Assignment 3
Control

Task 1

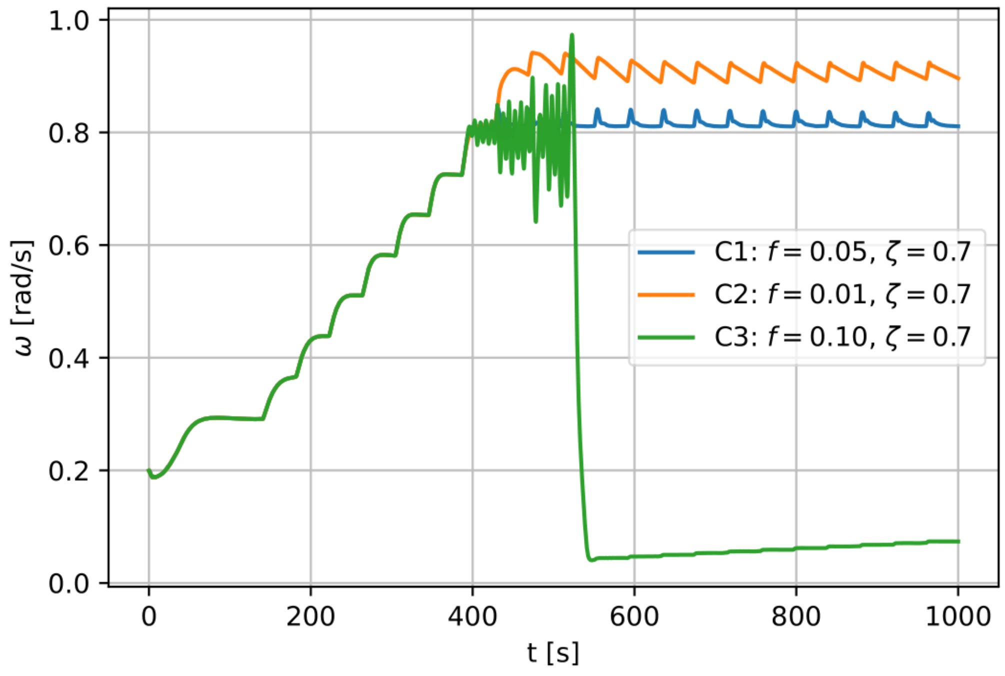
C1: 0.05 Hz and

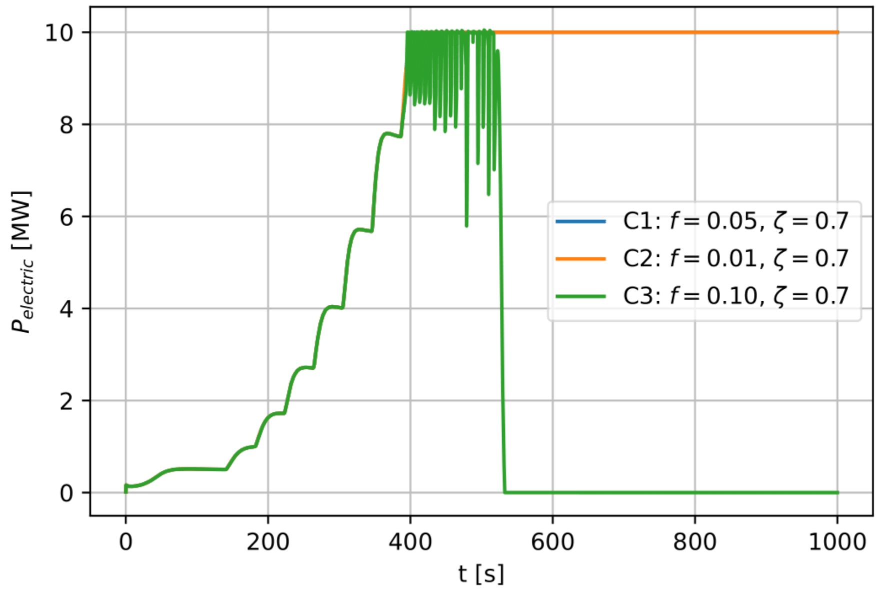
Parameter		Unit	HAWC2S
Optimal Cp tracking K factor	K	[N·m]/[rad·s^-2]	0.20411E+08
Proportional gain of torque controller	K_{Pg}	[N·m] / [rad]	0.111389E+09
Integral gain of torque controller	K_{Ig}	[N·m] / [rad s^-1]	0.249955E+08
Proportional gain of pitch controller	K_P	[rad] / [rad]	2.95133
Integral gain of pitch controller	K_I	[rad·s] / [rad]	0.577807
Coefficient of linear term in aerodynamic gain scheduling	KK_1	[rad]	4.31691
Coefficient of quadratic term in aerodynamic gain scheduling	KK_2	[rad^2]	-
Aerodynamic gain at pitch = 0 deg	$\partial Q_A/\partial\theta$	[N·m] / [rad]	-755.01842

