (1999), Evidence for nonlinear, diffusive sediment transport on hillslopes and implications for landscape morphology, *Water Resources Research*, *35*(3), 853–870, doi:10.1029/1998WR900090.

Rosenbloom, N. A., and R. S. Anderson (1994), Hillslope and channel evolution in a marine terraced landscape, Santa Cruz, California, *Journal of Geophysical Research: Solid Earth*, *99*(B7), 14,013–14,029, doi:10.1029/94JB00048.