Chapter 7, Solution 75.

In the circuit of Fig. 7.140, find v_o and i_o, given that $v_s = 4\,u(t)$ V and $v(0) = 1$ V.

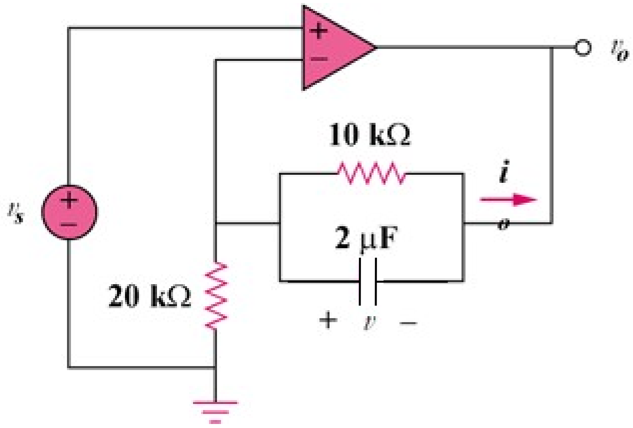

Figure 7.140
For Prob. 7.75.

Solution

Let v_1 = voltage at the noninverting terminal.

Let v_2 = voltage at the inverting terminal.

For t > 0, $v_1 = v_2 = v_s = 4$

$$\frac{0 - v_s}{R_1} = i_o, \qquad R_1 = 20\text{ k}\Omega$$

$$v_o = -i_o R \tag{1}$$

Also, $$i_o = \frac{v}{R_2} + C\frac{dv}{dt}, \qquad R_2 = 10\text{ k}\Omega, \quad C = 2\ \mu\text{F}$$

i.e. $$\frac{-v_s}{R_1} = \frac{v}{R_2} + C\frac{dv}{dt} \tag{2}$$

This is a step response.

$$v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}, \qquad v(0) = 1$$

where $$\tau = R_2 C = (10\times10^3)(2\times10^{-6}) = \frac{1}{50}$$

At steady state, the capacitor acts like an open circuit so that i_o passes through R_2. Hence, as $t \to \infty$

$$\frac{-v_s}{R_1} = i_o = \frac{v(\infty)}{R_2}$$

i.e.
$$v(\infty) = \frac{-R_2}{R_1} v_s = \frac{-10}{20}(4) = -2$$

$$v(t) = -2 + (1+2)e^{-50t}$$
$$v(t) = -2 + 3e^{-50t}$$

But
$$v = v_s - v_o$$

or
$$v_o = v_s - v = 4 + 2 - 3e^{-50t}$$
$$v_o = \mathbf{6 - 3}\boldsymbol{e^{-50t}}\ \boldsymbol{u(t)V}$$

= [6 – 3e–50t]

$$i_o = \frac{-v_s}{R_1} = \frac{-4}{20k} = -0.2 \text{ mA}$$

or
$$i_o = \frac{v}{R_2} + C\frac{dv}{dt} = \mathbf{-0.2\ mA}$$