Mie Scattering for PEC and Homogeneous Spheres Irradiated by a Linearly Polarized Plane Wave

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Abstarct

The provided Matlab package gives an accurate evaluation of the electromagnetic fields inside and outside a sphere, the bistatic and monostatic radar cross sections and also the extinction and scattered power efficiencies for an incident plane wave as a source. All aspects of the algorithms implemented in this package can be found easily in the relevant literature. Our goal is to provide a complete Mie Scattering package to help fellow scientists to validate their work. We note that the software is not free from bugs and it is not by any means optimal in programming sense, thus we are open to any suggestions and comments for improvment. Finally, please cite our github repository in case you use the software for your work.

I. Introduction

The Mie solution to Maxwell's equations (named after Gustav Mie, who developed it in 1908) describes the scattering of an electromagnetic wave by a homogeneous sphere. The solution takes the form of an infinite series of spherical multipole partial waves. Specifically, the electromagnetic wave is exapnded in a set where the basis functions are the following spherical harmonics

$$\vec{m}^{ie}_{mn} = \nabla \times \left[z^{(i)}_n(kr) P^m_n(\cos\theta)\cos(m\phi) \right]$$
$$\vec{m}^{io}_{mn} = \nabla \times \left[z^{(i)}_n(kr) P^m_n(\cos\theta)\sin(m\phi) \right]$$
$$\vec{n}^{ie}_{mn} = \frac{1}{k}\nabla \times \nabla \times \left[z^{(i)}_n(kr) P^m_n(\cos\theta)\cos(m\phi) \right]$$
$$\vec{n}^{io}_{mn} = \frac{1}{k}\nabla \times \nabla \times \left[z^{(i)}_n(kr) P^m_n(\cos\theta)\sin(m\phi) \right]$$

In the above, e, o denote the even and the odd parts of the function, z^n are the spherical Bessel functions, P^m_n are the associated Legendre functions, θ is the elevation angle, ϕ is the azimuthial angle, r is the distance, k is the wavenumber and the subscripts and superscripts i, n, m denote specific orders and kinds of the relevant functions.

II. Documentation

The distributed software is a Matlab class. Each method of the class calculates a specific electromagnetic value.

- **Constructor**
  The constructor of the class requires five inputs.
  1. **inp_N:** Integer, scalar. The number of terms of the series expansion (usually 40 terms are enough to reach machine precision error).
  2. **inp_radius:** Scalar. The radius of the sphere.
  3. **inp_Eo:** Scalar. The magnitude of the incident plane wave.
  4. **inp_ko:** Scalar. The wavenumber $k_0 = \frac{2\pi}{\lambda}$.
  5. **inp_ns:** Scalar. The refractive index $n_s = \sqrt{\epsilon_r \mu_r}$. For PEC, n_s should be equal to 0.
- **BistaticRCS**
  The method returns the bistatic radarc cross section for various elevation angles (on an arc of an infinite circle).
  1. **phi:** Scalar. The azimuthial angle.
  2. **theta:** Scalar or Vector. The elevation angles.
- **MonostaticRCS**
  The method returns the monostatic radarc cross section of the sphere.
- **Incident_Fields**
  The method returns the incident electrcic and magnetic field for various elevation angles (on an arc of a circle in space).
  1. **phi:** Scalar. The azimuthial angle.
  2. **theta:** Scalar or Vector. The elevation angles.
  3. **observation:** Scalar. The radius of the circle.
- **Interior_Fields_Line**
  The method returns the interior scattered electrcic and magnetic field of the sphere on one line inside the sphere (Implementation for plane and volume will be added in the future).
  1. **x:** Scalar or Vector. The points of the line on the xx' axis.
  2. **y:** Scalar or Vector. The points of the line on the yy' axis.
  3. **z:** Scalar or Vector. The points of the line on the zz' axis.
- **Scattered_Fields**
  The method returns the scattered electrcic and magnetic field of the sphere for various elevation angles (on an arc of a circle in space).
  1. **phi:** Scalar. The azimuthial angle.
  2. **theta:** Scalar or Vector. The elevation angles.
  3. **observation:** Scalar. The radius of the circle.