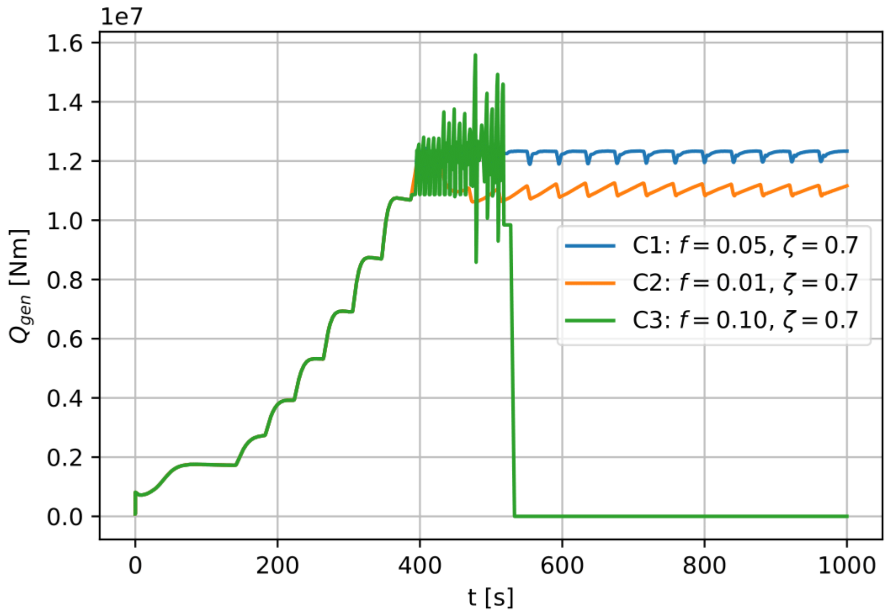
Task 2

Region 1			Region 2		Region 3		
CASE		K	KPg	KIg	KP	KI	KK1
Constant Power (CP)	C1 (f=0.05Hz)	2.0412E+07	1.1139E+08	2.4996E+07	2.9513E+00	5.7781E-01	4.3169
	C2 (f=0.01Hz)		2.2278E+07	9.9982E+05	8.9141E-01	2.3112E-02	
	C3 (f=0.1Hz)		2.2278E+08	9.9982E+09	5.5262E+00	2.3112E+00	
Constant Torque (CT)	C4 (f=0.05Hz)		1.1139E+08	2.4996E+07	2.5749E+00	5.7781E-01	
	C5 (f=0.01Hz)		2.2278E+07	9.9982E+05	5.1498E-01	2.3112E-02	
	C6 (f=0.1Hz)		2.2278E+08	9.9982E+07	5.1498E+00	2.3112E+00	
CP	C7 (f=0.05Hz)		0.9547E+08	2.4996E+07	2.58349	5.7781E-01	

