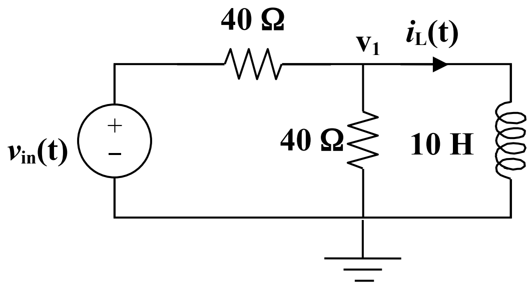
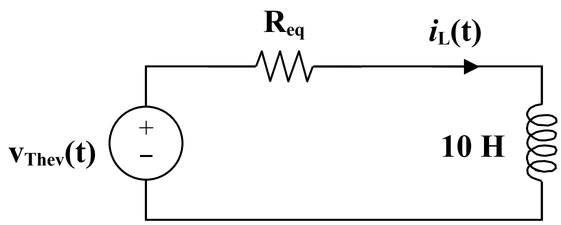
Chapter 7, Solution 64

Determine the value of $i_L(t)$ and the total energy dissipated by the circuit from $t = 0$ sec to $t = \infty$ sec. The value of $v_{in}(t)$ is equal to $[40–40u(t)]$ volts.





Solution

Step 1. Determine the Thevenin equivalent circuit to the left of the inductor. This means we need to find $v_{oc}(t)$ and $i_{sc}(t)$ which gives us $v_{Thev}(t) = v_{oc}(t)$ and $R_{eq} = v_{oc}(t)/i_{sc}(t)$ (note, this only works for resistor networks in the time domain). This leads to the second circuit shown above.

Now, with this circuit, we can use the generalized solution to a first order differential equation, $i_L(t) = Ae^{-(t-0)/\tau}+B$ where, $t_0 = 0$, $\tau = L/R$, $A+B = i_L(0)$ and $0+B = i_L(\infty)$. Finally, we can use $w = (1/2)Li_L(t)^2$ to calculate the energy dissipated by the circuit ($w = [(1/2)Li_L(\infty)^2–(1/2)Li_L(0)^2]$.

Step 2. We now determine the Thevenin equivalent circuit. First we need to pick a reference node and mark the unknown voltages, as seen above. With the inductor out of the circuit, the node equation is simply $[(v_1–v_{in}(t))/40] + [(v_1–0)/40] + 0$ (since the inductor is out of the circuit, there is an open circuit where it was) $= 0$. This leads to $[(1/40)+(1/40)]v_1 = (1/40)v_{in}(t)$ or $2v_1 = v_{in}(t)$ or $v_1 = 0.5v_{in}(t) = [20–20u(t)]$ $v_{oc}(t) = v_{Thev}(t)$. Now to short the open circuit which produces $v_1 = 0$ and $i_{sc} = –[(0–v_{in}(t))/40] = v_{in}(t)/40 = 0.025v_{in}(t)$ A.

Step 3. Now, everything comes together, $R_{eq} = v_{oc}(t)/i_{sc}(t) = 0.5v_{in}(t)/[0.025v_{in}(t)] = 0.5/0.025 = 20\ \Omega$. Next we find $\tau = L/R_{eq} = 10/20 = (1/2)$ sec. At $t = 0^-$, $v_{in}(0^-) = [10–0]$ V (note $u(t) = 0$ until $t = 0$). Since it has been at this value for a very long time, the inductor can be considered a short and the value of the current is equal to 20/20 or $i_L(0^-) = 1$ amp. Since you cannot change the current instantaneously, $i_L(0) = 1$ amp $= A+B$. Since $v_{Thev}(t) = 20–20 = 0$ for all $t > 0$, all the energy in the inductor will be dissipated by the circuit and $i_L(\infty) = 0 = B$ which means that $A = 1$ and $i_L(t) =$ **$[e^{-2t}]$ u(t) amps**. The total energy dissipated from $t = 0$ to ∞ sec is equal to $[(1/2)Li_L(0)^2–(1/2)Li_L(\infty)^2] = (0.5)10(1)^2–0 =$ **5 J**.