- **Power_Efficiency**
  The method returns the scattered and the extinction power efficiencies of the sphere.

III. Examples

For better understanding of the distributive software, four examples are offered.

- Example 1 (**Example_BRCS**)
  In the first example, we calculate the bistatic radar cross section of five dilectric ($\epsilon_r = 1.72 - j0.0028q, q = 0, 25, 50, 75, 100$) and one PEC sphere with radiuses $R = 1$, for 361 θ between $[-\pi, \pi]$ and $\phi = 0$. The wavenumber is $k_0 = 2\pi$, the magnitude of the incident plane wave is $E_0 = 1$ and the expansion terms are 40. The results are illustrated in Figure 1.

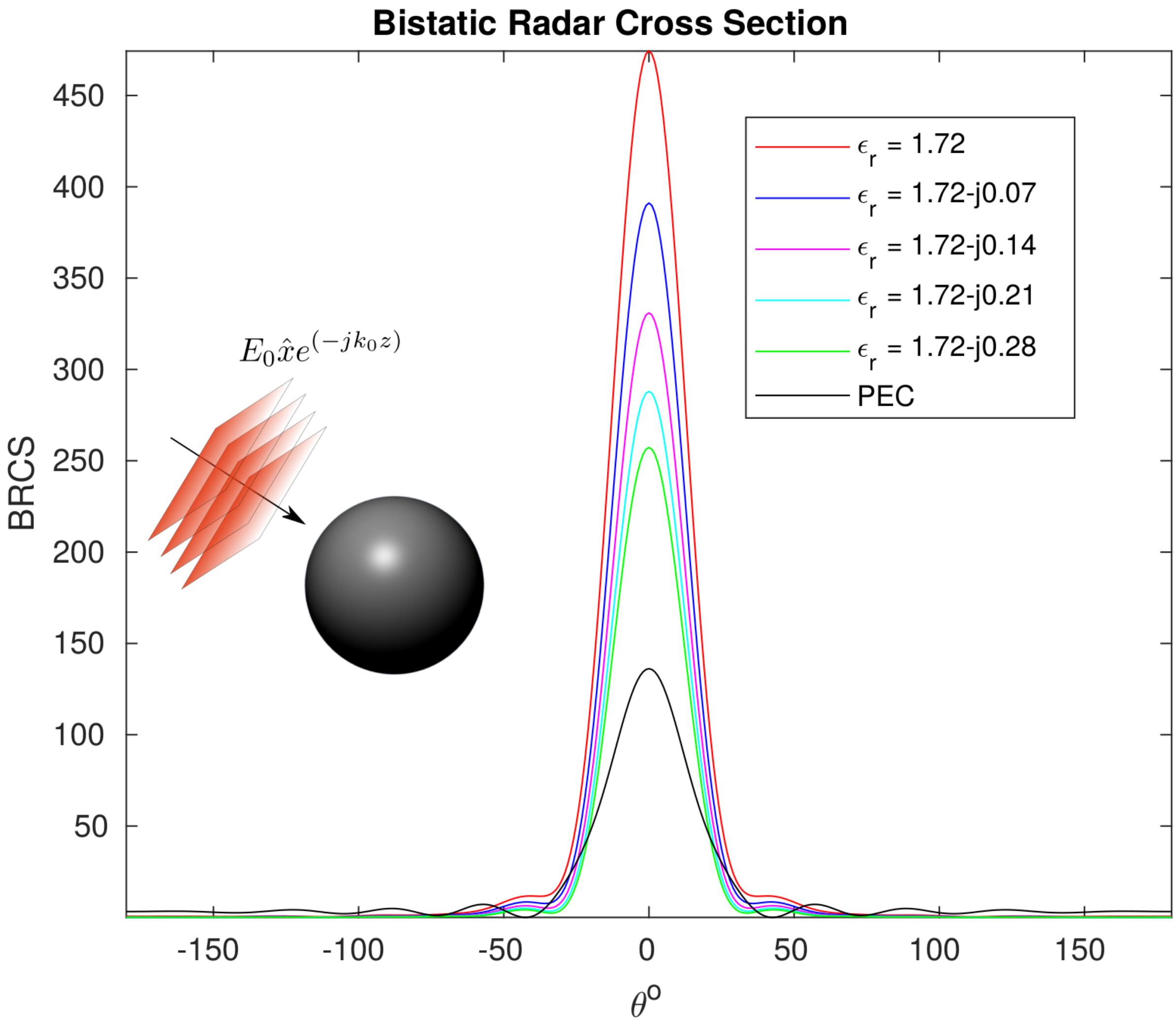

Figure 1: Bistatic Radar Cross Section for 6 different spheres

- Example 2 (**Example_MRCS**)
  In this example we calculate the monostatic radar cross section of one thousand PEC spheres with radiuses linearly scales between $0.01m$ and $20m$. The magnitude of the incident plane wave is $E_0 = 1$,the wavenumber is $k_0 = 1$ and the number of terms for the expansion is 40. The results are portrayed in Figure 2.