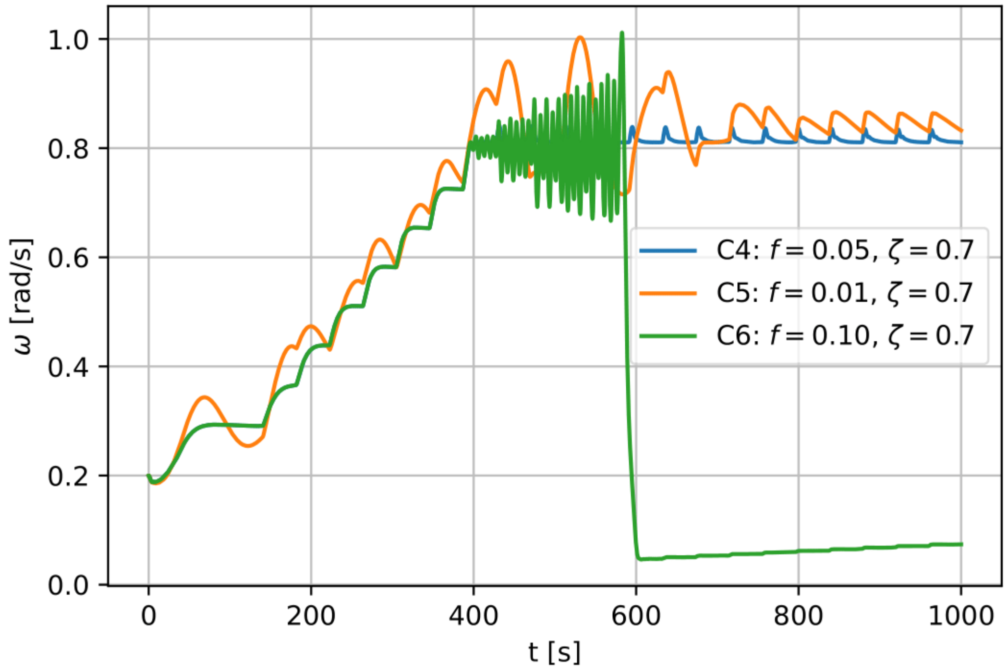
Constant Power Control

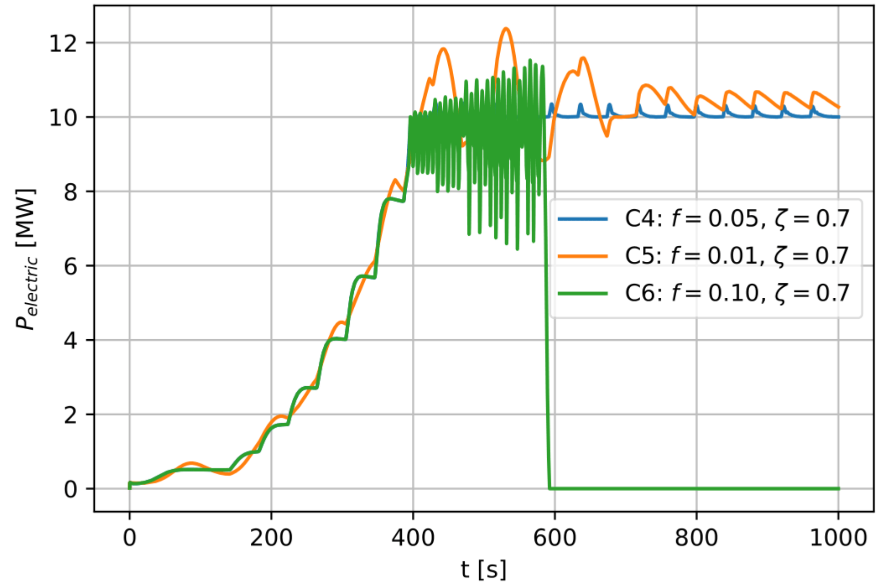


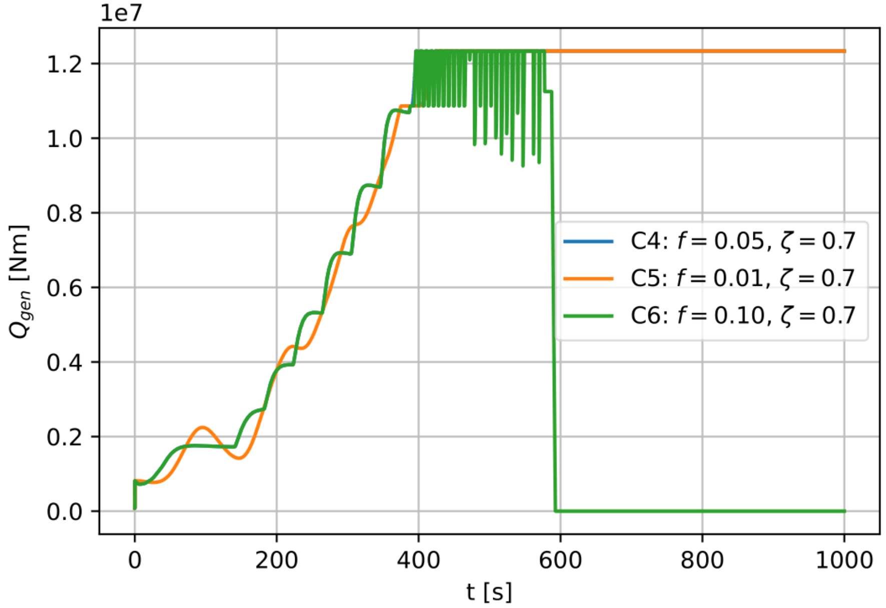




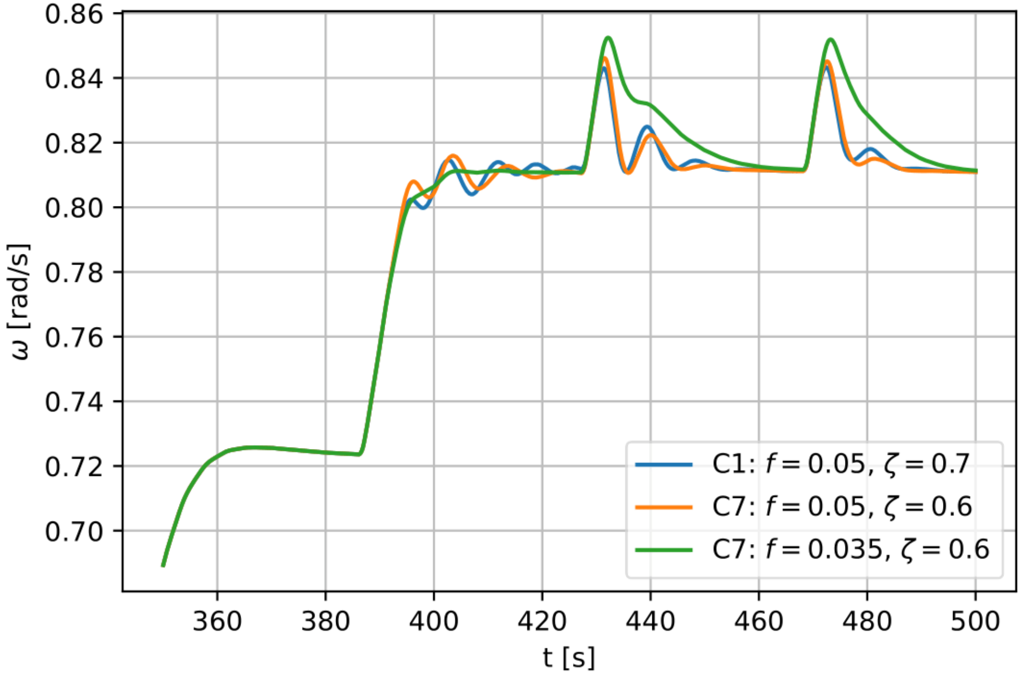
Constant Torque Control

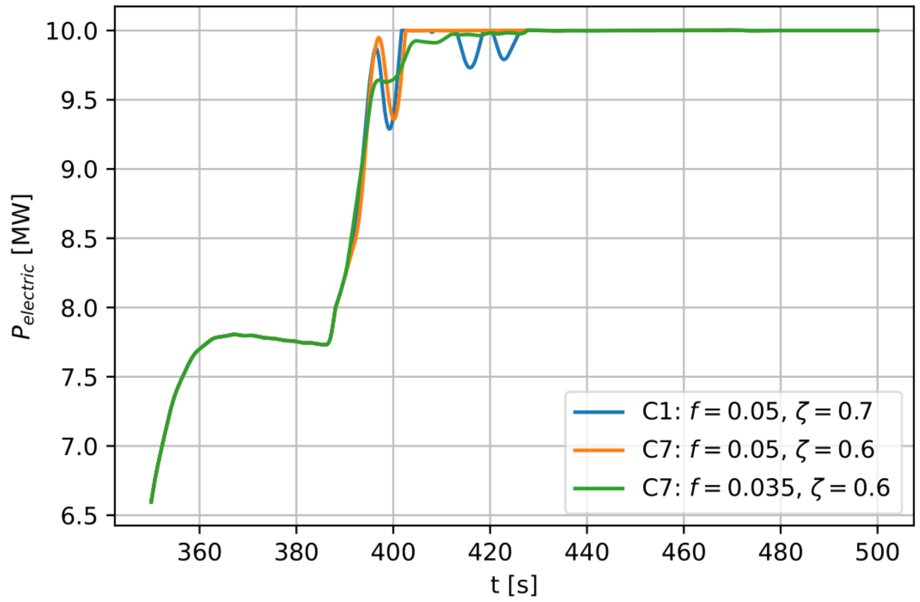


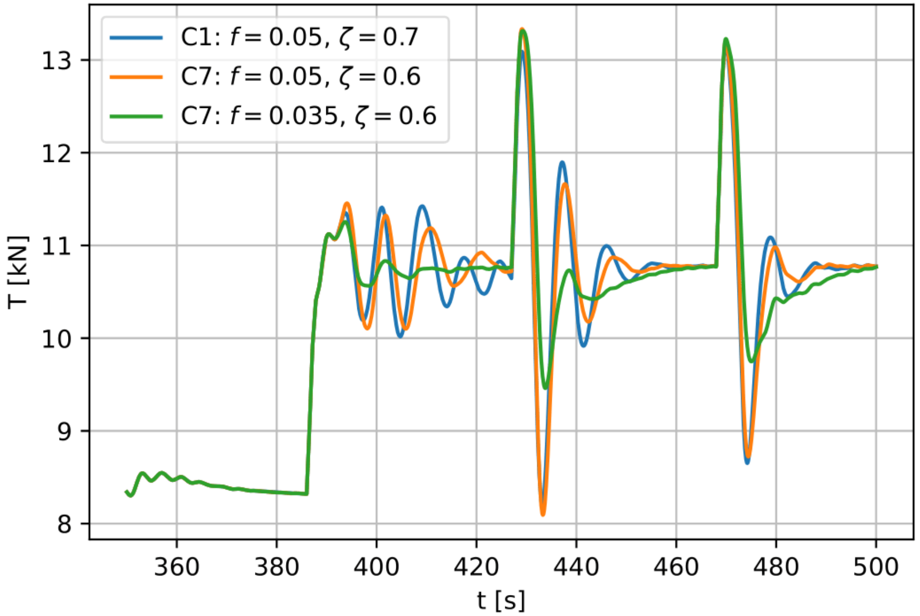




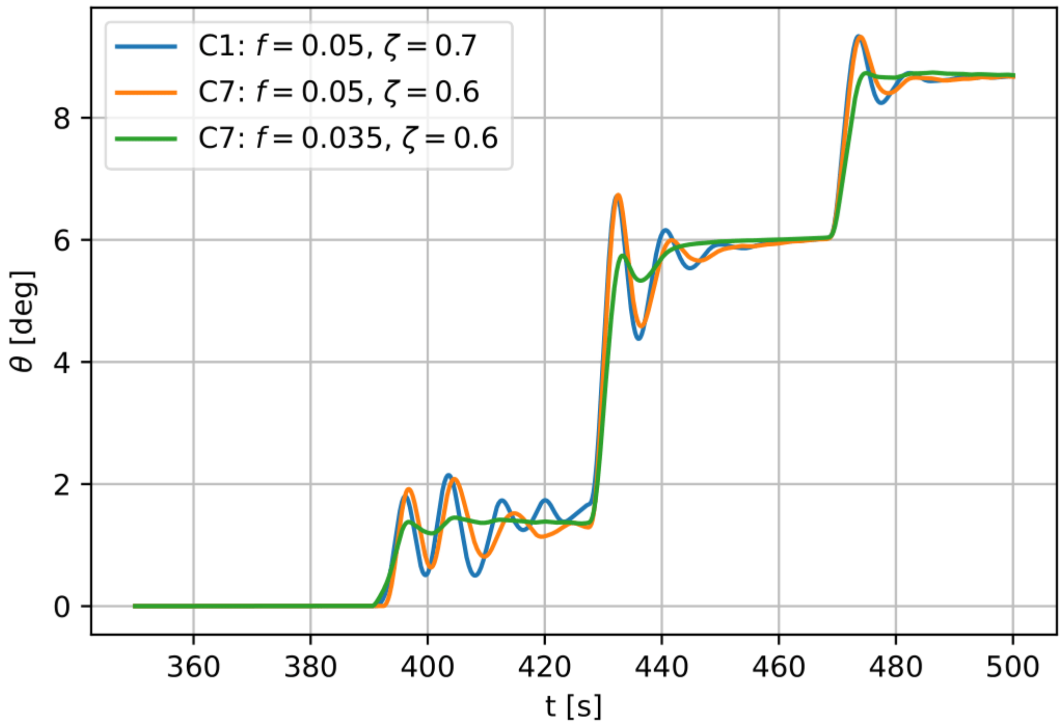
C7: Improved Controller Design



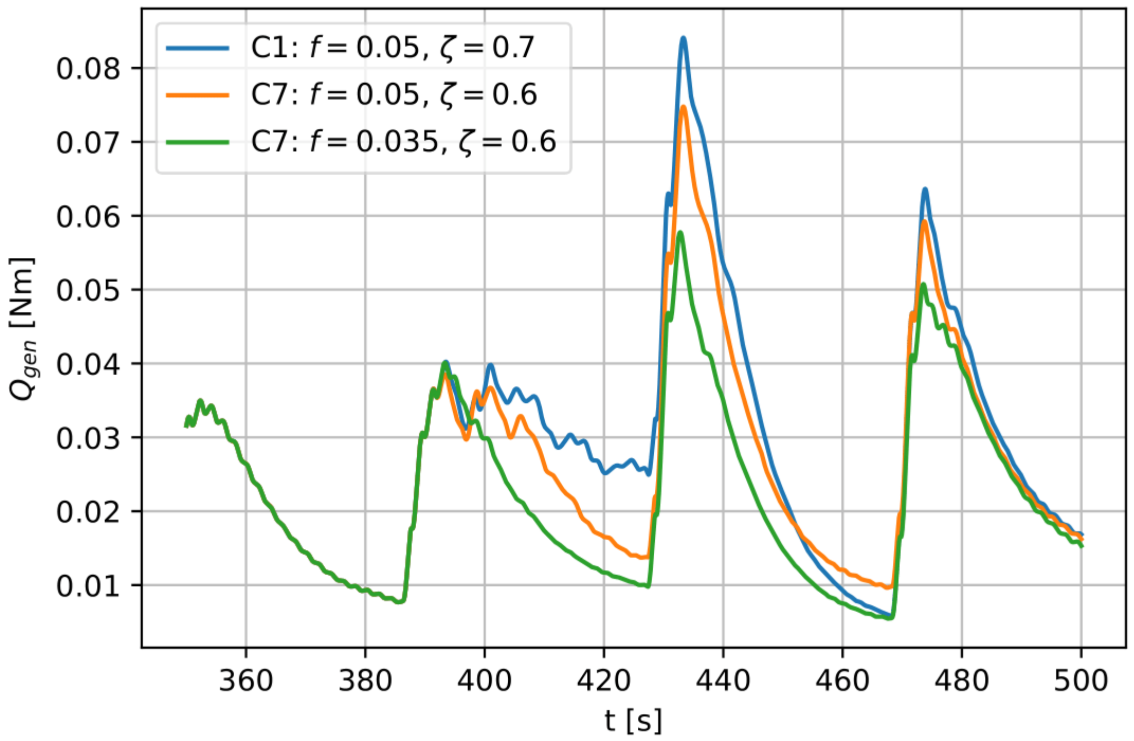




Case C7



C1: $f = 0.05$, $\zeta = 0.7$
C7: $f = 0.05$, $\zeta = 0.6$
C7: $f = 0.035$, $\zeta = 0.6$
θ [deg]
0
2
4
6
8
360
380
400
420
440
460
480
500
t [s]



C1: $f = 0.05$, $\zeta = 0.7$
C7: $f = 0.05$, $\zeta = 0.6$
C7: $f = 0.035$, $\zeta = 0.6$
Q_{gen} [Nm]
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
360
380
400
420
