Deep learning

Lecture 12

tl;dr Reinforcement learning

Yandex Data Factory

LAMBDA

British Hedgehog Preservation Society

Supervised learning

Given:

- objects and answers (x, y)
- algorithm family $a_\theta(x) \rightarrow y$
- loss function $L(y, a_\theta(x))$

Find:

$$\theta' \leftarrow argmin_\theta L(y, a_\theta(x))$$

Supervised learning

Given:

- objects and answers — **customer→ → give loan?** — (x, y)
- algorithm family — **linear / tree / NN** — $a_\theta(x) \rightarrow y$
- loss function — **crossentropy** — $L(y, a_\theta(x))$

Find:

$$\theta' \leftarrow argmin_\theta L(y, a_\theta(x))$$

Reinforcement learning

Given:

- Bank with some budget
- Influx of clients
- A month to make it work or you're fired

Find:

$$\theta' = argmax_{\theta} PROFIT!!!(a_{\theta}(client))$$

Reinforcement learning

Given:

- Bank with some budget
- Influx of clients
- A month to make it work or you're fired

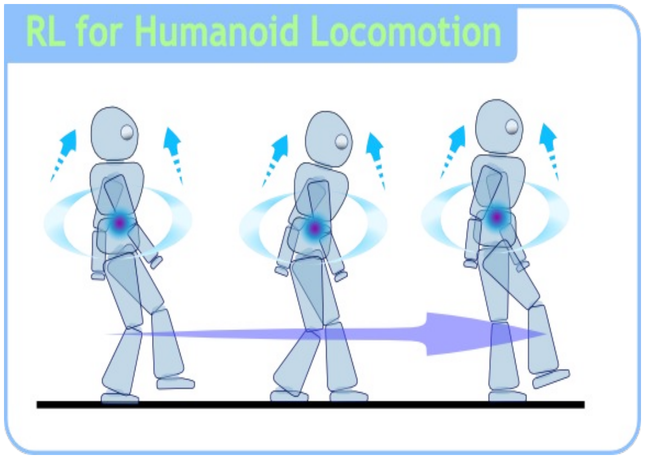
Find:

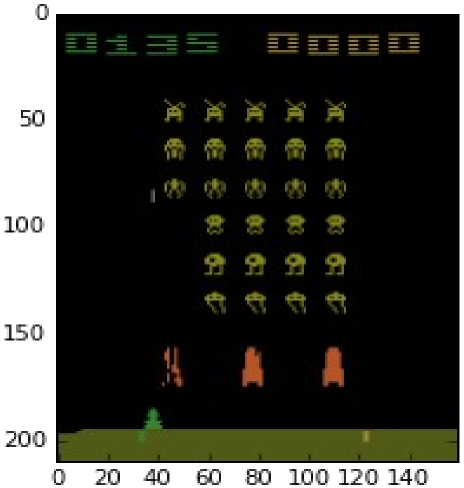
$$\theta' = argmax_{\theta} PROFIT!!!\left(a_{\theta}\left(client\right)\right)$$

Ideas?

Similar tasks

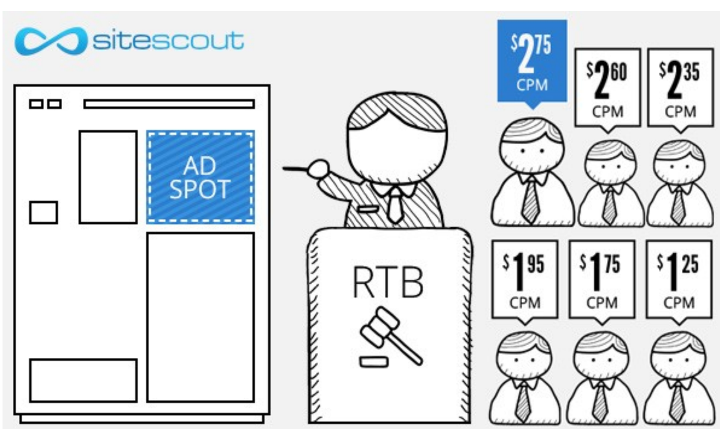
- Control robot to maximize speed
- Play game to maximize score
- Drive car to minimize human casualties

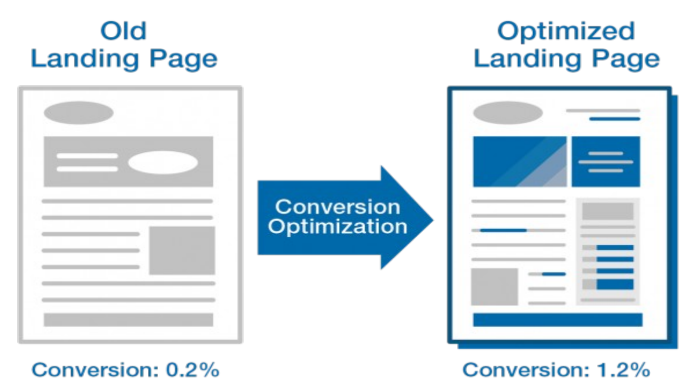




Similar tasks

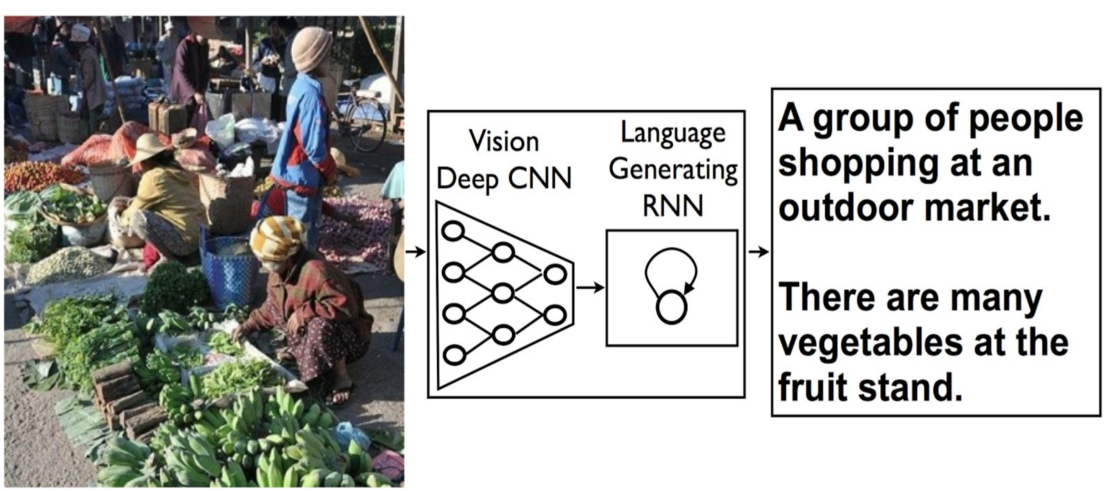
- Show banners to maximize clicks (or $)
- Recommend films to maximize happiness
- Find pages that maximize relevance to queries

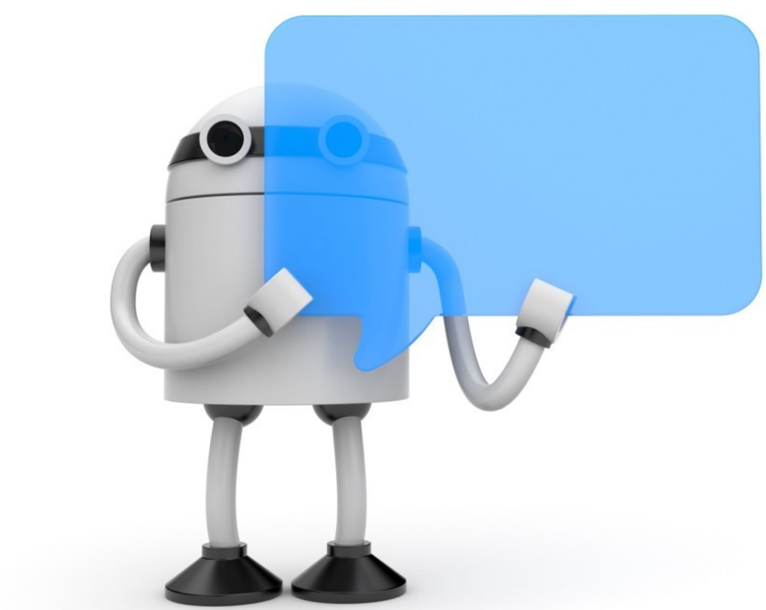




Similar tasks

- Talk to human to satisfy his goals/constraints
- Translate sentence to maximize BLEU
- Generate captions for image with max CIDEr



