Parallel Support Vector Machines in Practice

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Abstract

In this paper, we evaluate the performance of various parallel optimization methods for *Kernel Support Vector Machines* on multicore CPUs and GPUs. In particular, we provide the first comparison of algorithms with *explicit* and *implicit* parallelization. Most existing parallel implementations for multi-core or GPU architectures are based on *explicit* parallelization of Sequential Minimal Optimization (SMO)—the programmers identified parallelizable components and hand-parallelized them, specifically tuned for a particular architecture. We compare these approaches with each other and with *implicitly parallelized* algorithms—where the algorithm is expressed such that most of the work is done within few iterations with large dense linear algebra operations. These can be computed with highly-optimized libraries, that are carefully parallelized for a large variety of parallel platforms. We highlight the advantages and disadvantages of both approaches and compare them on various benchmark data sets. We find an approximate implicitly parallel algorithm which is surprisingly efficient, permits a much simpler implementation, and leads to unprecedented speedups in SVM training.

1 Introduction

Kernel support vector machines (SVM) are arguably among the most established machine learning algorithms. They can capture complex, nonlinear decision boundaries with good generalization to previously unseen data. Numerous specialized solvers exist [9, 21, 31], which take advantage of the sparseness inherent in the optimization, and are known to be effective on a large variety of classification problems.

Recently, trends in computer architecture have been moving toward increasingly parallel hardware. Most CPUs feature multiple cores, and general purpose graphics processing units (GPUs) can execute thousands of parallel threads on their hundreds of throughput-optimized cores. Both parallel frameworks offer enormous raw power, and have the potential to provide huge speedups; however, to utilize each type of parallel thread effectively, algorithms must be carefully decomposed and optimized in fundamentally different ways. For example, GPUs are based on a "same instruction multiple data" (SIMD) architecture, which requires all threads within one block to execute the exact same instructions, whereas multi-core CPUs have much fewer threads with no such restriction.

On a high level, there are two different approaches to parallelizing algorithms: *Explicit* and *implicit* approaches. In the explicit approach, an algorithm is parallelized by hand — that is, the programmer finds the independent components of the algorithm which can be run in parallel and encodes this

parallelism using some appropriate explicitly parallel language or library such as OpenMP (for multicores), MPI (for clusters), CUDA or OpenCL (for GPUs). In the implicit approach, the algorithm is expressed as a series of operations which are known to be highly parallel and for which highly optimized parallel libraries already exist for most platforms. Examples include libraries for dense linear algebra operations — such as PLASMA [2] and Intel's MKL [20] for multicores; MAGMA [2], Jacket [1], and CuBLAS [27] for GPUs — and PDE solvers such as PETSc [4].

Both approaches have advantages and disadvantages. The explicit approach can be applied to most algorithms; therefore, in particular, it can probably be applied to the exact algorithm of one's choice. However, it often requires a significant engineering effort and a fine-tuned tradeoff between parallel work and induced overhead—which needs to be calibrated specifically for any particular algorithm and parallel architecture. The implicit approach is only applicable if the algorithm in question can be formulated as operations of some well-optimized library (in our case, linear algebra operations), which may not always be possible or may require approximation or relaxation of the problem, potentially leading to a loss in accuracy. If it is possible, however, the implicit approach has two advantages. First, since researchers and engineers have carefully designed and optimized these linear algebra libraries for peak performance [2, 27], they typically provide great speedups as long as they are called on sufficiently large problems. Therefore, if we can express an algorithm in terms of linear algebra operations of large-enough granularity, implicit algorithms can provide great parallel speedups, often more so than explicit algorithms. Second, these libraries are maintained and ported to new hardware as it becomes available; therefore, there is no need to rewrite an implicit algorithm for each new generation. In light of these two options, we investigate the following question: Given recent changes in hardware design, which approach to kernel SVM parallelization is most efficient?