440
460
480
500
t [s]

Appendix

Task 1 - Derivations

Optimal Cp tracking K factor

$$K = \frac{\eta \rho A R^3 C_p\left(\theta_{\text{opt}}, \lambda_{\text{opt}}\right)}{2\lambda_{\text{opt}}^3}$$

$$= \frac{[-][kg.m^{-3}][m^2][m^3][-]}{2[-]}$$

$$= [kg][m^2]$$

$$= [kg][m][m][s^2][s^{-2}][rad^{-2}]$$

$$= \frac{[N.m]}{[rad.s^{-2}]}$$

Integral gain of torque Ctrl

$$k_{Ig} = \eta \left(I_r + n_g^2 I_g \right) \omega_{\Omega g}^2$$

$$k_{Ig} = [-] \left([\mathrm{kg.m^2}] + [-]^2 [\mathrm{kg.m^2}] \right) [\mathrm{rad\ s^{-1}}]^2$$

$$= \frac{[\mathrm{kg.rad^2.m^2}]}{[\mathrm{s^2}]} = [\mathrm{N.m.rad\ ^2}][\mathrm{rad^{-3}}] = \frac{[\mathrm{N.m}]}{[\mathrm{rad}]}$$

Proportional gain of torque Ctrl.

$$k_{Pg} = 2\eta\zeta_{\Omega g}\omega_{\Omega g}\left(I_r + n_g^2 I_g\right)$$

