

Chapter 9, Solution 85.

$$\text{Let } \mathbf{Z}_1 = R_1, \quad \mathbf{Z}_2 = R_2 + \frac{1}{j\omega C_2}, \quad \mathbf{Z}_3 = R_3, \text{ and } \mathbf{Z}_4 = R_4 \parallel \frac{1}{j\omega C_4}.$$

$$\mathbf{Z}_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{-jR_4}{\omega R_4 C_4 - j}$$

$$\text{Since } \mathbf{Z}_4 = \frac{\mathbf{Z}_3}{\mathbf{Z}_1} \mathbf{Z}_2 \longrightarrow \mathbf{Z}_1 \mathbf{Z}_4 = \mathbf{Z}_2 \mathbf{Z}_3,$$

$$\begin{aligned} \frac{-jR_4 R_1}{\omega R_4 C_4 - j} &= R_3 \left(R_2 - \frac{j}{\omega C_2} \right) \\ \frac{-jR_4 R_1 (\omega R_4 C_4 + j)}{\omega^2 R_4^2 C_4^2 + 1} &= R_3 R_2 - \frac{jR_3}{\omega C_2} \end{aligned}$$

Equating the real and imaginary components,

$$\frac{R_1 R_4}{\omega^2 R_4^2 C_4^2 + 1} = R_2 R_3 \quad (1)$$

$$\frac{\omega R_1 R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1} = \frac{R_3}{\omega C_2} \quad (2)$$

Dividing (1) by (2),

$$\begin{aligned} \frac{1}{\omega R_4 C_4} &= \omega R_2 C_2 \\ \omega^2 &= \frac{1}{R_2 C_2 R_4 C_4} \\ \omega = 2\pi f &= \frac{1}{\sqrt{R_2 C_2 R_4 C_4}} \\ \mathbf{f} &= \frac{1}{2\pi \sqrt{R_2 R_4 C_2 C_4}} \end{aligned}$$