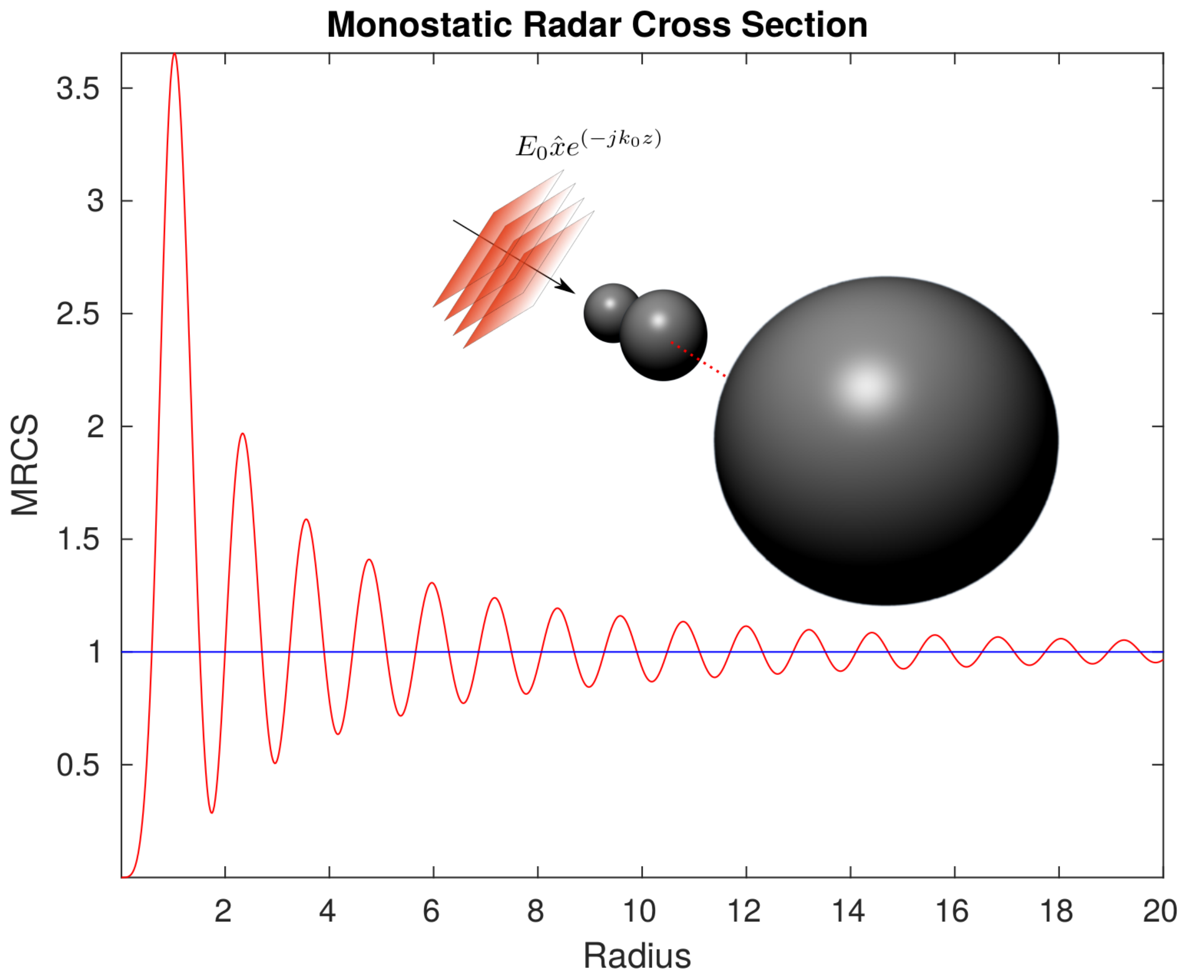

Figure 2: Monostatic Radar Cross Section for 1000 different spheres

- Example 3 (**Example_Power**)
  In this example we illustray the scattering and absorption power efficiencies of a golden lossy nanoparticle with radius $R = 1\mu m$, $\mu_r = 1$ and frequency-dependent permittivity.

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega(\omega + j\gamma)}$$

  where $\{\omega_p, \gamma\} = \{1.37\times10^{16}, 5.32\times10^{13}\}\frac{rad}{s}$ and $\omega = 2\pi f$ is the angular frequency. The calculations are implemented for 40 expansion terms and 100 logarithmic scaled frequencies. They are presented in Figure 3.