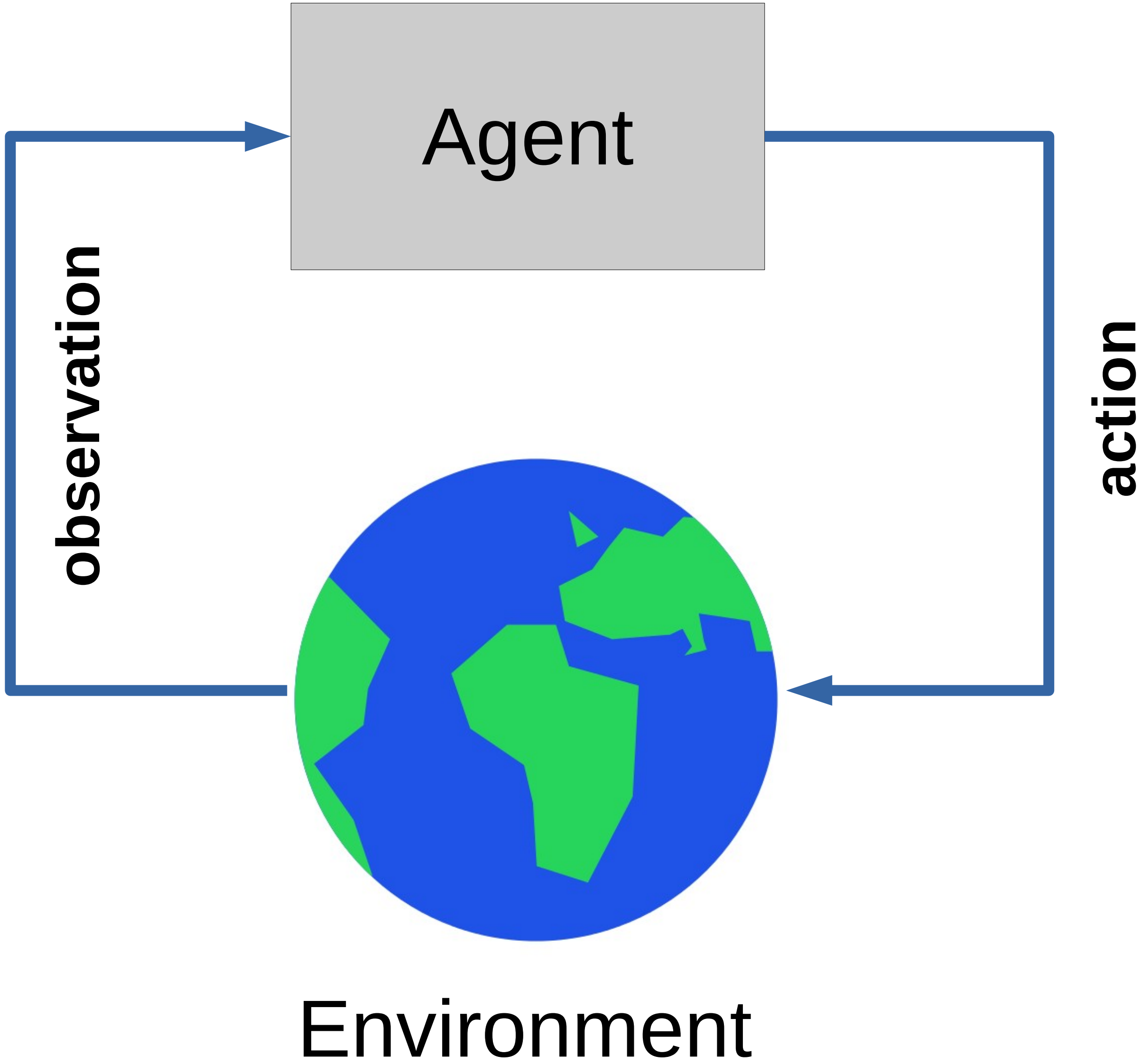


Similar tasks

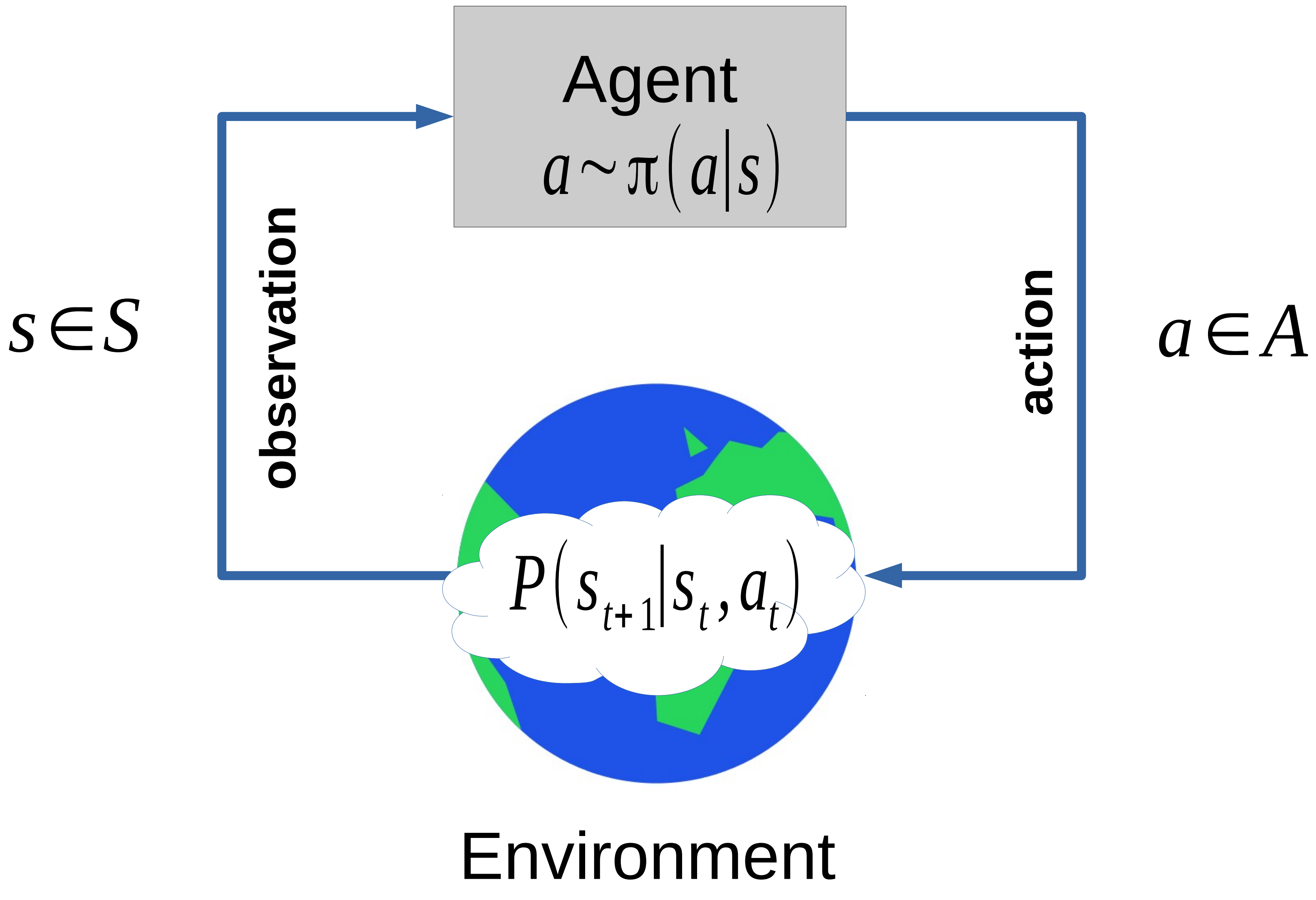
- Trade stocks
- Optimize datacenter usage
- ~~Bring you coffee~~

Image: I googled "whatsoever"

