The Dynamics of Flight of a Table Tennis Ball

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(Dated: May 11, 2013)

Table tennis is a popular sport invented in England. In it, two or four players hit a lightweight ball back and forth using table tennis rackets. The game takes place on a hard table divided by a net. The main challenge in the game is its high speed, which continuously tests the reaction time of the players. Also to further complicate the game, the players hit the ball in manners that cause the ball to spin, which in turn changes the ball's trajectory. The purpose of this report is to simulate the motion of a table tennis ball by evolving its differential equation of motion using the second order Taylor's approximation[6]. As a result of this investigation, we simulated the motion of the ball under variety of circumstances and showed that three simple physical forces govern the motion of table tennis ball and explain its oddest trajectories.

I. INTRODUCTION

Table tennis, or Ping-Pong, is a popular game similar to tennis played on a flat table by two players (four in double games). The game was invented in England in the early days of 20th century, and by 1920s was played in many countries [2]. The International Table Tennis Federation was founded in 1926, the founding members being England, Sweden, Hungary, India, Denmark, Germany, Czechoslovakia, Austria, and Wales. By the mid-1990s more than 165 national associations were members [2].

A game of table tennis is won by the player who first reaches 11 points . A point is initiated by a service performed by one of the players. The server tosses the ball upward from the palm of the free hand and strikes it as it descends so that it first bounces on the servers own court and then, passing over the net, bounces on the opponent's court. After that, each player aims to return the ball such that it strikes the opponent's court, and not his own. The rubber coat on table tennis rackets provides the friction that players use to give the ball spin. The spin in turn changes the trajectory of the ball while the ball passes through the air and when the ball hits the table. The purpose of this project is to simulate the trajectory of a standard table-tennis ball.

The motion of the ping-pong ball through the air is determined by three forces, gravity, air drag, and the magnus force [3]. The air drag is generally decomposed in two components, a quadratic resistance against the direction of the velocity of the ball, and torque that slows down the ball's angular velocity. The magnus force is equal an scalar multiple of the cross product between the angular velocity of the ball and its velocity relative to air [3]. The magnus force rises out of the high angular velocity of the ball, which rushes more air molecules to one side of the ball (through the push of friction) than to its other side. The resulting excess pressure in one side then curves the trajectory of the ball in direction of the spin [1, 3]. Figure-1 shows this effect on a tennis ball.

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TABLE I. Characteristics of a ping-pong ball. The coefficient of restitution in the table above refers to the fraction of speed that the ball retains after it bounces off a standard table tennis table [7].

Diameter of ball	40 mm
Weight of ball	2.7 g
Maximum speed of ball	28 m/s
Highest recorded angular velocity of the ball	940 rad/s
Coefficient of restitution	0.4
Formula of moment of inertia	$\frac{2}{3}MR^2$
Moment of inertia	$2.88 \times 10^{-6} kg \cdot m^2$
Loss of angular velocity per second	3%

The magnus-force is calculated by the formula $\mathbf{F_m} = S(\tilde{\omega} \times \mathbf{v})$, where ω is the angular velocity of the ball, v is the velocity of the fluid with respect to the object, and S is a positive coefficient [4].

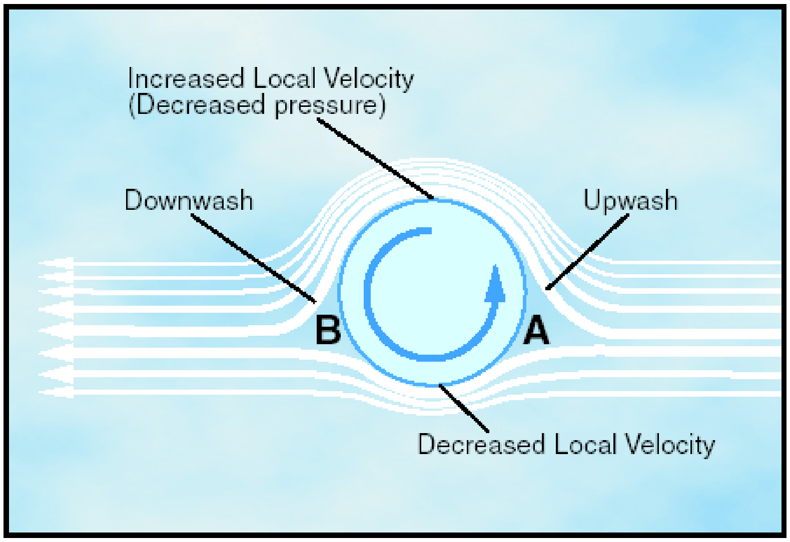
I use the information in table-I to calculate the magnitude of the forces on the ball. For air drag, it should be noted that due to the small diameter of the ball, the air drag may have a significant linear component at lower velocities. The ratio of the quadratic component of drag to its linear component (for a spherical body moving in air) is in general given by $\frac{f_{quad}}{f_{lin}} = (1.6 \times 10^3 s/m^2)Dv$, where D is the diameter of the ball and v is its velocity [9]. For a ping-pong ball, the linear drag component becomes important at low velocities, but I choose to ignore that, since at low velocity the effect of drag is very small. Knowing these equations we can write the Newton's law for the ping-pong ball as [5]:

