Differences of Hill Climbing strategies (First Improvement and Best Improvement) in finding global maximum of a cubic function

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Abstract

The most known strategies for Hill Climbing algorithms are First Improvement and Best Improvement. Usually, Best Improvement gives the best result in comparison to the other one. Testing these strategies on simple functions is a good way of understanding how they work. In this paper I compare them using the cubic function: $x^3 - 60x^2 + 900x + 100$ on natural numbers between 0 and 31, showing that Best Improvement is the best strategy for solving this problem.

1 Introduction

"**Hill climbing** method is an optimization technique that is able to build a search trajectory in the search space until reaching the local optima. It only accepts the uphill movement which leads it to easily get stuck in local optima."[1]
At every iteration of this algorithm, we can choose to go with the first or the best successor bigger than the candidate solution [2].
This choice has a big impact on performance and speed. The Best Improvement strategy gives the best result, but it takes a long time to reach it. First Improvement gives a mediocre average optima but within a short time frame. [3]
Even the endian[4] of the binary representation can affect the algorithm.

1.1 Definitions

- Let $f(x)$, a numeric function as follows:

$$f\colon \begin{array}{l} \mathbb{Z} \to \mathbb{Z} \\ x \mapsto f(x) = x^3 - 60x^2 + 900x + 100 \end{array}$$

- Let $b_n(x)$, a bijective function that returns the binary representation of x as a boolean array of size n

  $x \in \mathbb{N}$, $x \leq 2^n - 1$

  Example:

  $b_5(18) = [1, 0, 0, 1, 0]$, for Big Endian.

  $b_5(18) = [0, 1, 0, 0, 1]$, for Little Endian.

- Let d(`b_array`), a bijective function that returns the natural number represented by the binary array given as a parameter

  Example:

  d(`[1, 0, 0, 1, 0]`) $= 18$, for Big Endian

  d(`[0, 1, 0, 0, 1]`) $= 18$, for Little Endian

- Let first_improvement(x) be defined as follows:

Algorithm 1 First Improvement

```
Require: n ∈ ℕ*, x ∈ ℕ
Ensure: x ≤ 2^n − 1
  function FIRST_IMPROVEMENT(x)
    b_array ← b_5(x)
    for i ← 1 to n do
      candidate ← b_array
      candidate[i] = candidate[i] ⊕ 1                ▷ ⊕ is xor operator
      if d(candidate) is better than x then
        return d(candidate)
      end if
    end for
    return x
  end function
```

- Let best_improvement(x) be defined as follows:

Algorithm 2 Best Improvement

```
Require: n ∈ ℕ*, x ∈ ℕ
Ensure: x ≤ 2^n − 1
  function BEST_IMPROVEMENT(x)
    rv ← x
    b_array ← b_5(x)
    for i ← 1 to n do
      candidate ← b_array
      candidate[i] = candidate[i] ⊕ 1                ▷ ⊕ is xor operator
      if d(candidate) is better than rv then
        rv = d(candidate)
      end if
    end for
    return rv
  end function
```

- Let max_improvement(x), a function that returns the basin of attraction of x.

Algorithm 3 Max Improvement

```
Require: impr_func ∈ {first_improvement, best_improvement}
  function MAX_IMPROVEMENT(x)
    next ← x
    while next ≠ x do
      x ← next
      next ← impr_func(next)
    end while
    return x
  end function
```

- Let steps_improvement(x), a function that return the number of iterations of max_improvement

1.2 Describe Comparison Method

The strategies will be compared using:

- function $f(x)$, where $x \in [0, 31]$
- $n = 5$

Every strategy of obtaining global maxima using Hill Climbing will be characterised by:

- **maximum** value of steps_improvement(x)
- **average** value of steps_improvement(x)
- the **chance** of max_improvement(x) to be the global optima

Hypotheses:

1. First Improvement that uses Big Endian has a smaller average value of steps_improvement(x) compared to the one that uses Little Endian
2. Best Improvement has a smaller maximum value of steps_improvement(x) and a bigger chance of getting the global maximum compared to any First Improvement strategy.