To our knowledge, all existing (competitive) parallel SVM implementations for multi-core or GPU systems [3, 7, 8, 9, 13] use the explicit parallelization approach on dual decomposition methods, such as Platt's SMO algorithm [28]. Although implicit parallelization comes naturally for *e.g.* deep neural nets [24], it does not initially fit the SVM formulation and until this work there were no comparable SVM implementations of implicit parallelization. However, there exist at least three publications that reduce the kernel SVM optimization to dense linear algebra operations. Sha et al. [30] introduce a multiplicative update rule for the exact SVM optimization problem, which uses large matrix-vector multiplications in each iteration. Chapelle [10] proposes a primal formulation for the least squares hinge loss [32] which results in matrix-matrix and matrix-vector operations, and Keerthi et al. [23] approximates this approach by restricting the support vectors to a smaller subset (for reduced test-time complexity).

One advantage of the implicitly parallel approach is that, if done correctly, the algorithm spends almost all of its execution time in highly optimized routines and very little time in the remainder of the program, which therefore can be written in a high level language like MATLAB or Python. This enabled us to implement implicit parallel versions of all three approaches, which naturally work on both multi-core and GPU systems, by linking against appropriate algebra libraries [20, 27].

We apply an empirical approach and compare the various implementations with each other on several medium-sized classification data sets on GPU and multi-core architectures and arrive at an interesting conclusion: Although the multiplicative update rule [30] and the primal optimization [10] do not scale to our data set sizes due to their quadratic memory complexity, Keerthi's [22] sparse primal optimization appears to be an excellent compromise. Our MATLAB implementation tends to consistently outperform all highly optimized explicitly parallel algorithms and generally suffers no or little decrease in accuracy due to the problem relaxations.

In this paper we make two contributions: 1. We provide the first detailed empirical analysis of both explicit and implicit SVM parallelization for multi-core CPUs and GPU architectures; 2. We observe that implicit parallelization can be a much more efficient approach where possible. We believe that these insights are valuable to the machine learning community, which has so far focused almost entirely on explicit parallelism, and encourage further research into implicit approaches to parallelism.

2 Notation and Background

Throughout this paper we type vectors in bold (\mathbf{x}_i) , scalars in regular (C or b), matrices in capital bold (\mathbf{K}) and sets in cursive (\mathcal{J}) font. Specific entries in vectors or matrices are scalars and follow the corresponding convention, *i.e.* the i^{th}, j^{th} entry of matrix \mathbf{K} is written as K_{ij} and the i^{th} dimension of vector \mathbf{x} is x_i . In contrast, depending on the context, \mathbf{x}_i refers to the i^{th} vector within some ordered set $\mathbf{x}_1, \ldots, \mathbf{x}_n$ and \mathbf{k}_i refers to the i^{th} column in a matrix \mathbf{K} .

Kernelized SVMs. When training a support vector machine, we are given a training dataset $\mathcal{D} = \{(\mathbf{x}_1, y_1), ..., (\mathbf{x}_n, y_n)\}$ of feature vectors $\mathbf{x}_i \in \mathcal{R}^d$ with class labels $y_i \in \{-1, +1\}$. The goal of the optimization is to find a maximum margin hyperplane separating the two classes. (Binary classifiers can easily be extended to multiclass settings through pairwise coupling or similar approaches [29].) The primal formulation of the SVM optimization problem [12] learns a hyperplane parameterized by weight vector \mathbf{w} with a scalar offset b:

$$\min_{\mathbf{w},b} \frac{1}{2} ||\mathbf{w}||^2 + C \sum_{i=1}^n \max(0, 1 - y_i(\mathbf{w}^\top \mathbf{x}_i + b)).$$
 (1)