$$m\ddot{\mathbf{r}} = m\mathbf{g} - b\dot{\mathbf{r}}\dot{r} + S(\omega \times \mathbf{v}),$$

where $b = 1/2C_d \times \rho_{air} \times A = 2.1991 \times 10^{-4} \frac{Ns^2}{m^2}$.

In addition to the forces shown in equation above, the ball experiences a torque against its direction of rotation. Here, to simplify the matters, I assume that this torque has constant magnitude, so that the angular velocity of the ball decreases exponentially.

FIG. 1. The effect of the magnus force of a moving ball. The angular velocity of the ball slows down the air current in one side of the ball leading to increase of air pressure in that side which in turn deflects the ball's trajectory.



II. METHODS

The motion of the ball was simulated by evolving the equation of motion, as given in the introduction, through the use of second order Taylor's approximation in small time intervals [6]. In this method, the velocity of and the position of the ball at time $t + dt$ are given by:

$$\begin{cases} \dot{\boldsymbol{r}}(t+dt) = \dot{\boldsymbol{r}}(t) + \ddot{\boldsymbol{r}}(t)dt \\ \boldsymbol{r}(t+dt) = \boldsymbol{r}(t) + \dot{\boldsymbol{r}}(t)dt + \frac{1}{2}\ddot{\boldsymbol{r}}(t)dt^2 \end{cases}$$

The evolution was used on several different initial conditions in order to produce model different table tennis techniques. The codes used to simulate the motion of the table tennis ball took advantage of our gradually enhanced 3D vector objects. However, the evolution of the equation of motion was different in each case. Listing-1 shows a simple main method which simulates the motion of a ball, with some initial velocity and spin, as it falls on and bounces off the table.

Listing-1: The main method used for evolving the equation of motion of a ball as it bounces off of a table.

```
int index = 0;
        double time = 0.00;
        vec3 ballpos(-1.362,-1.07,0.3), ballvel(0.0,0.35,0.25),ep(0.0,0.0,0.0), spin(-20.0,0.0,0.0);
        ballvel = ballvel * invel; //invel is a scalar, fed to program by user.
        while (time < Tmax){
                if (index %mod == 0) //only print a fraction of the calculated ball positions
                        cout<<time<<"\t"<<ballpos<<ballvel<<"\t"<<spin.getl()<<endl;
                ++index;
                done--;
                if(ballpos.z > 0.011)
                        over = true;
                if(ballpos.z < 0.01 && done < 0 && over ==true) { //done is greater than zero if the ball has just hit the table
                        // over is not true if ball began it's motion from below the table and has not reached above it yet.
                        done = 200;
                        cerr<< "hit the table at " <<ballpos<<endl;
                        ballvel.z = -ballvel.z;
                        ballvel = ballvel * 0.4;
                        spin.y = spin.y * 0.4;
                        spin.x = spin.x * 0.4;
                        ballvel.x  += spin.y* RADIUS * J_FAC;   //spin gives the ball a velocity boost when it bounces
                        ballvel.y  += spin.x* RADIUS * J_FAC;
                }
                ep = ballAc(spin,ballvel);
                vec3 bvtmp = ballvel + ep * dT ;
                vec3 bptmp = ballpos + ballvel*dT + ep *0.5 * dT*dT;
                ballpos = bptmp;
                ballvel = bvtmp;
                time += dT;
                spin = spin * exp(-dT/TQ_EXP) ; // spin wanes logarithmically
        }
```

III. RESULTS AND DISCUSSION

I used programs similar to listing-1 to simulate the motion of the table tennis ball with several different initial conditions. The firsts in line are the planar motion of the ball whose angular velocity is in x direction (taking the longer side of the table as parallel to y direction), in other words ball had either a top or a back spin. These simulations are shown in figures-2&3. A strong top spin, causes the ball to curve downward, for which the players compensate by hitting the ball hard and giving it a good vertical velocity. This effect is important for fast shots for several reasons. In high velocity balls, a small change in angle of trajectory can mean that the ball flies out of the boundaries of the table. However, a ball with strong top spin will curve down after it passes the net and thus is much less unlikely to miss the table. Another reason, for the use of the top spin is that as the spinning ball hits the table, it gets a significant boost in velocity because of the friction between the ball and the table.