2 Basin of Attraction

2.1 First Improvement Big Endian

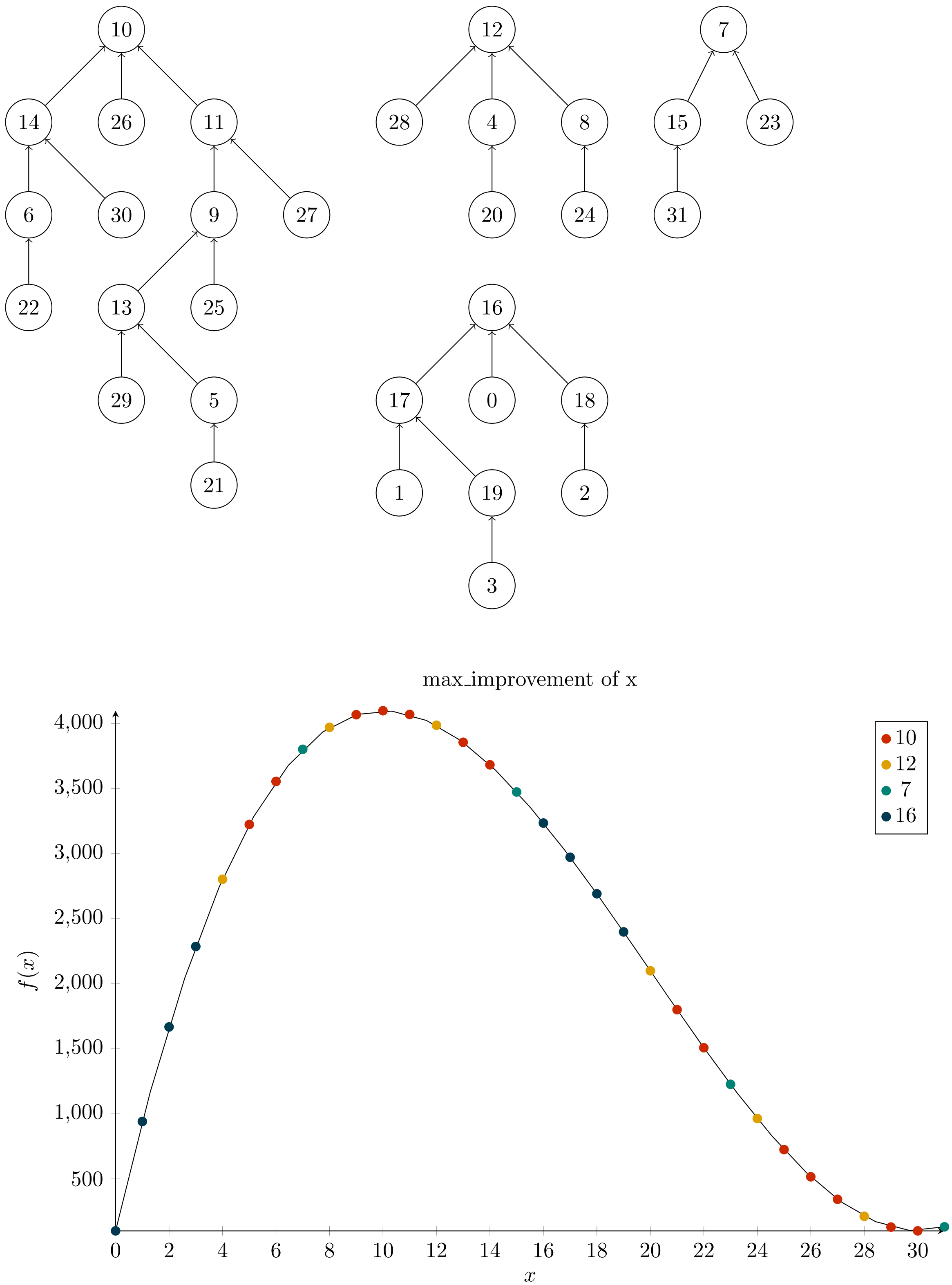


