Chapter 10, Solution 62.

Using Thevenin's theorem, find v_o in the circuit in Fig. 10.105.

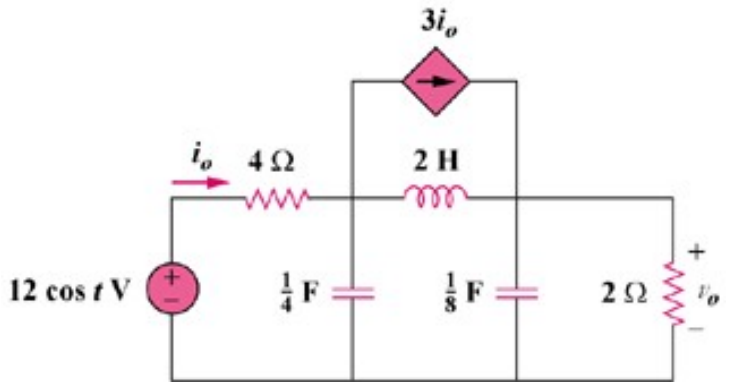


Figure 10.105
For Prob. 10.62.

Solution

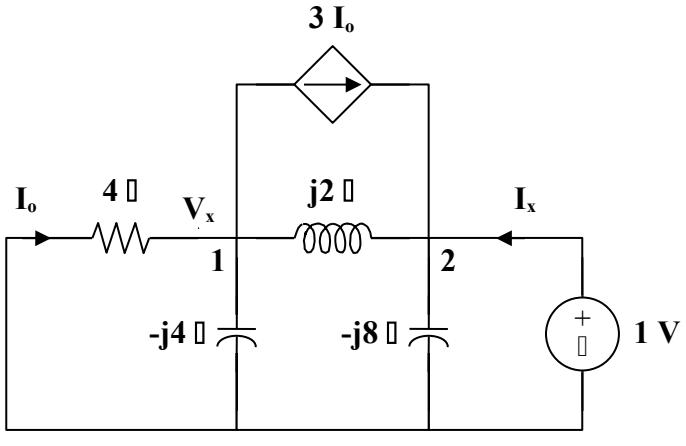
First, we transform the circuit to the frequency domain.

$$12\cos(t) \longrightarrow 12\angle 0°, \quad \omega = 1$$

$$2\text{ H} \longrightarrow j\omega L = j2$$

$$\frac{1}{4}\text{F} \longrightarrow \frac{1}{j\omega C} = -j4$$

$$\frac{1}{8}\text{F} \longrightarrow \frac{1}{j\omega C} = -j8$$



(a)

To find $\mathbf{Z}_{eq}$, consider the circuit in Fig. (a).
At node 1,

$$\frac{\mathbf{V}_x}{4} + \frac{\mathbf{V}_x}{-j4} + 3\mathbf{I}_o = \frac{1-\mathbf{V}_x}{j2}, \qquad \text{where } \mathbf{I}_o = \frac{-\mathbf{V}_x}{4}$$

Thus,

$$\frac{\mathbf{V}_x}{-j4} - \frac{2\mathbf{V}_x}{4} = \frac{1-\mathbf{V}_x}{j2}$$

$$\mathbf{V}_x = 0.4 + j0.8$$

At node 2,

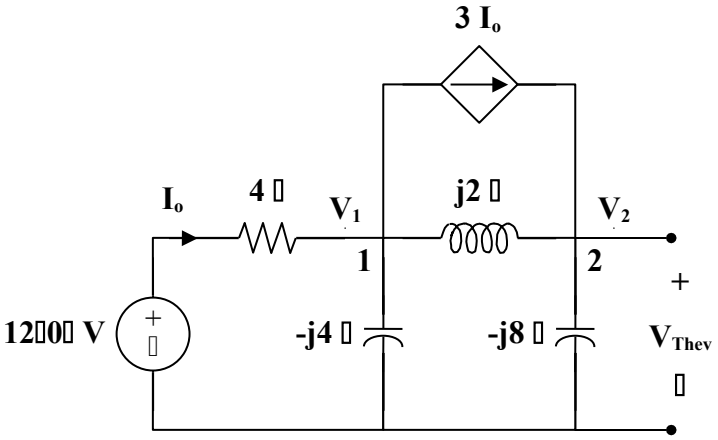
$$\mathbf{I}_x + 3\mathbf{I}_o = \frac{1}{-j8} + \frac{1-\mathbf{V}_x}{j2}$$

$$\mathbf{I}_x = (0.75 + j0.5)\mathbf{V}_x - j\frac{3}{8}$$

$$\mathbf{I}_x = -0.1 + j0.425$$

$$\mathbf{Z}_{eq} = \frac{1}{\mathbf{I}_x} = -0.5246 - j2.229 = 2.29\angle -103.24^\circ\ \Omega$$

To find $\mathbf{V}_{Thev}$, consider the circuit in Fig. (b).



(b)

At node 1,

$$\frac{12-\mathbf{V}_1}{4} = 3\mathbf{I}_o + \frac{\mathbf{V}_1}{-j4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j2}, \quad \text{where } \mathbf{I}_o = \frac{12-\mathbf{V}_1}{4}$$

$$24 = (2+j)\mathbf{V}_1 - j2\mathbf{V}_2 \qquad (1)$$

At node 2,

$$\frac{\mathbf{V}_1 - \mathbf{V}_2}{j2} + 3\mathbf{I}_o = \frac{\mathbf{V}_2}{-j8}$$

$$72 = (6 + j4)\mathbf{V}_1 - j3\mathbf{V}_2 \qquad (2)$$

From (1) and (2),

$$\begin{bmatrix} 24 \\ 72 \end{bmatrix} = \begin{bmatrix} 2 + j & -j2 \\ 6 + j4 & -j3 \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}$$

$$\Delta = -5 + j6, \qquad \Delta_2 = -j24$$

$$\mathbf{V}_{th} = \mathbf{V}_2 = \frac{\Delta_2}{\Delta} = 3.073\angle -219.8°$$

Thus,

$$\mathbf{V}_o = \frac{2}{2 + \mathbf{Z}_{th}} \mathbf{V}_{th} = \frac{(2)(3.073\angle -219.8°)}{1.4754 - j2.229}$$

$$\mathbf{V}_o = \frac{6.146\angle -219.8°}{2.673\angle -56.5°} = 2.3\angle -163.3°$$

Therefore,

$$v_o = \mathbf{2.3\cos(t - 163.3°)\ V}$$