Back-spins are another popular type of spin, which are used less frequently than the top-spin. Unlike the top-spin, back-spin is generally used in low velocity shots. At those velocities the magnus-effect is less significant significant, though players compensate for speed by giving the ball more angular velocity. The upward lifting effect of the magnus force causes the motion of the ball to appear "smoother" or more controlled than that of a simple spin less ball (since it reduces the effect of gravity and thus the acceleration of the ball). The more interesting effect of a back-spin is the sudden reduction of velocity, the moment the ball bounces off the table. Of the occasions in which back-spins are favored are those in which a player tries to deliver a ''near-net" ball to the opponent and so one gives the ball backspin to make sure that the ball does not travel far from the net after bouncing; figure-3 shows such a ball.

Every professional table tennis player takes advantage of the effects of side-spin while serving. The spins used in services have strong z component (the ball rotates fast around its vertical axis), and they may also have x and y components. The typical speed of a fast service is between 6.7 m/s and 10.7 m/s as I measured through timing videos of services; a spin-less ball with speed 7.2 barely hits the opponents court, but a spinning ball with speed 10.7 can hit both courts before rushing out of the table. In serving, a player throws the ball up between 0.3 to 1.5 meters then he strikes it such that the ball bounces first on his side of table and next, having passed the net, the opponent's side of table. Good servers deliver balls that are fast and spin fast; it's not unusual to see the served ball deflect from its original course by more than 20 degrees. In typical games, a good fraction of points are collected by services alone, since the opponent is unable to return the ball. Figures-4&5&6 show several services delivered with different spins and speeds.

The simulations presented so far show the effects that professional players can produce, while serving or returning the balls. At this point, we go on to discover the moves that although physically correct, are unlikely to be seen in table tennis competitions.

The simulations presented so far show the effects that professional players can produce, while serving or returning the balls. At this point, we go on to discover the moves that although physically correct, are unlikely to be seen in table tennis competitions. Here, I consider the movement of the ball with with very strong spins. Figure-7 shows such a ball whose spin is in z direction. The magnus effect is perpendicular to the velocity of the ball, creating a helical trajectory that we know from magnetism. The radius of the helix increases as the ball loses its angular velocity. Figure-8 shows a ball with relatively strong back-spin. The spin is enough to neutralize the effect of gravity and lift the ball. Figure-9 shows a ball with strong top spin. The spin of the ball in this figure is enough to have it return from bellow the table.

IV. CONCLUSIONS

Table tennis is a fast sport where the principles of aerodynamics are employed frequently. Experienced table tennis players rarely return a ball without significant spin. The effect of spin is especially accentuated in services, where excellent servers take advantage of the optimal situations to deliver very fast balls that deviate from their original course of movement by up to 20 degrees. Or in other cases, very slow balls that do not advance more than a few inches beyond the net.

The game of table tennis is mostly about trying to surprise the opponent by return the ball in unexpected ways, and trying to not get surprised by the opponent. The simulations here show a few of the ways that table tennis players can use the spin to reach that goal.

[1] Magnus effect (physics). http://www.britannica.com/EBchecked/topic/357684/Magnus-effect.

