Chapter 10, Solution 31.

Use mesh analysis to determine current $\mathbf{I}_o$ in the circuit of Fig. 10.79 below.

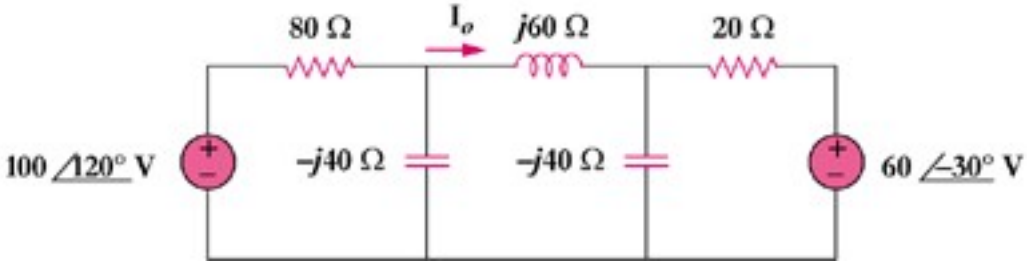
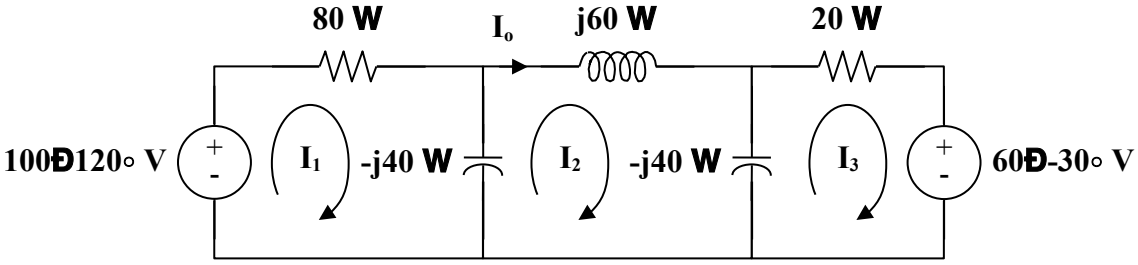


Figure 10.79
For Prob. 10.31.

Solution

Consider the network shown below.



For loop 1,

$$-100\angle 120^\circ + (80 - j40)\mathbf{I}_1 + j40\mathbf{I}_2 = 0$$
$$10\angle 20^\circ = 4(2 - j)\mathbf{I}_1 + j4\mathbf{I}_2 \qquad (1)$$

For loop 2,

$$j40\mathbf{I}_1 + (j60 - j80)\mathbf{I}_2 + j40\mathbf{I}_3 = 0$$
$$0 = 2\mathbf{I}_1 - \mathbf{I}_2 + 2\mathbf{I}_3 \qquad (2)$$

For loop 3,

$$60\angle -30^\circ + (20 - j40)\mathbf{I}_3 + j40\mathbf{I}_2 = 0$$
$$-6\angle -30^\circ = j4\mathbf{I}_2 + 2(1 - j2)\mathbf{I}_3 \qquad (3)$$

From (2),

$$2\mathbf{I}_3 = \mathbf{I}_2 - 2\mathbf{I}_1$$

Substituting this equation into (3),

$$-6\angle -30° = -2(1-j2)\mathbf{I}_1 + (1+j2)\mathbf{I}_2 \qquad (4)$$

From (1) and (4),

$$\begin{bmatrix} 10\angle 120° \\ -6\angle -30° \end{bmatrix} = \begin{bmatrix} 4(2-j) & j4 \\ -2(1-j2) & 1+j2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 8-j4 & -j4 \\ -2+j4 & 1+j2 \end{vmatrix} = 32 + j20 = 37.74\angle 32°$$

$$\Delta_2 = \begin{vmatrix} 8-j4 & 10\angle 120° \\ -2+j4 & -6\angle -30° \end{vmatrix} = -4.928 + j82.11 = 82.25\angle 93.44°$$

$$\mathbf{I}_o = \mathbf{I}_2 = \frac{\Delta_2}{\Delta} = \mathbf{2.179Ð61.44◦\ A}$$