$$= 2[-][-][rad.\,s^{-1}][kg.\,m^2]$$

$$= [rad.\,s^{-1}][kg.\,m][m][s^2][s^{-2}]$$

$$= \frac{[N][m]}{rad.\,s^{-1}}$$

Integral gain of pitch Ctrl.

$$k_I = \frac{\omega_\Omega^2 \left(I_r + n_g^2 I_g\right)}{-\left.\frac{\partial Q}{\partial \theta}\right|_0}$$

$$= \frac{[rad/s]^2 \left([Kg - m^2] + [-]^2[Kg - m^2]\right)}{-[N - m/deg]}$$

$$= \frac{[rad/s]^2[Kg - m^2]}{-[Kg - m^2/s^2 * deg]}$$

$$= [rad^2/rad]$$

$$= [rad/rad]$$

Proportional gain of pitch Ctrl.

$$k_P = \frac{2\zeta_\Omega \omega_\Omega \left(I_r + n_g^2 I_g\right) - \frac{1}{\eta}\left.\frac{\partial Q_g}{\partial \Omega}\right|_0}{-\left.\frac{\partial Q}{\partial \theta}\right|_0}$$

$$k_P = \frac{[-][\mathrm{rad.s^{-1}}]\left([\mathrm{kg.m^2}] + [-]^2[\mathrm{kg.m^2}]\right) - \frac{1}{[-]}[\mathrm{kg.m^2.s^{-1}.rad^{-1}}]}{-[\mathrm{kg.m^2.s^{-2}.deg^{-1}}]}$$

$$= \frac{[\mathrm{rad}][\mathrm{s}]}{[\mathrm{rad}]}$$

Aerodynamic Gain

- Aerodynamic damping equation: $$\frac{\partial Q_A}{\partial \theta} = \left.\frac{\partial Q_A}{\partial \theta}\right|_{\theta=0} \left(1 + \frac{\theta}{KK_1} + \frac{\theta^2}{KK_2}\right)$$

- Aerodynamic gain is $\frac{\partial Q_A}{\partial \theta}$ and has units $\frac{Nm}{rad}$
- $KK1$ must have units rad
- $KK2$ must have units rad^2
- $\left.\frac{\partial Q_A}{\partial \theta}\right|_{\theta=0}$ also has units $\frac{Nm}{rad}$

For the same $\omega\Omega$, why is the kP different but kI is the same when using Constant Power and Constant Torque?