The simple linear case can be solved very efficiently with special purpose algorithms [17]. In this paper we focus on non-linear SVMs, which map the inputs into a new feature space $\mathbf{x}_i \to \phi(\mathbf{x}_i)$ prior to optimizing, where $\phi(\mathbf{x}_i)$ is a nonlinear transformation of \mathbf{x}_i . This mapping is generally to a higher (possibly infinite) dimensional representation. As inputs are only accessed through pairwise inner products in the dual formulation of the optimization, the mapping can be computed implicitly with the *kernel-trick* [29] through a positive semi-definite kernel function $k(\mathbf{x}_i, \mathbf{x}_j) = \phi(\mathbf{x}_i)^{\top} \phi(\mathbf{x}_j)$. The (dual) optimization to find the large-margin hyperplane becomes

$$\max_{C \ge \alpha_i \ge 0} -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i \alpha_j y_i y_j k(\mathbf{x}_i, \mathbf{x}_j) + \sum_{i=1}^n \alpha_i, \tag{2}$$

where a Lagrange multiplier variable α_i corresponds to each training input. At the end of the optimization, only some variables α_i are nonzero, which are referred to as *support vectors*. (For convenience, henceforth, we omit the bias term b, which can be solved for in a straight-forward fashion from the solution of (2) [29]. Throughout this paper we will focus primarily on the Radial Basis Function (RBF) kernel: $k(\mathbf{x}_i, \mathbf{x}_j) = e^{-\gamma ||\mathbf{x}_i - \mathbf{x}_j||^2}$. The RBF kernel is particularly interesting because of its universal approximation properties [29] and its wide-spread application.

Although solving the SVM optimization in the dual formulation (2) avoids the explicit feature computation $\phi(\mathbf{x}_i)$, it is still significantly slower than solving the linear formulation. In particular, it requires either precomputing the kernel matrix \mathbf{K} where $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$, requiring $O(n^2)$ space, or recomputing $k(\mathbf{x}_i, \mathbf{x}_j)$ as it is needed, with space or time complexity that is too great for ever increasing data set sizes. This motivates the adoption of SVM-specific optimization procedures.

3 Explicitly Parallel SVM Optimization

To our knowledge, all competitive implementations of parallel SVMs (for multi-core CPUs or GPU architectures) are based on explicit parallelization of dual decomposition approaches. Dual decomposition methods, which include Sequential Minimal Optimization (SMO) [28], are among the most efficient sequential algorithms for solving the dual formulation. They operate on a small *working set* of Lagrange multiplier variables in each iteration, holding others constant. For example, in each iteration, SMO heuristically selects two dual variables α_i , α_j and optimizes them analytically. Lib-SVM, a very popular tool for training SVMs, implements a variant of this method [9]. In general, any small number of dual variables may be optimized at once with working set size representing a tradeoff between work per iteration and number of iterations required. Explicit parallelization approaches parallelize the computation within each iteration as well as parallelizing kernel computations. A common theme among explicitly parallel methods is high code complexity, making it hard to verify correctness or port the code to new or updated hardware platforms.

Multi-core. There are several parallel implementations of dual decomposition-based SVM solvers targeted toward multi-cores. Some methods attempt to extract existing parallelism from SMO-based

approaches [14, 16], including a simple modification to LibSVM that computes kernel matrix entries in parallel with OpenMP. Other approaches attempt some restructuring of the problem. Increasing the working set size (originally two variables in SMO) exposes additional parallelism, as several dual variables are optimized at each iteration [5, 15, 35], as does optimizing over nested working sets [37]. Another common approach is to partition the training set, optimize over the partitions in parallel, and combine the resulting solutions [6, 11, 18, 19, 36]. We were only able to obtain source code for two of these methods — namely LibSVM with OpenMP and PSVM[38]. We only report the results of the former, since the latter was not designed for multi-core CPUs and consumed an infeasible amount of memory for medium-scale datasets. However, a comparison of published training times (with consideration of the various architectures) makes us believe that most other approaches are comparable or (more often) less competitive in practice.

GPU. Likewise, all previous attempts to accelerate the training of kernelized SVMs on GPUs have been direct implementations of a dual decomposition method such as SMO. GPU SVM [8] offloads computation of kernel matrix rows to the GPU using the CUBLAS library and computes KKT condition updates on the GPU with explicitly parallelized routines. A similar approach and results were demonstrated by [7]. GTSVM [13] takes the strategy of increasing the working set size of dual variables to 16 to better utilize GPU resources. The method features built-in support for both multi-class SVMs and sparse training vectors. GTSVM achieves the best previously published kernel SVM training times of which we are aware. Other GPU implementations include solvers especially optimized for multi-class problems [26] and a specialized implementation in R [34].