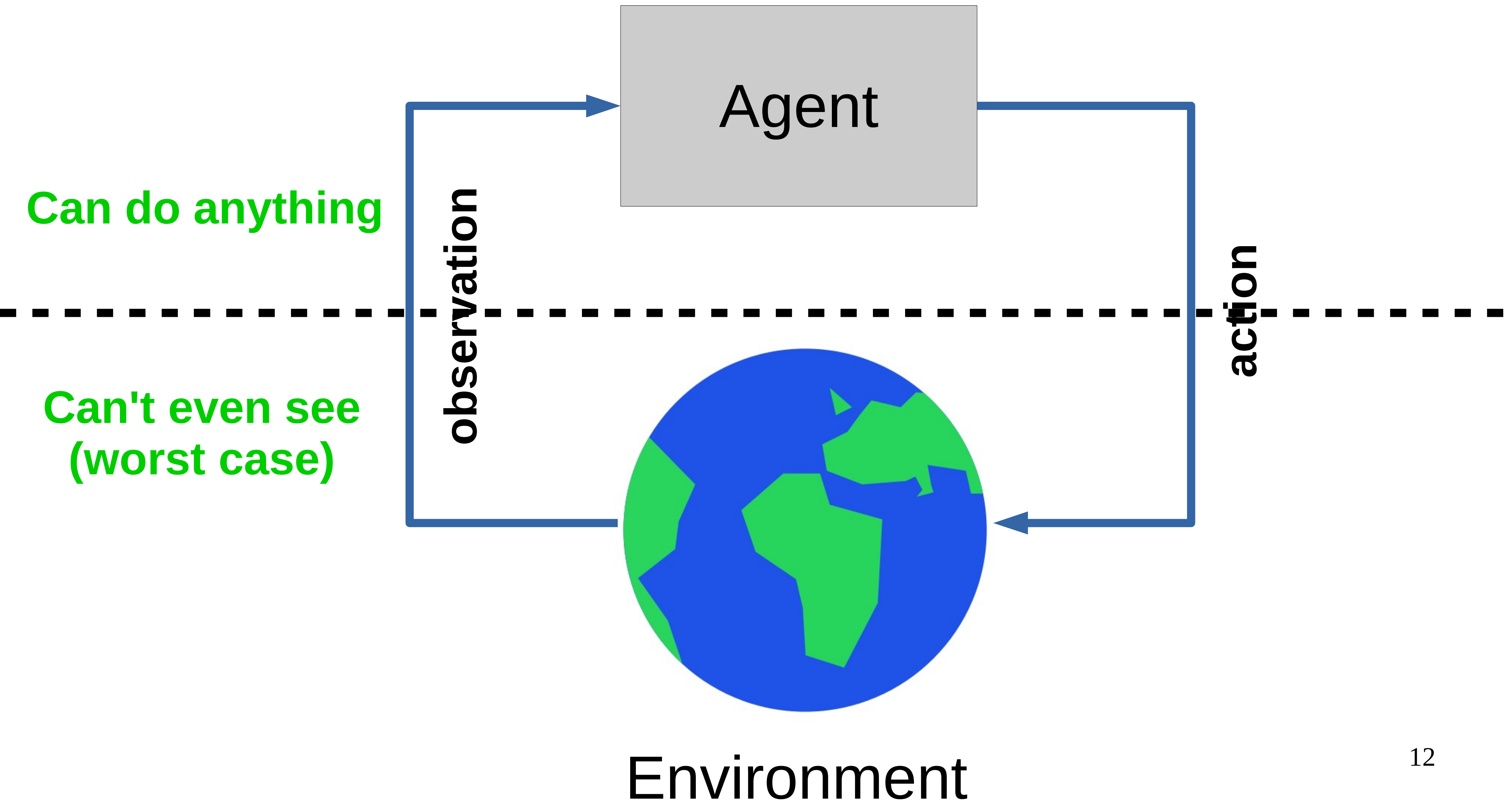
Reinforcement learning formalism



Reinforcement learning formalism



Reinforcement learning formalism



Objective:

- $R(s_{0,} a_0, s_1, a_1, s_2, a_2 \ldots s_n, a_n)$ – reward for session
  - E.g. CIDEr metric of your captioning or total score
  - Total distance your robot walked in 1 minute

- Maximize reward

$$\pi' = \underset{\pi}{argmax} \underset{s_0, a_0, s_1, a_1, \ldots}{E} R(s_0, a_0, s_1, a_1, \ldots)$$

Objective:

- $R(s_{0,} a_0, s_1, a_1, s_2, a_2 \dots s_n, a_n)$ – reward for session
  - E.g. CIDEr metric of your captioning or total score
  - Total distance your robot walked in 1 minute

- Maximize reward

$$\pi' = \underset{\pi}{argmax} \underset{s_0, a_0, s_1, a_1, \dots}{E} R(s_0, a_0, s_1, a_1, \dots)$$

Where is pi used in the argmax-ed expression?

Objective:

- $R(s_{0,} a_0, s_1, a_1, s_2, a_2 \ldots s_n, a_n)$ – reward for session
  - E.g. CIDEr metric of your captioning or total score
  - Total distance your robot walked in 1 minute

$$\pi' = \underset{\pi}{argmax} \underset{\substack{s_0 \sim P(s_0) \\ a_0 \sim \pi(a|s_0) \\ s_1 \sim P(s'|s_0, a_0) \\ a1 \sim \ldots}}{E} R(s_0, a_0, s_1, a_1, \ldots)$$

Agent's policy

- Policy ~ whatever is used to choose actions
- Table of probabilities for each s
- Linear model that learns p(a|s)
- Guess what?

Agent's policy

- Policy ~ whatever is used to choose actions
- Table of probabilities for each s
- Linear model that learns p(a|s)
- Neural network that learns p(a|s)

Objective

- For simplicity, consider single-step process

Agent sees one state, takes one action and it's over

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a) \, da \, ds$$

Objective

Agent's policy

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a)\, da\, ds$$

state visitation frequency

True action value

Trivia: how do we compute that?

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a) \, da \, ds$$

True action value

$$J \approx \frac{1}{N} \sum_{i=0}^{N} R(s_i, a_i)$$

sample N sessions

Objective

$$J = \mathop{E}_{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a)\, da\, ds$$

True action value

$$J \approx \frac{1}{N} \sum_{i=0}^{N} R(s_i, a_i)$$

sample N sessions

Can we optimize policy now?

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a)\, da\, ds$$

parameters “sit” here

$$J \approx \frac{1}{N} \sum_{i=0}^{N} R(s_i, a_i)$$

We don't know how to compute dJ/dtheta

Optimization

Finite differences

– Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

Optimization

- Finite differences
  - Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

Trivia: any problems with this?

Optimization

Finite differences

- Change policy a little, evaluate

$$\nabla J \approx \frac{J_{\theta+\epsilon} - J_{\theta}}{\epsilon}$$

VERY noizy

especially if both J are sampled

Objective

$$J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} R(s,a) = \int_s p(s) \int_a \pi_\theta(a|s) R(s,a)\, da\, ds$$

Wish list:

- Analytical gradient
- Easy/stable approximations

Logderivative trick

Simple math

$$\nabla \log \pi(z) = ???$$

(try chain rule)

Logderivative trick

Simple math

$$\nabla \log \pi(z) = \frac{1}{\pi(z)} \cdot \nabla \pi(z)$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) R(s,a) da\, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) R(s,a) \, da \, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \int_s p(s) \int_a \pi_\theta(a|s) \nabla \log \pi_\theta(a|s) R(s,a) \, da \, ds$$

Trivia: anything curious about that formula?

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) R(s,a) \, da \, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \underbrace{\int_s p(s) \int_a \pi_\theta(a|s)}_{\text{that's expectation :)}} \nabla \log \pi_\theta(a|s) R(s,a) \, da \, ds$$

Policy gradient

Analytical inference

$$\nabla J = \int_s p(s) \int_a \nabla \pi_\theta(a|s) R(s,a) \, da \, ds$$

$$\pi \cdot \nabla \log \pi(z) = \nabla \pi(z)$$

$$\nabla J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} \nabla \log \pi_\theta(a|s) \cdot R(s,a)$$

Policy gradient (1-step)

- Policy gradient

$$\nabla J = \underset{\substack{s \sim p(s) \\ a \sim \pi_\theta(s|a)}}{E} \nabla \log \pi_\theta(a|s) \cdot R(s,a)$$

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s_i, a_i} \nabla \log \pi_\theta(a_i|s_i) \cdot R(s_i, a_i)$$

Policy gradient (REINFORCE)

- Policy gradient

$$\nabla J = \underset{\substack{s_0 \sim p(s_0) \\ a \sim \pi_\theta(a_0|s_0) \\ s_1 \sim P(s_1|s_0, a_0)}}{E} \sum_t \nabla \log \pi_\theta(a_t|s_t) \cdot R(s_0, a_0, s_1, a_{1,} \ldots a_n)$$

- Approximate with sampling

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s_t, a_t \in z_i} \nabla \log \pi_\theta(a_t|s_t) \cdot R(\ldots)$$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$
- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot R(s,a)$$

  - Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

REINFORCE algorithm

- Initialize NN weights $\theta_0 \leftarrow random$
- Loop:

  actions under current policy = on-policy

  – Sample N sessions **z** under current $\pi_\theta(a|s)$

  – Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot R(s,a)$$

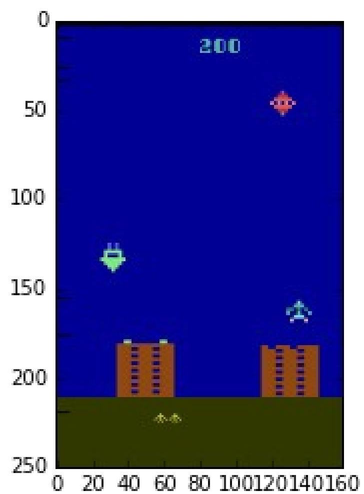
  – Ascend $\theta_{i+1} \leftarrow \theta_i + \alpha \cdot \nabla J$

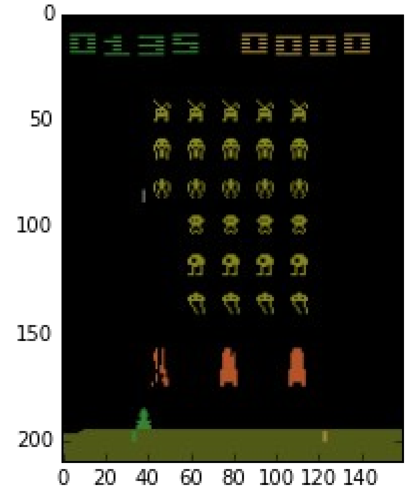
Problems so far

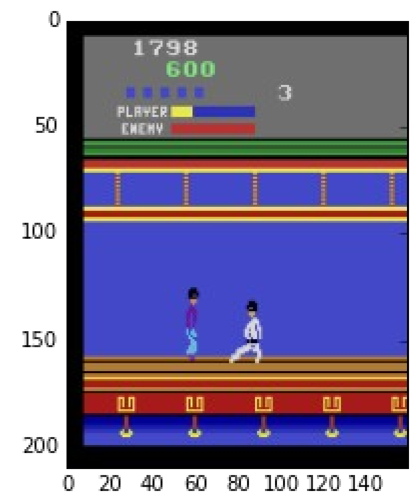
- We assume that process is finite
- Okay, you just did 1000 steps and got R=10

  What of 1000 actions caused that reward?
- Can we define immediate rewards for
  - E.g. chess?
  - Atari game?

~~Reality~~ check: videogames







- **Trivia: how to measure reward before game ends**?

Discounted reward MDP

Objective:

Total action value

$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots + \gamma^n \cdot r_{t+n}$$

$$R_t = \sum_i \gamma^i \cdot r_{t+i} \qquad \gamma \in (0,1)\, const$$

γ ~ patience

Cake tomorrow is γ as good as now

Reinforcement learning:

- Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[R] \rightarrow max$$

Discounted reward MDP

Objective:
Total action value

$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots + \gamma^n \cdot r_{t+n}$$

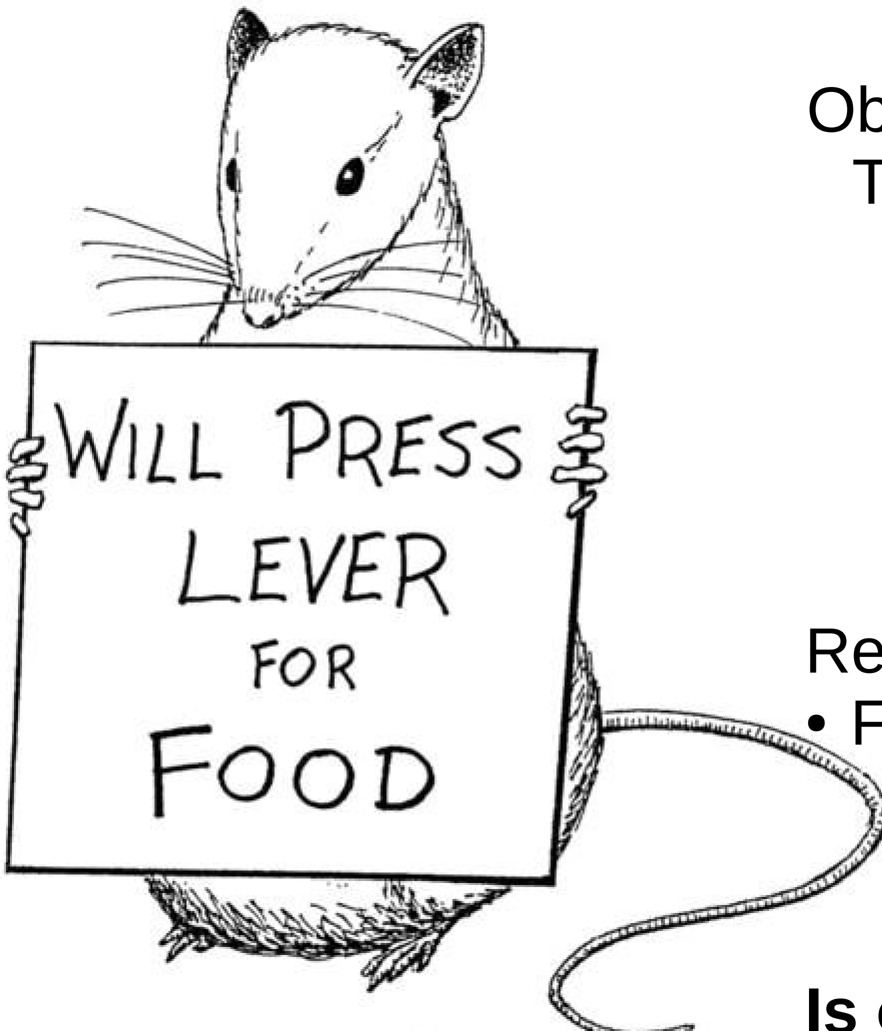
$$R_t = \sum_i \gamma^i \cdot r_{t+i} \qquad \gamma \in (0,1)\, const$$

Trivia: which γ corresponds to "only current reward matters"?

Reinforcement learning:
- Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[R] \rightarrow max$$

Discounted reward MDP



Objective:
Total reward

$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots + \gamma^n \cdot r_{t+n}$$

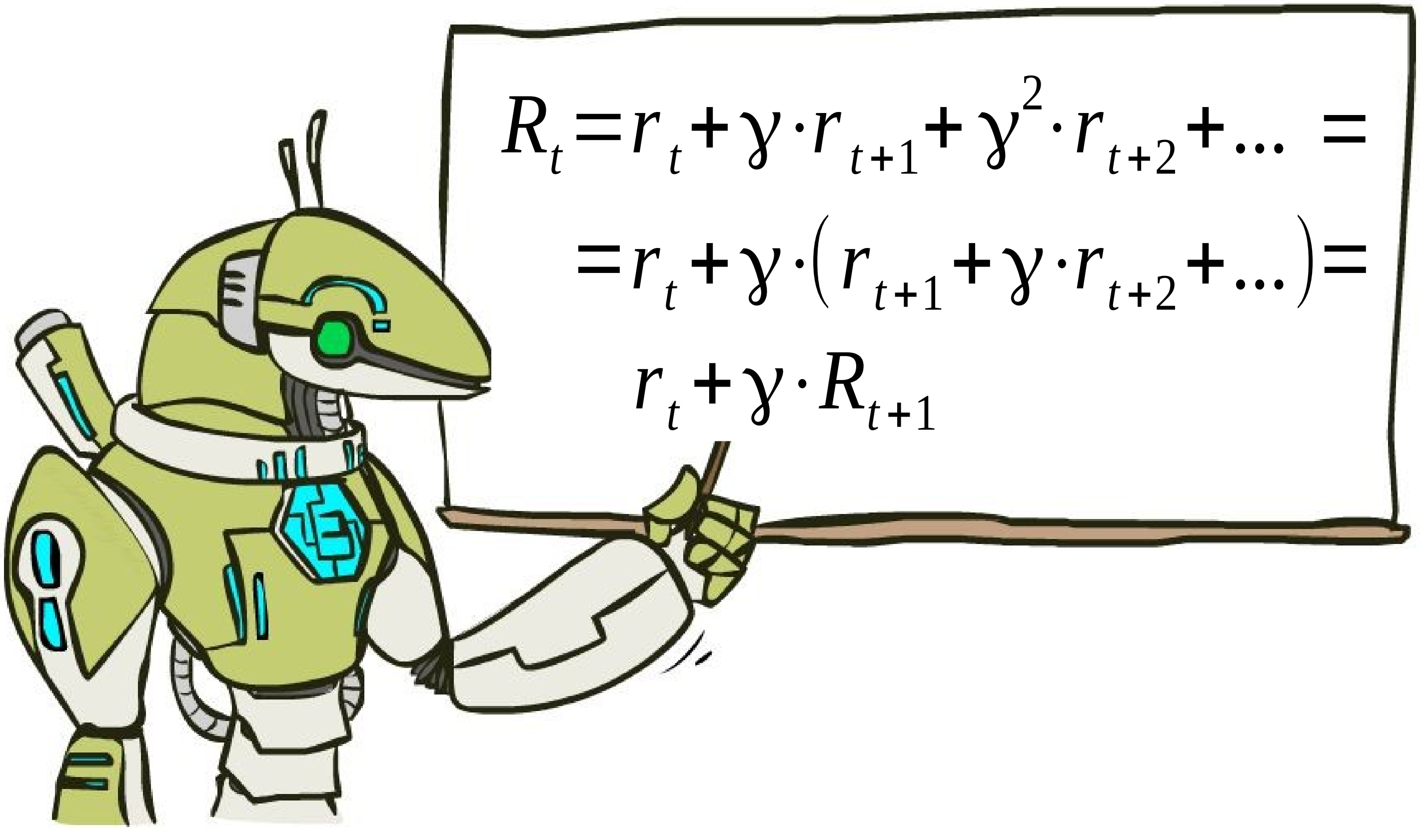
$$R_t = \sum_i \gamma^i \cdot r_{t+i} \qquad \gamma \in (0,1)\ const$$