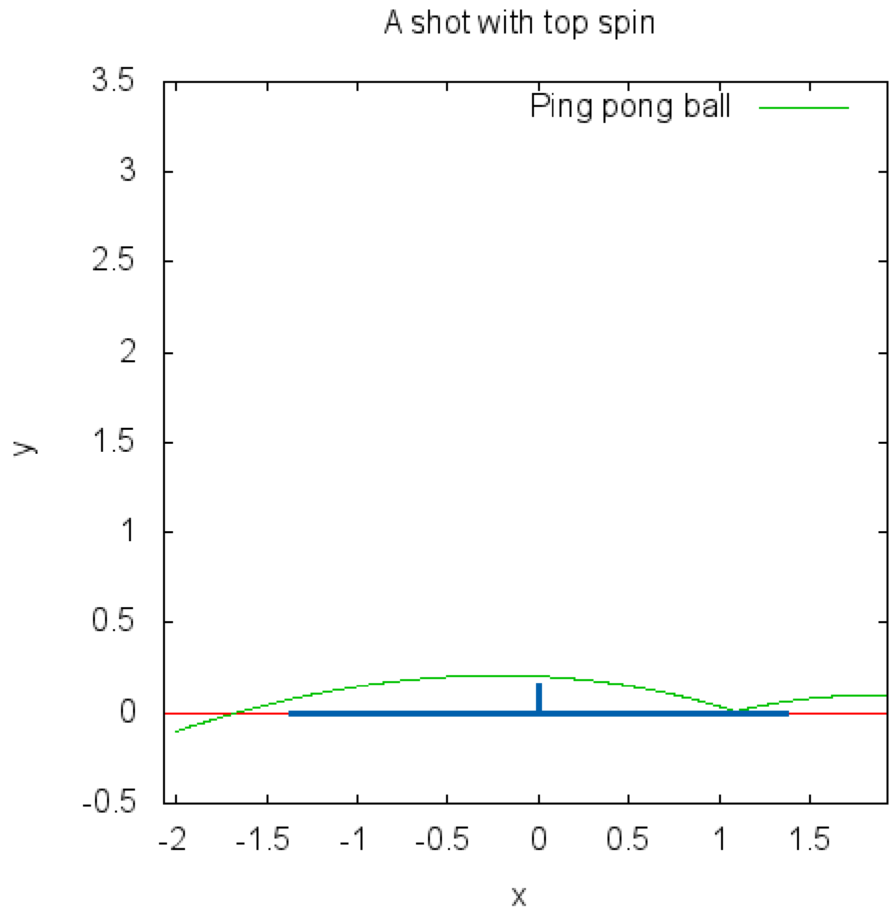
[2] table tennis (sport). http://www.britannica.com/EBchecked/topic/579773/table-tennis.

[3] Lyman J. Briggs. Effect of spin and speed on the lateral deflection (curve) of a baseball; and the magnus effect for smooth spheres. *American Journal of Physics*, 27(8):589, 1959.

[4] S.R. Goodwill, S.B. Chin, and S.J. Haake. Aerodynamics of spinning and non-spinning tennis balls. *Journal of Wind Engineering and Industrial Aerodynamics*, 92(11):935–958, September 2004.

[5] Iwan Griffiths, Colin Evans, and Neil Griffiths. Tracking the flight of a spinning football in three dimensions. *Mea-*

FIG. 2. A fast ping-pong ball with top spin. The spin of the ball gives it a boost in velocity as the ball bounces the table. A strong top-spin is the only way to deliver a fast ball from below the surface of the table, because of the curve that the spin creates in the trajectory of the ball.



surement Science and Technology, 16(10):2056, October 2005.

[6] Rubin Landau and Mejia Paez. *Computational physics: problem solving with computers*. Wiley, New York, 1997.

[7] Rod Cross. Measurements of the horizontal coefficient of restitution for a superball and a tennis ball. *American Journal of Physics*, 70(5):482, May 2002.

[8] J. Smith, editor. *AIP Conf. Proc.*, volume 841. 2007.

[9] John R. Taylor. *Classical mechanics*. University Science Books, Sausalito, CA, 1 edition, 2005.

FIG. 3. A slower ping-pong ball with bach spin. The spin of the ball prevents it from getting far from the net.

