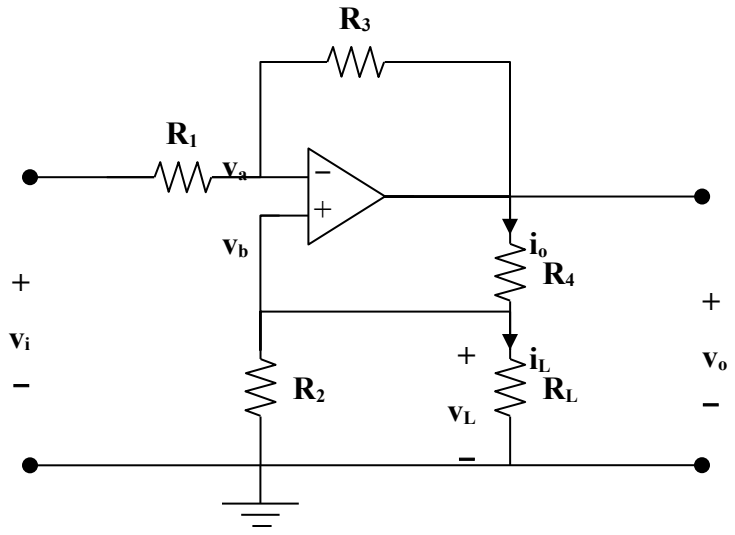
Chapter 5, Solution 93.



At node a, $(v_i - v_a)/R_1 = (v_a - v_o)/R_3$

$$v_i - v_a = (R_1/R_3)(v_a - v_o)$$

$$v_i + (R_1/R_3)v_o = (1 + R_1/R_3)v_a \qquad (1)$$

But $v_a = v_b = v_L$. Hence, (1) becomes

$$v_i = (1 + R_1/R_3)v_L - (R_1/R_3)v_o \qquad (2)$$

$i_o = v_o/(R_4 + R_2||R_L)$, $i_L = (R_2/(R_2 + R_L))i_o = (R_2/(R_2 + R_L))(v_o/(R_4 + R_2||R_L))$

Or, $$v_o = i_L[(R_2 + R_L)(R_4 + R_2||R_L)/R_2 \qquad (3)$$

But, $$v_L = i_L R_L \qquad (4)$$

Substituting (3) and (4) into (2),

$$v_i = (1 + R_1/R_3)\, i_L R_L - R_1[(R_2 + R_L)/(R_2 R_3)](R_4 + R_2||R_L)i_L$$

$$= [((R_3 + R_1)/R_3)R_L - R_1((R_2 + R_L)/(R_2 R_3)(R_4 + (R_2 R_L/(R_2 + R_L))]i_L$$

$$= (1/A)i_L$$

Thus,

$$A = \frac{1}{\left(1+\frac{R_1}{R_3}\right)R_L - R_1\left(\frac{R_2+R_L}{R_2R_3}\right)\left(R_4+\frac{R_2R_L}{R_2+R_L}\right)}$$

Please note that A has the units of mhos. An easy check is to let every resistor equal 1-ohm and v_i equal to one amp. Going through the circuit produces $i_L = 1A$. Plugging into the above equation produces the same answer so the answer does check.