Chapter 7, Solution 85.

(a) The light is on from 75 volts until 30 volts. During that time we essentially have a 120-ohm resistor in parallel with a 6-μF capacitor.

$v(0) = 75$, $v(\infty) = 0$, $\tau = 120\text{x}6\text{x}10^{-6} = 0.72$ ms

$v(t_1) = 75\, e^{-t_1/\tau} = 30$ which leads to $t_1 = -0.72\ln(0.4)$ ms = **659.7 μs** of lamp on time.

(b) $\tau = RC = (4\times10^{6})(6\times10^{-6}) = 24$ s

Since $v(t) = v(\infty) + \left[\, v(0) - v(\infty)\right] e^{-t/\tau}$

$$v(t_1) - v(\infty) = \left[\, v(0) - v(\infty)\right] e^{-t_1/\tau} \qquad (1)$$

$$v(t_2) - v(\infty) = \left[\, v(0) - v(\infty)\right] e^{-t_2/\tau} \qquad (2)$$

Dividing (1) by (2),

$$\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)} = e^{(t_2 - t_1)/\tau}$$

$$t_0 = t_2 - t_1 = \tau \ln\left(\frac{v(t_1) - v(\infty)}{v(t_2) - v(\infty)}\right)$$

$$t_0 = 24\ln\left(\frac{75-120}{30-120}\right) = 24\ln(2) = \mathbf{16.636\ s}$$