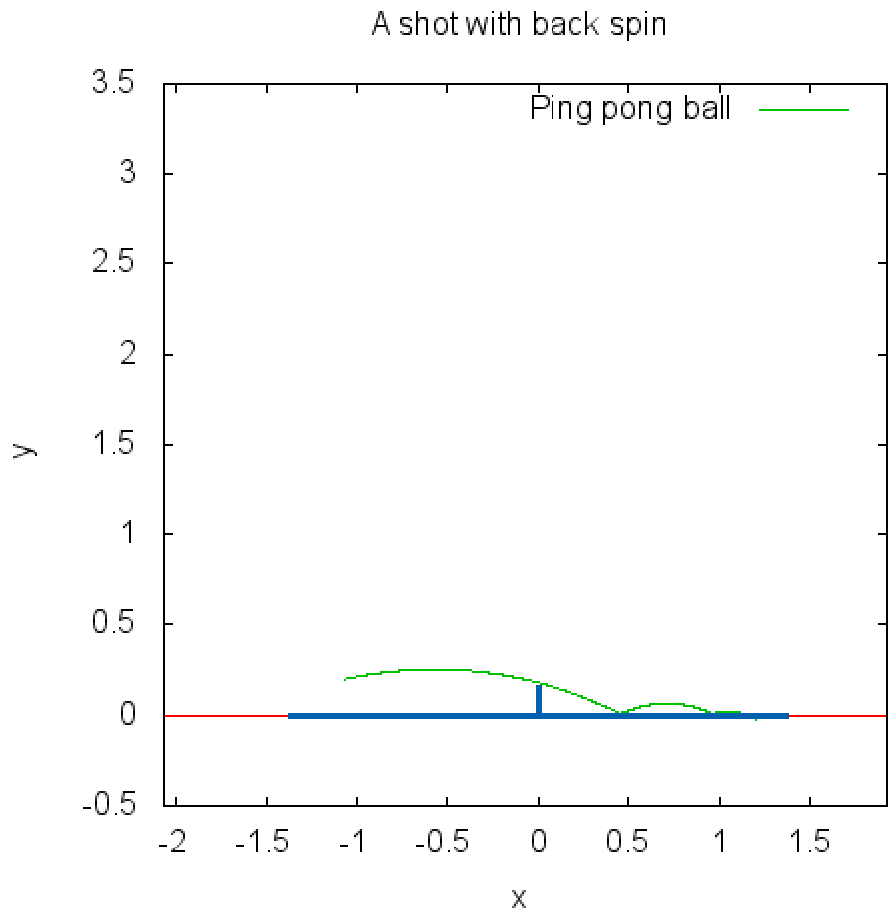


FIG. 4. A fast service with pure side-spin. The ball is deflected from its original course such that the opponent is likely to miss it.

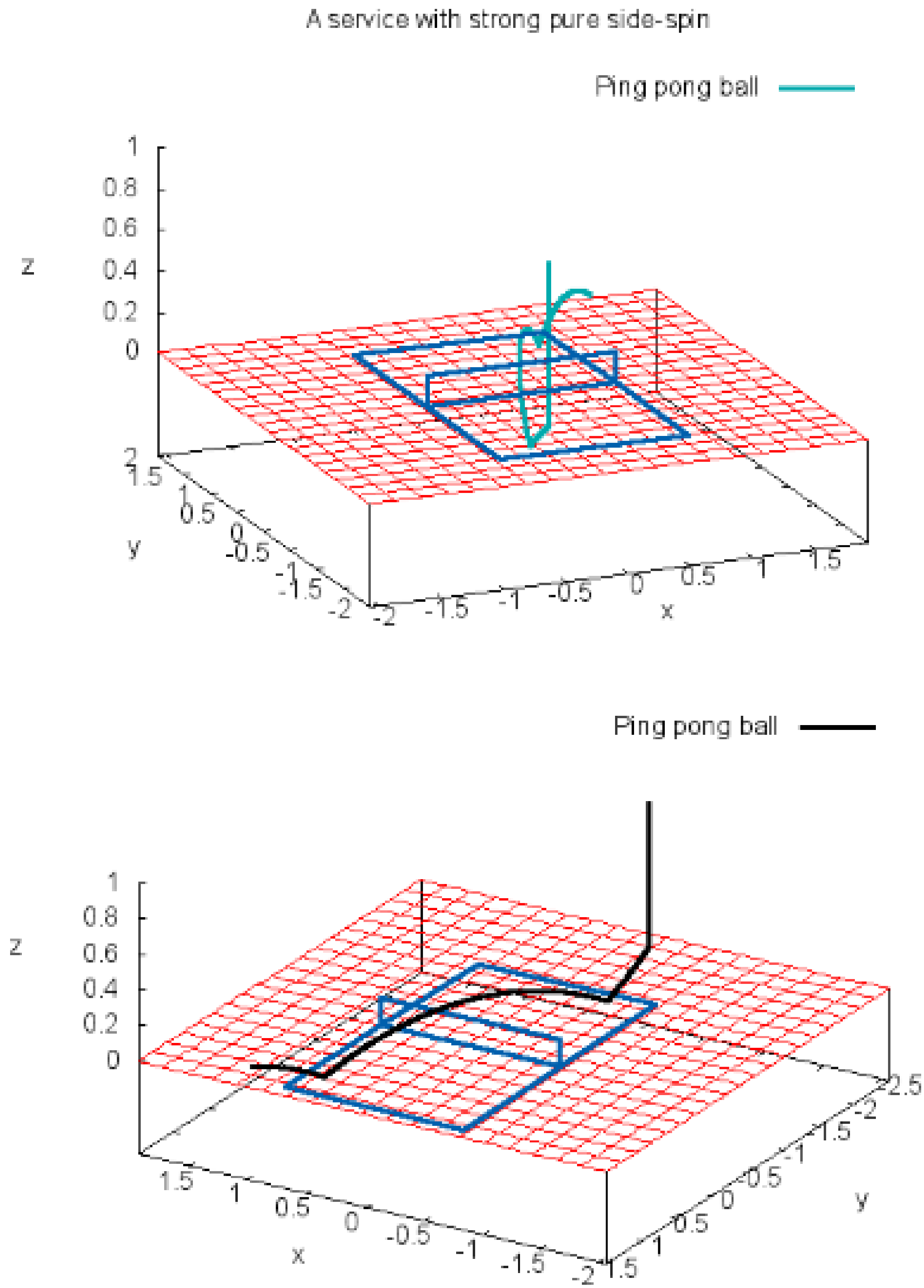


FIG. 5. A fast service. The spin of the ball has a strong z component and a smaller y component. The ball bounces off the table with different horizontal velocity.

