Chapter 11, Solution 16.

For the circuit in Fig. 11.47, find the maximum power that can be delivered to the load \mathbf{Z}_L .

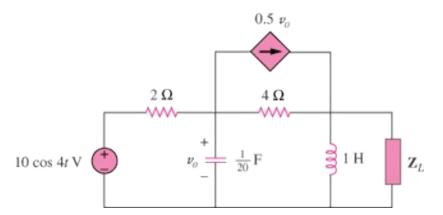
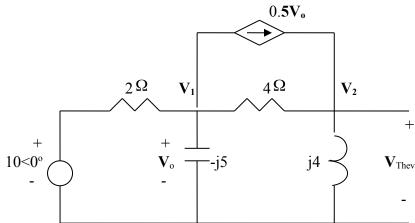


Figure 11.47 For Prob. 11.16.

Solution

$$\omega = 4$$
, 1H $\longrightarrow j\omega L = j4$, $1/20$ F $\longrightarrow \frac{1}{j\omega C} = \frac{1}{j4x1/20} = -j5$

We find the Thevenin equivalent at the terminals of Z_L . To find V_{Thev} , we use the circuit shown below.



At node 1,

$$\frac{10 - V_1}{2} = \frac{V_1}{-j5} + 0.5V_1 + \frac{V_1 - V_2}{4} \longrightarrow 5 = V_1(1.25 + j0.2) - 0.25V_2$$
 (1)

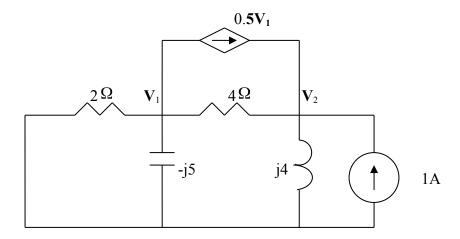
At node 2,

$$\frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow 0 = 0.5V_1 + V_2(-0.25 + j0.25)$$
(2)

Solving (1) and (2) leads to

$$V_{\textit{Thev}} = V_2 = 6.1947 + j7.0796 = 9.4072 \angle 48.81^\circ$$

To obtain R_{eq} , consider the circuit shown below. We replace Z_L by a 1-A current source.



At node 1,

$$\frac{V_1}{2} + \frac{V_1}{-j5} + 0.25V_1 + \frac{V_1 - V_2}{4} = 0 \longrightarrow 0 = V_1(1+j0.2) - 0.25V_2$$
(3)

At node 2,

$$1 + \frac{V_1 - V_2}{4} + 0.25V_1 = \frac{V_2}{j4} \longrightarrow -1 = 0.5V_1 + V_2(-0.25 + j0.25)$$
(4)

Solving (1) and (2) gives

$$Z_{eq} = \frac{V_2}{1} = 1.9115 + j3.3274 = 3.837 \angle 60.12^{\circ}$$
 and $\mathbf{Z}_L = 3.837 \angle -60.12^{\circ} \Omega$

$$P_{\text{max}} = \frac{|V_{Th}|^2}{2|Z_{eq} - Z_L|^2} 1.9115 = \frac{9.4072^2}{2x4x1.9115} = \frac{9.787 \text{ W}}{2x4x1.9115}$$