**Chapter 11, Solution 20.**

The load resistance $R_L$ in Fig. 11.51 is adjusted until it absorbs the maximum average power. Calculate the value of $R_L$ and the maximum average power.

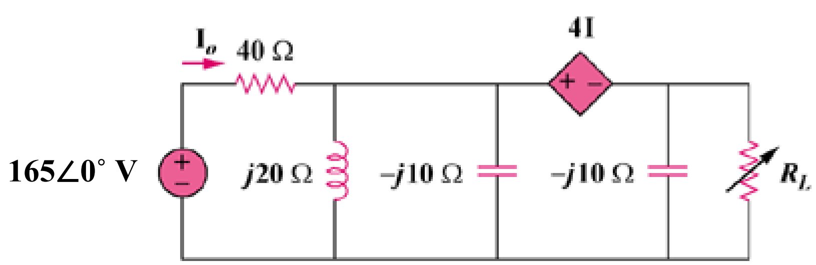
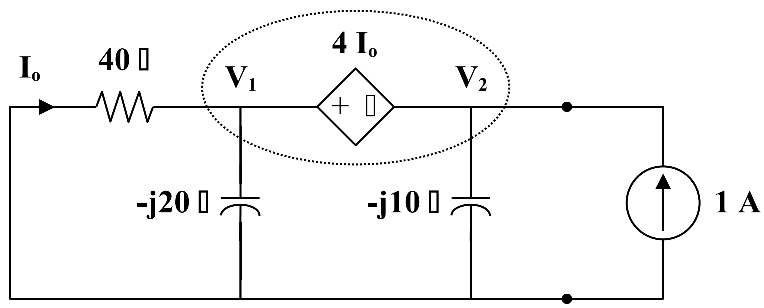



Figure 11.51
For Prob. 11.20.

**Solution**

Combine j20 Ω and -j10 Ω to get $j20 \parallel -j10 = -j20$.

To find $\mathbf{Z}_{Th}$, insert a 1-A current source at the terminals of $R_L$, as shown in Fig. (a).




**(a)**

At the supernode,

$$1 = \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{-j20} + \frac{\mathbf{V}_2}{-j10}$$

$$40 = (1 + j2)\,\mathbf{V}_1 + j4\,\mathbf{V}_2 \qquad (1)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \dfrac{-\mathbf{V}_1}{40}$

$$1.1\mathbf{V}_1 = \mathbf{V}_2 \longrightarrow \mathbf{V}_1 = \frac{\mathbf{V}_2}{1.1} \qquad (2)$$

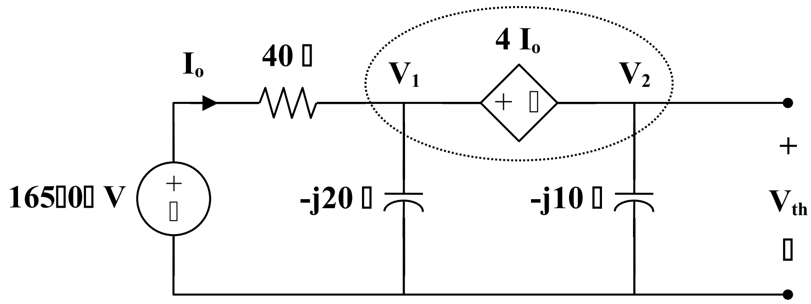
Substituting (2) into (1),

$$40 = (1 + j2)\left(\frac{\mathbf{V}_2}{1.1}\right) + j4\,\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{44}{1 + j6.4}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{1} = 1.05 - j6.71\,\Omega$$

$$R_L = \left|\mathbf{Z}_{Th}\right| = \mathbf{6.792\,\Omega}$$




(b)

To find $\mathbf{V}_{Th}$, consider the circuit in Fig. (b).

At the supernode,

$$\frac{165 - \mathbf{V}_1}{40} = \frac{\mathbf{V}_1}{-j20} + \frac{\mathbf{V}_2}{-j10}$$

$$165 = (1 + j2)\mathbf{V}_1 + j4\mathbf{V}_2 \qquad (3)$$

Also, $\mathbf{V}_1 = \mathbf{V}_2 + 4\mathbf{I}_o$, where $\mathbf{I}_o = \dfrac{165 - \mathbf{V}_1}{40}$

$$\mathbf{V}_1 = \frac{\mathbf{V}_2 + 16.5}{1.1} \qquad (4)$$

Substituting (4) into (3),

$$150 - j30 = (0.9091 + j5.818)\mathbf{V}_2$$

$$\mathbf{V}_{Th} = \mathbf{V}_2 = \frac{150 - j30}{0.9091 + j5.818} = \frac{152.97\angle -11.31^\circ}{5.889\angle 81.12^\circ} = 25.98\angle -92.43^\circ$$

$$P_{max} = \left|\frac{25.98}{1.05 - j6.71 + 6.792}\right|^2 \frac{6.792}{2} = \mathbf{21.51\ W}$$