Figure 1: First Improvement Big Endian - Basin of Attraction

2.2 First Improvement Little Endian

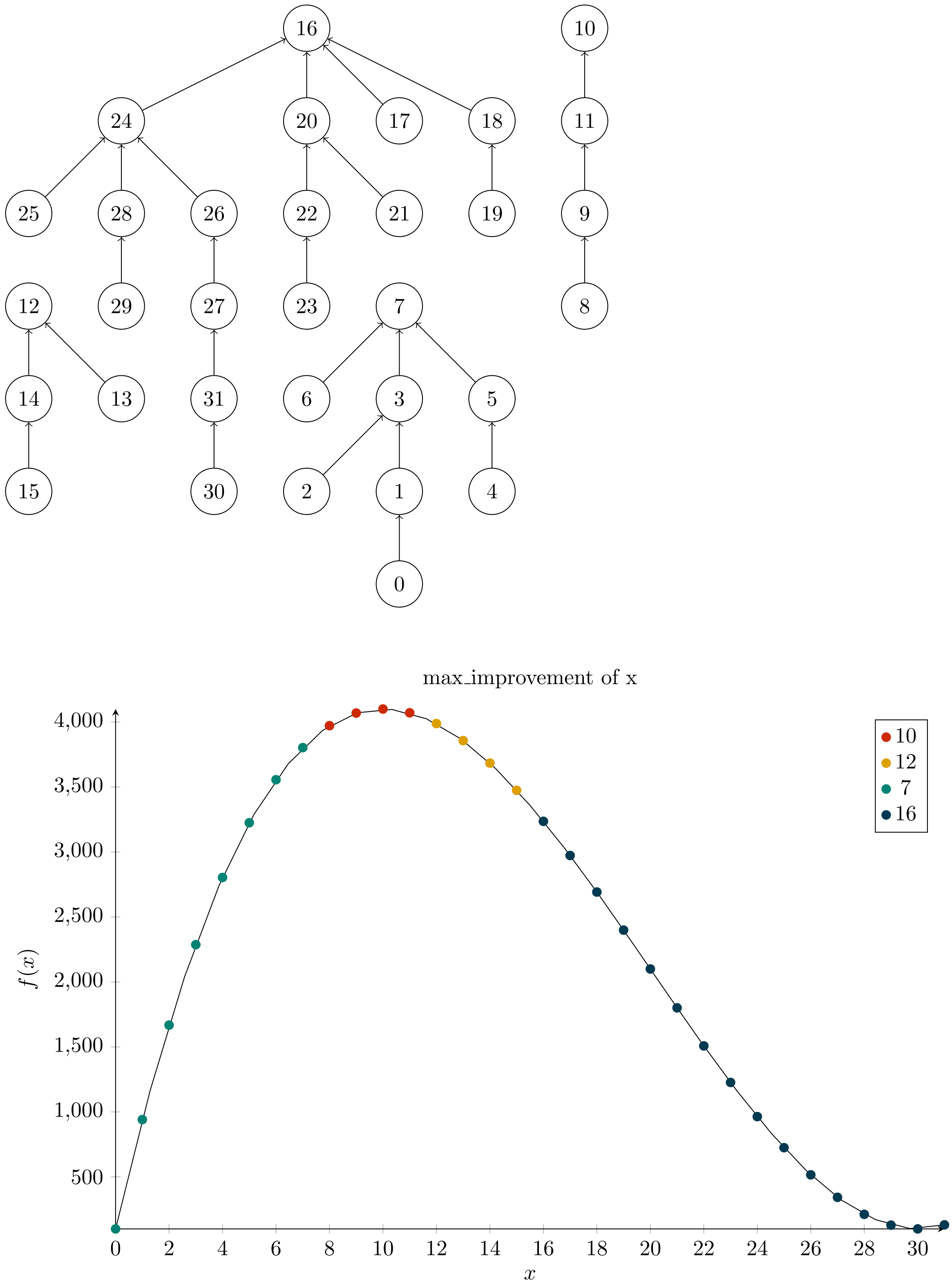
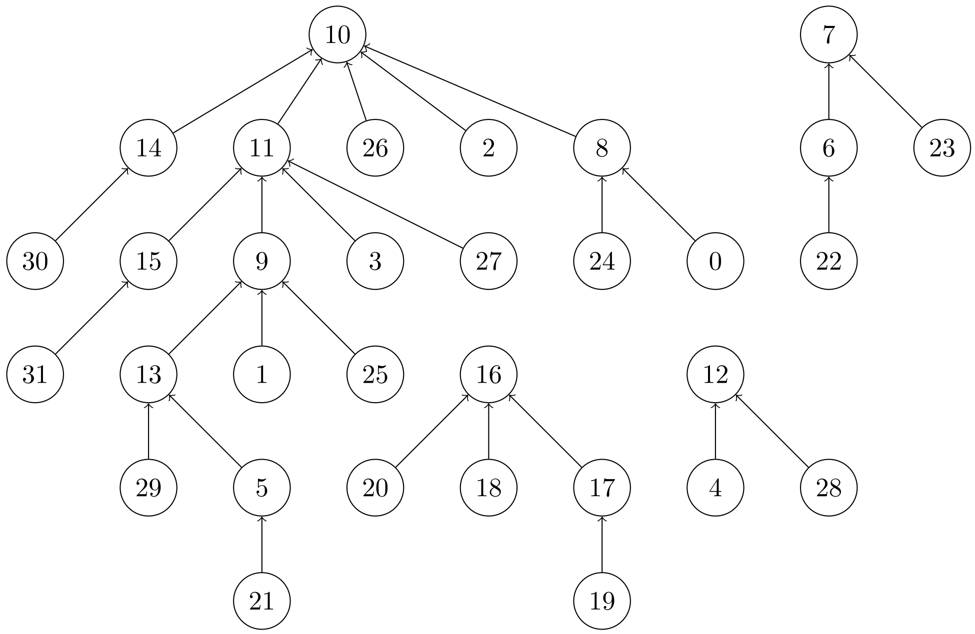