4 Implicitly Parallel SVM Optimization

As an alternative to explicitly parallelized SMO-type optimization methods, we also investigate algorithms that are amenable to implicit parallelization; that is, algorithms where the majority of the work can be expressed in few iterations with dense linear algebra computations, which can then be performed using optimized libraries. We identify three re-formulations of the SVM problem that lend themselves towards this approach, while noting that none of these methods were explicitly developed for increased parallelism. It is important to point out that in all formulations in this section, the linear algebra computations are dense irrespective of the sparsity of the data, as they operate on the dense kernel matrix, *e.g.* computing Hessian updates. One downside of this implicit approach is that it sometimes requires a reformulation or relaxation of the SVM optimization in (2), which can impact accuracy and memory efficiency.

Multiplicative update. Sha et al.[30] proposed the multiplicative update rule, which updates all dual variables α_i in each iteration, to solve the dual optimization (2). This approach relies on matrix-vector multiplication which can be readily parallelized; the authors remark in their original publication that the algorithm could potentially be used for parallel implementations. While our implementation demonstrated some speedups when linked against parallel libraries, the method was ultimately considered not competitive (and is not included in our experimental section) for two reasons: 1. The entire kernel matrix must be stored in memory at all times, which renders the method infeasible for typical medium-sized data sets; and 2. the convergence rate of the multiplicative update is too slow in practice, requiring too many iterations.

Primal optimization. Chapelle introduced a method for solving a kernel SVM optimization problem in the primal [10]. The SVM classifier can be expressed as $h(\mathbf{x}) = \mathbf{w}^{\top} \phi(\mathbf{x}) + b$, where $\mathbf{w} = \sum_{i=1}^{n} \alpha_i y_i \phi(\mathbf{x}_i)$ (and with bias b). After the transformation $\mathbf{x} \to \phi(\mathbf{x})$, solving (1) with respect to \mathbf{w} directly is impractical, due to the high (possibly infinite) dimensionality of $\phi(\mathbf{x})$. However, after a change of variable, with $\beta_i = \alpha_i y_i$ and $\beta \in \mathcal{R}^n$, (1) can be rewritten as follows:

$$\min_{\boldsymbol{\beta}, b} \frac{1}{2} \boldsymbol{\beta}^{\mathsf{T}} \mathbf{K} \boldsymbol{\beta} + \frac{C}{2} \sum_{i=1}^{n} \max(0, 1 - y_i (\boldsymbol{\beta}^{\mathsf{T}} \mathbf{k}_i + b))^2$$
 (3)

where \mathbf{k}_i is the kernel matrix row corresponding to the i^{th} training example. Notice that there are two relaxations: 1. the β_i are unconstrained, in contrast to α_i in (2), which must satisfy $0 \le \alpha_i \le C$; and 2. the squared hinge loss is used in place of the more common absolute hinge loss. These changes allow the use of second order optimization methods. In particular Newton's method yields

very fast convergence with computations expressed as dense linear algebra operations. As noted in [10], the squared hinge loss leads to almost identical results as the absolute hinge loss—a claim that we confirm in our experimental results. Similar to the multiplicative approach, this method requires the computation of the entire kernel matrix, which renders it impractical for larger data sets. We therefore do not include it in our experimental result section, which focuses on data sets with prohibitively large sizes.