Reinforcement learning:
- Find policy that maximizes the expected reward

$$\pi = P(a|s) : E[R] \rightarrow max$$

Is optimal policy same as it would be in R(s0a0..sn) (if we add-up all r_t)?

Optimal policy



$$R_t = r_t + \gamma \cdot r_{t+1} + \gamma^2 \cdot r_{t+2} + \ldots =$$

$$= r_t + \gamma \cdot \left(r_{t+1} + \gamma \cdot r_{t+2} + \ldots \right) =$$

$$r_t + \gamma \cdot R_{t+1}$$

We rewrite R with sheer power of math!

Value iteration (Temporal Difference)

Idea:
- For each state, obtain V(state)

$$V(s) = \underset{a \sim \pi(a|s)}{E} R(s, a)$$

Definition V(s) – expected total reward R that can be obtained starting from state s under **current** policy

Note: some algorithms define V differently,
e.g. using not current but optimal policy

Value iteration (Temporal Difference)

Idea:
- For each state, obtain V(state)

$$V(s) = \underset{a \sim \pi(a|s)}{E} R(s,a) = \underset{a \sim \pi(a|s)}{E} [r(s,a) + \gamma \cdot V(s'(s,a))]$$

Definition V(s) – expected total reward R that can be obtained starting from state s under **current** policy

Note: some algorithms define V differently,
e.g. using not current but optimal policy

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$

- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot R(s,a)$$

R(s,a) = V(s) + A(s,a)

Actions influence A(s,a) only, so V(s) is irrelevant

REINFORCE baseline

- Initialize NN weights $\theta_0 \leftarrow random$
- Loop:
  - Sample N sessions **z** under current $\pi_\theta(a|s)$
  - Evaluate policy gradient

$$\nabla J \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot (R(s,a) - V(s))$$

Anything that doesn't depend on action

Actor-critic

- Learn both V(s) and $\pi_\theta(a|s)$
- Hope for best of both worlds :)

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

Non-**trivia:** how can we estimate **A(s,a)** from (s,a,r,s') and V(s) function?

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a)=R(s,a)-V(s)$$

$$R(s,a)=r+\gamma\cdot V(s')$$

$$A(s,a)=r+\gamma\cdot V(s')-V(s)$$

Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

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Advantage actor-critic

Idea: learn both $\pi_\theta(a|s)$ and $V_\theta(s)$

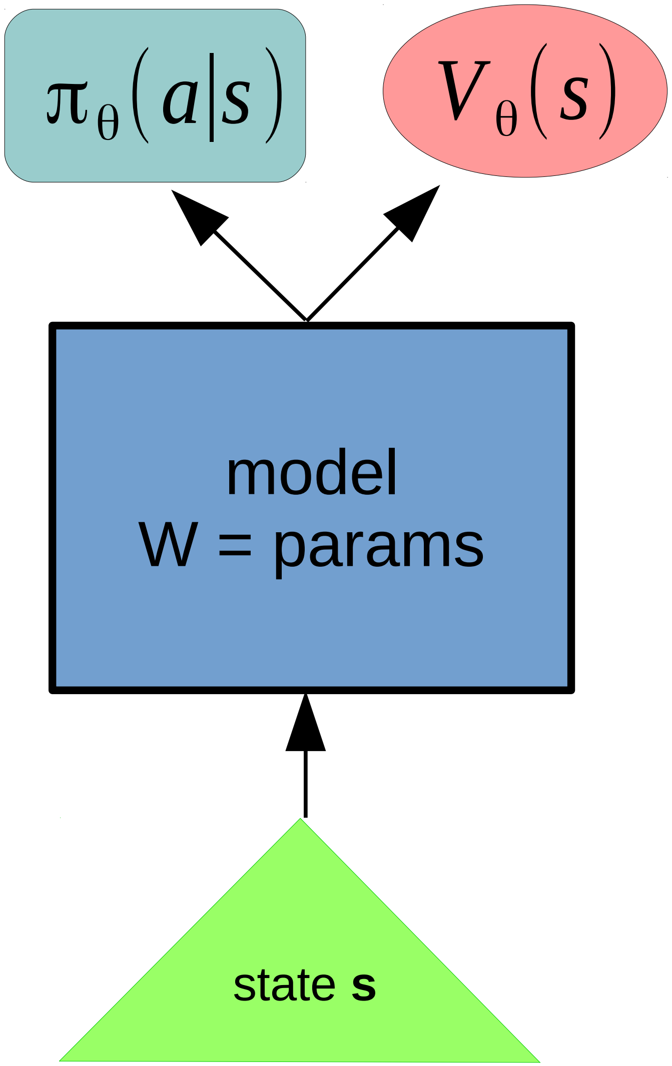
Use $V_\theta(s)$ to learn $\pi_\theta(a|s)$ faster!

$$A(s,a) = r + \gamma \cdot V(s') - V(s)$$

$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot \underbrace{A(s,a)}_{\text{consider const}}$$

Trivia: how do we train V then?