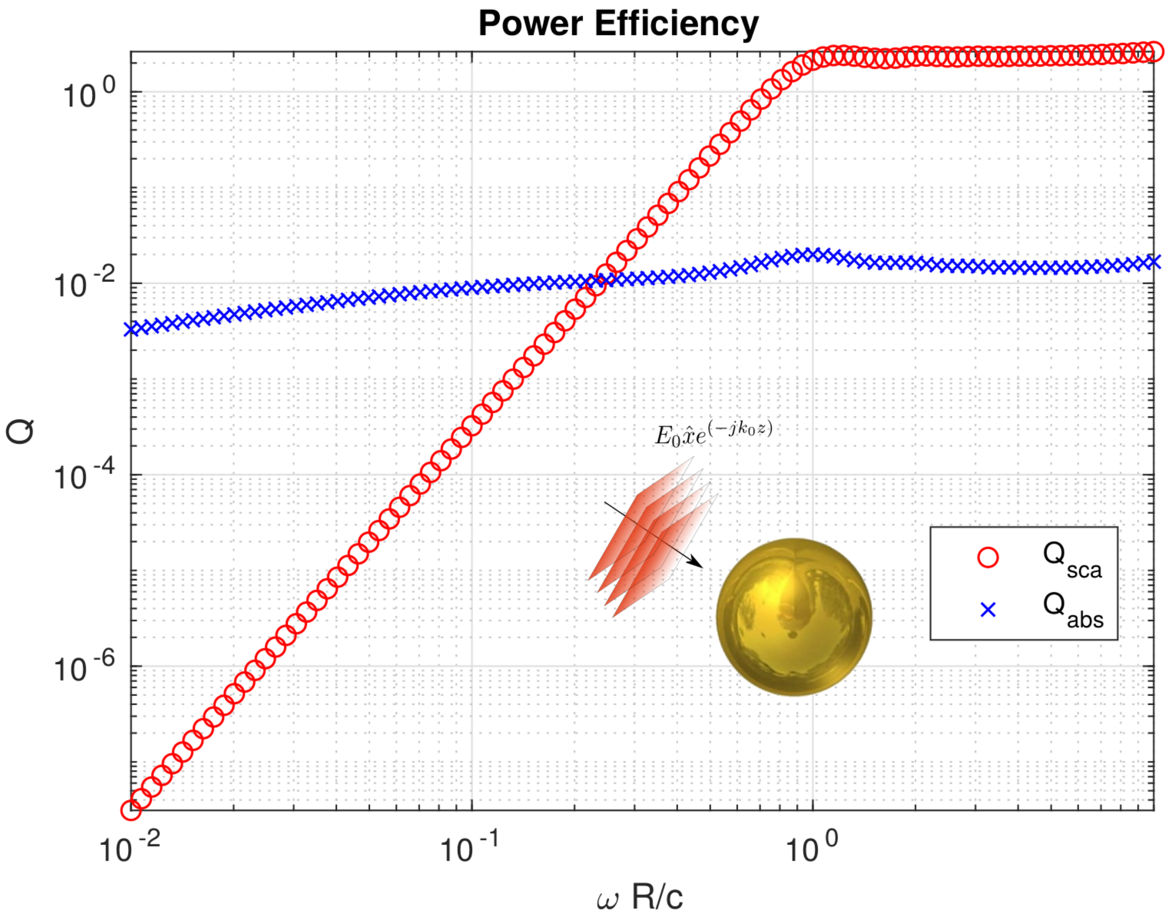

Figure 3: Power efficiencies for a Golden nanoparticle

- Example 4 (**Example_EM_fields_on_xyz_axis**)
  We measure the electromagnetic field on xx', yy' and zz' axis inside a dielectric sphere with radius $R = 0.5$ and relative permittivity $\epsilon_r = 20 - j29.76$. The magnitude of the incident wave is $E_0 = 1$, the wavenumber is $k_0 = 1$ and the expansion terms are 10. The relevant figures are presented in Figure 4.