Sparse primal optimization. Keerthi et al. proposed a method to reduce the complexity of Chapelle's primal approach by restricting the support vectors to some subset of *basis vectors* $\mathcal{J} \subset \{1, \ldots, n\}$ so that $j \notin \mathcal{J} \Rightarrow \beta_j = 0$. Then equation (3) becomes:

$$\min_{\boldsymbol{\beta}, b} \frac{1}{2} \boldsymbol{\beta}^{\mathsf{T}} \mathbf{K}_{\mathcal{J}\mathcal{J}} \boldsymbol{\beta} + \frac{C}{2} \sum_{i=1}^{n} \max(0, 1 - y_i (\boldsymbol{\beta}^{\mathsf{T}} \mathbf{k}_{\mathcal{J}i} + b))^2.$$
(4)

Here, β has been restricted to contain only those β_i with $i \in \mathcal{J}$. $\mathbf{K}_{\mathcal{J}\mathcal{J}}$ is the kernel matrix between only basis vectors, and $\mathbf{k}_{\mathcal{J}i}$ is the kernel row of the i^{th} training example with all basis vectors (i.e., the vector $\mathbf{k}(\mathbf{x}_k, \mathbf{x}_i)$ for each $k \in \mathcal{J}$). As the set \mathcal{J} is originally unknown, Keerthi et al. propose to grow \mathcal{J} with a heuristic. Initially, \mathcal{J} is empty and the algorithm then has two distinct stages that are cycled. Basis vector selection: A small subset of the training set is randomly sampled, and then a heuristic is used to estimate the reduction in loss from adding each input to \mathcal{J} . The highest scoring point is then greedily added to \mathcal{J} to get \mathcal{J}' . Reoptimization: After a certain number of basis vectors have been added to \mathcal{J}' , (4) is optimized using \mathcal{J}' as the basis vector set. This whole process of gradually selecting basis vectors and then re-optimizing repeats until some stopping criterion is met. The resulting algorithm performs only a few iterations in total, each of which make use of intensive linear algebra computation. This method still requires the kernel matrix of basis vectors with all training examples, requiring $O(|\mathcal{J}|n)$ space. In practice, $|\mathcal{J}| \ll n$; however, this may still be a concern, particularly on GPUs where memory availability is more limited than RAM.

We reimplement this sparse primal SVM (SP-SVM) in MATLAB. For linear algebra operations on multicores, we use a combination of built-in linear algebra functions and Intel MKL. For linear algebra operations on the GPU, we use Jacket[1], a MATLAB toolkit for accelerating computations on GPUs. Additionally, we incorporate the freely available C++/CUDA package CUBLAS [27] in cases where Jacket proves to be inefficient or lacks desired functionality. Because no stopping criterion is suggested in the original publication [22], our implementation stops when, after re-optimization, the change in training error divided by the number of basis vectors added in the previous selection stage is less than some threshold ϵ . We have released an optimized C++ version of SP-SVM, called WU-SVM, for both multicore and GPU architectures at http://tinyurl.com/wu-svm.

5 Results

This section presents an empirical evaluation of several of the algorithms described in sections 3 and 4 on two modern parallel architectures: multi-core CPUs (MC) and graphics processing units (GPUs). Running time and accuracy statistics on seven datasets show the benefits and drawbacks of the approaches included in our evaluation.

Hardware. Experiments are run on a 12-core machine with Intel Xeon X5650 processors at 2.67 GHz with hyperthreading enabled and 96 GB of RAM. The attached NVIDIA Tesla C2075 graphics card contains 448 cores and 6 GB of global memory.

Methods evaluated. The *single-threaded* CPU baseline method is LibSVM [9], a popular implementation of SMO, which we use as the baseline for classification accuracy. On *multi-cores* we evaluate a modified version of LibSVM which performs kernel computations in parallel with OpenMP¹. Further, we evaluate our implementation of SP-SVM in MATLAB with Intel MKL BLAS functions for matrix operations. For the *GPU* settings, we compare two explicitly parallel GPU adaptations of dual decomposition: GPU SVM [8], an adaptation of LibSVM for GPUs, and GTSVM [13]. We also