Advantage actor-critic

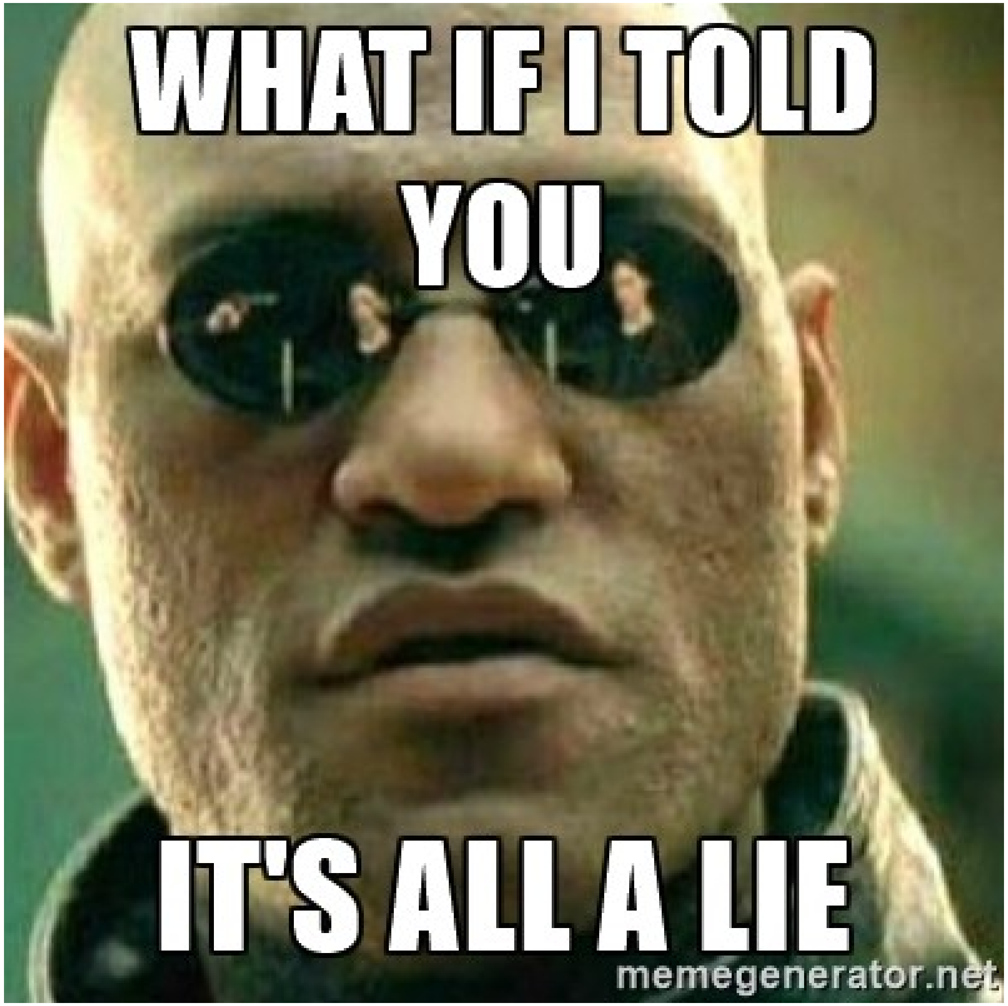


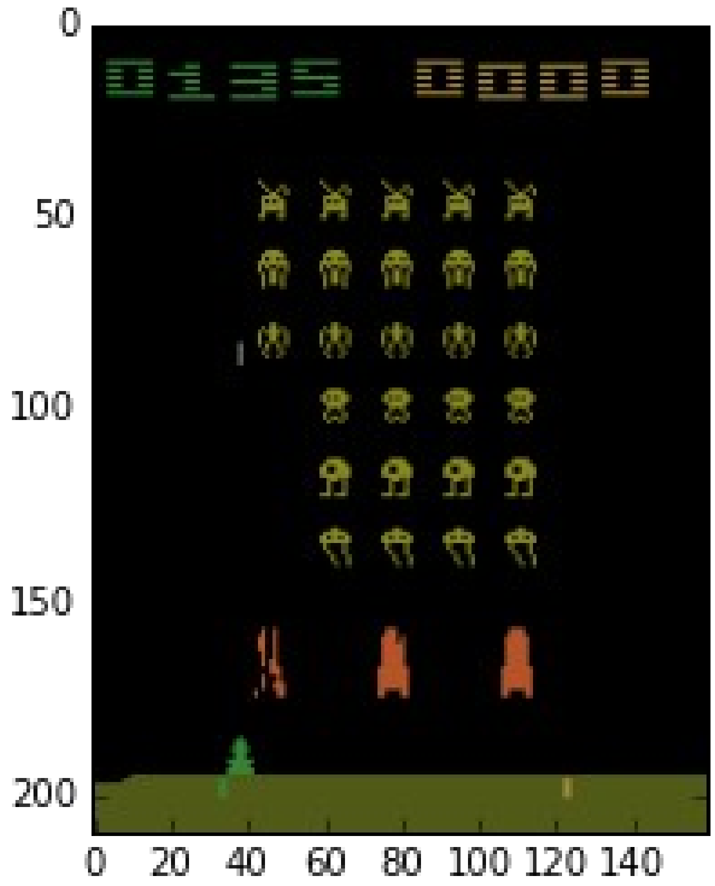
Improve policy:

$$\nabla J_{actor} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \nabla \log \pi_\theta(a|s) \cdot A(s,a)$$

Improve value:

$$L_{critic} \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{s,a \in z_i} \left(V_\theta(s) - [r + \gamma \cdot V(s')]\right)^2$$


WHAT IF I TOLD YOU
IT'S ALL A LIE
memegenerator.net

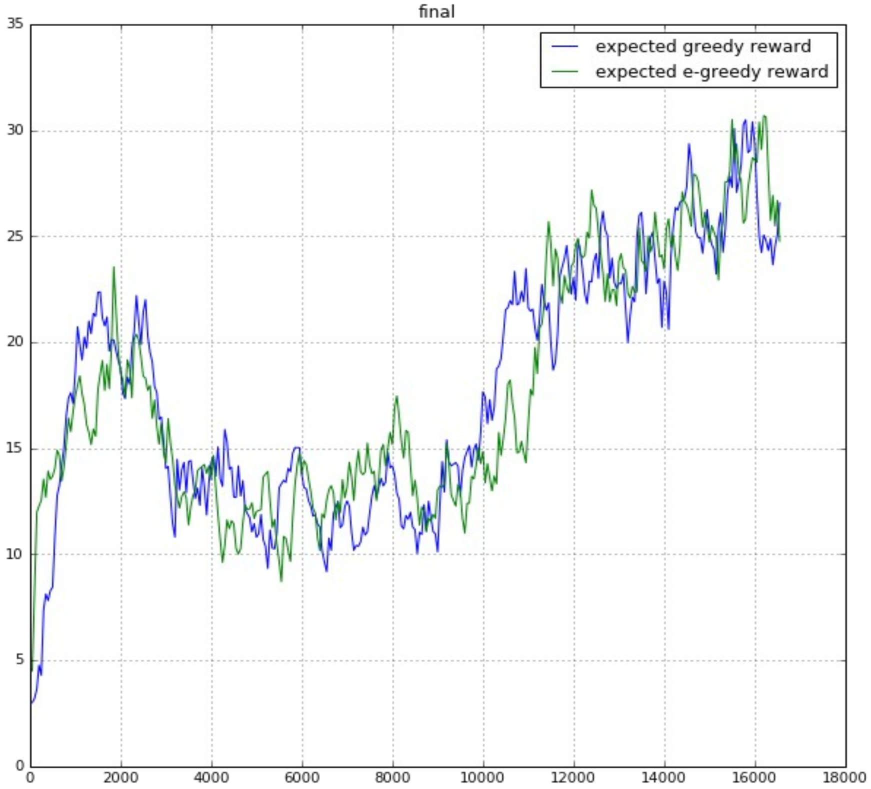


How bad it is if agent spends

next 1000 ticks under the left rock?
(while training)

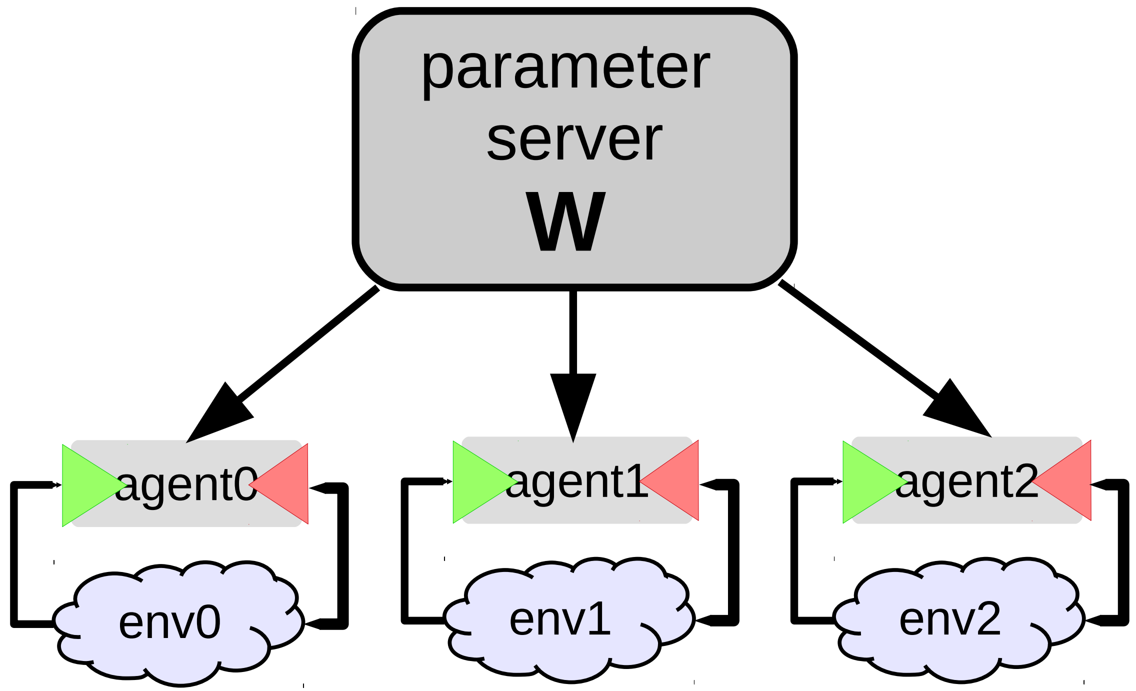
Problem

- Training samples are **not “i.i.d”**,
- Model forgets parts of environment it hasn't visited for some time
- Drops on learning curve
- **Any ideas?**