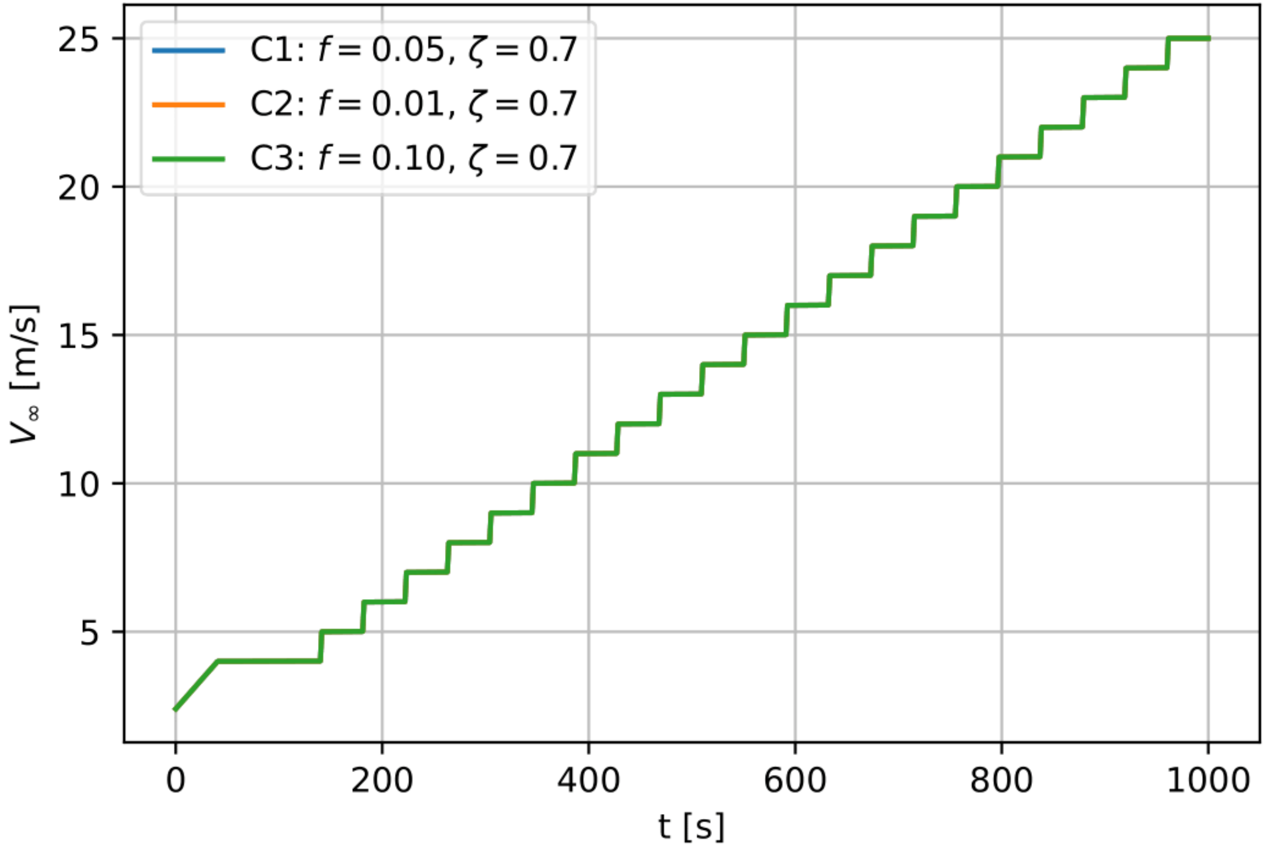
$$k_P = \frac{2\zeta_\Omega\omega_\Omega(I_r + n_g^2 I_g) - \frac{1}{\eta}\left.\frac{\partial Q_g}{\partial\Omega}\right|_0}{-\left.\frac{\partial Q}{\partial\theta}\right|_0};$$

It can be seen that the expression of kp depends on the partial derivative of the generator torque wrt. angular velocity. Therefore, this term is 0 for constant torque case but not for constant power.