http://www.csie.ntu.edu.tw/~cjlin/libsvm/faq.html

Data Set		Method	Test Error (%)	Training Time	Speedup
Adult	SC	LibSVM	14.9	1m 6s	1×
7мв	MC	LibSVM	14.9	10.5s	18×
n = 31562, d = 123	Σ	SP-SVM	14.8	15.2s	13×
$C = 1, \gamma = 0.05$ [13]		GPU SVM	14.9	6s	32×
	GPU	GTSVM	14.8	1 s	190×
		SP-SVM	14.8	11.3s	17×
Covertype/Forest	SC	LibSVM	13.9	5h 1m 19s	1×
96мв	MC	LibSVM	13.9	1h 5m 46s	5×
n = 522911, d = 54	Σ	SP-SVM	13.7	10m 10s	29 ×
$C = 3, \gamma = 1$ [13]		GPU SVM	13.9	7m 32s	40×
	GPU	GTSVM	36.8	5m 15s	57×
		SP-SVM	13.8	4m 38s	65×
KDDCup99	SC	LibSVM	7.4	3h 0m 29s	1×
970мв	MC	LibSVM	7.4	26m 37s	7×
n = 4898431, d = 127	Σ	SP-SVM	7.9	56s	193×
$C = 10^6, \gamma = 0.137$ [33]		GPU SVM	_	_	_
	GPU	GTSVM	19.9	1h 15m 39s	$2\times$
		SP-SVM	<u> </u>	_	_
MITFaces	SC	LibSVM	5.6 [†]	34m 22s	1×
1.3Gв	MC	LibSVM	5.6 [†]	4m 8s	8 ×
n = 489410, d = 361	Ž	SP-SVM	7.4^{\dagger}	20s	103×
$C = 20, \gamma = 0.02$ [33]		GPU SVM	5.7 [†]	33s	61×
	GPU	GTSVM	5.6 [†]	1m 34s	22×
	0	SP-SVM	7.4^{\dagger}	10s	200×
FD	SC	LibSVM	1.4	2h 6m 50s	1×
1.3GB	MC	LibSVM	1.4	27m 54s	5×
$n = 200000^*, d = 900$	Ž	SP-SVM	1.5	1m 22s	92 ×
$C=10, \gamma=1$		GPU SVM	1.4	6m 20s	$20 \times$
	GPU	GTSVM	1.5	2m 26s	52×
		SP-SVM	1.5	29s	262×
Epsilon	SC	LibSVM	10.9	19h 12m 27s	1×
2.4GB	MC	LibSVM	_	_	_
$n = 160000^*, d = 2000$	Σ	SP-SVM	10.8	8m 10s	141×
$C\!=\!1, \gamma\!=\!0.125$	_	GPU SVM	10.9	29m 1s	40×
	GPU	GTSVM	10.9	4m 33s	253×
		SP-SVM	10.8	1m 55s	601×
MNIST8M (24GB)	SC	LibSVM	1.0	12d 15h 21m 31s	1×
n = 8100000, d = 784	MC	LibSVM	1.0	1d 23h 12m 8s	6×
$C = 1000, \gamma = 0.006$ [25]	Σ	SP-SVM	1.4	2h 37m 50s	115×

Table 1: Comparison of test error, training time, and speedup of kernelized SVM training methods. The first column indicates dataset file size, number of instances, dimensionality, and SVM hyperparameters C and γ (with a citation for previously published values, otherwise derived by cross-validation using GTSVM). Results for SP-SVM are the average of five runs with different randomly sampled candidate sets (see text for standard deviations). Row background colors indicate implementation architecture: single-core (SC), multi-core (MC), GPU. Red font color indicates poor test error results. Bold typeface indicates the best timing results for each dataset and architecture. Symbol † indicates accuracy metric is (1-AUC)%. Symbol - indicates a data set/method pair that was unable to be run, as explained in the text.

include the implicitly parallel MATLAB implementation of SP-SVM, linked against the appropriate libraries for GPU linear algebra computations. With the exception of SP-SVM, all implementations are written in $\mathbb{C}/\mathbb{C}++$ by the authors of the respective publications.