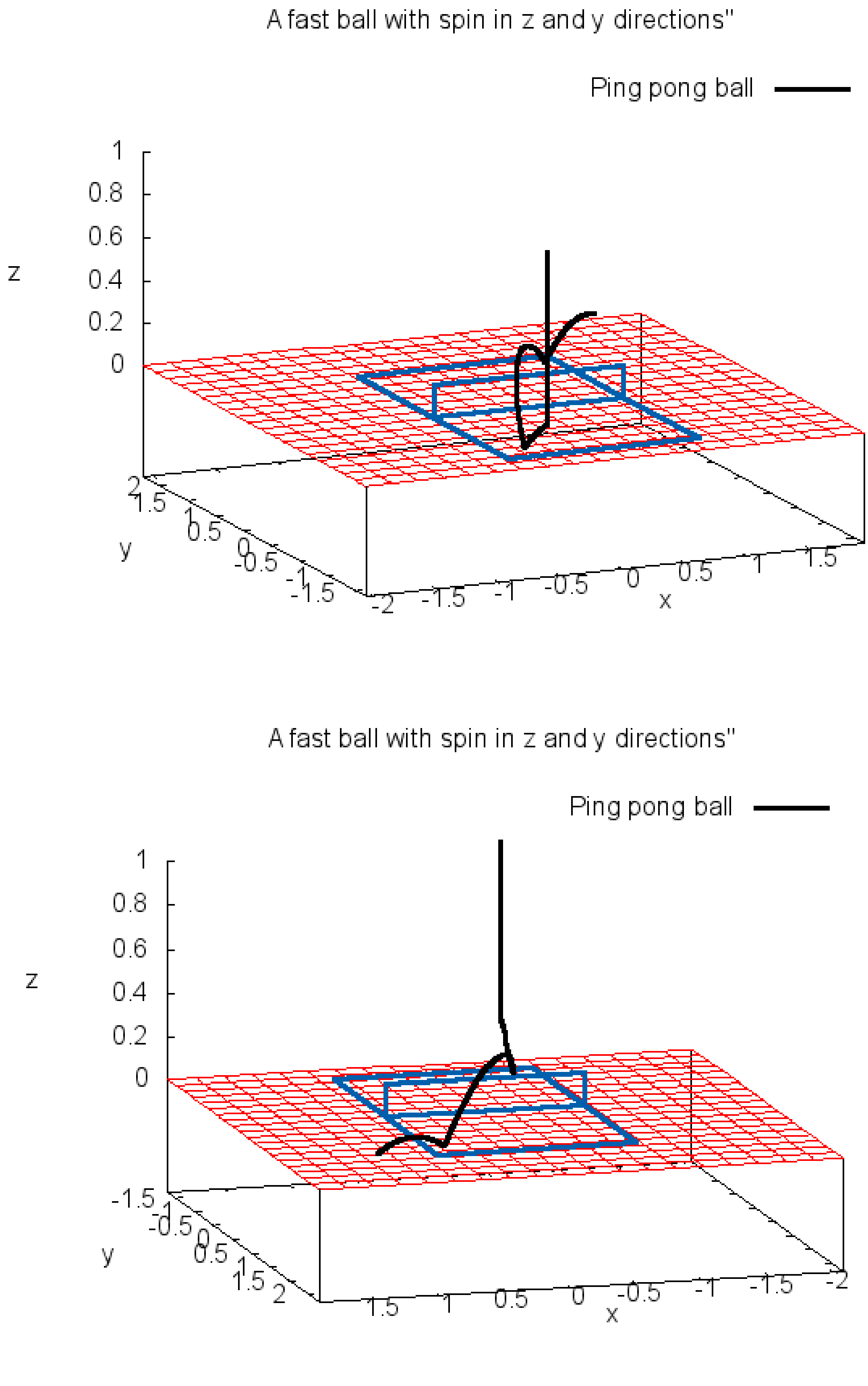


FIG. 6. A slow service. The movement of the ball in this service is determined more by the x and y components of its spin than its initial velocity. The service may be used as a strategy. The opponent will have to return the ball from the middle of the table with more vertical velocity and little spin. The server can then reply in variety of ways since the returned ball has optimal position and speed.