Figure 2: First Improvement Little Endian - Basin of Attraction

2.3 Best Improvement



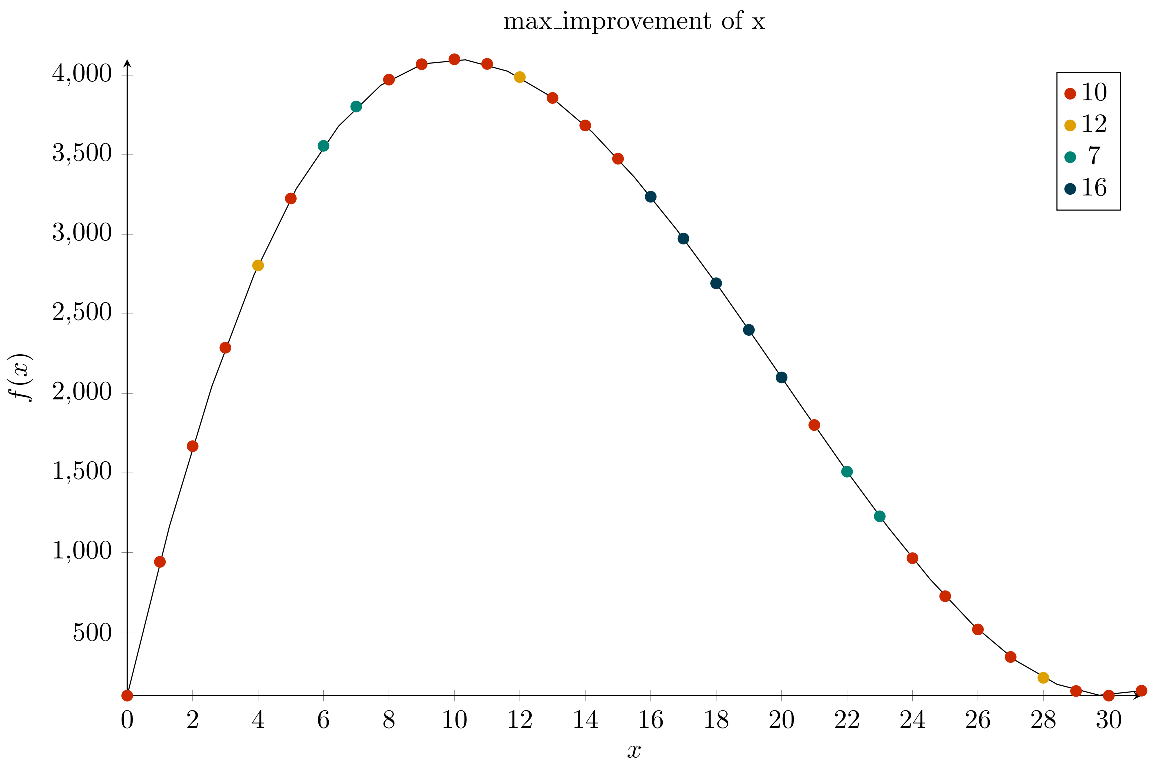


Figure 3: Best Improvement - Basin of Attraction

3 Characterisation

3.1 First Improvement Big Endian

- **maximum** value of steps_improvement(x) is 5 for $x = 21$
- **average** value of steps_improvement(x) is 1.75
- the **chance** of max_improvement(x) to be the global optima is 43.75%

3.2 First Improvement Little Endian

- **maximum** value of steps_improvement(x) is 5 for $x = 30$
- **average** value of steps_improvement(x) is 1.75
- the **chance** of max_improvement(x) to be the global optima is 12.15%

3.3 Best Improvement

- **maximum** value of steps_improvement(x) is 5 for $x = 21$
- **average** value of steps_improvement(x) is 1.71875
- the **chance** of max_improvement(x) to be the global optima is 62.5%

4 Conclusion

1. *"First Improvement that uses Big Endian has a smaller average value of steps_improvement(x) compared to the one that uses Little Endian"*

   Conclusion: The hypothesis is false, at least for this case.

   Correction: Both strategies have an average of 1.75. First Improvement Big Endian has a bigger chance of achieving global maxima.

2. *"Best Improvement has a smaller maximum value of steps_improvement(x) and a bigger chance of getting the global maximum compared to any First Improvement strategy"*

   Conclusion: The hypothesis is false, at least for this case.

   Correction: Both strategies have the same maximum value of steps_improvement(x). Best Improvement has a bigger chance of achieving global maxima.

As the result, it was established that Best Improvement does show the best result for each characteristic, which indicates that it is the best possible strategy in this scenario.

References

[1] M. A. Al-Betar, "β-hill climbing: an exploratory local search," *Neural Computing and Applications*, vol. 28, no. 1, pp. 153–168, 2017.

[2] P. Hansen and N. Mladenović, "First vs. best improvement: An empirical study," *Discrete Applied Mathematics*, vol. 154, no. 5, pp. 802–817, 2006.

[3] G. Buțco, "Differences between hill climbing algorithm (first improvement, best improvement) and simulated annealing algorithm in finding global minimum of numeric functions," 2021.

[4] D. Cohen, "On holy wars and a plea for peace," *Computer*, vol. 14, no. 10, pp. 48–54, 1981.