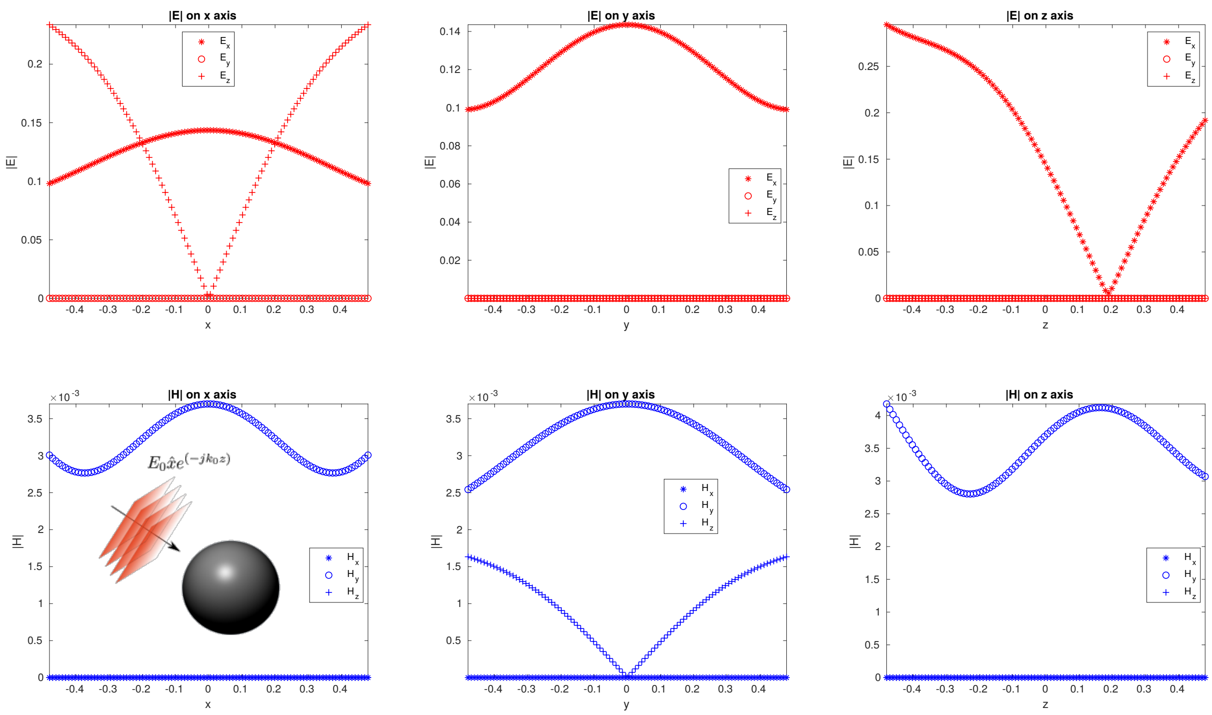

Figure 4: Interior electromagnetic field on all the axial cuts of a dielectric sphere

- Example 5 (**Example_EM_fields_on_volume**)
  We measure the total electromagnetic field on $100 \times 100 \times 100$ voxels in a normal 3D grid that surrounds a sphere of radius $R = 0.5$ and relative permittivity $\epsilon_r = 20 - j29.76$. The magnitude of the incident wave is $E_0 = 1$, the wavenumber is $k_0 = 1$ and the expansion terms are 10. The center of the sphere is at $(0, 0, 0)$ and the domain expands from -0.75 to 0.75 in each direction. The electric and magnetic field are presented in Figure 5 for the middle axial, sagittal, and coronal cut.

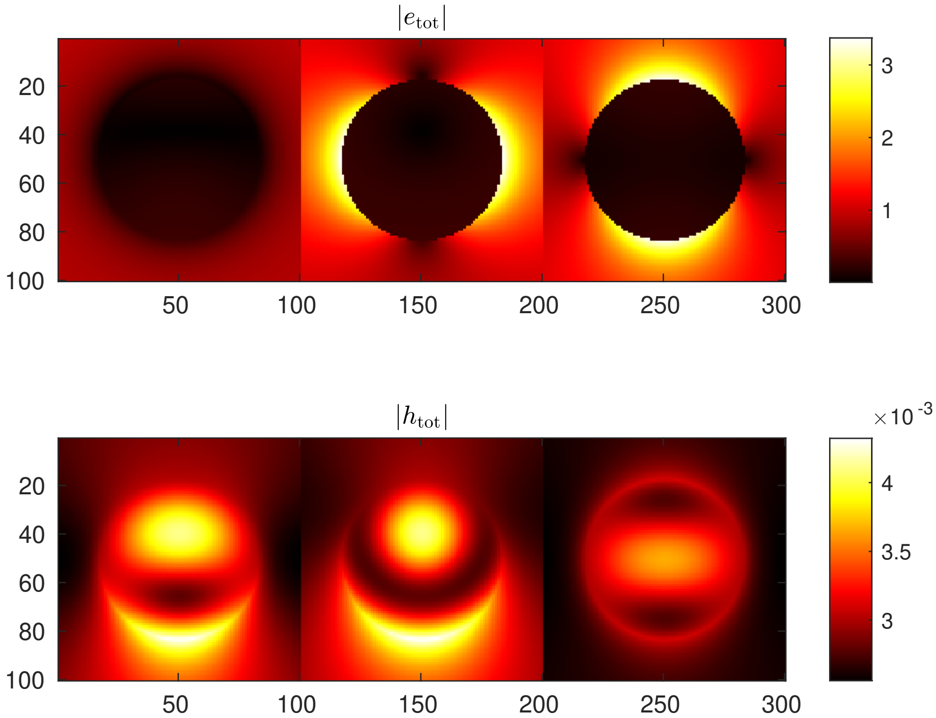

Figure 5: Total electric (top) and magnetic (bottom) fields on (from left to right) the middle axial, sagittal, and coronal cuts of a dielectric sphere