# Testing Anuga-Sed against analytical and experimental solutions

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We wish to test Anuga-Sed against analytical solutions for its equations as well as agaisnt experimental datasets for sediment transport, flow through vegetation, and sediment transport through vegetation. This will confirm that the model can simulate these processes with a certain degree of accuracy before using the model for more complex simulations and numerical experiments.

## 1 Sediment transport

### 1.1 Analytical solution to transport equations

The governing equation for sediment transport in Anuga-Sed is the mass balance of sediment in the water column and bed layer:

$$\frac{\partial Ch}{\partial t} = \dot{E} - \dot{D} - \left( \frac{\partial q_{s_x}}{\partial x} + \frac{\partial q_{s_y}}{\partial y} \right) \tag{1}$$

where $h$ is the flow depth, $\dot{E}$ is entrainment flux, $\dot{D}$ is deposition flux, $C$ is the sediment concentration, and $q_{s_x}$ and $q_{s_y}$ are specific volumetric sediment fluxes in the $x$ and $y$ directions [e.g.,][]davy2009fluvial. Sediment flux is given by $q_{s_{x,y}} = \beta C q_{x,y}$, where $q_{x,y}$ is specific discharge in the $x$ or $y$ direction and $\beta$ ($\leq 1$) describes the effective speed of sediment relative to that of the water.

Assuming that transport of water on sediment only occurs in $x$, the equation can be simplified to:

$$\frac{\partial Ch}{\partial t} = \dot{E} - \dot{D} - \frac{\partial q_{s_x}}{\partial x} \tag{2}$$

This assumption is reasonable for simulations of flow over ramps that slope only in the $x$ direction. By assuming that the volume of sediment in the water column had reached a steady state, we further simplified the mass balance to write:

$$\frac{dq_{sx}}{dx} = \dot{E} - \dot{D} \tag{3}$$

Given $q_{sx} = Cq_x$, we can write

$$\frac{dC}{dx} = \frac{\dot{E}}{q_x} - \frac{\dot{D}}{q_x} \tag{4}$$

The deposition rate is $\dot{D} = d^* v_s C$, where $d^*$ is a parameter that describes the distribution of sediment in the water column and $v_s$ is the settling velocity of the sediment. Both of these can be assumed to be constants (or near constants).

Integrating with respect to distance downstream $x$, we find an expression for the volumetric sediment concentration with distance downstream from the sediment source.

$$C(x) = \left( C_o - \frac{\dot{E}}{d^* v_s} \right) \mathrm{e}^{-\frac{d^* v_s x}{q}} + \frac{\dot{E}}{d^* v_s} \tag{5}$$

where $C_o$ is the sediment concentration at the inlet.

The rate of change of bed elevation $\eta$ is given by

$$\frac{d\eta}{dt} = \frac{\dot{D} - \dot{E}}{1 - \phi} \tag{6}$$

where $\phi$ is the bed porosity. Combining both equations we find:

$$\eta(x,t) = \frac{t}{1-\phi} \left( d^* v_s C_o - \dot{E} \right) \mathrm{e}^{-\frac{d^* v_s x}{q}} + \eta_o(x) \tag{7}$$

where $\eta_o(x)$ is the initial profile of the bed.

Table 1 summarizes the constants used in these simulations.

We ran a simulation "plane1" with the parameters summarized in table 2. By the end of the simulation, the volume of sediment in the water column had reached steady state (defined as maximum and mean differences in sediment volume between two consecutive timesteps of less than 0.0001).

Table 1: Model constants

| | |
|---|---|
| $\phi$ | 0.3 |
| g | 9.81 |
| $\kappa$ | 0.408 |
| $\rho_w$ | 1000 |
| $\tau_c{}^*$ | 0.06 |
| $\tau_c$ | 0.126126 |
| $v_s$ | 0.13809968 |

Table 2: Simulation "plane1"

| | | |
|---|---|---|
| Flow algorithm | DE0 | |
| Length | 15 | |
| Width | 2 | |
| dx, dy | 0.2 | |
| Initial topography | $z = 10 - x/50$ | |
| Initial stage | topography | |
| Friction | 0.0 | |
| Inflow boundary | [11,0,0] | Dirichlet boundary |
| Side boundaries | | Reflective boundary |
| Outflow boundary | | Transmissive boundary |
| Inflow concentration | 0.005 | |
| Yieldstep | 1 | |
| Finaltime | 60 | |