Chapter 10, Solution 48.

Find i_o in the circuit in Fig. 10.93 using superposition.

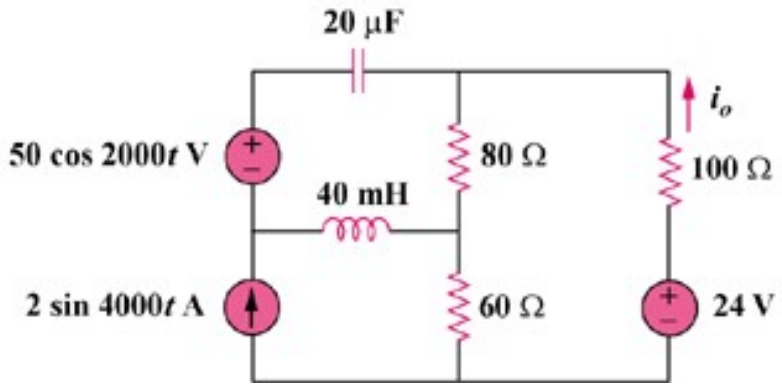


Figure 10.93
For Prob. 10.48.

Solution

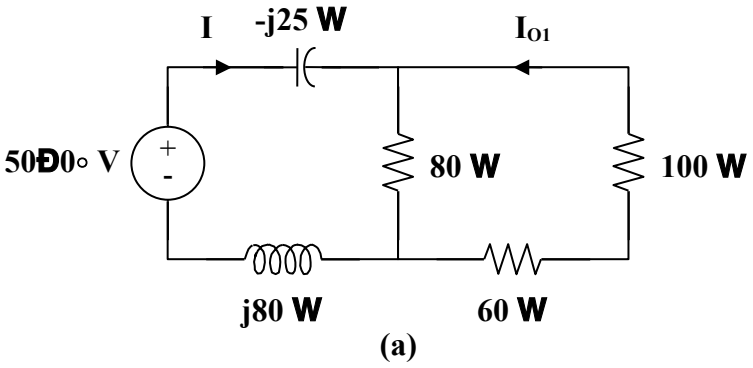
Let $i_O = i_{O1} + i_{O2} + i_{O3}$, where i_{O1} is due to the ac voltage source, i_{O2} is due to the dc voltage source, and i_{O3} is due to the ac current source. For i_{O1}, consider the circuit in Fig. (a).

$$\omega = 2000$$

$$50\cos(2000t) \longrightarrow 50\angle 0°$$

$$40 \text{ mH} \longrightarrow j\omega L = j(2000)(40\times10^{-3}) = j80$$

$$20 \ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2000)(20\times10^{-6})} = -j25$$



(a)

$$80 \parallel (60 + 100) = 160/3$$

$$\mathbf{I} = \frac{50}{160/3 + j80 - j25} = \frac{30}{32 + j33}$$

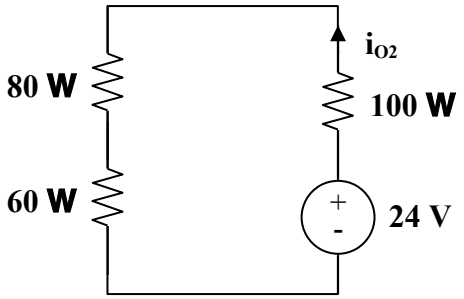
Using current division,

$$\mathbf{I}_{O1} = \frac{-80\mathbf{I}}{80+160} = \frac{-1}{3}\mathbf{I} = \frac{10\angle 180^\circ}{46\angle 45.9^\circ}$$

$$\mathbf{I}_{O1} = 0.217\angle 134.1^\circ$$

Hence,
$$i_{O1} = 0.217\cos(2000t + 134.1^\circ)\ \text{A}$$

For i_{O2}, consider the circuit in Fig. (b).



(b)

$$i_{O2} = \frac{24}{80 + 60 + 100} = 0.1\ \text{A}$$

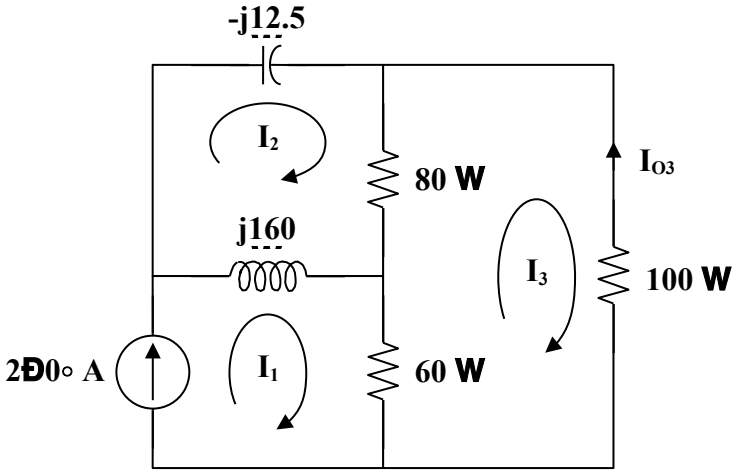
For i_{O3}, consider the circuit in Fig. (c).

$$\omega = 4000$$

$$2\cos(4000t) \longrightarrow 2\angle 0^\circ$$

$$40\ \text{mH} \longrightarrow j\omega L = j(4000)(40\times 10^{-3}) = j160$$

$$20\ \mu\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(4000)(20\times 10^{-6})} = -j12.5$$



(c)

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For mesh 1,

$$\mathbf{I}_1 = 2 \qquad (1)$$

For mesh 2,

$$(80 + j160 - j12.5)\mathbf{I}_2 - j160\mathbf{I}_1 - 80\mathbf{I}_3 = 0$$

Simplifying and substituting (1) into this equation yields

$$(8 + j14.75)\mathbf{I}_2 - 8\mathbf{I}_3 = j32 \qquad (2)$$

For mesh 3,

$$240\mathbf{I}_3 - 60\mathbf{I}_1 - 80\mathbf{I}_2 = 0$$

Simplifying and substituting (1) into this equation yields

$$\mathbf{I}_2 = 3\mathbf{I}_3 - 1.5 \qquad (3)$$

Substituting (3) into (2) yields

$$(16 + j44.25)\mathbf{I}_3 = 12 + j54.125$$

$$\mathbf{I}_3 = \frac{12 + j54.125}{16 + j44.25} = 1.1782\angle 7.38°$$

$$\mathbf{I}_{O3} = -\mathbf{I}_3 = -1.1782\angle 7.38°$$

Hence, $$i_{O3} = -1.1782\sin(4000t + 7.38°)\ \text{A}$$

Therefore,

$$i_O = \{0.1 + 0.217 \cos(2000t + 134.1\circ) - 1.1782 \sin(4000t + 7.38\circ)\} \text{ A}$$