Datasets. We evaluate all methods on several medium scale data sets, each involving classification tasks. Medium scale datasets are chosen because parallel runtimes with small datasets tend to be dominated by overhead while large-scale datasets generally require an exorbitant amount of system memory. The datasets are as follows: Adult²—an annual income prediction task (greater or less than \$50K) based on census data; Covertype/Forest³—a tree cover prediction task based on geographical and climate features (predicting class 2 versus the rest); KDDCup99⁴—a classification task for intrusion detection in network traffic; MITFaces⁵—a face detection task from raw images (with accuracy presented in (1-AUC)% due to an extreme class imbalance); Epsilon⁶—a synthetic classification task from the 2008 PASCAL Large Scale Learning Challenge; FD⁶—another face detection task (without heavy class imbalance); and MNIST8M⁷—a multiclass handwritten digit recognition task based on label invariant transformations of images from the MNIST data set. We use the one-versus-one classifier approach to multi-class classification, as also adopted by LibSVM [9].

Features for the datasets Adult, Covertype/Forest, KDDCup99, MITFaces, and MNIST8M are scaled to [0,1] before training. In addition, we subsample two of the largest data sets, Epsilon and FD, uniformly at random from 400,000 to 160,000 and 5,469,800 to 200,000 respectively for two reasons. First, single core algorithms require prohibitively long training times on the full sets. Second, on GPUs, if the data does not fit into GPU memory the running time is dominated by memory transfer, which is not the focus of this study.

Hyper parameters. The left column of Table 1 provides details of the size and dimensionality of each data set. In addition, it also indicates the regularization parameter C and inverse Gaussian kernel width γ used throughout the experiments. These parameters are derived from cited works for most datasets, as indicated in the table. For Epsilon and FD, a thorough cross-validation grid search was conducted using GTSVM as it is an exact implementation and tends to behave identically to LibSVM in terms of hyper parameters but does not have the large time requirement of cross validating with LibSVM. This approach does a slight disservice to SP-SVM, however it may be viewed as a fair compromise as LibSVM is the gold standard and our main focus is the speedup. Throughout all experiments with SP-SVM we set the stopping criterion to $\epsilon = 5 \times 10^{-6}$.

Evaluation. Table 1 shows test error, training time, and speedup versus single-core LibSVM for all methods on each of the seven data sets. The training times omit both loading data from disk and computing test predictions for all methods. As MNIST8M is multi-class, the times reported are the accumulative time for each one-versus-one classifier trained individually.⁸

Since SP-SVM deploys a heuristic based on random sampling of basis vectors, we computed five runs for each setting and report the average runtime and test error. Standard deviations on SP-SVM test error are less than 0.001 for all datasets except for the multicore implementation on KDDCup99 (0.0023). Similarly, standard deviations for SP-SVM training time are on the order of seconds for each run. (For increased readability, we omit them from the table.)

Not all algorithms converge on all data sets. GTSVM is the only GPU method that runs on KDD-Cup99 (which is 90% sparse). GPU SVM and SP-SVM both store the inputs in dense format on the

²http://archive.ics.uci.edu/ml/datasets/Adult

³http://archive.ics.uci.edu/ml/datasets/Covertype

⁴http://kdd.ics.uci.edu/databases/kddcup99/kddcup99.html

⁵http://c2inet.sce.ntu.edu.sg/ivor/cvm.html

⁶http://largescale.ml.tu-berlin.de/instructions/

⁷http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/multiclass.html

⁸Shared memory computers, such as multi-core CPUs and GPUs, are arguably less suited for this kind of multi-class classification, since one-versus-one classifiers are "embarrassingly parallel" for problems with many classes and can be solved on (cheaper) distributed memory machines (clusters) with near-perfect speedup.

GPU, which exceed its memory. The dense MNIST8M data is too large for all GPU algorithms. Also, LibSVM with OpenMP failed to converge on Epsilon in less time than single-core LibSVM.