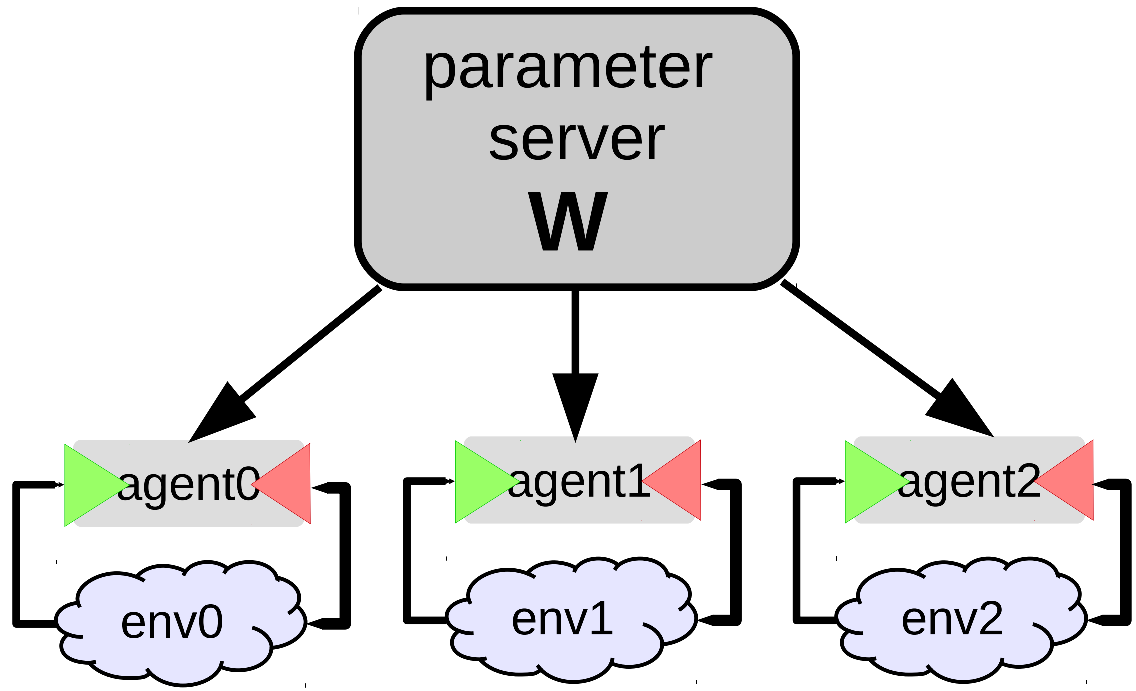
Multiple agent trick

Idea: Throw in several agents with shared **W**.



Multiple agent trick

Idea: Throw in several agents with shared **W**.



- Chances are, they will be exploring different parts of the environment,
- More stable training,
- Requires a lot of interaction

Trivia: your agent is a real robot car. Any problems?

Final problem

Left or right?

Problem:

Most practical cases are partially observable:

Agent observation does not hold all information about process state (e.g. human field of view).

Any ideas?

Problem:

Most practical cases are partially observable:

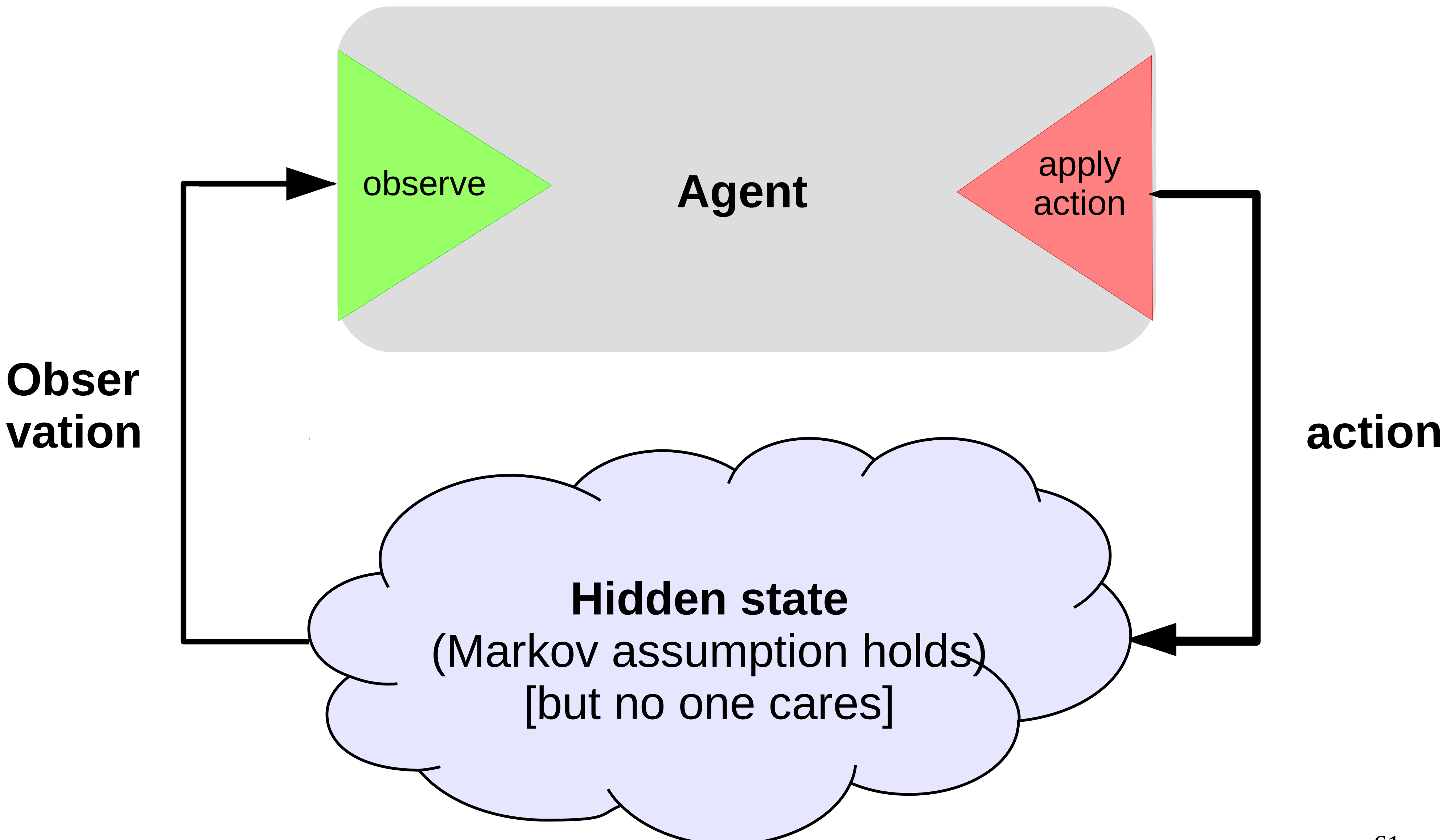
Agent observation does not hold all information about process state (e.g. human field of view).

- However, we can try to infer hidden states from sequences of observations.

$$s_t \simeq m_t : P\left(m_t | o_t, m_{t-1}\right)$$

- Intuitively that's agent memory state.

Partially observable MDP



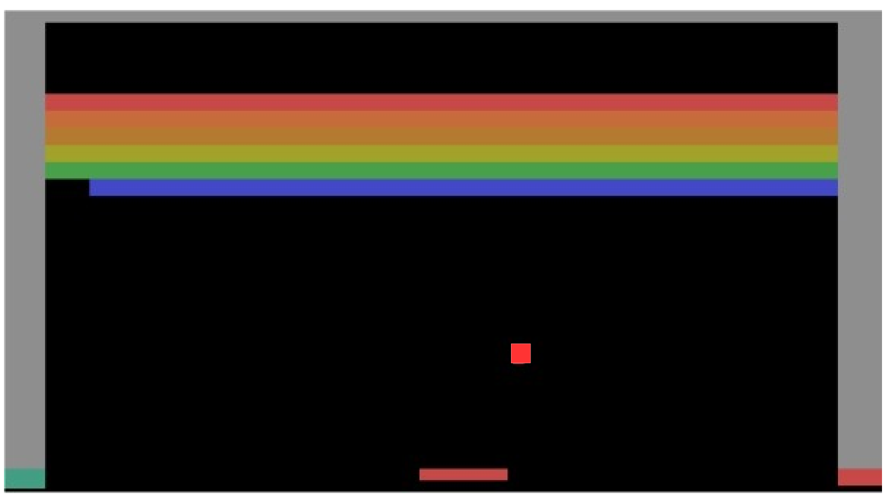
N-gram heuristic

Idea:

$$s_t \neq o(s_t)$$

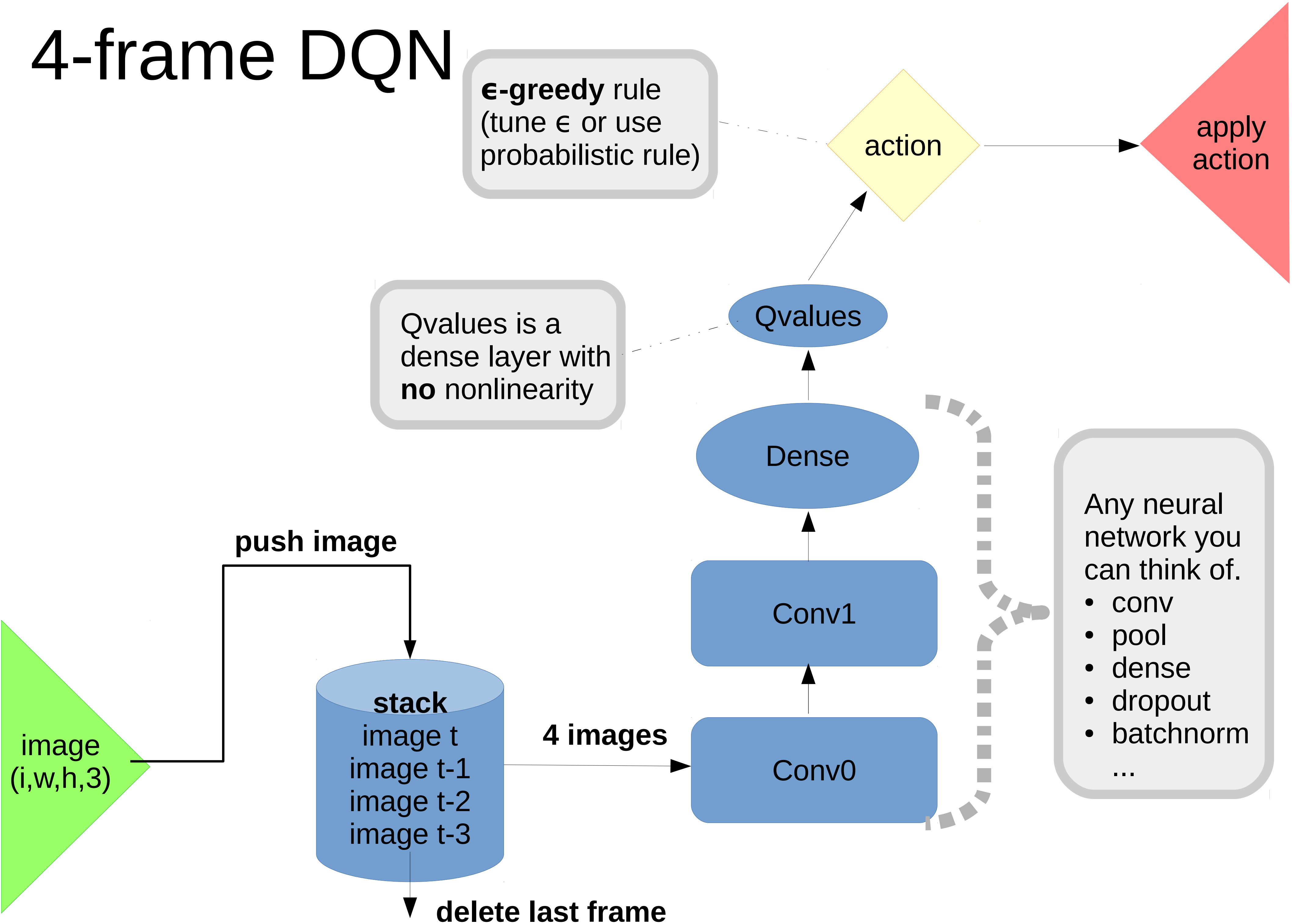
$$s_t \approx (o(s_{t-n}), a_{t-n}, \ldots, o(s_{t-1}), a_{t-1}, o(s_t))$$

e.g. ball movement in breakout



- One frame
- Several frames

4-frame DQN



ϵ-greedy rule
(tune ϵ or use
probabilistic rule)
action
apply
action
Qvalues is a
dense layer with
no nonlinearity
Qvalues
Dense
Conv1
Conv0
Any neural
network you
can think of.
• conv
• pool
• dense
• dropout
• batchnorm
...
push image
image
(i,w,h,3)
stack
image t
image t-1
image t-2
image t-3
4 images
delete last frame

N-gram heuristic

Idea:

$$s_t \neq o(s_t)$$

$$s_t \approx (o(s_{t-n}), a_{t-n}, \ldots, o(s_{t-1}), a_{t-1}, o(s_t))$$

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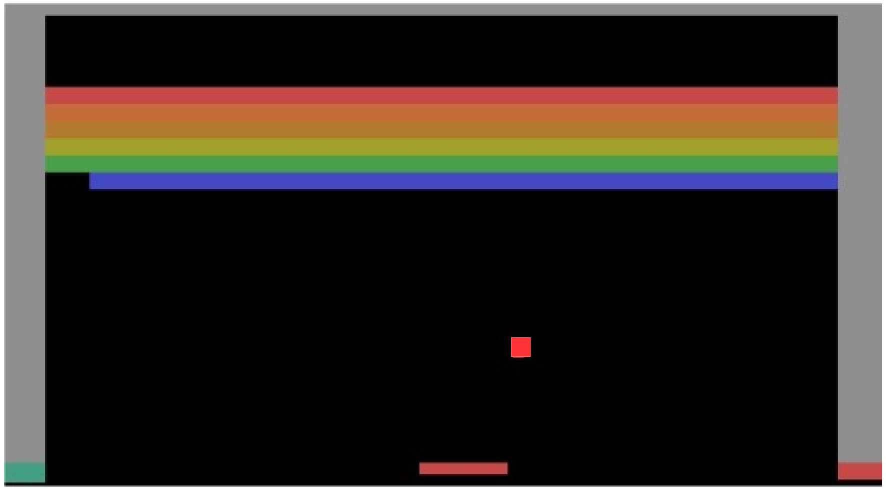
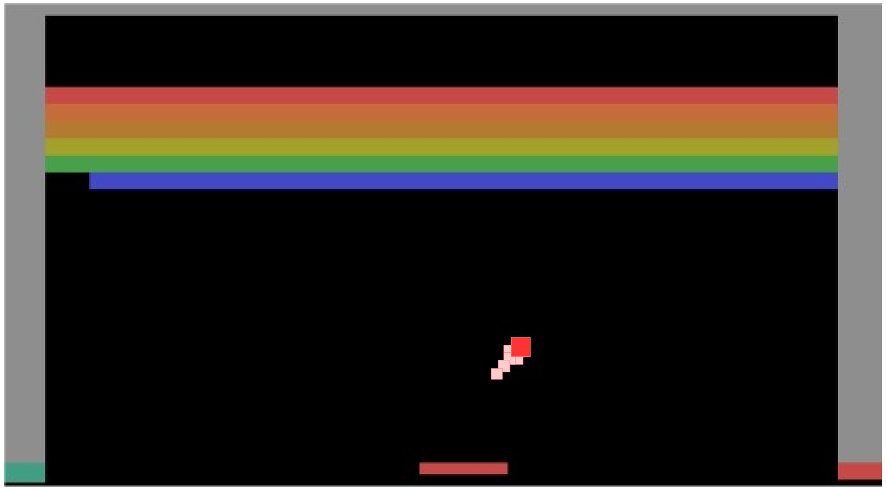


- One frame



- Several frames

Alternatives

Ngrams:

- Nth-order markov assumption
- Works for velocity/timers
- Fails for anything longer that N frames
- Impractical for large N

Alternative approach:

- Infer hidden variables given observation sequence
- Kalman Filters, Recurrent Neural Networks
- More on that in a few lectures

Continuous action spaces

What if there's continuously many actions?

- Robot control: control motor voltage
- Trading: assign money to equity

How does the algorithm change?

Continuous action spaces

What if there's continuously many actions?

- Robot control: control motor voltage
- Trading: assign money to equity

How does the algorithm change?

it doesn't :)
Just plug in a different formula for pi(a|s), e.g. normal distribution

Duct tape zone

- V(s) errors less important than in Q-learning
  - actor still learns even if critic is random, just slower
- Regularize with entropy
  - to prevent premature convergence
- Learn on parallel sessions
  - Or super-small experience replay
- Use logsoftmax for numerical stability

Let's code!