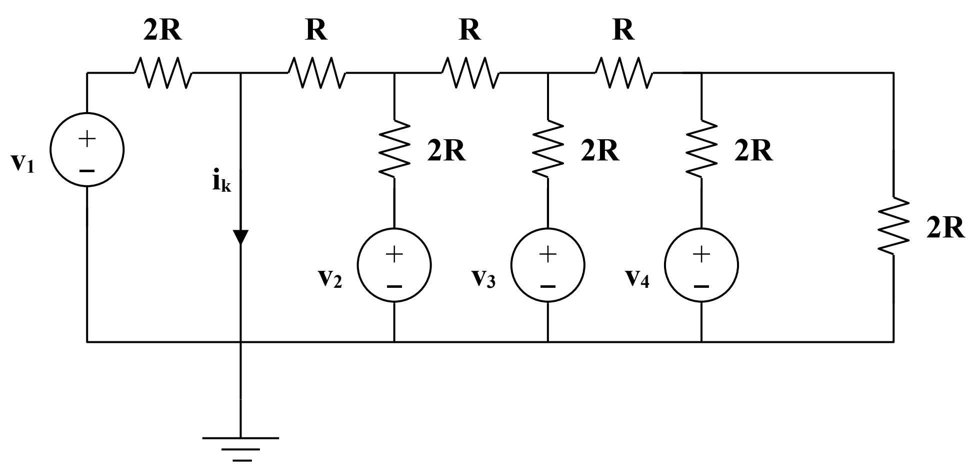
Chapter 5, Solution 84.

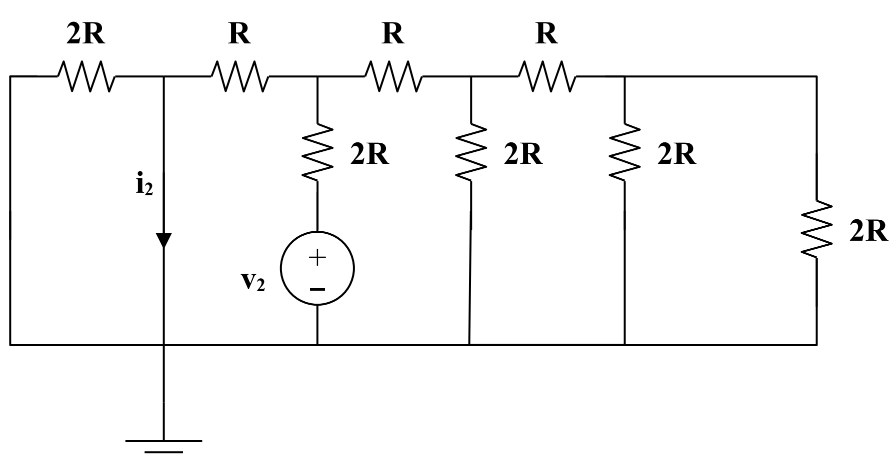
(a) The easiest way to solve this problem is to use superposition and to solve for each term letting all of the corresponding voltages be equal to zero. Also, starting with each current contribution (i_k) equal to one amp and working backwards is easiest.



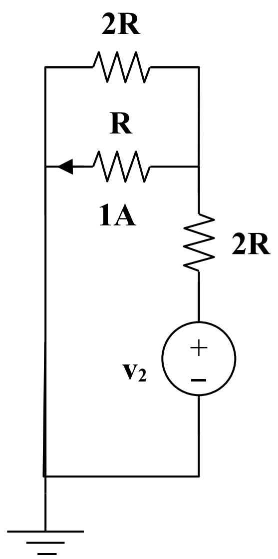
For the first case, let $v_2 = v_3 = v_4 = 0$, and $i_1 = 1A$.

Therefore, $v_1 = 2R$ volts or $i_1 = v_1/(2R)$.

Second case, let $v_1 = v_3 = v_4 = 0$, and $i_2 = 1A$.

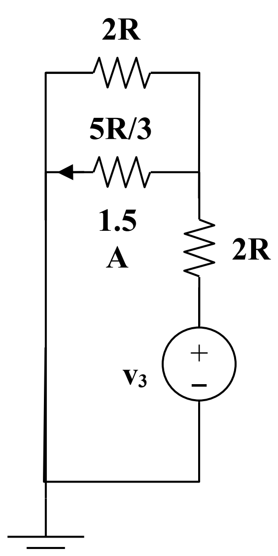


Simplifying, we get,



Therefore, $v_2 = 1xR + (3/2)(2R) = 4R$ volts or $i_2 = v_2/(4R)$ or $i_2 = 0.25v_2/R$. Clearly this is equal to the desired $1/4^{th}$.