Accuracy. For most datasets and methods, test errors are remarkably consistent, even between exact and approximate methods. However there are a few notable exceptions, highlighted in red in Table 1. GTSVM fails on Covertype/Forest and we hypothesize that this anomaly may be due a floating point precision error as the method converges when run on smaller subsets of the training data. On KDDCup99 GTSVM obtains an error rate of 19.9%, which is not significantly better than a constant predicting the most common class (no GPU method in our evaluation could successfully learn from this data). SP-SVM performs slightly worse on KDDCup99 (7.9% vs. 7.4%) and noticeably worse on MITFaces (7.4% 1-AUC vs. 5.6%) and MNIST8M (1.4% vs. 1.0%). The approximation error may be more pronounced on MITFaces due to the large class imbalance (a few additional false positives have a strong effect on the final area under the curve) and also for MNIST8M, where the approximation error is being aggregated across the many (45) one-versus-one classifiers.

Speedup. The most basic method of speedup is LibSVM on multicores. This involves a trivial change directly to the source of LibSVM, allowing for the use of OpenMP parallel for-loops in kernel computations. Because kernel computations account for such a significant portion of LibSVM's runtime, this baseline improvement results in a $5-8\times$ speedup on twelve cores.

GPU SVM achieves $20-40\times$ speedups over single-core LibSVM by performing kernel computations and KKT condition updates directly on the GPU. GTSVM achieves the largest speedups among the dual decomposition methods, by also increasing the working set size to 16 (compared to 2 used by LibSVM and GPU SVM), leading to $2.5-6.5\times$ speedup over GPU SVM, and $2-250\times$ speedup over LibSVM. This highlights the correlation between speedup and the amount of hand-crafted parallelism that is included in the algorithm design for the explicit parallel approaches.

In comparison to single core LibSVM, SP-SVM achieves $13\times$ to $193\times$ speedup on multi-cores, and $17\times$ up to $601\times$ speedup on GPUs. On both architectures, the speedup of SP-SVM tends to increase with data set size, which reflects the increasing time spent inside parallelized library operations. The smallest speedup for both architectures is on the smallest data set, Adult—however, by a mere 11s or 15s compared to the fastest algorithm (GTSVM). It is surprising just how effective the parallelism derived from the dense linear algebra in SP-SVM proves to be on both architectures. SP-SVM is particularly effective on GPUs where it outperforms all other GPU methods by $1.5\times$ to $5\times$ on all but Adult, and achieves a $1.3-4.3\times$ speedup over multi-core SP-SVM. However even on multi-cores, SP-SVM outperforms GPU SVM and GTSVM significantly on MITFaces and FD. SP-SVM requires only 11 minutes on average across all binary classification datasets, compared to the several hours often required by LibSVM.

6 Discussion

One trend clearly follows from our study: massive speedups are possible when the parallelism of modern hardware is leveraged. Although explicit parallelization is by far the most dominant approach to SVM parallelization, our results demonstrate that implicit parallelization can be more efficient and deserves some attention. We believe that the community can benefit from our findings in two ways: first, practitioners will obtain an easy to use implementation of SP-SVM with unprecedented training speed that can readily be used on most platforms with BLAS compatible libraries; second, researchers working on parallel machine learning algorithms may reconsider spending days in agony on C/C++ programming of parallel code and may instead focus on relaxing or reformulating their algorithm to rely more heavily on dense linear Algebra routines.

One downside of implicit parallelization for SVMs is that the exact reformulations are too memory intensive, and SP-SVM enforces a reduced basis vector set. We show in our results that in practice this effect might be small, however it will be interesting to see if there are exact formulations that avoid such restrictions. We predict that relaxations into implicit parallelization may become increas-

⁹As GPU memory sizes grow, this limitation will become less important. In addition, GPUs and CPUs might eventually converge on using a single memory space. For sparse data sets one might also consider special purpose libraries, such as CUSPARSE (https://developer.nvidia.com/cusparse), for the kernel computation.

ingly important as multi-cores and GPUs establish themselves as the common computing platforms, similar to relaxing optimization problems into convexity, as has been common practice for years.

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