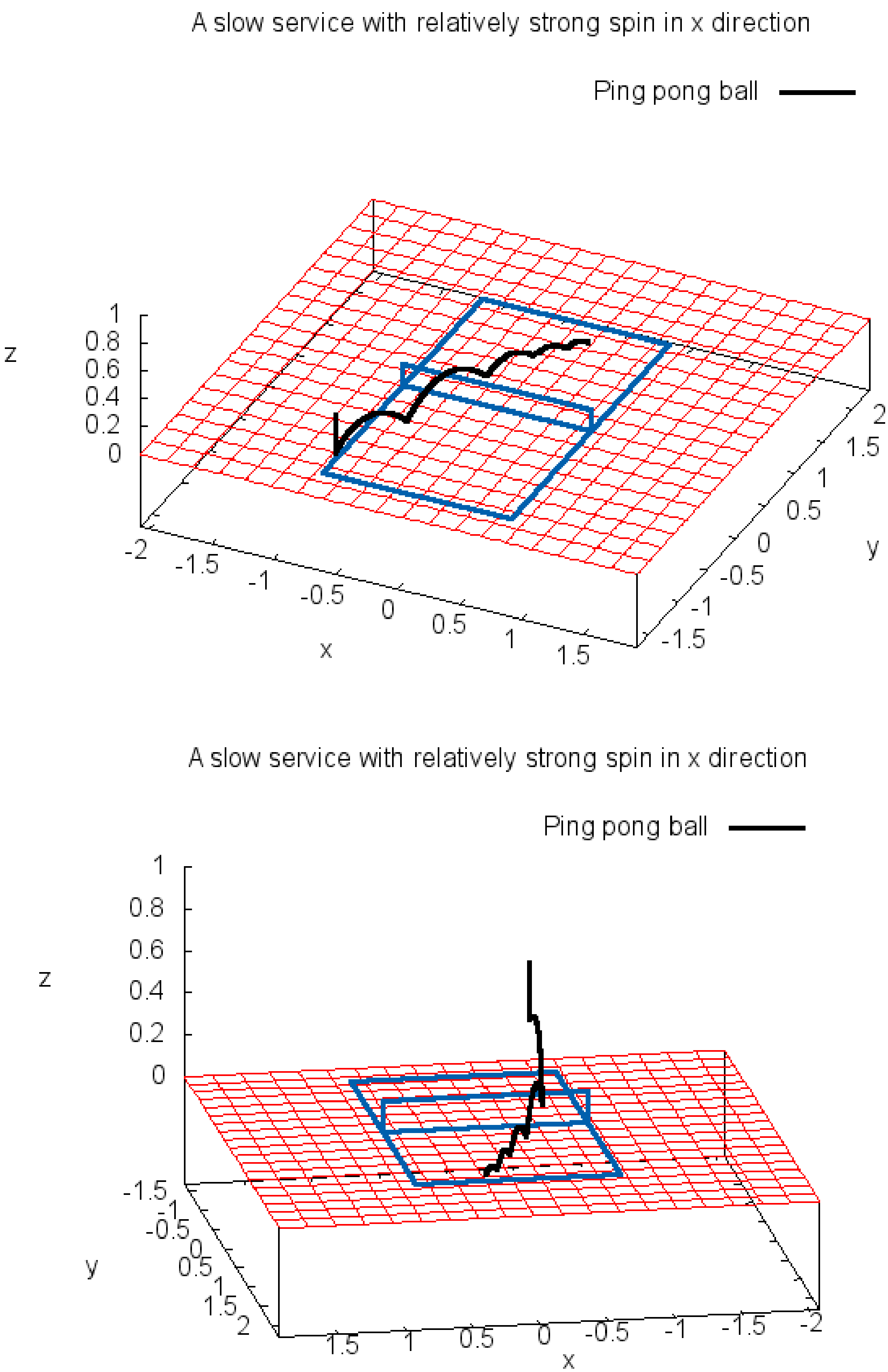


FIG. 7. The motion of a ball with unrealistically strong side spin. The ball was released initially with velocity $(20, 20, 15)$ m/s and spin $(0, 0, 1000)$ rev/s.

