Chapter 10, Solution 61.

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.

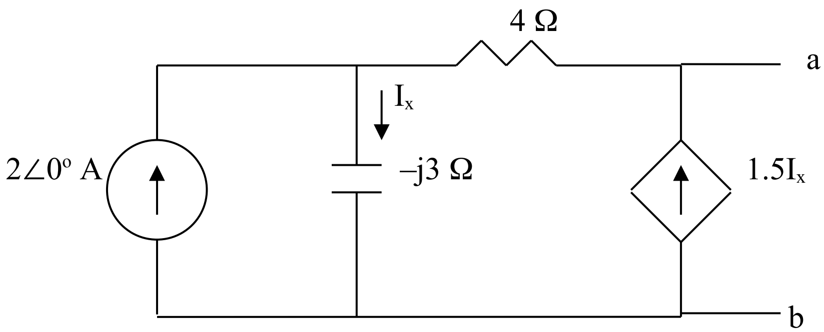

Figure 10.104
For Prob. 10.61.

Solution

Find the Thevenin equivalent at terminals a-b of the circuit in Fig. 10.104.

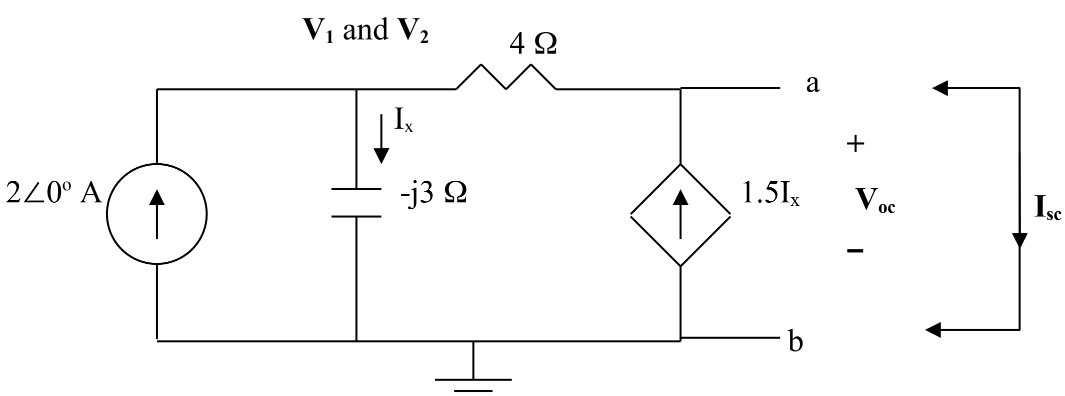

Figure 10.104
For Prob. 10.61.

Solution

Step 1. First we solve for the open circuit voltage using the above circuit and writing two node equations. Then we solve for the short circuit current which only need one node equation. For being able to solve for $\mathbf{V_{oc}}$, we need to solve these three equations,

$$-2 + [(\mathbf{V_1}-0)/(-j3)] + [(\mathbf{V_1}-\mathbf{V_{oc}})/4] = 0 \text{ and}$$

$$[(\mathbf{V_{oc}}-\mathbf{V_1})/4] - 1.5\mathbf{I_x} = 0 \text{ where } \mathbf{I_x} = [(\mathbf{V_1}-0)/(-j3)].$$

To solve for $\mathbf{I}_{sc}$, all we need to do is to solve these three equations,

$-2 + [(\mathbf{V}_2 - 0)/(-j3)] + [(\mathbf{V}_2 - 0)/4] = 0$, $\mathbf{I}_{sc} = [\mathbf{V}_2/4] + 1.5\mathbf{I}_x$, and

$\mathbf{I}_x = [\mathbf{V}_2/-j3]$.

Finally, $\mathbf{V}_{Thev} = \mathbf{V}_{oc}$ and $\mathbf{Z}_{eq} = \mathbf{V}_{oc}/\mathbf{I}_{sc}$.

Step 2. Now all we need to do is to solve for the unknowns. For $\mathbf{V}_{oc}$,

$\mathbf{I}_x = j0.33333\mathbf{V}_1$ and $(0.25+(1.5)(j0.33333))\mathbf{V}_1 = 0.25\mathbf{V}_{oc}$ or
$(0.25+j0.5)\mathbf{V}_1 = (0.55902\angle 63.43°)\mathbf{V}_1 = 0.25\mathbf{V}_{oc}$ or
$\mathbf{V}_1 = (0.44721\angle -63.43°)\mathbf{V}_{oc}$ which leads to,

$(0.25+j0.33333)\mathbf{V}_1 - 0.25\mathbf{V}_{oc} = 2$
$= (0.41666\angle +53.13°)(0.44721\angle -63.43°)V_{oc} - 0.25\mathbf{V}_{oc}$
$= (0.186335\angle -10.3°)\mathbf{V}_{oc} - 0.25\mathbf{V}_{oc} = (0.183333-0.25-j0.033333)\mathbf{V}_{oc}$
$= (-0.066667-j0.033333)\mathbf{V}_{oc} = (0.074536\angle -153.435°)\mathbf{V}_{oc} = 2$ or

$$\mathbf{V}_{oc} = \mathbf{V}_{Thev} = \mathbf{26.83\angle 153.44°\ V = (-24+j12)}\ \text{V}.$$

Now for $\mathbf{I}_{sc}$,

$\mathbf{I}_{sc} = [\mathbf{V}_2/4] + 1.5\mathbf{I}_x = (0.25+(1.5)(j0.33333))\mathbf{V}_2 = (0.25+j0.5)\mathbf{V}_2$.

$[(\mathbf{V}_2 - 0)/(-j3)] + [(\mathbf{V}_2 - 0)/4] = 2 = (0.25+j0.3333)\mathbf{V}_2$
$= (0.41667\angle 53.13°)\mathbf{V}_2 = 2$ or $\mathbf{V}_2 = 4.8\angle -53.13°$

$\mathbf{I}_{sc} = (0.25+j0.5)\mathbf{V}_2 = (0.55901\angle 63.435°)(4.8\angle -53.13°)$
$= 2.6832\angle 10.305°$ A

Finally,

$\mathbf{Z}_{eq} = \mathbf{V}_{oc}/\mathbf{I}_{sc} = 26.833\angle 153.435°/2.6832\angle 10.305°$

$$= \mathbf{10\angle 143.13°\ \Omega} \text{ or } = \mathbf{(-8+j6)\ \Omega}.$$