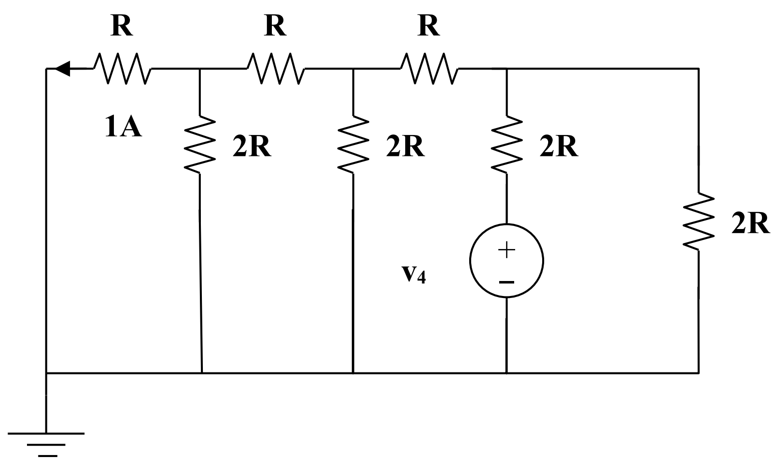
Now for the third case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1A$.

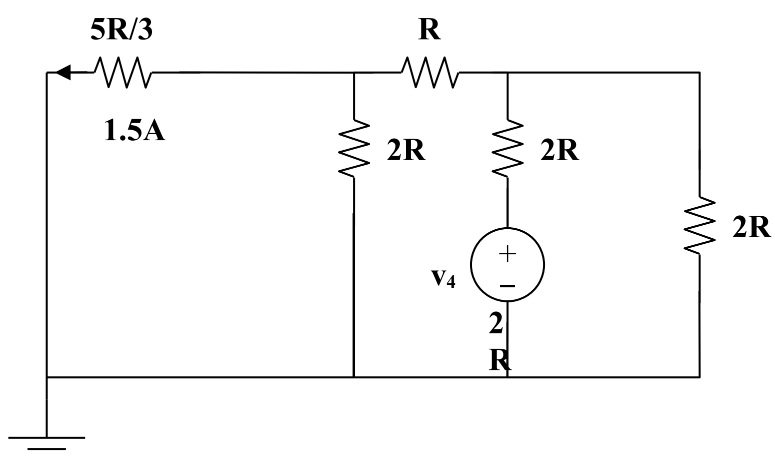


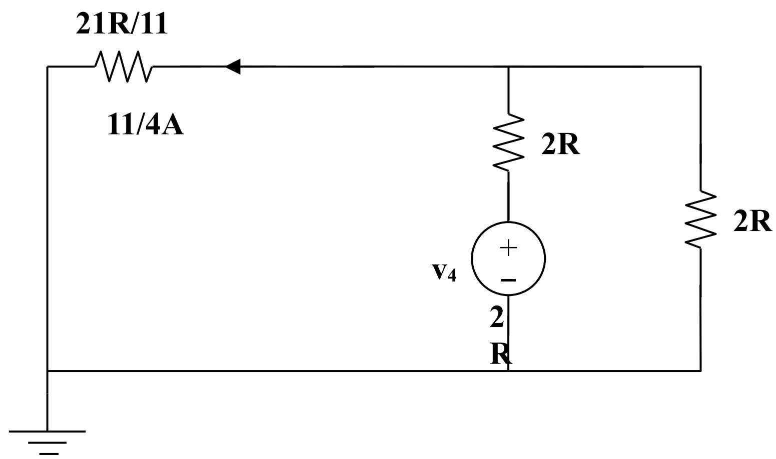
The voltage across the 5R/3-ohm resistor is 5R/2 volts. The current through the 2R resistor at the top is equal to (5/4) A and the current through the 2R-ohm resistor in series with the source is (3/2) + (5/4) = (11/4) A. Thus,

$v_3 = (11/2)R + (5/2)R = (16/2)R = 8R$ volts or $i_3 = v_3/(8R)$ or $0.125v_3/R$. Again, we have the desired result.

For the last case, $v_1 = v_2 = v_3$ and $i_4 = 1A$. Simplifying the circuit we get,







Since the current through the equivalent 21R/11-ohm resistor is (11/4) amps, the voltage across the 2R-ohm resistor on the right is (21/4)R volts. This means the current going through the 2R-ohm resistor is (21/8) A. Finally, the current going through the 2R resistor in series with the source is ((11/4)+(21/8)) = (43/8) A.

Now, $v_4 = (21/4)R + (86/8)R = (128/8)R = 16R$ volts or $i_4 = v_4/(16R)$ or $0.0625v_4/R$. This is just what we wanted.

(b) If $R_f = 12$ k ohms and $R = 10$ k ohms,

$$-v_o = (12/20)[v_1 + (v_2/2) + (v_3/4) + (v_4/8)]$$

$$= 0.6[v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4]$$

For $[v_1\ v_2\ v_3\ v_4] = [1\ 0\ 11]$,

$$|v_o| = 0.6[1 + 0.25 + 0.125] = \mathbf{825\ mV}$$

For $[v_1\ v_2\ v_3\ v_4] = [0\ 1\ 0\ 1]$,

$$|v_o| = 0.6[0.5 + 0.125] = \mathbf{375\ mV}$$