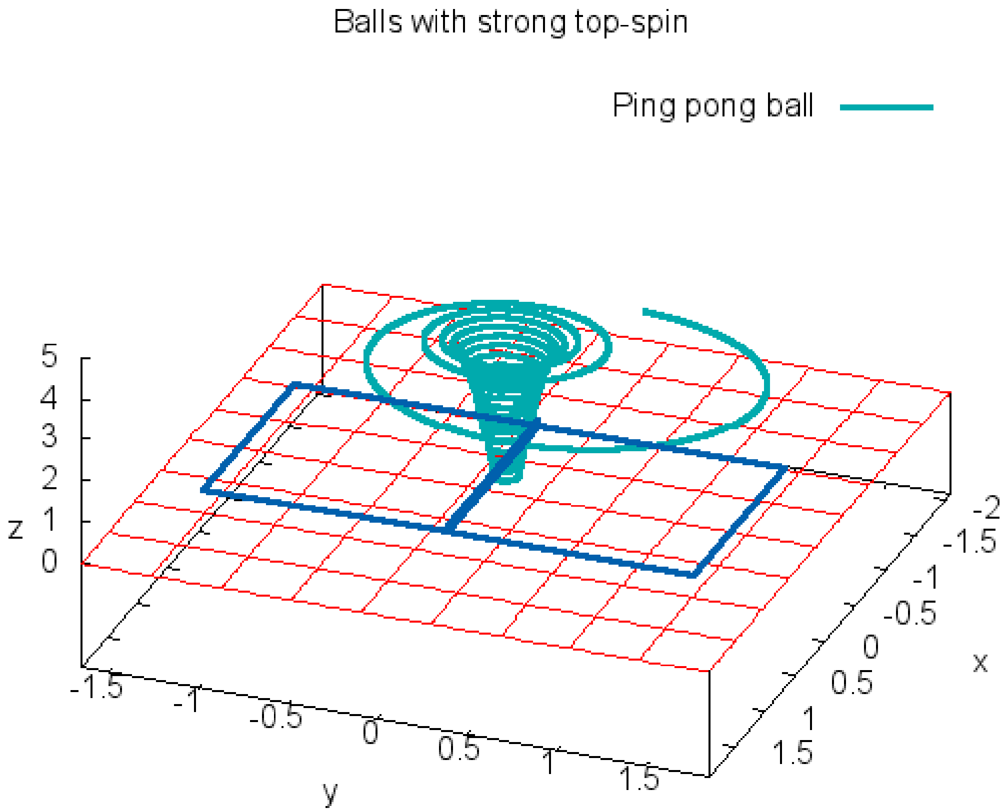


FIG. 8. The motion of a ball with very strong back spin. The ball initiated it's flight in the middle of table with 20 m/s speed and 15 rev/s spin in x direction. The spin of the ball is enough to lift it against the effect of gravity.

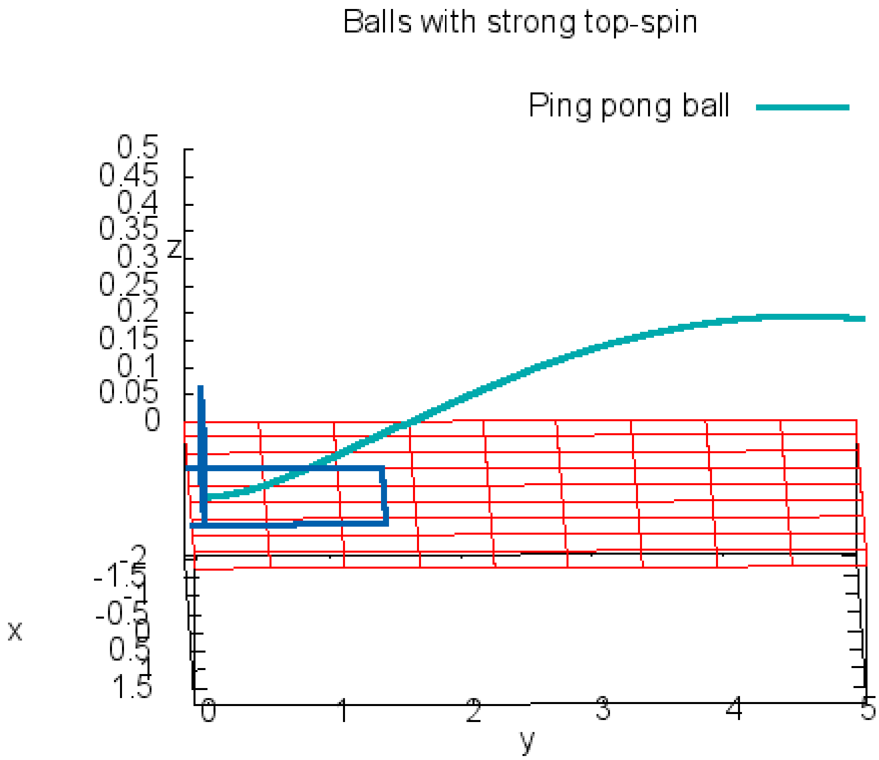


FIG. 9. The motion of balls with very strong top spin. The balls were thrown with 180 rev/s spin in x direction. The initial velocity of the balls were different in y direction leading to vastly different outcomes.