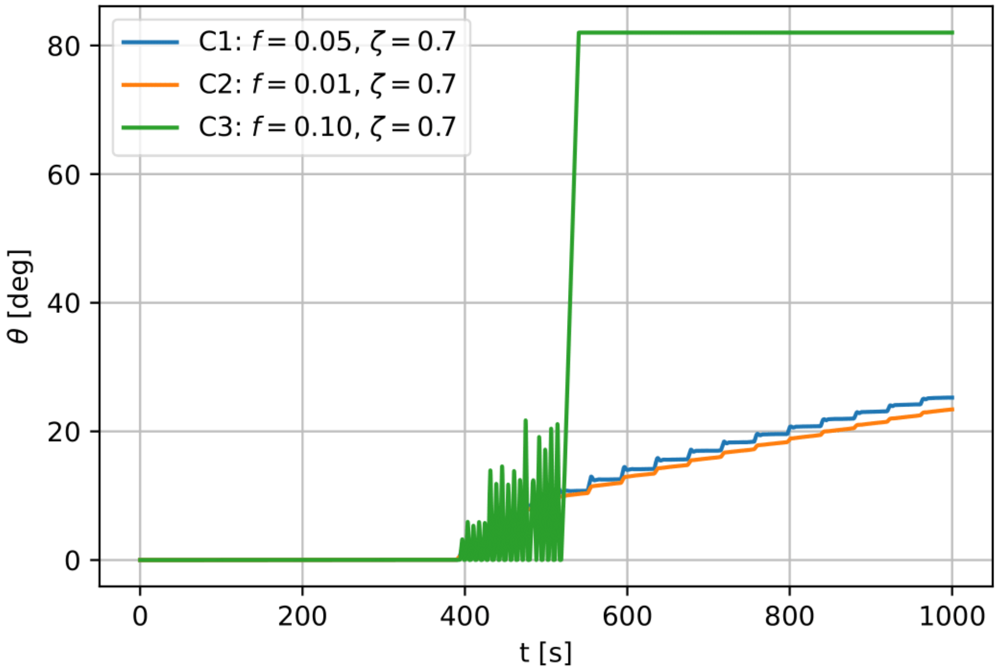
$$k_I = \frac{\omega_\Omega^2 \left(I_r + n_g^2 I_g\right)}{-\frac{\partial Q}{\partial \theta}\Big|_0}$$

The expression of kI does not depend on the partial derivative of the generator torque wrt. angular velocity, and so it's the same in both cases. (it just depends on the total inertia of the system, the frequency and the partial derivative of the the aerodynamic torque)

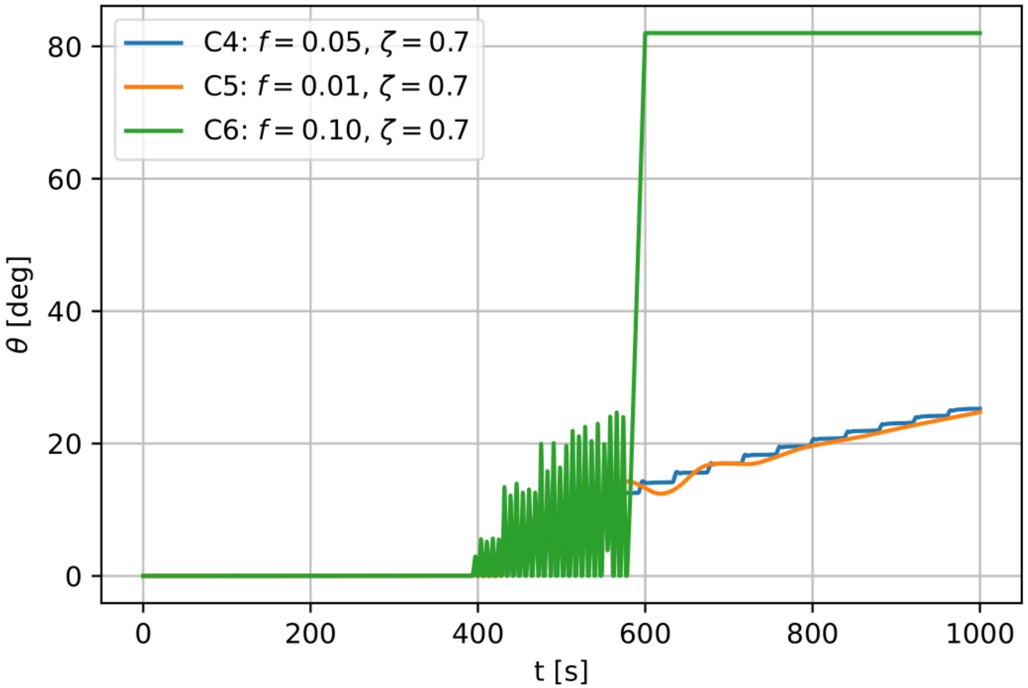
Step Wind Simulation



Pitch Angles



Constant Power



Constant Torque

Thanks for your attention