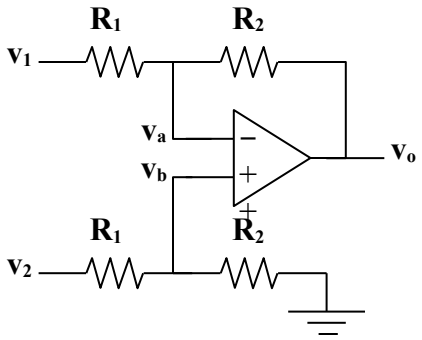
Chapter 5, Solution 53.

(a)



At node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \longrightarrow v_a = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2} \qquad (1)$$

At node b, $$v_b = \frac{R_2}{R_1 + R_2} v_2 \qquad (2)$$

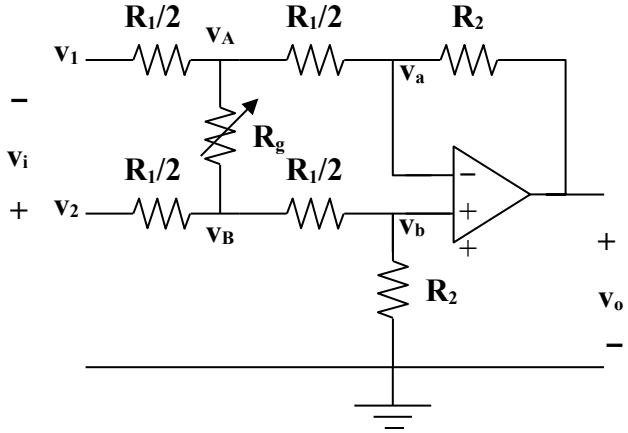
But $v_a = v_b$. Setting (1) and (2) equal gives

$$\frac{R_2}{R_1 + R_2} v_2 = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2}$$

$$v_2 - v_1 = \frac{R_1}{R_2} v_o = v_i$$

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

(b)



At node A, $$\frac{v_1 - v_A}{R_1/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_a}{R_1/2}$$

or $$v_1 - v_A + \frac{R_1}{2R_g}(v_B - v_A) = v_A - v_a \qquad (1)$$

At node B, $$\frac{v_2 - v_B}{R_1/2} = \frac{v_B - v_A}{R_1/2} + \frac{v_B - v_b}{R_g}$$

or $$v_2 - v_B - \frac{R_1}{2R_g}(v_B - v_A) = v_B - v_b \qquad (2)$$

Subtracting (1) from (2),

$$v_2 - v_1 - v_B + v_A - \frac{2R_1}{2R_g}(v_B - v_A) = v_B - v_A - v_b + v_a$$

Since, $v_a = v_b$,

$$\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_g}\right)(v_B - v_A) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{v_i}{2} \cdot \frac{1}{1 + \frac{R_1}{2R_g}} \qquad (3)$$

But for the difference amplifier,

$$v_o = \frac{R_2}{R_1/2}(v_B - v_A)$$

or $$v_B - v_A = \frac{R_1}{2R_2} v_o \qquad (4)$$

Equating (3) and (4), $$\frac{R_1}{2R_2} v_o = \frac{v_i}{2} \cdot \frac{1}{1 + \frac{R_1}{2R_g}}$$

$$\frac{v_o}{v_i} = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{R_1}{2R_g}}$$

(c) At node a, $$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_A}{R_2/2}$$

$$v_1 - v_a = \frac{2R_1}{R_2} v_a - \frac{2R_1}{R_2} v_A \qquad (1)$$

At node b, $$v_2 - v_b = \frac{2R_1}{R_2} v_b - \frac{2R_1}{R_2} v_B \qquad (2)$$

Since $v_a = v_b$, we subtract (1) from (2),

$$v_2 - v_1 = \frac{-2R_1}{R_2}(v_B - v_A) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{-R_2}{2R_1} v_i \qquad (3)$$

At node A,

$$\frac{v_a - v_A}{R_2/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_o}{R/2}$$

$$v_a - v_A + \frac{R_2}{2R_g}(v_B - v_A) = v_A - v_o \qquad (4)$$

At node B, $$\frac{v_b - v_B}{R/2} - \frac{v_B - v_A}{R_g} = \frac{v_B - 0}{R/2}$$

$$v_b - v_B - \frac{R_2}{2R_g}(v_B - v_A) = v_B \qquad (5)$$

Subtracting (5) from (4),

$$v_B - v_A + \frac{R_2}{R_g}(v_B - v_A) = v_A - v_B - v_o$$

$$2(v_B - v_A)\left(1 + \frac{R_2}{2R_g}\right) = -v_o \qquad (6)$$

Combining (3) and (6),

$$\frac{-R_2}{R_1} v_i \left(1 + \frac{R_2}{2R_g}\right) = -v_o$$

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_g}\right)$$