Chapter 7, Solution 73.

For the op amp circuit of Fig. 7.138, let $R_1 = 10$ kΩ, $R_f = 20$ kΩ, $C = 20\ \mu F$, and $v(0) = 1$ V. Find v_o.

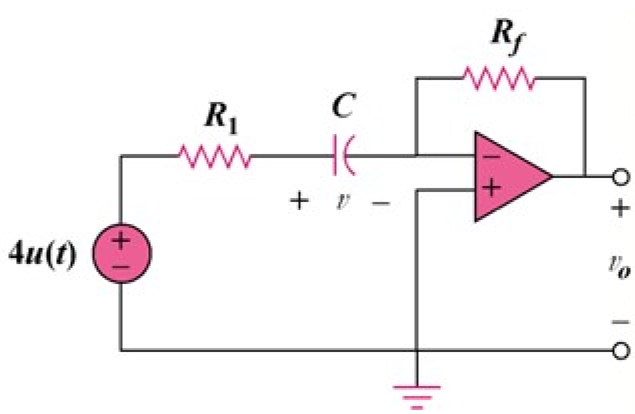
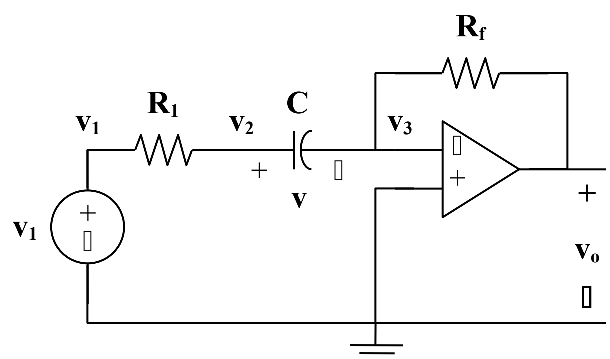


Figure 7.138
For Prob. 7.73.

Solution

Consider the circuit below.



At node 2,

$$\frac{v_1 - v_2}{R_1} = C\frac{dv}{dt} \qquad (1)$$

At node 3,

$$C\frac{dv}{dt} = \frac{v_3 - v_o}{R_f} \qquad (2)$$

But $v_3 = 0$ and $v = v_2 - v_3 = v_2$. Hence, (1) becomes

$$\frac{v_1 - v}{R_1} = C\frac{dv}{dt}$$

$$v_1 - v = R_1 C\frac{dv}{dt}$$

or

$$\frac{dv}{dt} + \frac{v}{R_1 C} = \frac{v_1}{R_1 C}$$

which is similar to Eq. (7.42). Hence,

$$v(t) = \begin{cases} v_T & t < 0 \\ v_1 + \left(v_T - v_1\right)e^{-t/\tau} & t > 0 \end{cases}$$

where $v_T = v(0) = 1$ and $v_1 = 4$

$$\tau = R_1 C = (10\times10^3)(20\times10^{-6}) = 0.2$$

$$v(t) = \begin{cases} 1 & t < 0 \\ 4 - 3e^{-5t} & t > 0 \end{cases}$$

From (2),

$$v_o = -R_f C\frac{dv}{dt} = (20\times10^3)(20\times10^{-6})(15e^{-5t})$$

$$v_o = -6e^{-5t}, \quad t > 0$$

$$v_o = -6e^{-5t}u(t)V$$